

# University of Pretoria Yearbook 2022

# BScHons Applied Mathematics (02240172)

**Department** Mathematics and Applied Mathematics

Minimum duration of

1 year

study

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Total credits

135

NQF level

80

## Programme information

## Admission requirements

- 1. BSc (Mathematics) degree **or** BSc (Applied Mathematics) degree **or** relevant bachelor's degree
- 2. At least 60% in all mathematics and applied mathematics modules at final-year level
- 3. At least four (4) of the following modules/subjects (or equivalent) with at least 60% at final-year level:
  - · Partial differential equations
  - · Dynamical systems (ordinary differential equations)
  - Real analysis
  - · Complex analysis
  - Numerical analysis
  - Continuum mechanics



## Curriculum: Final year

Minimum credits: 135

#### **Additional information:**

- The programme compilation consists of seven honours modules of 15 credits each as well as the mandatory project (WTW 795 30 credits).
- It is required that students select the stream and modules according to the prerequisites of the modules.
- WTW 795 is a compulsory module for both streams.
- The modules to be selected for each stream, are set out below.

## **Stream 1: Applied analysis**

Core credits: 75 credits Elective credits: 60 credits

Core modules: WTW 795, WTW 710, WTW 734 and WTW 776

Elective modules: Four (4) electives must be chosen from the list below. The selection must contain at least one of WTW 782 or WTW 764 and at least one of WTW 733 or WTW 763. (Students are also allowed to register for all 4 these modules.)

4 these modules.)

#### Stream 2: Differential equations and modelling

Core credits: 135 credits

Core modules: WTW 795, WTW 733, WTW 735, WTW 750, WTW 763, WTW 772, WTW 776 and WTW 782.

### **Core modules**

## **Functional analysis 710 (WTW 710)**

Module credits	15.00
NQF Level	08
Prerequisites	Real analysis on third-year level
Contact time	2 lectures per week
Language of tuition	Module is presented in English
Department	Mathematics and Applied Mathematics
Period of presentation	Semester 1

#### **Module content**

An introduction to the basic mathematical objects of linear functional analysis will be presented. These include metric spaces, Hilbert spaces and Banach spaces. Subspaces, linear operators and functionals will be discussed in detail. The fundamental theorems for normed spaces: The Hahn-Banach theorem, Banach-Steinhaus theorem, open mapping theorem and closed graph theorem. Hilbert space theory: Riesz' theorem, the basics of projections and orthonormal sets.

#### **Numerical analysis 733 (WTW 733)**

Module credits	15.00
NQF Level	08



**Prerequisites** No prerequisites.

**Contact time** 2 lectures per week

**Language of tuition** Module is presented in English

**Department** Mathematics and Applied Mathematics

**Period of presentation** Semester 1

#### Module content

An analysis as well as an implementation (including computer programs) of methods are covered. Numerical linear algebra: Direct and iterative methods for linear systems and matrix eigenvalue problems: Iterative methods for nonlinear systems of equations. Finite difference method for partial differential equations: Linear elliptic, parabolic, hyperbolic and eigenvalue problems. Introduction to nonlinear problems. Numerical stability, error estimates and convergence are dealt with.

## Measure theory and probability 734 (WTW 734)

Module credits 15.00

NQF Level 08

**Prerequisites** Real analysis on third-year level

**Contact time** 2 lectures per week

**Language of tuition** Module is presented in English

**Department** Mathematics and Applied Mathematics

**Period of presentation** Semester 1

#### Module content

Measure and integration theory: The Caratheodory extension procedure for measures defined on a ring, measurable functions, integration with respect to a measure on a  $\sigma$ -ring, in particular the Lebesgue integral, convergence theorems and Fubini's theorem.

Probability theory: Measure theoretic modelling, random variables, expectation values and independence, the Borel-Cantelli lemmas, the law of large numbers. L¹-theory, L²-theory and the geometry of Hilbert space, Fourier series and the Fourier transform as an operator on L², applications of Fourier analysis to random walks, the central limit theorem.

## Main principles of analysis in application 735 (WTW 735)

Module credits	15.00
NQF Level	08
Prerequisites	Calculus at 2nd-year level (eg WTW 218) and one 3rd-year level module on analysis or applications of analysis (eg WTW 310, WTW 382, WTW 383 or WTW 386)
Contact time	2 lectures per week
Language of tuition	Module is presented in English
Department	Mathematics and Applied Mathematics



#### **Period of presentation** Semester 1

#### **Module content**

Study of main principles of analysis in the context of their applications to modelling, differential equations and numerical computation. Specific principles to be considered are those related to mathematical biology, continuum mechanics and mathematical physics as presented in the modules WTW 772, WTW 787 and WTW 776, respectively.

## Mathematical optimisation 750 (WTW 750)

Module credits	15.00
NQF Level	08
Prerequisites	Multivariate Calculus on 2nd-year level; Linear Algebra on 2nd-year level
Contact time	2 lectures per week
Language of tuition	Module is presented in English
Department	Mathematics and Applied Mathematics

## **Period of presentation** Semester 1

#### Module content

Classical optimisation: Necessary and sufficient conditions for local minima. Equality constraints and Lagrange multipliers. Inequality constraints and the Kuhn-Tucker conditions. Application of saddle point theorems to the solutions of the dual problem. One-dimensional search techniques. Gradient methods for unconstrained optimisation. Quadratically terminating search algorithms. The conjugate gradient method. Fletcher-Reeves. Second order variable metric methods: DFP and BFCS. Boundary following and penalty function methods for constrained problems. Modern multiplier methods and sequential quadratic programming methods. Practical design optimisation project.

## Finite element method 763 (WTW 763)

Module credits	15.00
NQF Level	08
Prerequisites	WTW 733 is strongly recommended
Contact time	2 lectures per week
Language of tuition	Module is presented in English
Department	Mathematics and Applied Mathematics
Period of presentation	Semester 2

#### **Module content**

An analysis as well as an implementation (including computer programs) of methods is covered. Introduction to the theory of Sobolev spaces. Variational and weak formulation of elliptic, parabolic, hyperbolic and eigenvalue problems. Finite element approximation of problems in variational form, interpolation theory in Sobolev spaces, convergence and error estimates.



## Mathematical methods and models 772 (WTW 772)

Module credits 15.00

NQF Level 08

**Prerequisites** No prerequisites.

**Contact time** 2 lectures per week

**Language of tuition** Module is presented in English

**Department** Mathematics and Applied Mathematics

**Period of presentation** Semester 2

#### **Module content**

This module aims at using advanced undergraduate mathematics and rigorously applying mathematical methods to concrete problems in various areas of natural science and engineering.

The module will be taught by several lecturers from UP, industry and public sector. The content of the module may vary from year to year and is determined by relevant focus areas within the Department. The list of areas from which topics to be covered will be selected, includes: Systems of differential equations; dynamical systems; discrete structures; Fourier analysis; methods of optimisation; numerical methods; mathematical models in biology, finance, physics, etc.

## Partial differential equations of mathematical physics 776 (WTW 776)

Module credits 15.00

NQF Level 08

**Prerequisites** WTW 710 or WTW 735

**Contact time** 2 lectures per week

**Language of tuition** Module is presented in English

**Department** Mathematics and Applied Mathematics

**Period of presentation** Semester 2

#### **Module content**

Field-theoretic and material models of mathematical physics. The Friedrichs-Sobolev spaces. Energy methods and Hilbert spaces, weak solutions – existence and uniqueness. Separation of variables, Laplace transform, eigenvalue problems and eigenfunction expansions. The regularity theorems for elliptic forms (without proofs) and their applications. Weak solutions for the heat/diffusion and related equations.

## **Dynamical systems 782 (WTW 782)**

Module credits 15.00

NQF Level 08

**Prerequisites** No prerequisites.

**Contact time** 2 lectures per week

**Language of tuition** Module is presented in English



**Department** Mathematics and Applied Mathematics

**Period of presentation** Semester 1

#### **Module content**

Introduction to the general theory of dynamical systems and to the theory of dynamical systems represented via systems of ODEs. Quantitative and qualitative analyses of linear systems. Qualitative analysis of nonlinear systems: domain, invariant sets, stability of equilibria, Hartman-Grobman theorem, centre manifold theorem, Lyapunov method. Structural stability and bifurcation. Bifurcation of equilibria. Hopf bifurcation. Applications: population models, chemical reactions, circuits.

## **Project 795 (WTW 795)**

Module credits 30.00

NQF Level 08

**Prerequisites** No prerequisites.

**Language of tuition** Module is presented in English

**Department** Mathematics and Applied Mathematics

**Period of presentation** Year

#### Module content

Consult Department.

## **Elective modules**

#### Special topics 727 (WTW 727)

Module credits 15.00

NQF Level 08

**Prerequisites** As required by specific topical content.

**Contact time** 1 lecture per week

**Language of tuition** Module is presented in English

**Department** Mathematics and Applied Mathematics

**Period of presentation** Semester 2

#### **Module content**

A selection of special topics will be presented that reflects the expertise of researchers in the Department. The presentation of a specific topic is contingent on student numbers. Consult the website of the Department of Mathematics and Applied Mathematics for more details.

## **Numerical analysis 733 (WTW 733)**

Module credits 15.00

NQF Level 08

**Prerequisites** No prerequisites.



**Contact time** 2 lectures per week

**Language of tuition** Module is presented in English

**Department** Mathematics and Applied Mathematics

**Period of presentation** Semester 1

#### Module content

An analysis as well as an implementation (including computer programs) of methods are covered. Numerical linear algebra: Direct and iterative methods for linear systems and matrix eigenvalue problems: Iterative methods for nonlinear systems of equations. Finite difference method for partial differential equations: Linear elliptic, parabolic, hyperbolic and eigenvalue problems. Introduction to nonlinear problems. Numerical stability, error estimates and convergence are dealt with.

#### Mathematical optimisation 750 (WTW 750)

Module credits 15.00

NQF Level 08

**Prerequisites** Multivariate Calculus on 2nd-year level; Linear Algebra on 2nd-year level

**Contact time** 2 lectures per week

**Language of tuition** Module is presented in English

**Department** Mathematics and Applied Mathematics

**Period of presentation** Semester 1

#### **Module content**

Classical optimisation: Necessary and sufficient conditions for local minima. Equality constraints and Lagrange multipliers. Inequality constraints and the Kuhn-Tucker conditions. Application of saddle point theorems to the solutions of the dual problem. One-dimensional search techniques. Gradient methods for unconstrained optimisation. Quadratically terminating search algorithms. The conjugate gradient method. Fletcher-Reeves. Second order variable metric methods: DFP and BFCS. Boundary following and penalty function methods for constrained problems. Modern multiplier methods and sequential quadratic programming methods. Practical design optimisation project.

## Finite element method 763 (WTW 763)

Module credits	15.00
NQF Level	08

**Prerequisites** WTW 733 is strongly recommended

**Contact time** 2 lectures per week

**Language of tuition** Module is presented in English

**Department** Mathematics and Applied Mathematics

**Period of presentation** Semester 2



#### **Module content**

An analysis as well as an implementation (including computer programs) of methods is covered. Introduction to the theory of Sobolev spaces. Variational and weak formulation of elliptic, parabolic, hyperbolic and eigenvalue problems. Finite element approximation of problems in variational form, interpolation theory in Sobolev spaces, convergence and error estimates.

### **Stochastic calculus 764 (WTW 764)**

Module credits 15.00

NQF Level 08

**Prerequisites** WTW 734 or WTW 735

**Contact time** 2 lectures per week

**Language of tuition** Module is presented in English

**Department** Mathematics and Applied Mathematics

**Period of presentation** Semester 2

### **Module content**

Mathematical modelling of Random walk. Conditional expectation and Martingales. Brownian motion and other Lévy processes. Stochastic integration. Ito's Lemma. Stochastic differential equations. Application to finance.

#### Mathematical methods and models 772 (WTW 772)

Module credits 15.00

NQF Level 08

**Prerequisites** No prerequisites.

**Contact time** 2 lectures per week

**Language of tuition** Module is presented in English

**Department** Mathematics and Applied Mathematics

**Period of presentation** Semester 2

#### **Module content**

This module aims at using advanced undergraduate mathematics and rigorously applying mathematical methods to concrete problems in various areas of natural science and engineering.

The module will be taught by several lecturers from UP, industry and public sector. The content of the module may vary from year to year and is determined by relevant focus areas within the Department. The list of areas from which topics to be covered will be selected, includes: Systems of differential equations; dynamical systems; discrete structures; Fourier analysis; methods of optimisation; numerical methods; mathematical models in biology, finance, physics, etc.

### **Dynamical systems 782 (WTW 782)**

**Module credits** 15.00

NQF Level 08



**Prerequisites** No prerequisites.

**Contact time** 2 lectures per week

**Language of tuition** Module is presented in English

**Department** Mathematics and Applied Mathematics

**Period of presentation** Semester 1

#### Module content

Introduction to the general theory of dynamical systems and to the theory of dynamical systems represented via systems of ODEs. Quantitative and qualitative analyses of linear systems. Qualitative analysis of nonlinear systems: domain, invariant sets, stability of equilibria, Hartman-Grobman theorem, centre manifold theorem, Lyapunov method. Structural stability and bifurcation. Bifurcation of equilibria. Hopf bifurcation. Applications: population models, chemical reactions, circuits.

The regulations and rules for the degrees published here are subject to change and may be amended after the publication of this information.

The General Academic Regulations (G Regulations) and General Student Rules apply to all faculties and registered students of the University, as well as all prospective students who have accepted an offer of a place at the University of Pretoria. On registering for a programme, the student bears the responsibility of ensuring that they familiarise themselves with the General Academic Regulations applicable to their registration, as well as the relevant faculty-specific and programme-specific regulations and information as stipulated in the relevant yearbook. Ignorance concerning these regulations will not be accepted as an excuse for any transgression, or basis for an exception to any of the aforementioned regulations.