Discrete breathers in silicate layers

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Collaborators: MD Alba, M Naranjo, JM Trillo, Materials Science Institute (MSG) V Dubinko, P Selyshchev, FM Russell, J Cuevas, Y Kosevich

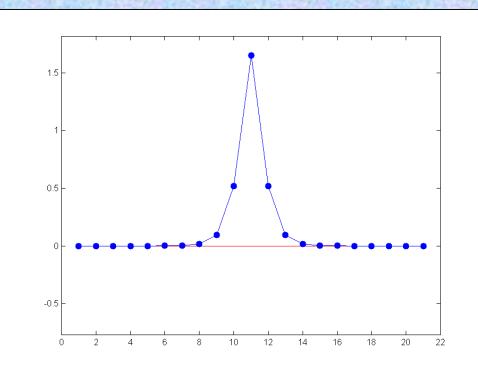
NEMI 2012: 1st International Workshop on Nonlinear Effects in Materials under Irradiation 12-17 February, 2012. Pretoria, South Africa



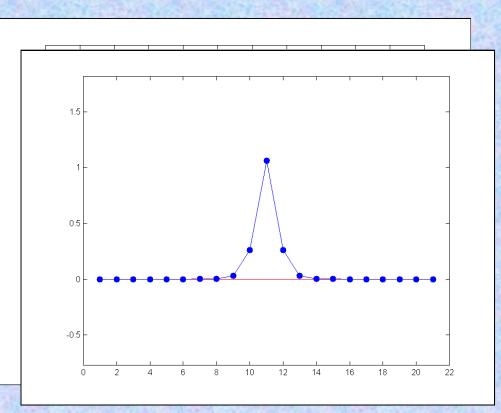




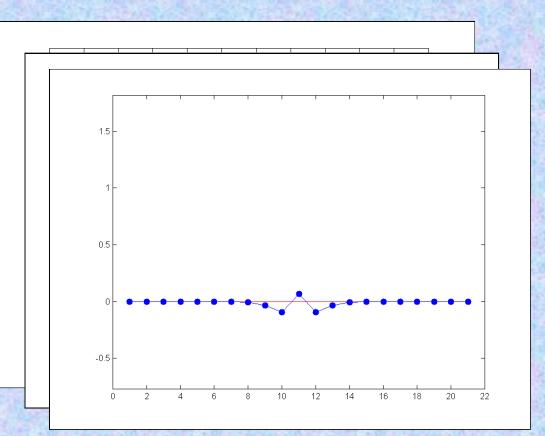
- In systems of coupled nonlinear oscillators.
 - What are they?
 - Vibrations
 - Localized
 - Exact



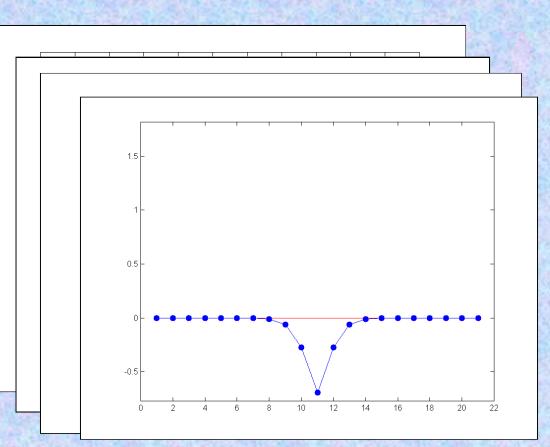
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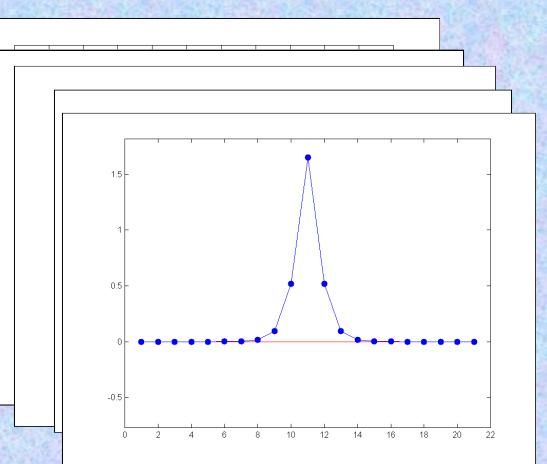
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Theoreticians in breathers

An experimentalist knows the question but not the anwer.

A theoretician knows the answer but doesn't know the question.

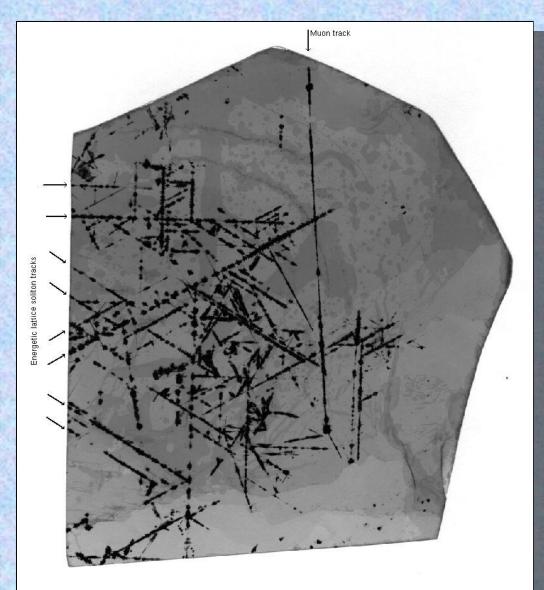
If breathers are the answer, what is the question? GP Tsironis, Chaos 13, 657 (2003)

Two questions on mica

• Dark tracks: Russell, Eilbeck

 Low Temperature Reconstructive Transformations (LTRT).
 Sevilla Materials Science Group:Alba, Becerro, Naranjo, Trillo (MSG)

Dark traks in mica moscovite: Quodons (Russell)



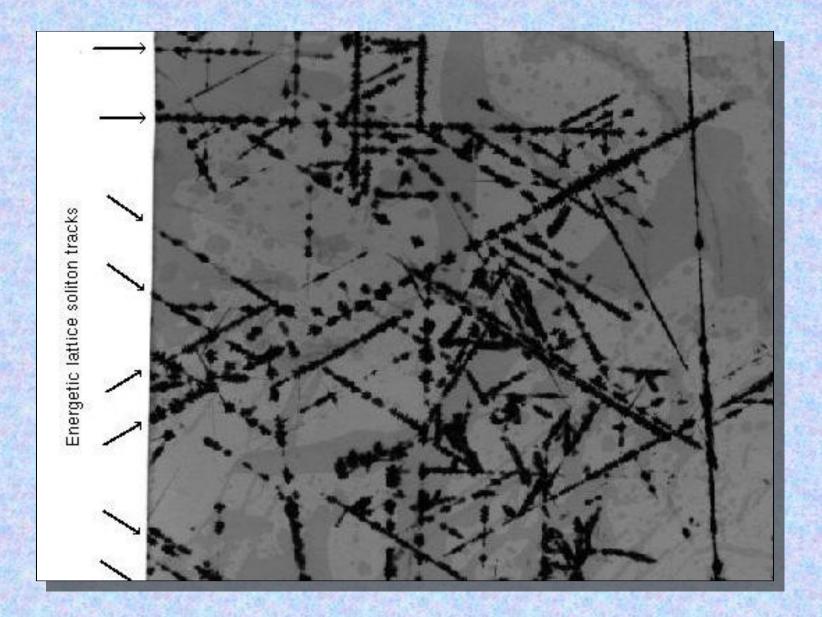
Black tracks: Fe₃O₄

Cause: • 0.1% Particles: • muons: produced by interaction with neutrinos • Positrons: produced by muons' electromagnetic interaction and K decay

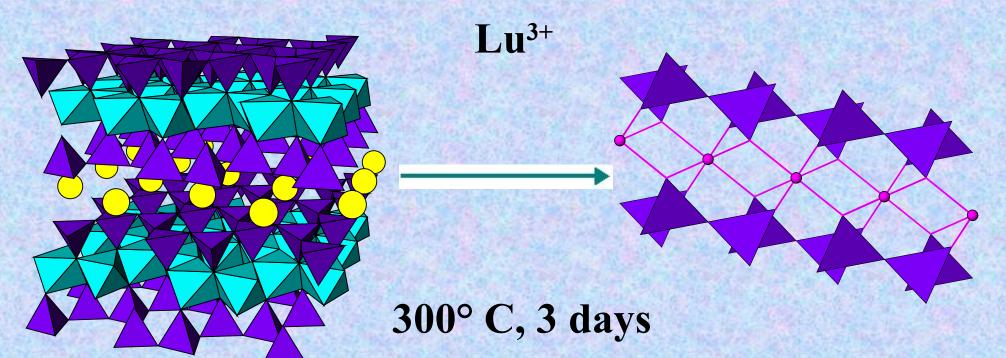
• 99.9% Unknown ¿Lattice localized vibrations: quodons?

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Black traks are alogn lattice directions within the K⁺ layer



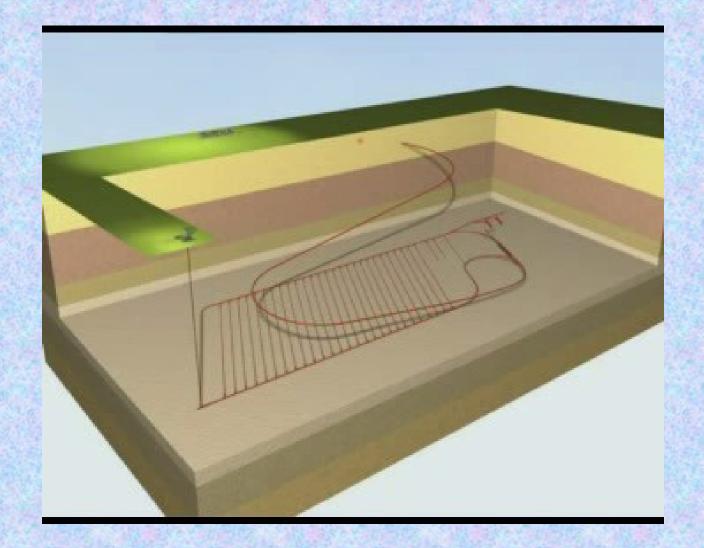
Reconstructive transformation of muscoviteMuscovite $K_2[Si_6Al_2]^{IV}[Al_4]^{VI}O_{20}(OH)_4$ Disilicate of LutetiumLu_2Si_0O_7



About 36% of muscovite is transformed

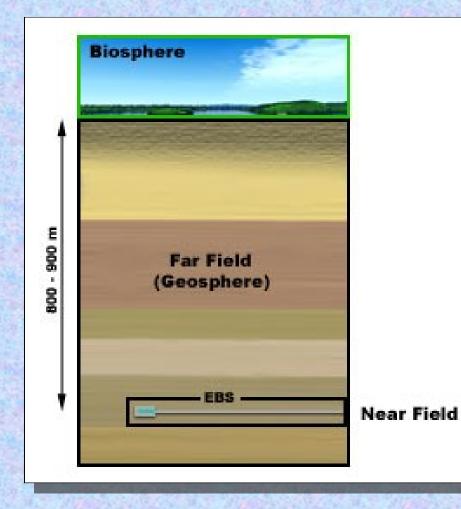


Why LTRT can be interesting?



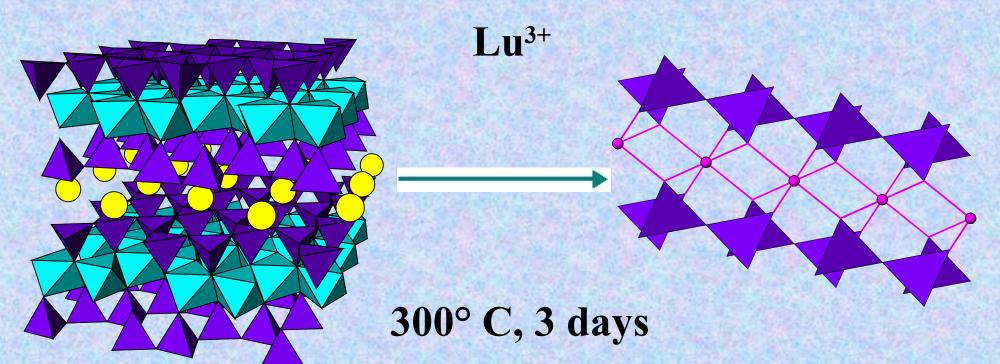
Deeep geological depositories for nuclear waste.

Reconstructive transformations trap the radionuclides



EBS: Engineered barrier system

• In laboratory lutetium substitutes to heavy radionuclides Reconstructive transformation of muscoviteMuscovite $K_2[Si_6Al_2]^{IV}[Al_4]^{VI}O_{20}(OH)_4$ Disilicate of LutetiumLu_2Si_0O_7



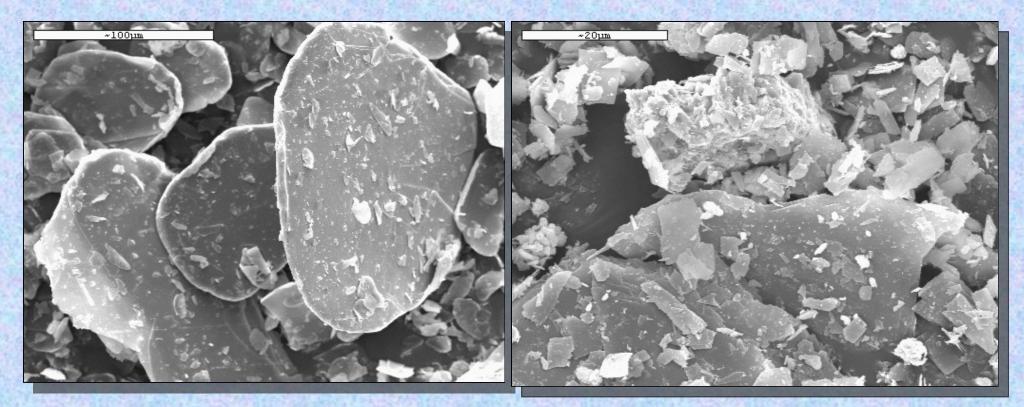
About 36% of muscovite is transformed



Scanning electron microscopy with energy dispersive X-ray (EDX) analysis

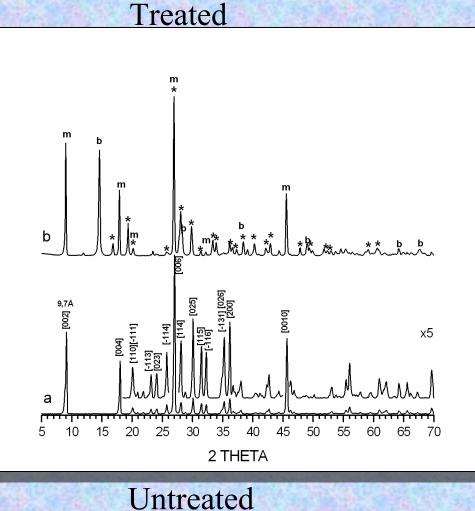
Untreated muscovite

Treated muscovite



Three different types of particles: muscovite, Lu2Si2O7 and bohemite

X-Ray powder diffraction



m=muscovite, b=bohemite, *Lu₂Si₂O₇ Consistent with: •Untreated: Perfect ordering Treated •Two new phases:

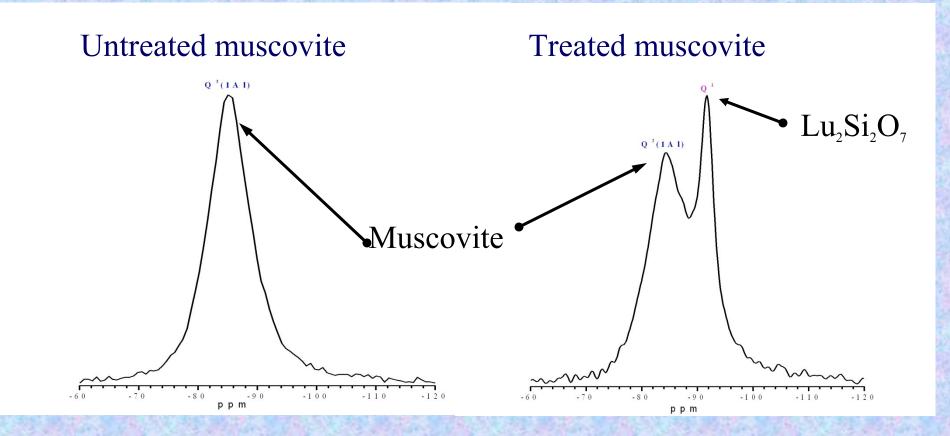
 $Lu_2Si_2O_7$ Bohemite

• Uncomplete transformation

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[Alba and Chain, Clays Clay Min. 53. 39 (2005)]

Nuclear Magnetic Resonance Magic Angle Spinning for silicon



36.6% of Si has changed to the Lu₂Si₂O₇ phase

Reconstructive transformations in layered silicates

- In the laboratory the long times of ageing are simulated with higher temperatures
- Activation energies range typically about 200-400 kJ/mol
- They involve the breaking of the Si-O bond, stronger than that between any other element and oxygen and are observed in silicates only above 1000 C
- A condition for the transformation to take place is that sufficient atoms have enough energy to achieve a transition *activated state*.
- Low temperature reconstructive transformations (LTRT) in layered silicates was achieved by MSG at temperatures 500 C lower than the lowest temperature reported before [Becerro et al, J. Mater. Chem 13, (2003)]
- LTRT take place in the presence of the cation layer
- Possible application in engineered barriers for nuclear waste in deep geological repositories.

Some facts about LTRT

LTRT can be described by:

- Breaking of the Arrhenius law
- An increase of the reaction speed
- A diminution of the activation energy

No explanation had been provided for LTRT **Could breathers be?** Mackay and Aubry [Nonlinearity, 7, 1623 (1994)] suggested the breaking of Arrhenius law as a consequence of discrete breathers.

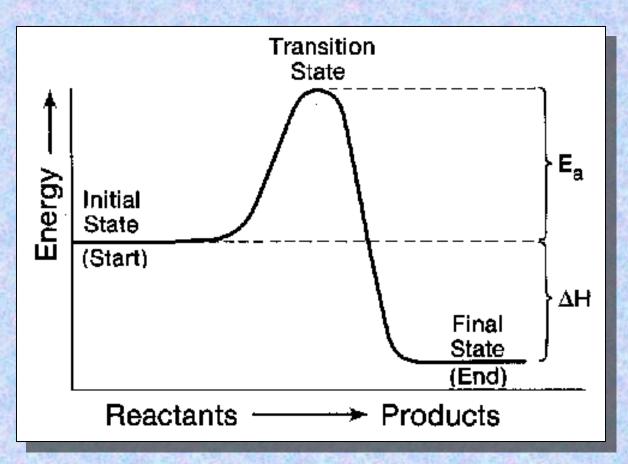
Reaction speed and statistics

Arrhenius law:

$$\kappa = A \exp(-E_a/RT)$$

Transition state theory

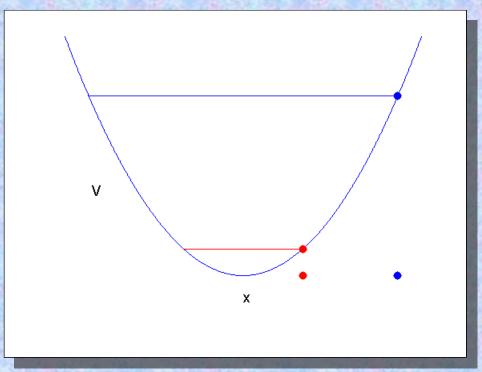
Ea~100-200 KJ/mol



Outline of what follows:

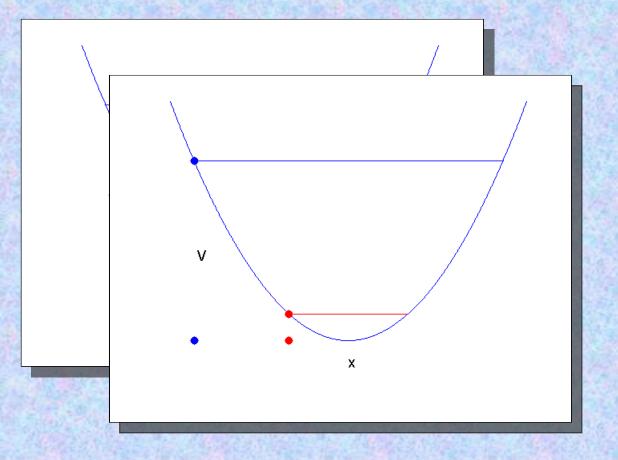
Breather review with application to mica Breathers in mica. Breather statistics with modification Effect of breathers on the reaction rate Effect of breathers on the reaction rate theory

Linear oscillator: F=-k x, $V = \frac{1}{2} k x^2$



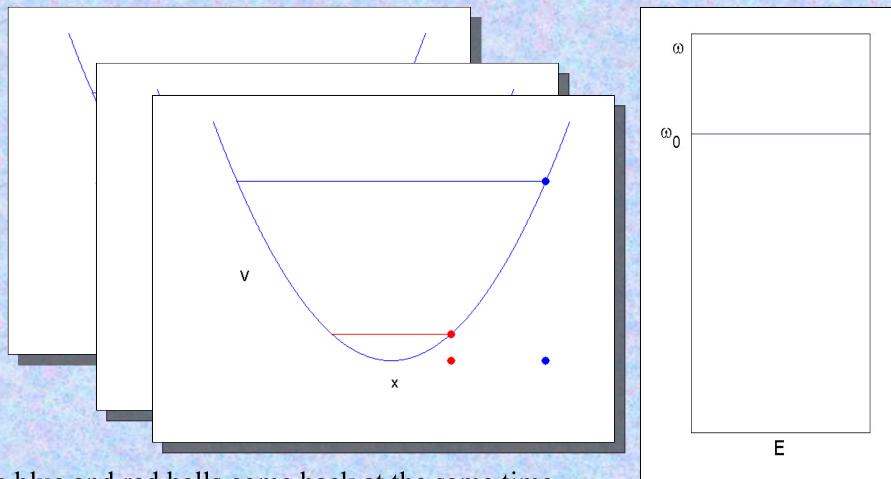
$$x = A \cos(\omega_0 t + \phi_0)$$

Linear oscillator: F=-k x, $V = \frac{1}{2} k x^2$



 $x=A\cos(\omega_0 t+\varphi_0)$,

Linear oscillator: F=-k x, $V=\frac{1}{2} k x^2$

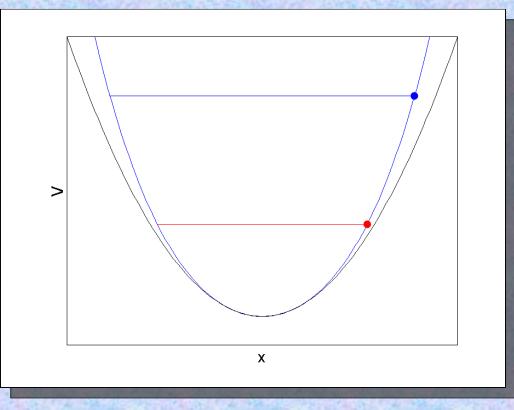


The blue and red balls come back at the same time

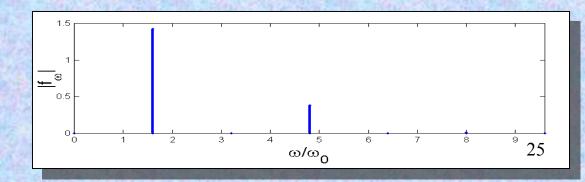
 $x=A\cos(\omega_0 t+\varphi_0)$,

 $\omega_0 \neq \omega_0(E)^{-24}$

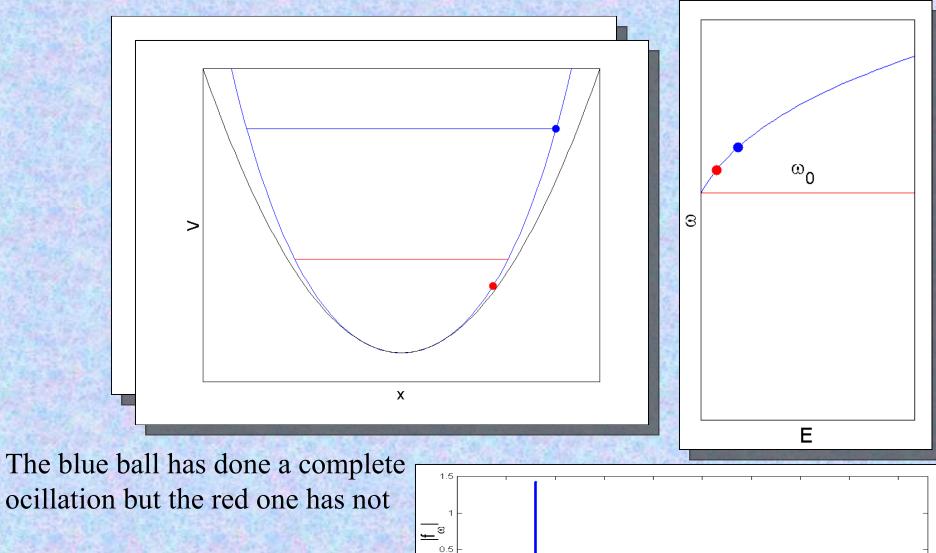
Hard nonlinear oscillator



$V = \frac{1}{2} (\omega_0)^2 x^2 + \frac{1}{4} x^4$



Hard nonlinear oscillator



4

 ω/ω_0

2

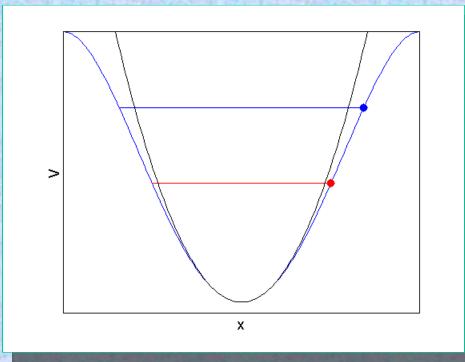
⁹ 26

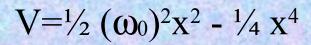
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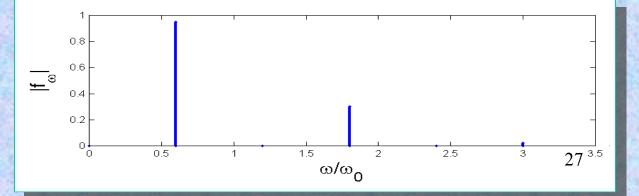
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 $V = \frac{1}{2} (\omega_0)^2 x^2 + \frac{1}{4} x^4$

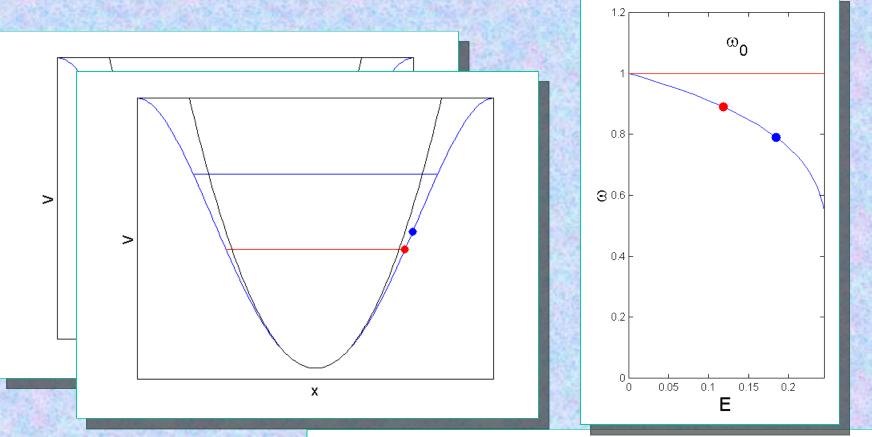
Soft nonlinear oscillator





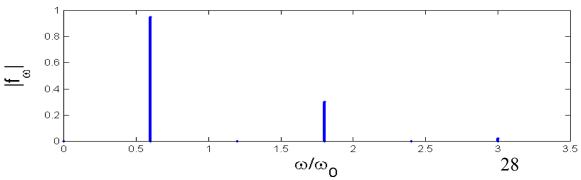


Soft nonlinear oscillator

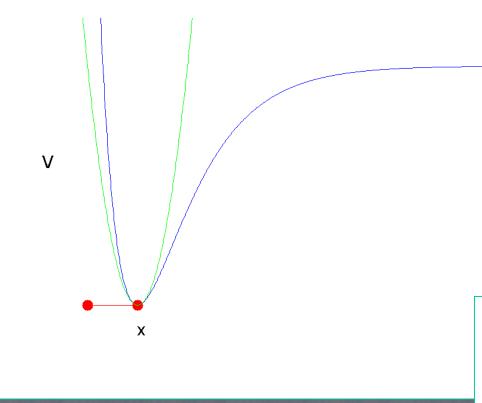


The red ball has done a complete oscillation but the blue one has not

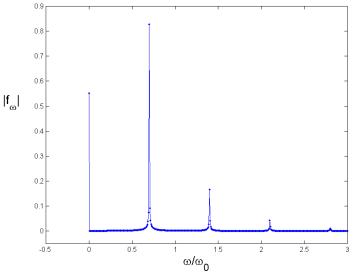
$$V = \frac{1}{2} (\omega_0)^2 x^2 - \frac{1}{4} x^4$$



Asymmetric soft nonlinear oscillator



Morse potential $V=\frac{1}{2}(\omega_0)^2(1-\exp(-x))^2$



The nonlinear oscillator

Potential: V(x)~1/2 m (ω_0)² x² + a x³+b x⁴+...

Fuerza: $F = -V'(x) = -m (\omega_0)^2 x + 3a x^2 + 4b x^3 \neq -k x$

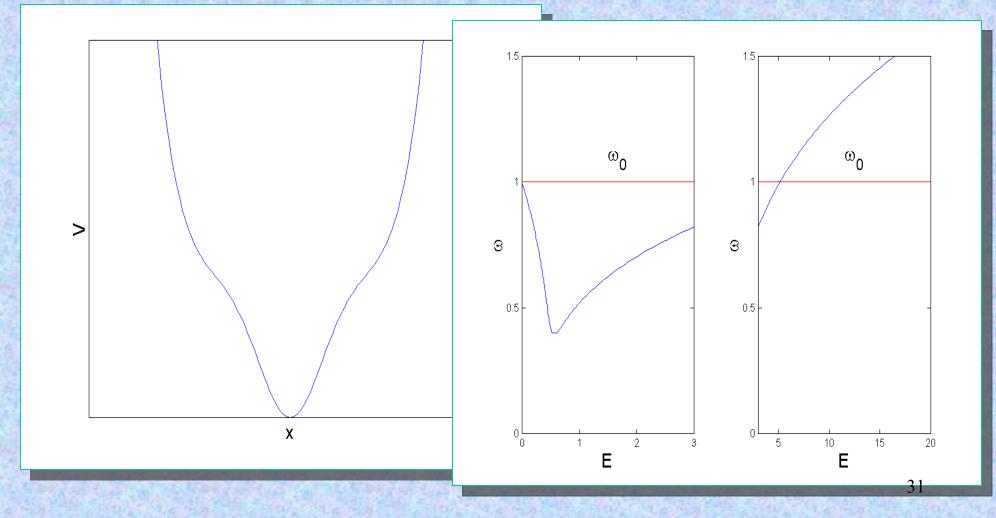
Solution: $x=g(\omega_b t + \varphi_0)$; g: 2π periodic

 $\mathbf{x} = \mathbf{a}_0 + \mathbf{a}_1 \cos(\omega_b \mathbf{t} + \boldsymbol{\varphi}_1) + \mathbf{a}_2 \cos(2\omega_b \mathbf{t} + \boldsymbol{\varphi}_2) + \cdots$

Breather frequency ω_b depends on E: $\omega_b = \omega_b(E)$

- Hard: $\omega_b'(E) > 0$, $\omega_b > \omega_0$
- Soft: $\omega_b'(E) < 0$, $\omega_b < \omega_0$

Nonlinear oscillator: soft-hard potential Potential $V(x)=D(1-e^{-bx^2})+\gamma x^6$



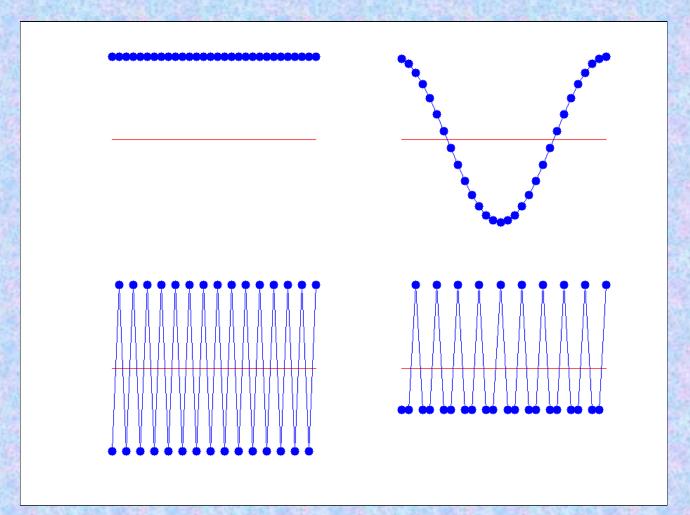
Lattice of coupled nonlinear oscillators

Equation:

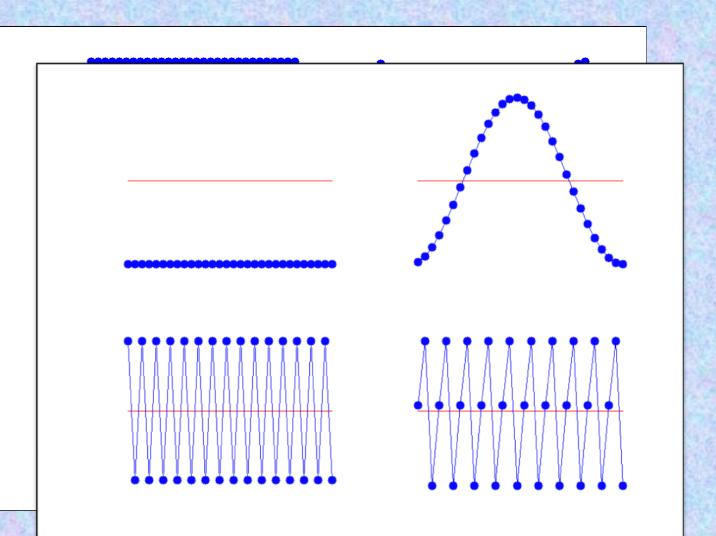
 $x_{n'}(t) = -V''(x_{n}) + \varepsilon (x_{n+1}-x_{n}) - \varepsilon (x_{n}-x_{n-1})$

For small oscillations or linear potentials: $x_n''(t) = -\omega_0^2 x_n^2 + \varepsilon (x_{n+1} - x_n) - \varepsilon (x_n - x_{n-1})$ Well known solutions: **phonons**

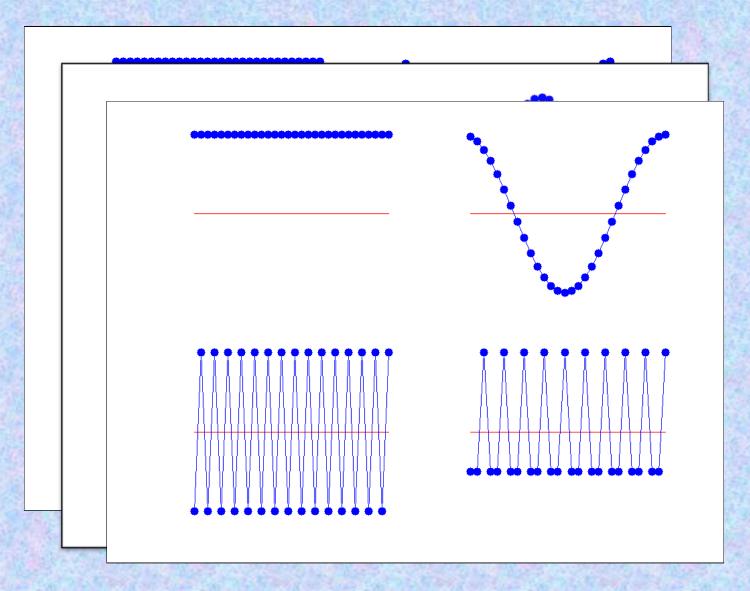
Phonons: $x_n = A \cos(q n - \omega_q t)$



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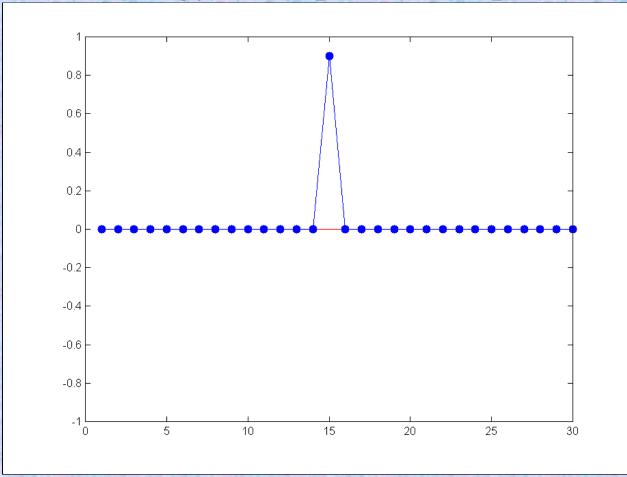
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Phonon characteristics

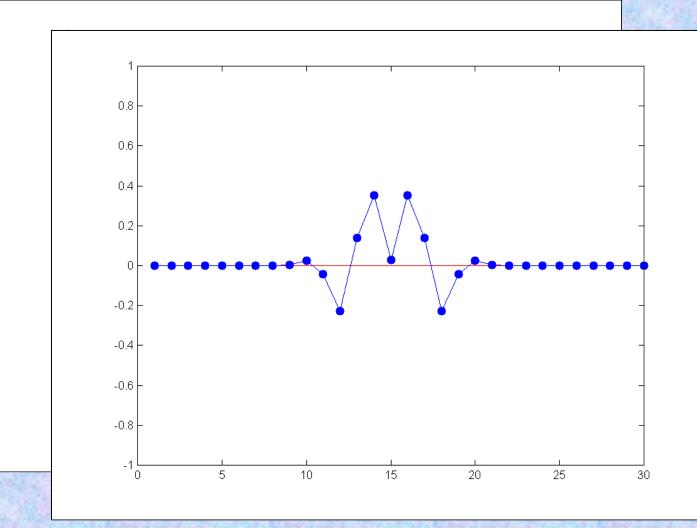
- Extended with uniform amplitude
- Frquency band:



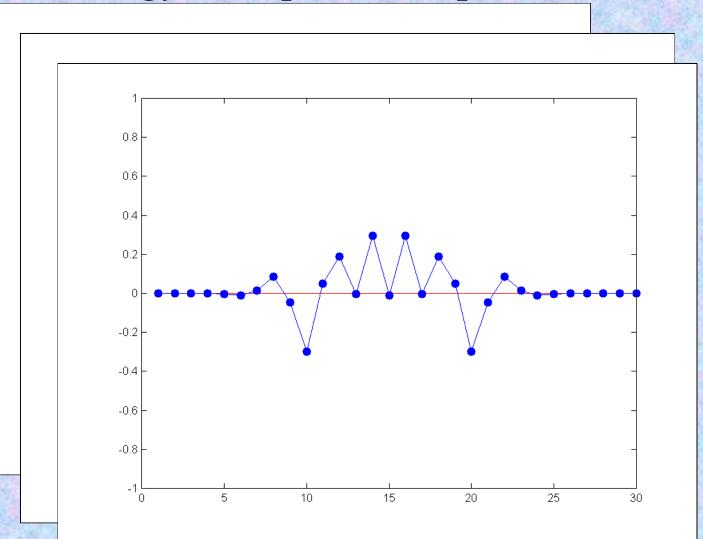
• The energy is dispersed on phonons



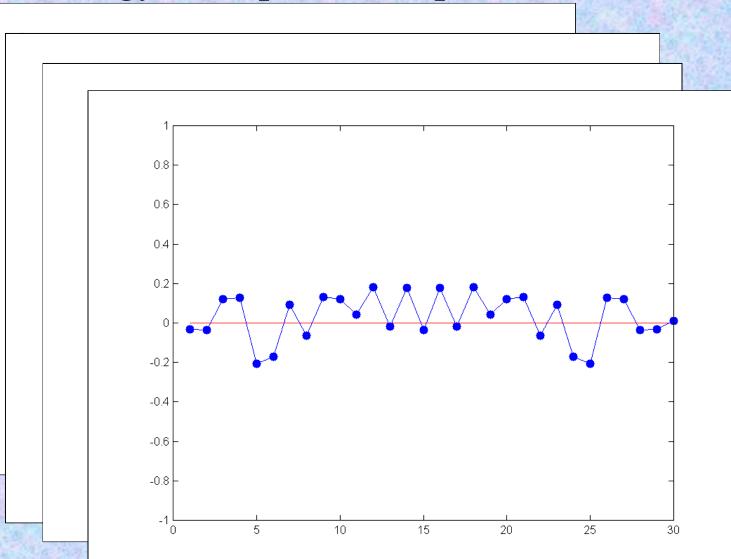
• The energy is dispersed on phonons



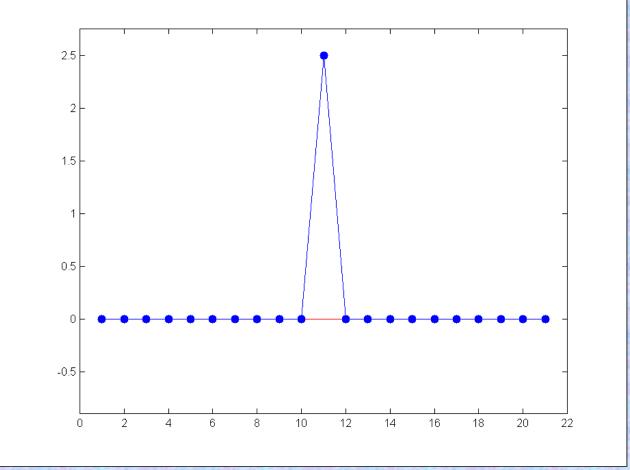
The energy is dispersed on phonons



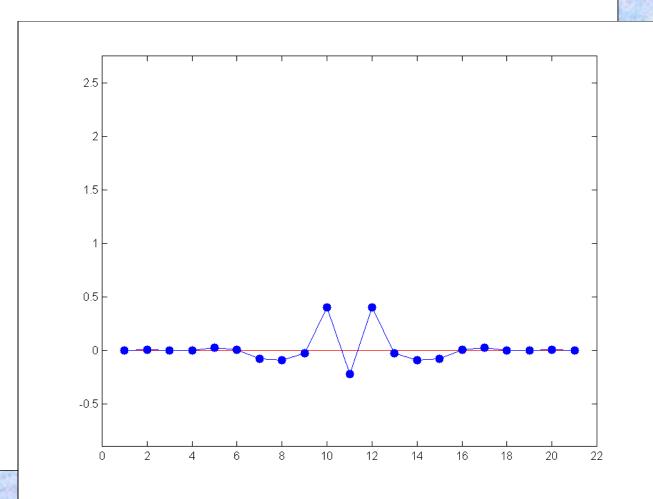
• The energy is dispersed on phonons



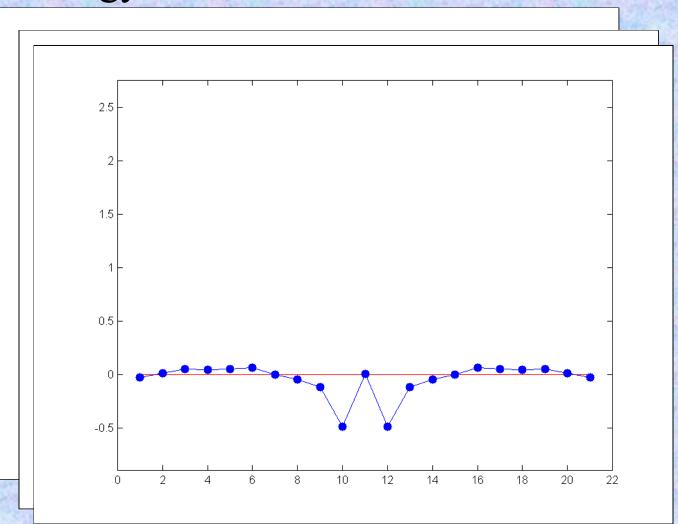
• Energy remains localized



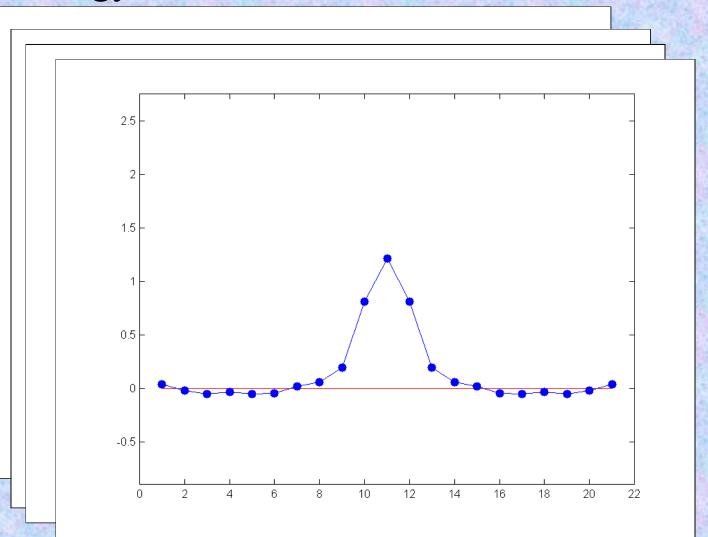
• Energy remains localized

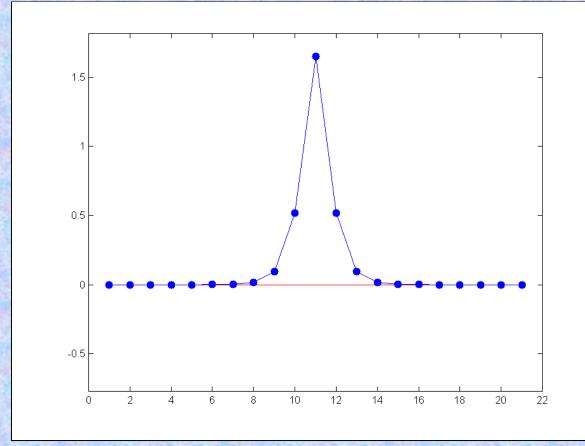


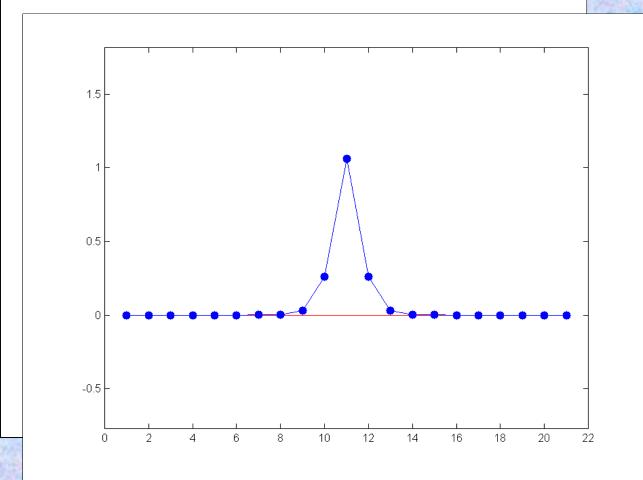
• Energy remains localized

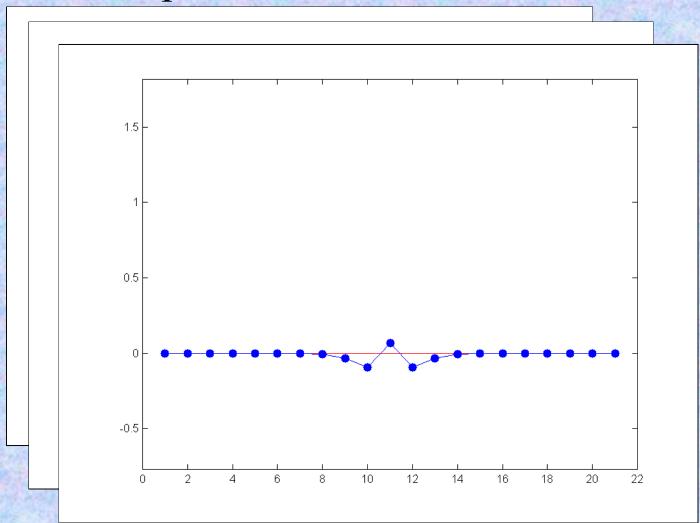


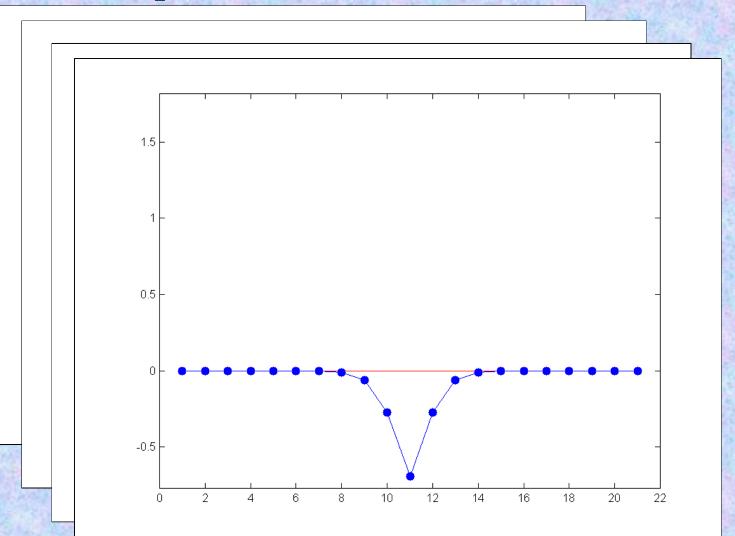
• Energy remains localized



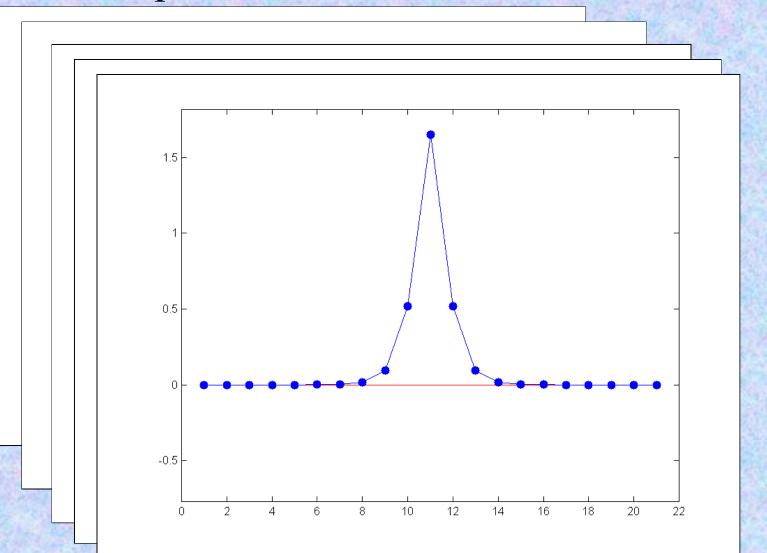




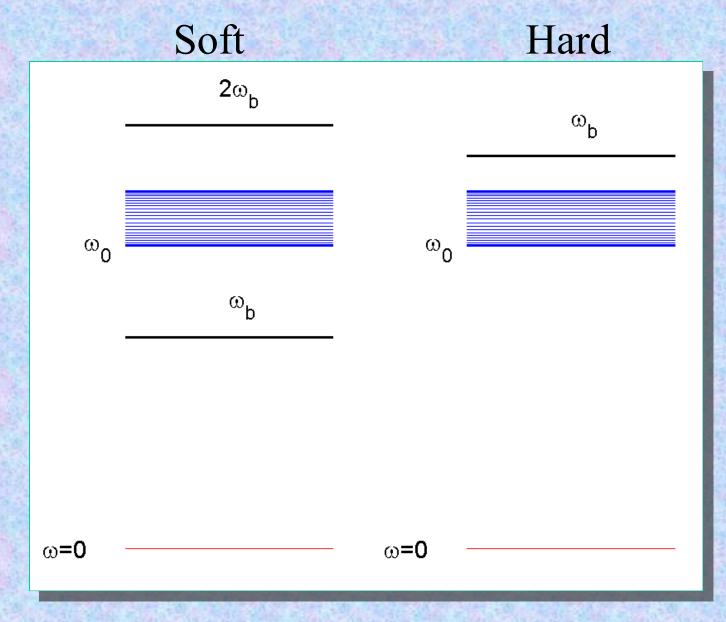




• Exact, periodic, localized solution



Breather frequency and phonon band

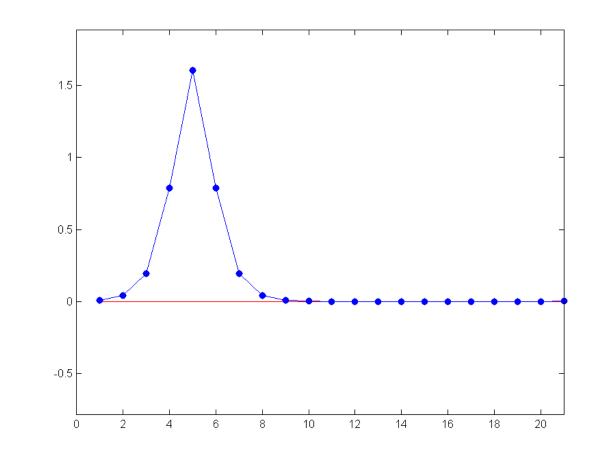


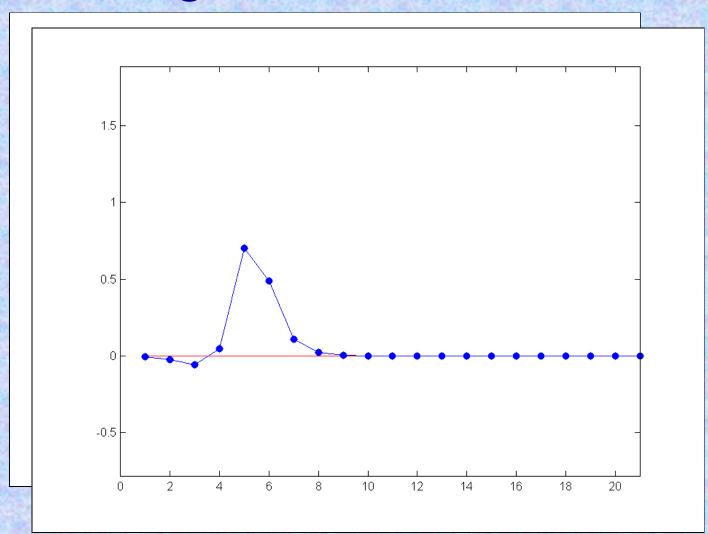
Conditions for breather existence

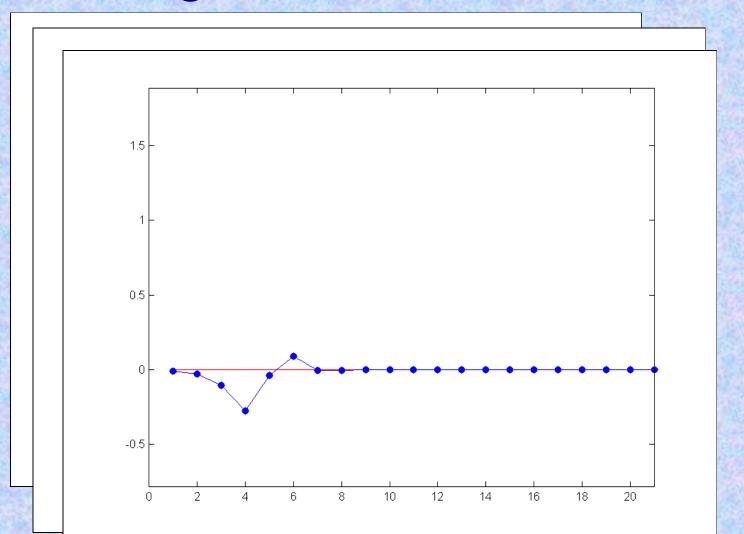
• The breather frequency and its harmonics have to be outside the phonon band.

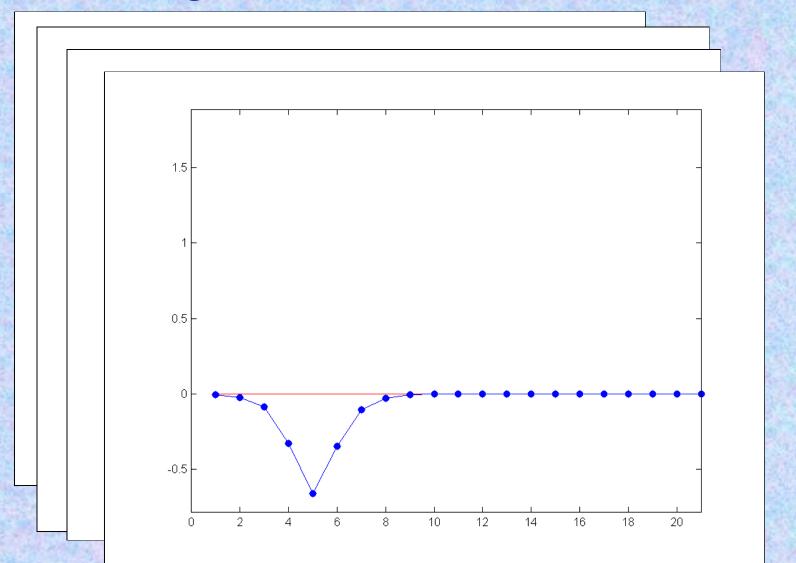
 $\mathbf{n} \, \boldsymbol{\omega}_{\mathrm{b}} \notin \left[\boldsymbol{\omega}_{\mathrm{0}}, \boldsymbol{\omega}_{\mathrm{ph,max}} \right]$

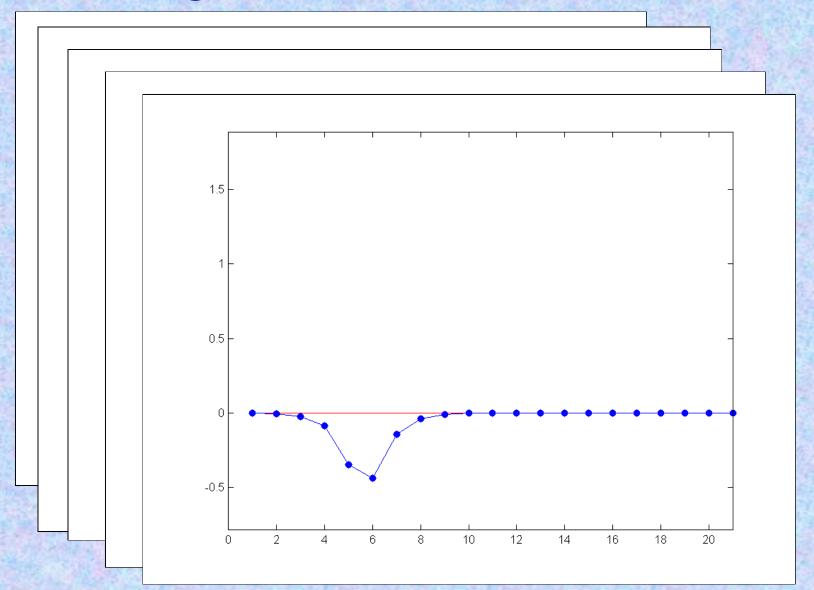
• The oscillator has to be nonlinear for the given amplitude or energy $\omega_{\rm h}'(E) \neq 0$

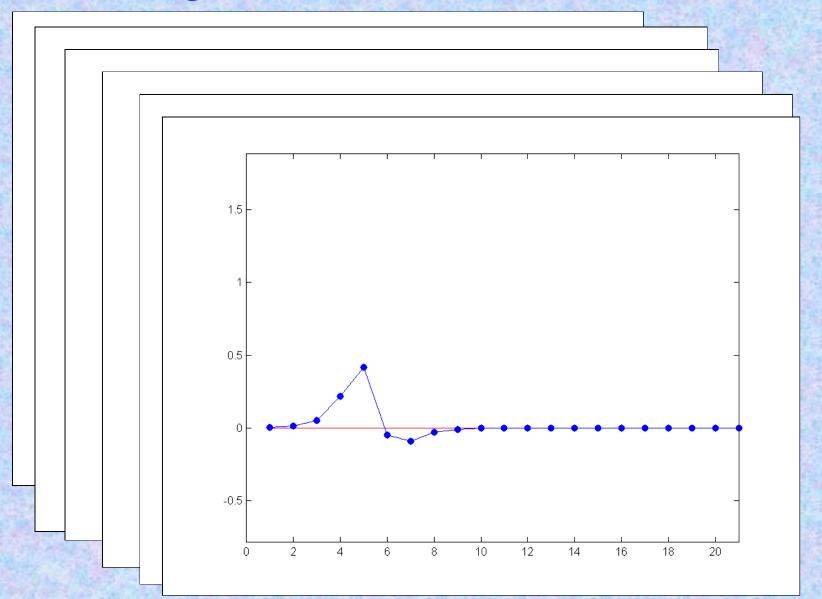


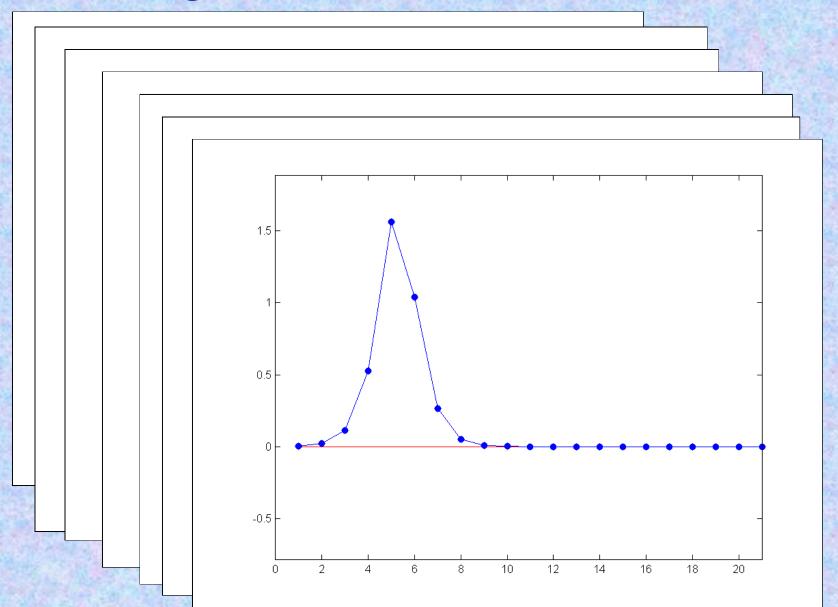


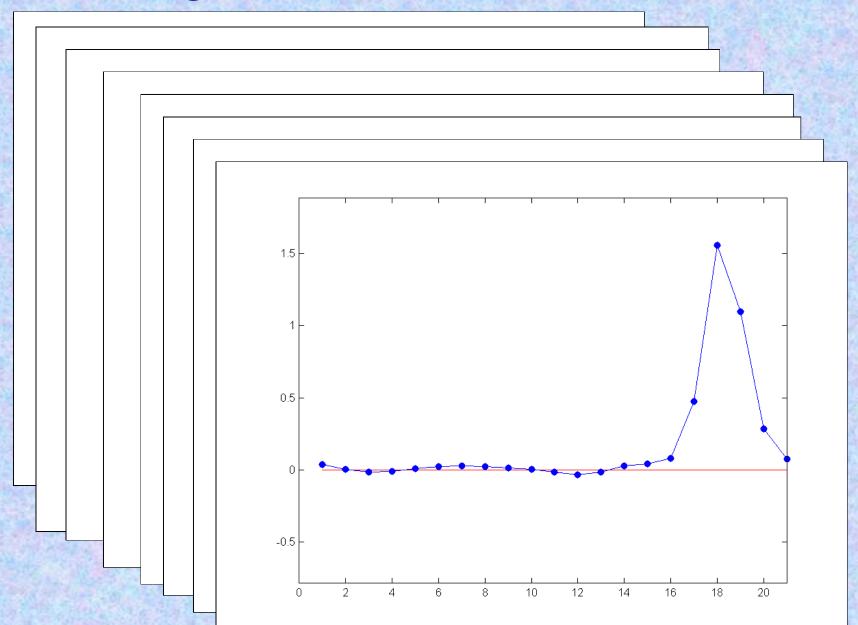




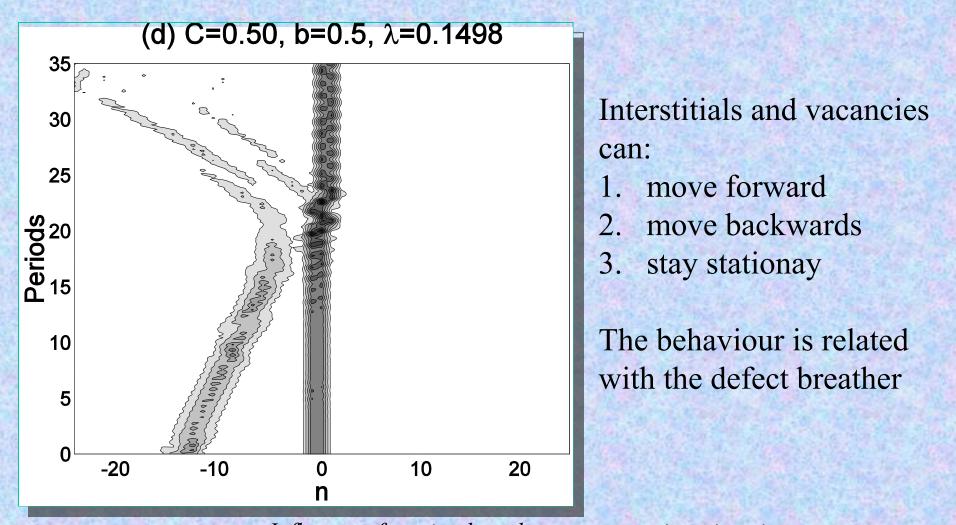






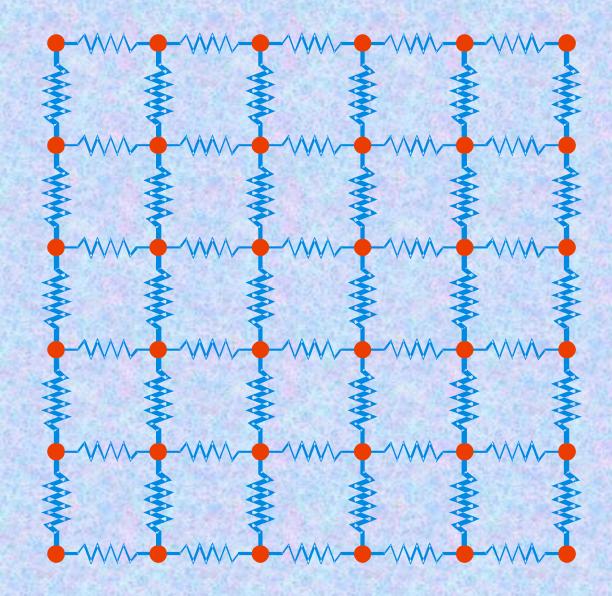


Moving breather sent against a vacancy

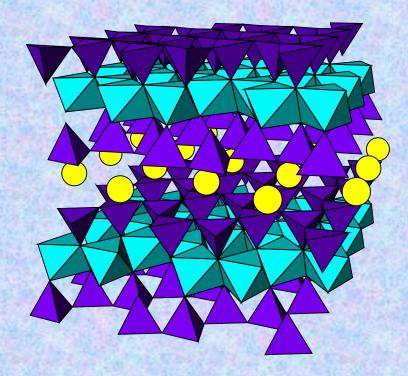


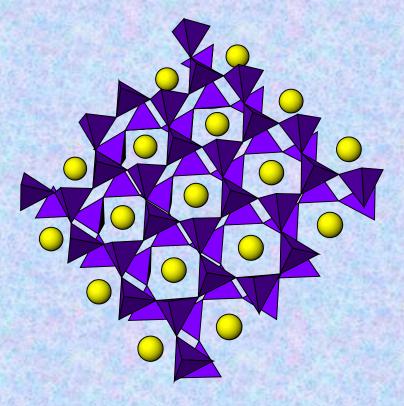
Influence of moving breathers on vacancies migration. J Cuevas, C Katerji, JFR Archilla, JC Eilbeck and FM Russell Phys. Lett. A 315(5):364-371, 2003

Two dimensional networks

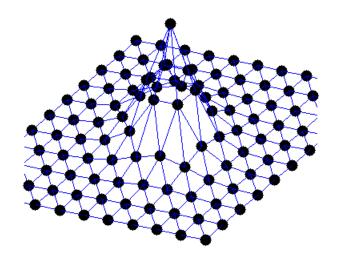


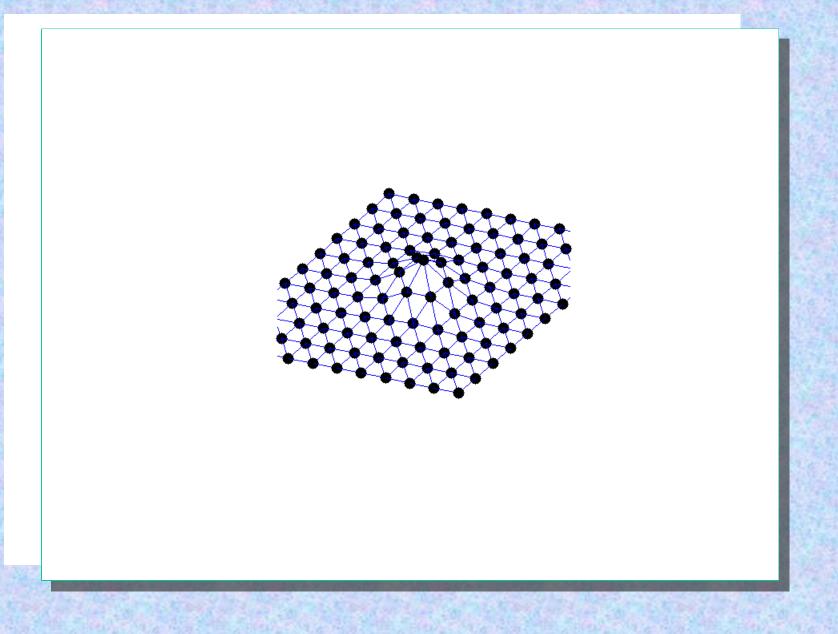
Example: moscovite mica

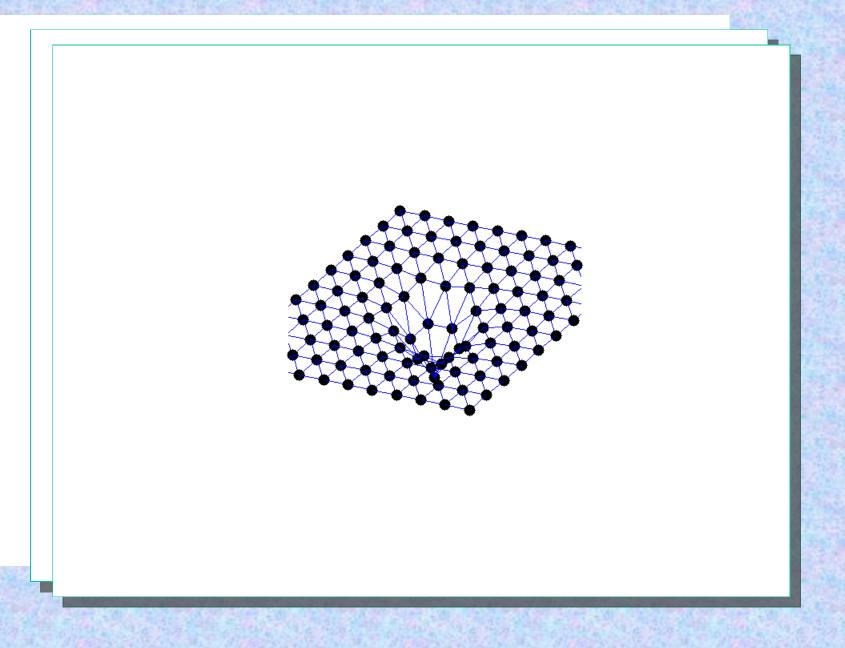


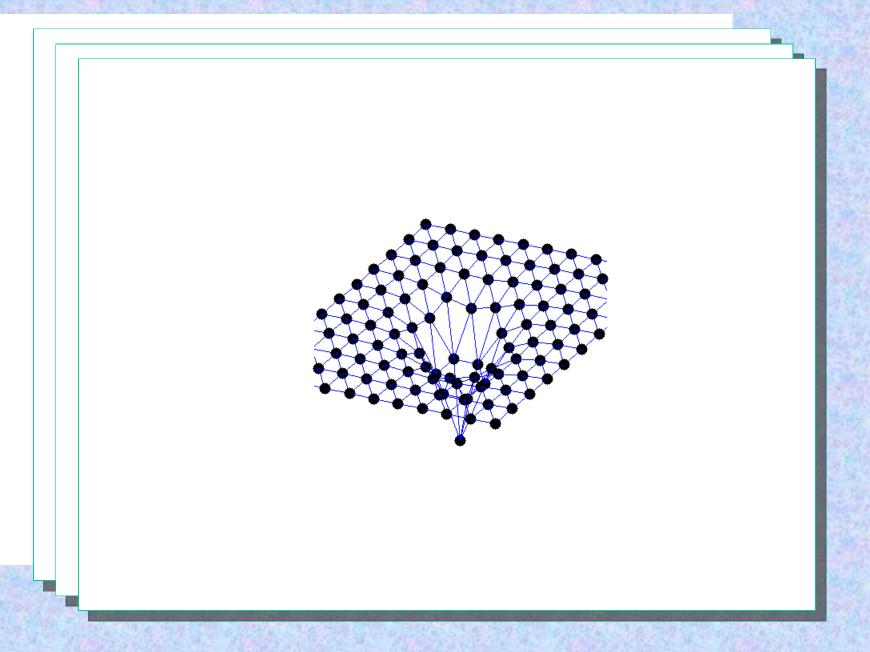


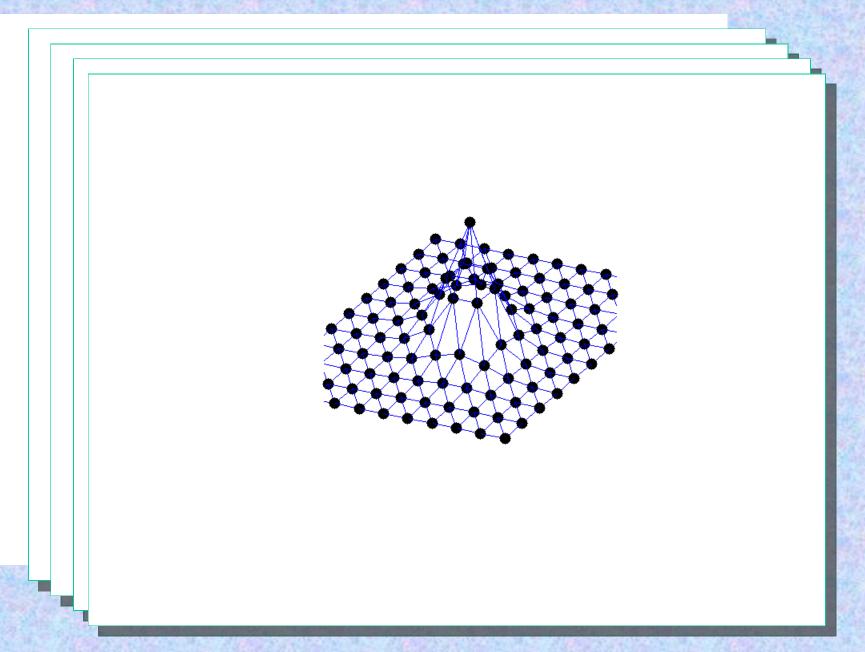
○ K+



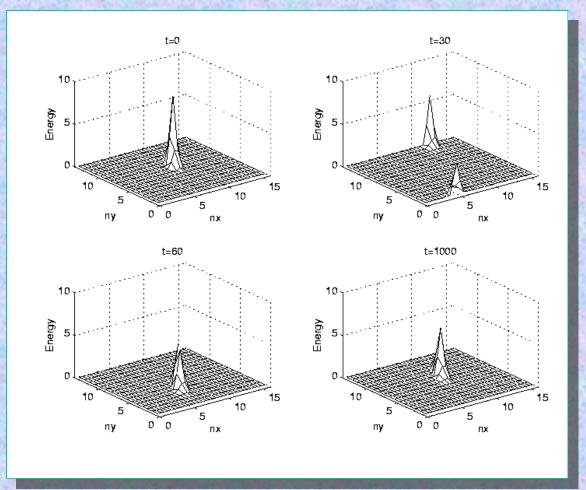








Moving breathers in a 2D hexagonal lattice



No apparent dispersion in 1000~10000 lattice units

Localized moving breathers in a 2D hexagonal lattice. JL Marín, JC Eilbeck, FM Russell, Phys. Lett A 248 (1998) 225

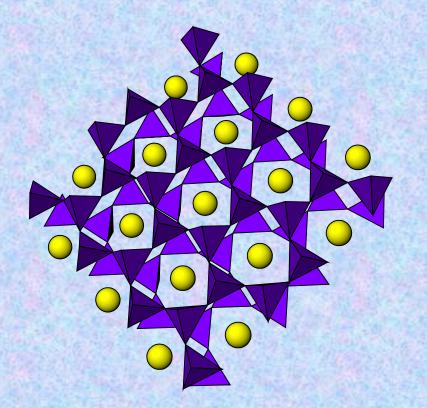
Steps:

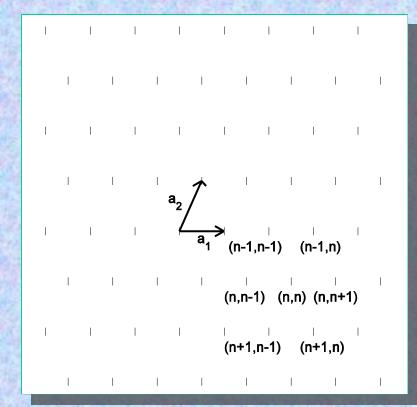
- Find the vibration mode
- Construct the model
- Obtain parameter values
- Obtain breather energies and frequencies

Later:

- Are their energies high enough to influence the reaction rate?
- Are there enough of them?

Mode: vibration of K⁺ normal to the cation layer





Mathematical model

Hamiltonian

$$H = \sum_{\vec{n}} \left[\frac{1}{2} m \dot{u}_{\vec{n}}^2 + V(u_{\vec{n}}) + \frac{1}{2} k \sum_{\vec{n}'} (u_{\vec{n}} - u_{\vec{n}'})^2 \right]$$

Harmonic coupling

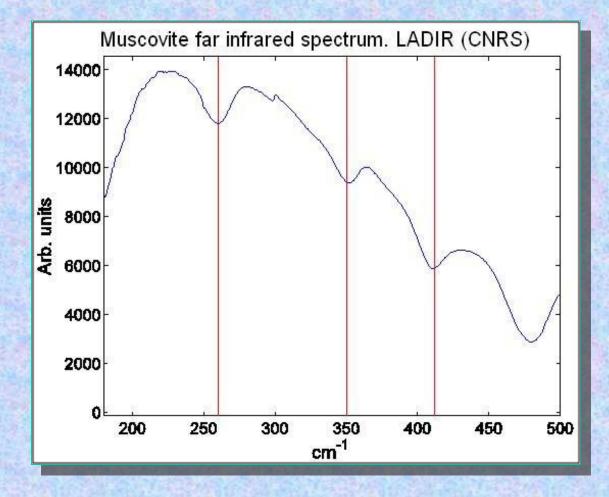
• k=10±1 N/m (D. R. Lide Ed., *Handbook of Chemistry and Physics,* CRC press 2003-2004)

Local potential V

 Assignement of far infrared absortion bands of K⁺ in muscovite, [Diaz et al, *Clays Clay Miner.*, **48**, 433 (2000)] with a band at 143 cm^{-1.}

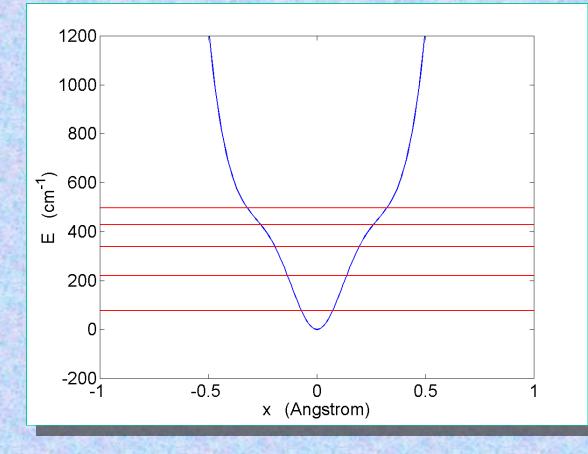
The nonlinear potential has to be obtained.

Mica far infrared spectrum obtained at LADIR-CNRS



Bands at 143, 260, 350 and 420 cm⁻¹ are assigned to transitions of K⁺ vibrations

Fitting the nonlinear potential $\hat{H}\Psi = E\Psi$

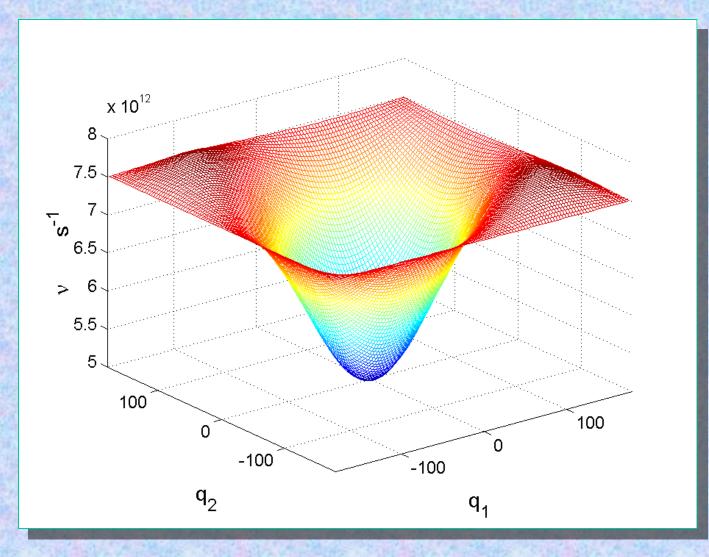


 $V(x) = D([1 - \exp(-b^2 x^2)] + \gamma x^6)$

 $D = 453 \text{ cm}^{-1}$ b² = 36 Å⁻² $\gamma = 49884 \text{ cm}^{-1} \text{ Å}^{-6}$

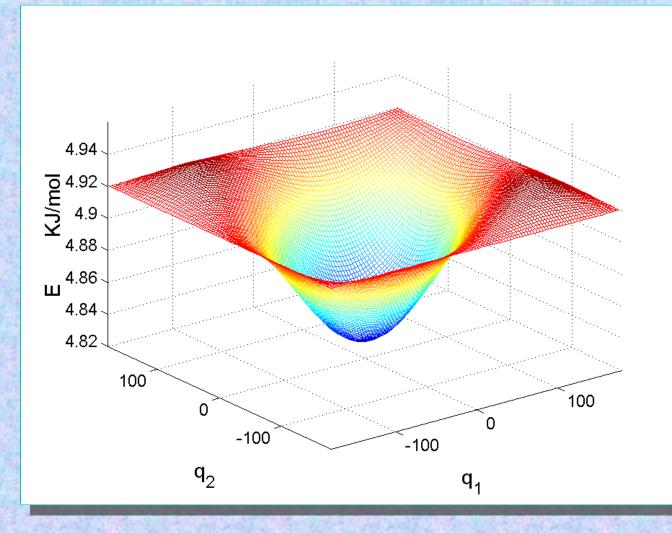
cm⁻¹~1.24x10⁻⁴ eV 1eV~8000 cm⁻¹

Consistent with the available space for K^+ 2x1.45 Å



 $v^2 = (v_0)^2 [1+4 \epsilon(sen^2(q_1/2)+sen^2(q_1/2)+sen^2(q_2/2-q_1/2))]$ 74

Mean energy of each phonon mode



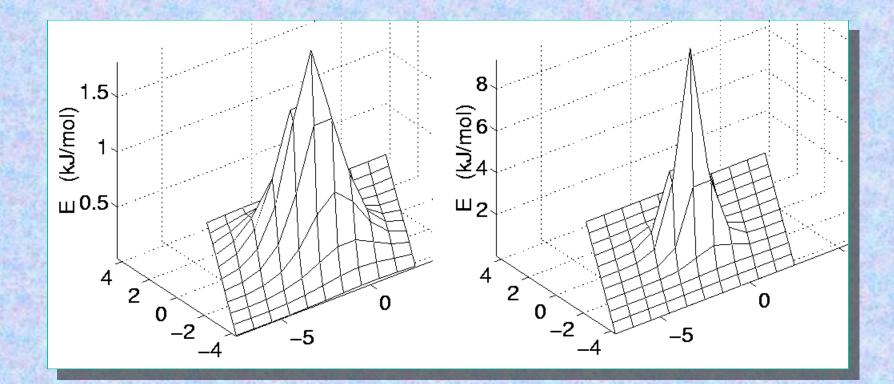
$$< E_{ph} >= (n+0.5) hv$$

 $n=1/(e^{\beta hv} -1)$

T=573 K

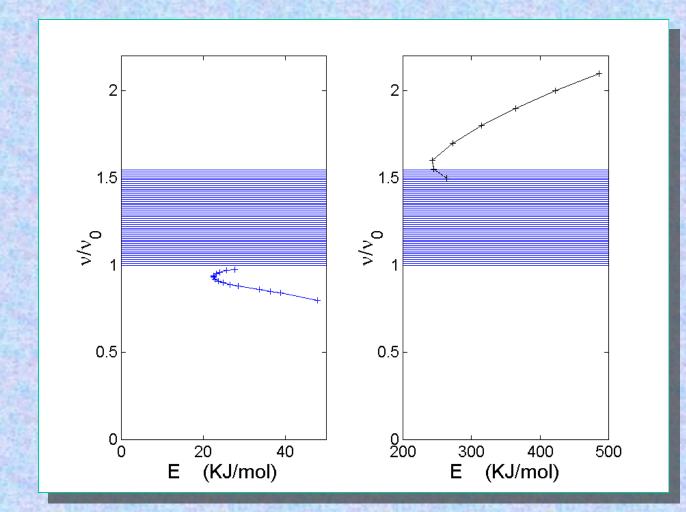
1eV~100 KJ/mol

Energy density profiles for two soft breathers



 $v_b = 0.97v_0$, E =25.6 kJ/mol $v_b = 0.85 v_0$, E =36.3 kJ/mol $v_0 = 167.5 \text{ cm}^{-1} \sim 5 \cdot 10^{12} \text{ Hz}$

Breather frequency versus energy



 $v_0 = 167.5 \text{ cm}^{-1}$ ~ 5.10¹² Hz

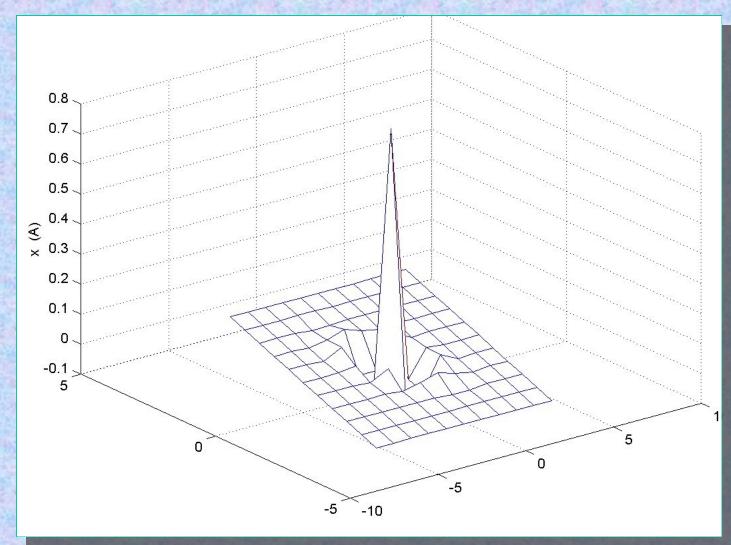
Mimimum energies $\Delta_s = 22.4 \text{ kJ/mol}$

 $\Delta_{\rm h} = 240 \text{ kJ/mol}$

Activation energy estimated in 100-200 kJ/mol

BREATHERS HAVE LARGER ENERGIES THAN THE ACTIVATION ENERGY

Profile of a hard breather



 $v=1.7 v_0=$ 8.54 THz E=272 KJ/mol

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¿How many phonons? ¿How many breathers? ¿With which energies?

Phonons: fraction of phonons per site with energy larger than Ea : $C_{ph}(E_a) = \exp(-E_a/RT)$

Breathers:

•Numerically: $< n_B > ~ 10^{-3}$ por K⁺

•Theory: Piazza et al, Chaos 13, 589 (2003)]

2D breather statistics: Piazza et al, 2003

1.- They have a minimum energy: Δ

- 2.- Rate of breather creation: B(E) $\alpha \exp(-\beta E)$, $\beta = 1/k_BT$
- 3.- Rate of breather destruction: D(E) α 1/(E- Δ) ^z Large breathers live longer.
- 4.- Thermal equilibrium: if P_b(E) dE is the probability that a breather energy is between E and E+dE:
 D(E) P_b(E) dE=A B(E)dE, A≠A(E)
 5.- Normalization: ∫₀[∞] P_b(E) dE=1

Breathers statistics. Results. 1.- $P_b(E) = \beta^{z+1} (E - \Delta)^z \exp[-\beta(E - \Delta)]/\Gamma(z+1)$ 2.- $\langle E \rangle = \Delta + (z+1) k_B T$

3.- Most probable energy: $E_p = \Delta + z k_B T$

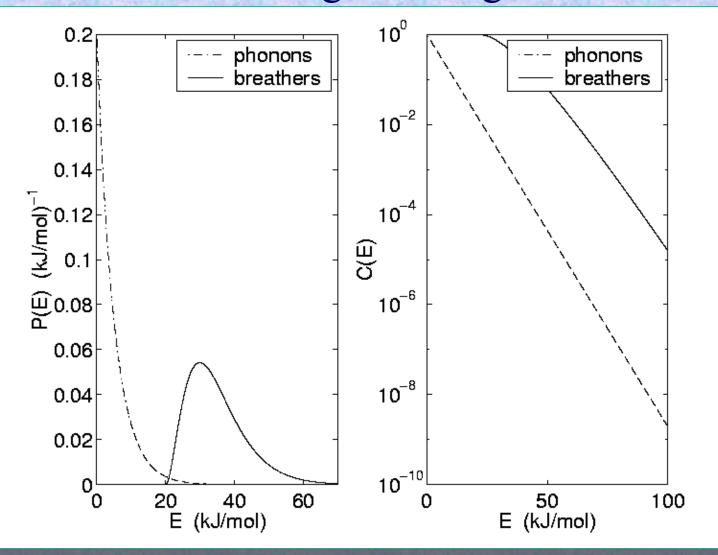
3.-Fraction of breathers with energy above E:

C_b(*E*)=Γ(z+1)⁻¹ Γ(z+1, β[*E*-Δ])

4.- Mean number of breathers per site with energy above E: $n_b(E) = \langle n_b \rangle C_b(E)$

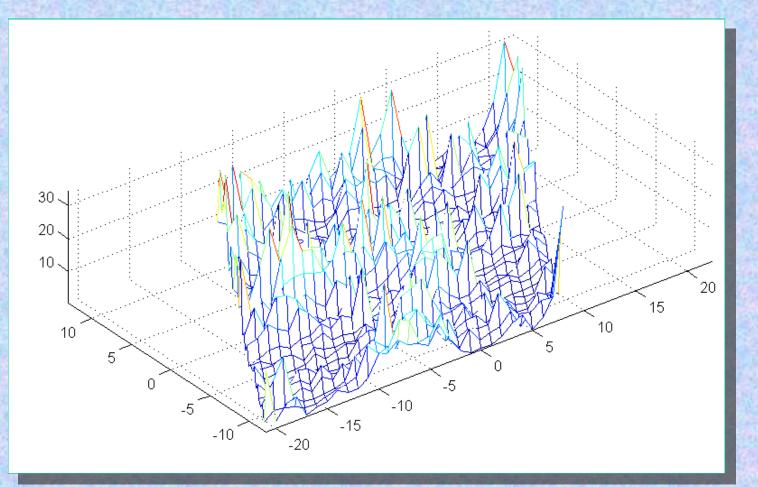
<n_b>=mean number of breathers per site ~10⁻³
-Function gamma and first incomplete gamma function: $\Gamma(z+1) = \int_{0}^{\infty} y^{z} \exp(-y) dy, \quad \Gamma(z+1,x) = \int_{x}^{\infty} y^{z} \exp(-y) dy$

Probability density and cumulative probability. Breathers accumulate at higher energies

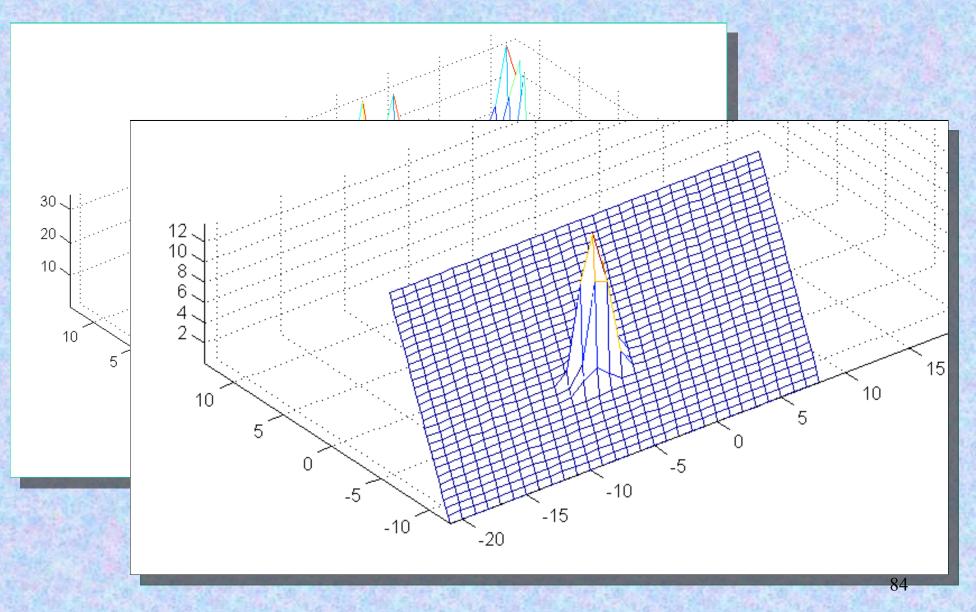


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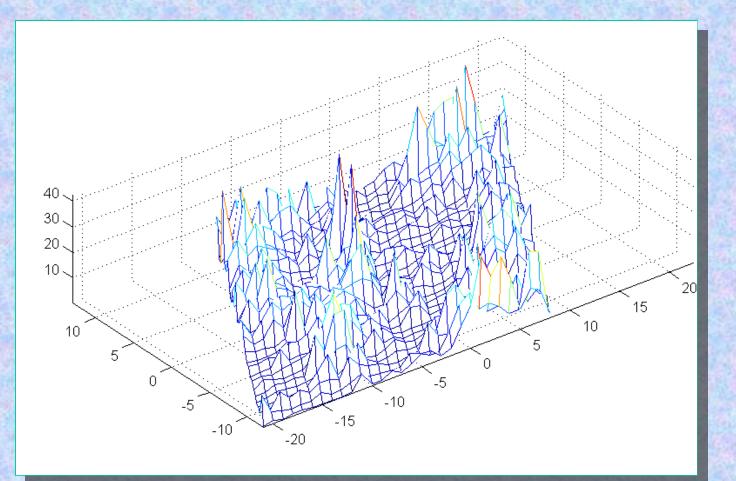
Numerical simulations in mica (1)



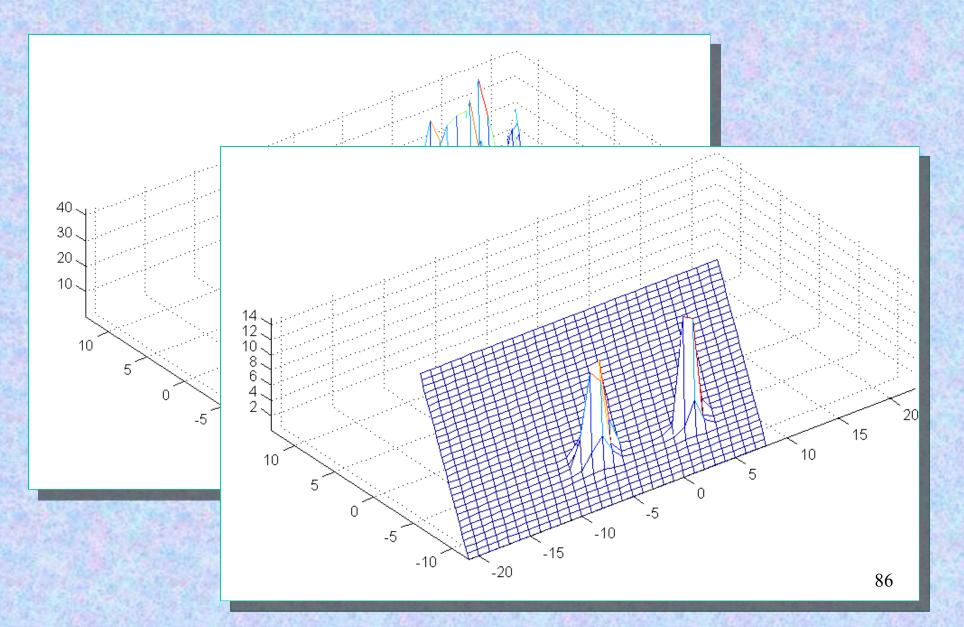
Numerical simulations in mica (1)



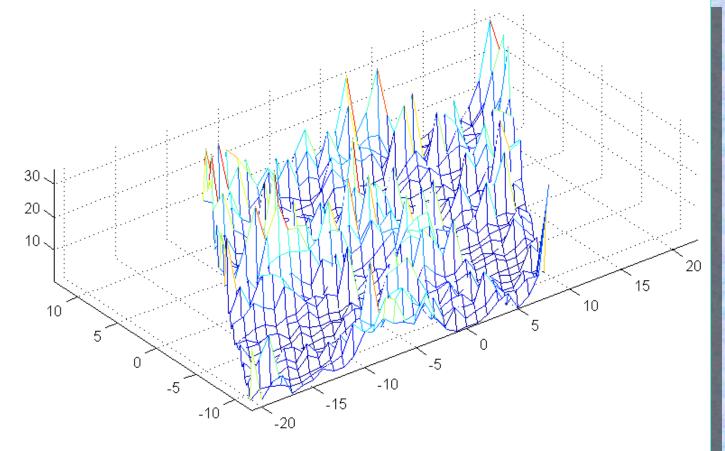
Numerical simulations in mica (2)



Numerical simulations in mica (2)

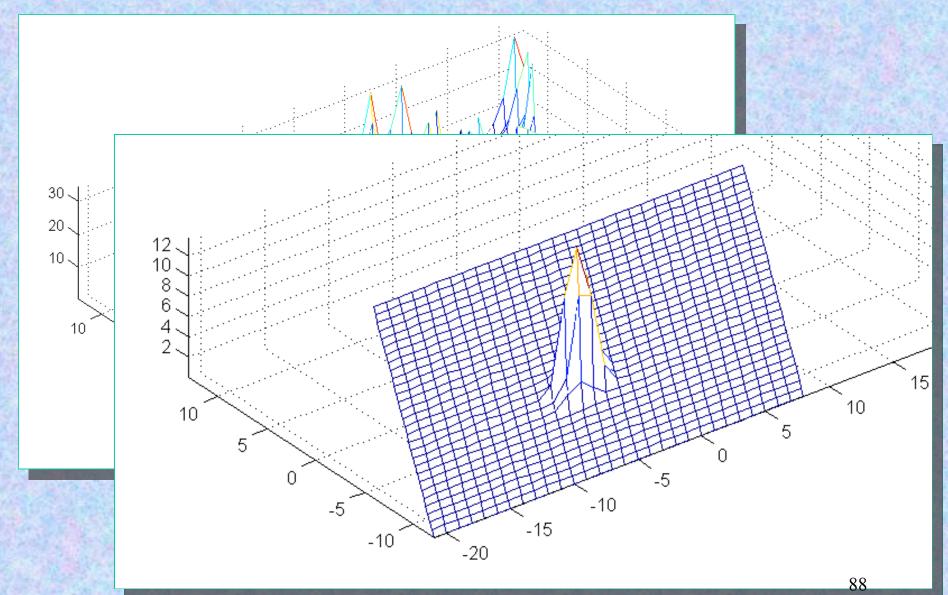


Comparison with numerical simulations in mica. Before cooling.

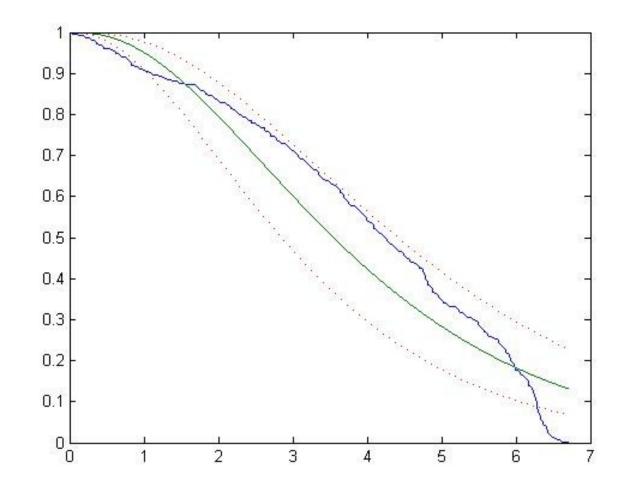


Random velocities and positions. Thermal equilibrium. Cooling at the borders.

Numerical simulations in mica. After cooling.

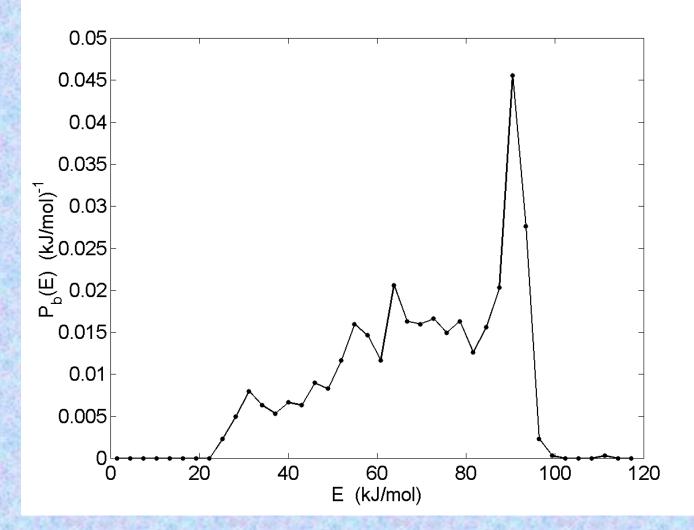


Attemp to fit $C_B(E)$: failure.

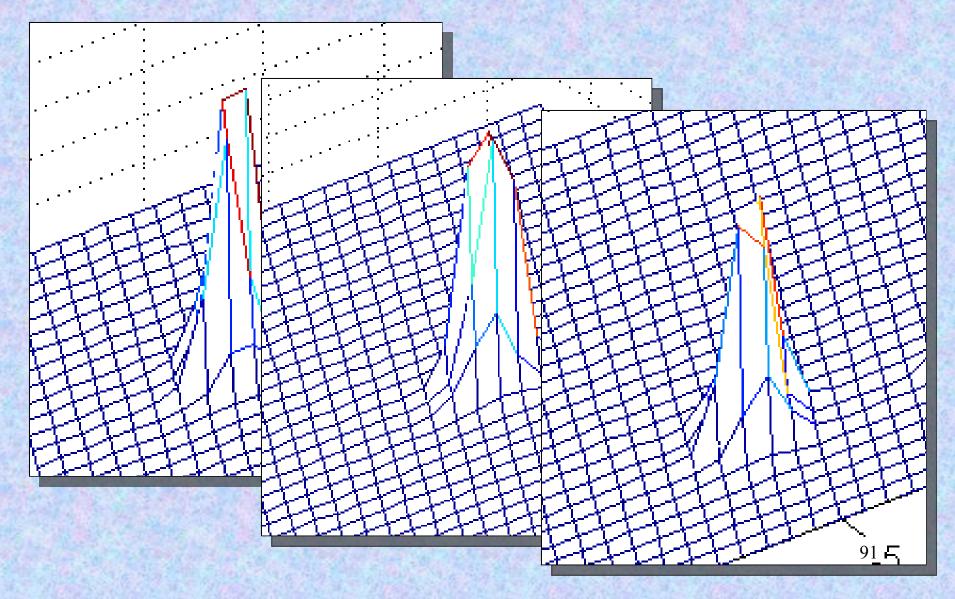


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Total failure: $P_b(E)$



Reason: different breathers and multibreathers



Modification of the theory. Breathers with maximum energy 1.- Multiple breather types

- 2.- Differences:
 - Minimum energy Δ
 - Parameter z
 - Maximum energy E_M !! : E_M
 - Normalization:

 $\oint P_b(E) dE=1$

• Different probability for each type of breather:

$$P(\Delta, z, E_M, ?)$$

Breathers with maximum energy. Results.

1.- Probability density:

 $P_{b}(E) = \beta^{z+1} (E - \Delta)^{z} \exp[-\beta(E - \Delta)] / \gamma(z+1, \beta[E_{M} - \Delta])$

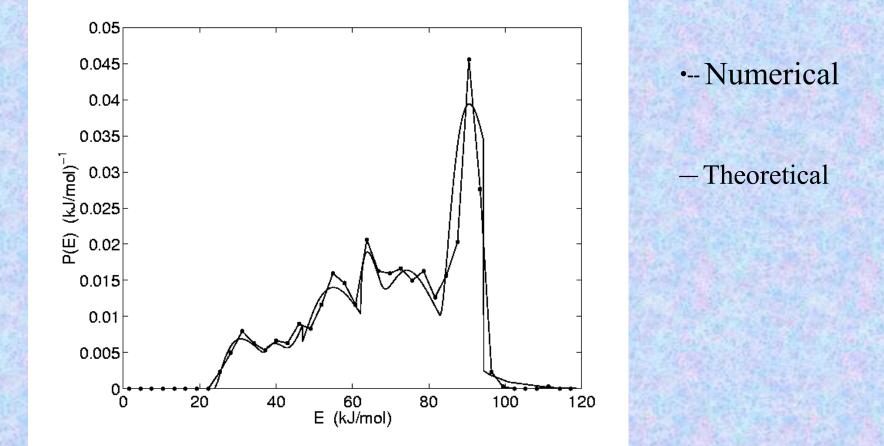
3.- Fraction of breathers with energy above E:

 $C_{b}(E)=1-\gamma(z+1,\beta[E-\Delta])/\gamma(z+1,\beta[E_{M}-\Delta])$

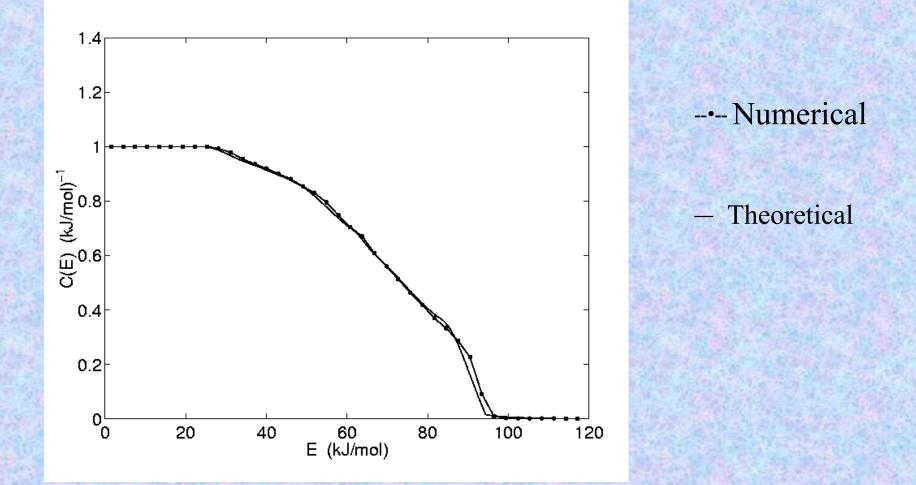
- Second incomplete gamma function:

 $\gamma(z+1,x) = \int_0^x y^z \exp(-y) dy$

Density probability for breathers in mica



Cumulative probability: Fraction of breathers with energy equal or larger than E



Breather energy spectrum

36.6 41.4 62.2 67.3 Δ (kJ/mol) 23.9 82.9 3.00 0.52 2.07 1.50 1.17 1.80 Z46.9 $E_{\rm M}$ (kJ/mol)) 94.4 ---0.281 0.097 0.202 0.290 probability 0.103 0.026

Estimations

For $E_a \sim 100-200 \text{ kJ/mol}$, T=573 K:

 $\frac{\text{Number of breathers}}{\text{Number of phonons}} = 10^4 - 10^5 \qquad (\text{with } \text{E} \ge \text{Ea})$

Reaction time without breathers: 80 a 800 años,

Moreover, breather can localize more the energy delivered, which will increse further the reaction speed

THERE ARE MUCH LESS BREATHERS THAN LINEAR MODES, BUT MUCH MORE WITH ENERGY ABOVE THE ACTIVATION ENERGY

Discrete breathers for understanding reconstructive mineral processes at low temperatures JFR Archilla, J Cuevas, MD Alba, M Naranjo and JM Trillo, J. Phys. Chem. B 110 (47): 24112-24120 (2006) . 97

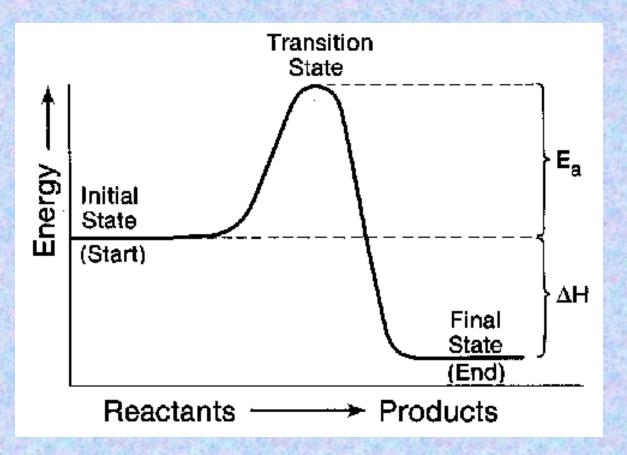
Kramer's theory revisited

Arrhenius law:

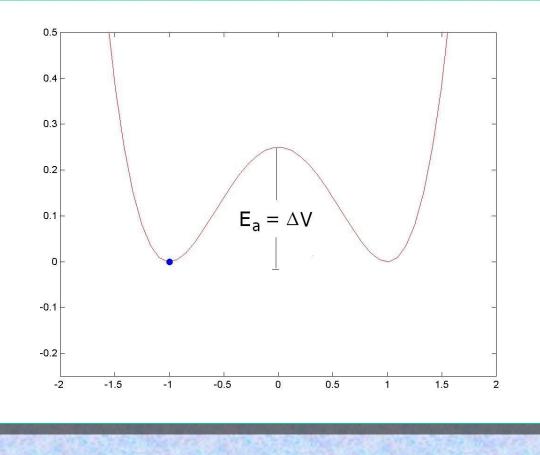
$$\kappa = A \exp(-E_a/RT)$$

Transition state theory

Ea~100-200 KJ/mol



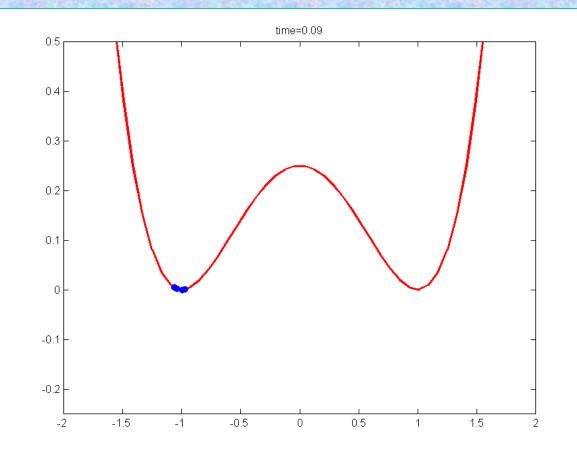
Kramers theory of reaction rate (1)



Reactants

Products

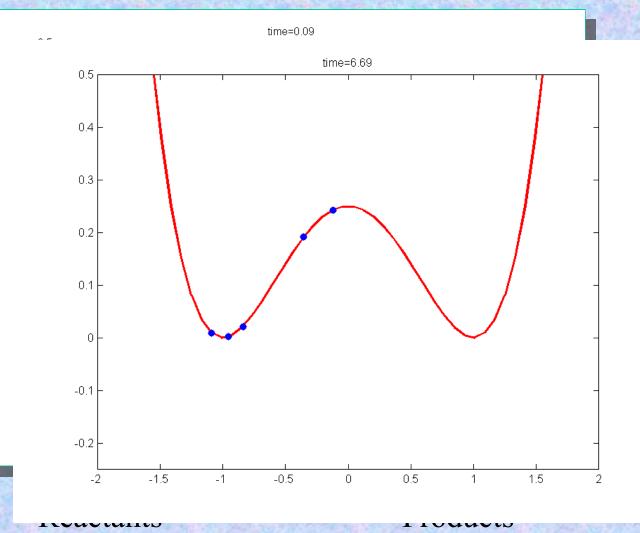
Kramers theory of reaction rate (2)



Reactants

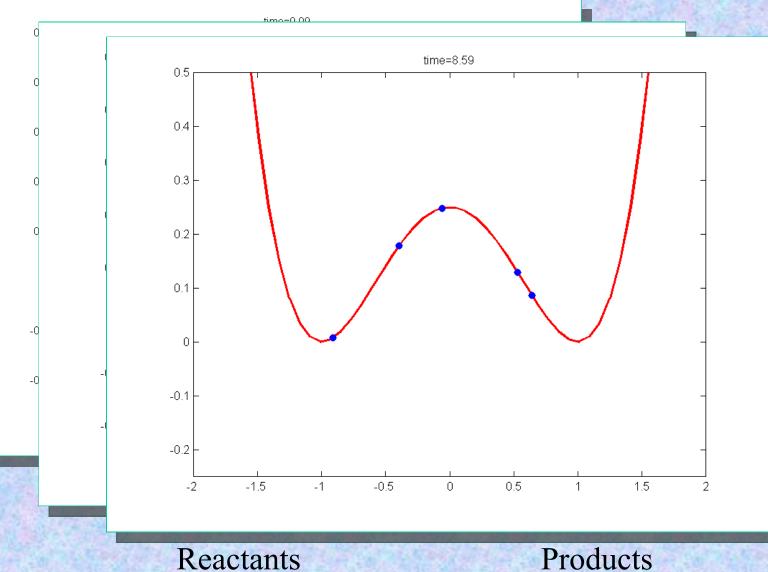
Products

Kramers theory of reaction rate (2)



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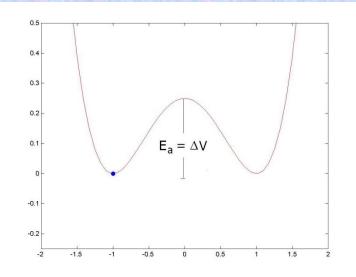
Kramers theory of reaction rate (2)



Reactants

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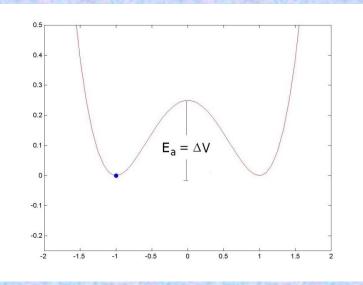
Kramers theory of reaction rate (3)



 $V(x) = (1/4)bx^4 - (1/2)ax^2$ Minima at $\pm x_m = \pm (a/b)^{1/2}$ Barrier height=activation energy: $E_a \equiv \Delta V = a^2/4b$ Frequencies: $\omega_0^2 = V''(x_m)/m$ $\omega_b^2 = |V''(x_b)/m|$

 $\ddot{x} = -V''(x) - \gamma \dot{x} + F(t)$ Stochastic equation: Stochastic force: $\langle F(t) \rangle = 0$ $\langle F(t)F(t') \rangle = 2\pi \gamma k_{\rm B} T \delta (t'-t)$

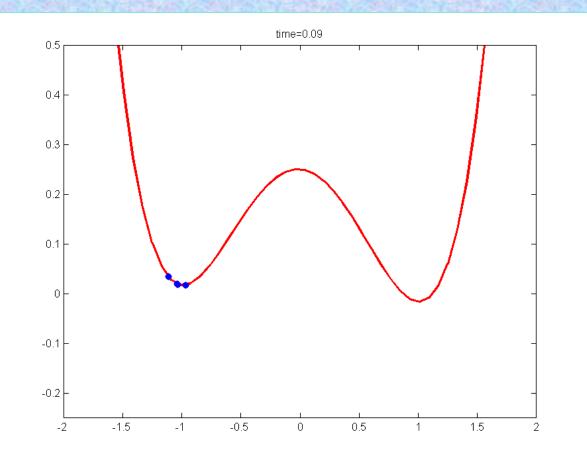
Kramers theory of reaction rate (4)

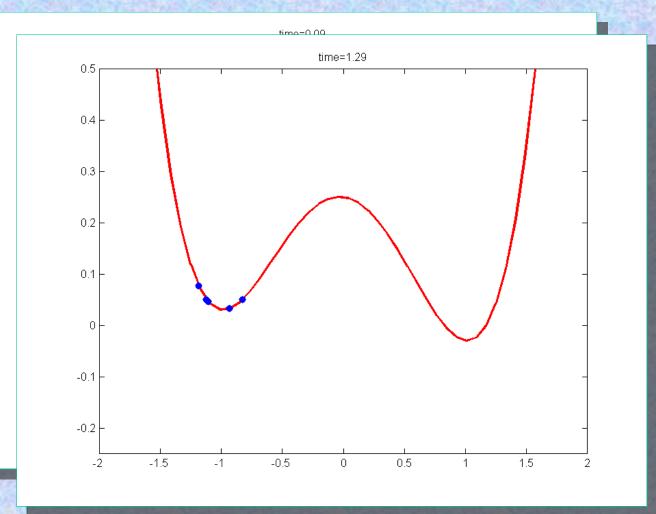


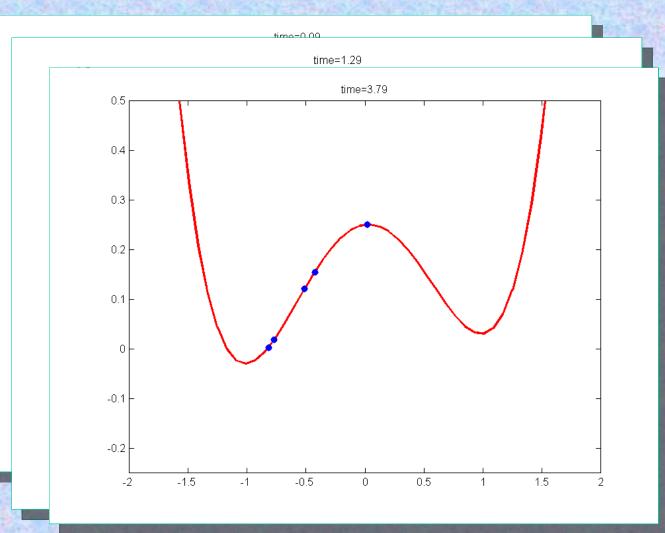
Reaction $A + B \rightarrow C$ Reaction rate constant: k_R $\frac{\mathrm{d} C}{\mathrm{d} t} = k_R [A]^n [B]^m$ Arrhenius' law: $k_{R} = A \exp(-E_{a}/k_{B}T)$

Kramers reaction rate constant:

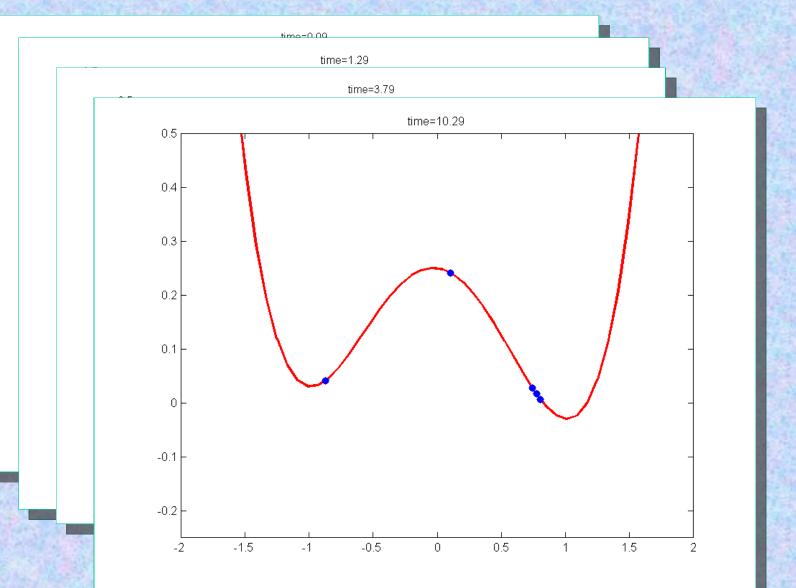
 $k_{R} = \frac{\omega_{b}\omega_{0}}{2\pi\gamma} \exp(-E_{a}/k_{B}T)$







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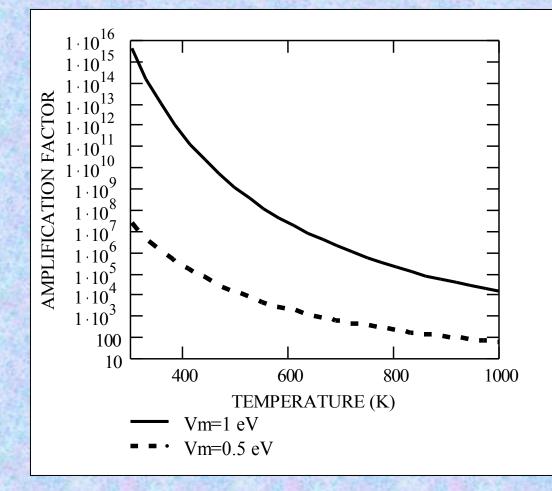
$$V(x,t) = V(x) - (x/x_m)V_m \cos(\Omega t)$$

If $\Omega \ll \omega_0$ (adiabatic assumption):

$$\dot{R}(t) = \dot{R}_{K} \exp\left(\frac{V_{m} \cos(\Omega t)}{k_{B}T}\right), \quad \text{with mean value:}$$
$$\left\langle \dot{R}(t) \right\rangle = \dot{R}_{K} \frac{\Omega}{2\pi} \int_{0}^{2\pi/\Omega} \exp\left(\frac{V_{m} \cos(\Omega t)}{k_{B}T}\right) dt = \dot{R}_{K} I_{0} \left(\frac{V_{m}}{k_{B}T}\right)$$

 I_0 is the modified Bessel function of the first kind

Breather effect: modulation amplification factor Amplification factor: $I_0 (V_m/k_BT)$

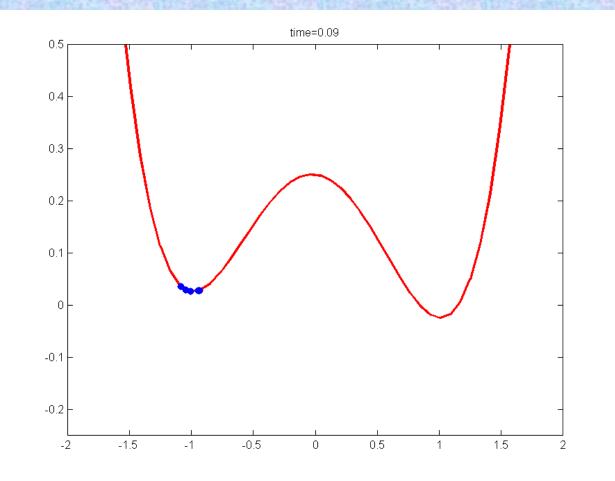


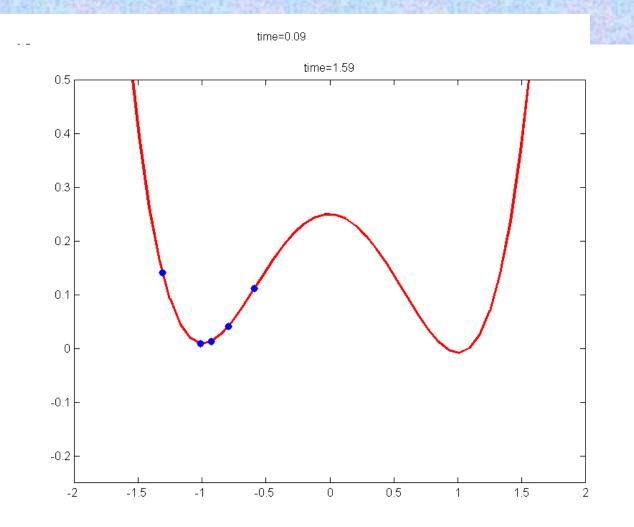
Probability of escape from the reactants well: $\exp(-\widetilde{V}/k_{\rm B}T)$

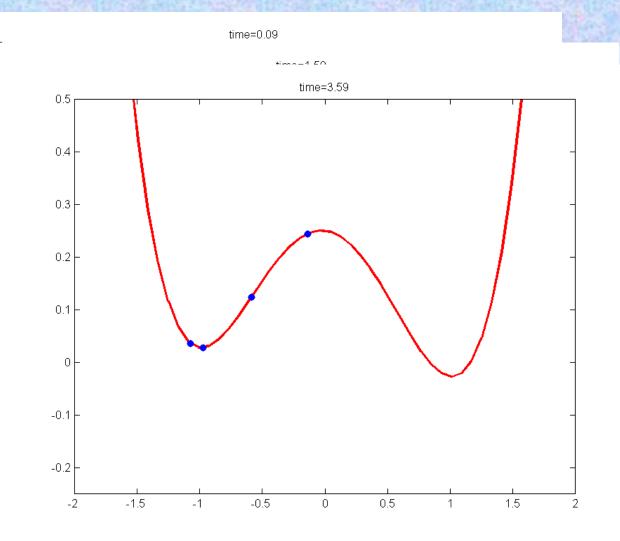
with
$$\widetilde{V} = E_a + V_m \cos(\widetilde{\varphi})$$

and $\tilde{\varphi}$ a random variable with probability density $1/2\pi$, leads to

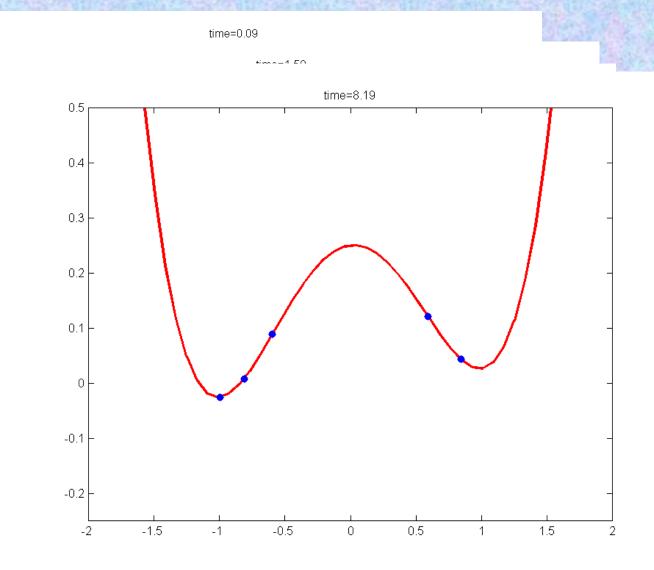
$$\left\langle \dot{R}(\tilde{\varphi}) \right\rangle = \dot{R}_{K} \frac{1}{2\pi} \int_{0}^{2\pi} \exp\left(\frac{V_{m} \cos(\tilde{\varphi})}{k_{B}T}\right) d\tilde{\varphi} = \dot{R}_{K} I_{0} \left(\frac{V_{m}}{k_{B}T}\right)$$







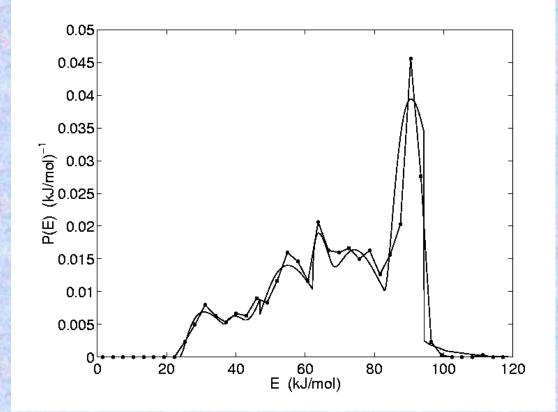
114



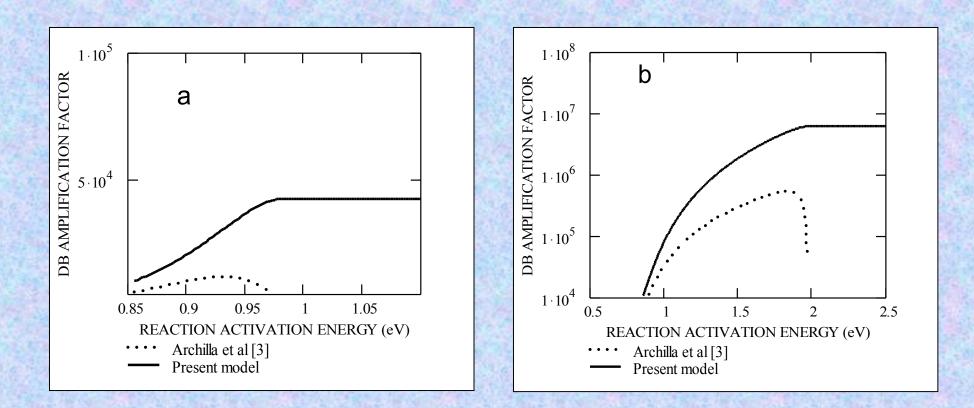
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Amplification factor for breathers in the mica model

$$\left\langle \dot{R}_{B} \right\rangle = \dot{R}_{K} \int_{E_{\min}} f_{B}(E) I_{0}(E/k_{B}T) dE$$

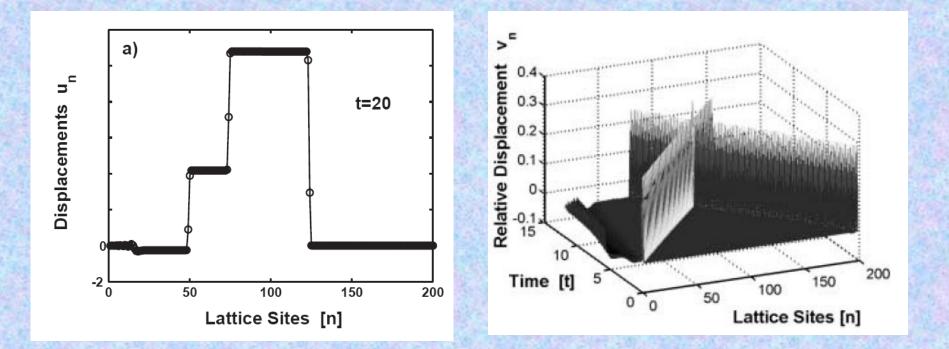


Amplification factor for breathers in the mica model



Reaction rate theory with account of the crystal anharmonicity VI Dubinko, PA Selyshchev and JFR Archilla Phys Rev E 83, 041124 (2011)

Transversal breathers move slowly but we pan to study supersonics kinks along the lattice directions



Supersonic discrete kink-solitons and sinusoidal patterns with "magic" wave number in anharmonic lattices. Yu A Kosevich, R Khomeriki and S Ruffo. Europhys. Lett., 66 (1), pp. 21–27 (2004)

SUMMARY

- 1. Breathers do not need to have an energy larger that the activation energy to influence reconstructive transformations
- 2. A breather modulates the potential barrier in Kramers theory which introduces an amplification factor in the reaction rate.
- 3. Different types of breathers appear in simulations for the cation layer in muscovite
- 4. The amplification factor increases several order of magnitude the reaction rate according with the observed low temperature reconstructive transformations
- 5. They move slowly, then probably longitudinal kinks are more appropriate for quodons.