A study of fatigue loading on automotive and transport structures

Johann Wannenburg
George Mallory: “Because it’s there”
JW: “Because I started it”
Introduction

- Defective structural designs often caused by insufficient knowledge of input data, not inadequate analysis or testing methods.
- Loads associated with automotive and transport structures nontrivial to quantify.
- Arise from stochastic, ill-defined processes such as driver/operator actions, structure-terrain interaction.
Introduction

- Measurements
- Surveys
- Simulation
- Failure data

Determination of Input Loading

Design criteria
Testing requirements

- Generalise & unify new & existing techniques into cohesive methodology
- Combining current theory & best practices, with lessons learned during application on, as well as new techniques developed for, a number of complex case studies
Fundamental theory and methodologies

- Framework for summary of fatigue design methods
Fundamental theory and methodologies

- Populated framework
- Quasi-static finite element analysis:
  - Calculating the stress response $\sigma_{ij}$ for all (or critical) elements $i$, caused by applying static unit loads $L_{j\text{-unit}}$ at nodes $j$, one at a time, for all significant loads
  - Establish a quasi-static transfer matrix $[K]$ between element stresses and loads. The known time histories of each load $L_j(t)$ are then multiplied by the inverse of the transfer matrix, achieving, through superposition, the stress time histories $\sigma_i(t)$ at each element $i$:

$$[K] = \begin{bmatrix} \sigma_{ij} \end{bmatrix}$$

$$\{\sigma_i(t)\} = [K]^{-1}\{L_j(t)\}$$
Fundamental theory and methodologies

- Co-variance method:
  - Stress load matrix $[B]$ calculated for only critical areas, with $[\sigma_c(t)]$ stress tensors at critical locations, $[L(t)]$ input loads (forces and accelerations) and $[B]$ contains stress tensor results from static unit load analyses for each input load, as well as modal stresses from eigenvalue finite element analysis.
  - Time-independent stress load matrix allows load covariance matrix to be transformed into stress covariance matrix

$$\{\sigma(t)\} = [B][L(t)]$$

$$[P(\sigma)] = [B][P(L)][B]^T$$
Fundamental theory and methodologies

- Direct integration
- Modal superposition

  - Solution for equation of motion:
    \[ \{u\} = [k]^{-1}\{p\} - \sum_{r=1}^{\hat{N}} \frac{1}{\omega_r} \phi_r \ddot{\hat{n}}_r \]

  - First term represents quasi-static (or non-modal) response. Second term superimposes the effects of the truncated modes (\(N\)) that may participate in the full dynamic response
Fundamental theory and methodologies

- Force inputs: wheel loadcells, CARLOS
- Acceleration inputs: methods to use as inputs to direct integration
- Strain inputs: remote parameters
- Multi-body dynamic simulation

### MULTI-BODY DYNAMIC SIMULATION

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Type 1</th>
<th>Type 2</th>
<th>Type 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>To obtain FEA load inputs when measurements are not available</td>
<td>To obtain FEA load inputs on suspension hard points</td>
<td>To obtain FEA load inputs on suspension hard points</td>
</tr>
<tr>
<td>Input</td>
<td>Digitized road profiles</td>
<td>Measured spindle loads</td>
<td>Measured wheel accelerations</td>
</tr>
<tr>
<td>Difficulty</td>
<td>Require a complex tyre model</td>
<td>Require a specialized loadcell</td>
<td>Double integration of measured accelerations presents problems</td>
</tr>
</tbody>
</table>
Fundamental theory and methodologies

- Rainflow cycle counting

- Miner damage accumulation:

\[ D = \sum \frac{n_i}{N_i} \]
Fundamental theory and methodologies

- *Dirlik formula* estimates probability density function (PDF) of rainflow ranges as function of moments of PSD, empirically derived from results of IFFTs of a number of PSDs with random phases, allowing closed-form estimation of fatigue damage from PSD data (assuming time data is stationary).

- Fracture mechanics approach not widely applied in automotive industry.
Fundamental theory and methodologies

- Stress-life approach:
  \[ \Delta \sigma = S_f N^b \]

- Welding SN curves

- Nominal stress, hot-spot stress, geometrical stress, fracture mechanics methods comparison

- Spotwelds

- Relative fatigue: estimate of b
Fundamental theory and methodologies

- Strain-life approach

\[ \frac{\Delta \varepsilon}{2} = \frac{S_f}{E} (2N)^b + \varepsilon_f (2N)^c \]

- Iterative solution

- Considered better model of fundamental mechanism of fatigue initiation since it takes account of the notch root plasticity, with cyclic plastic strain being driving force behind fatigue mechanism.

- High-cycle fatigue applications stress-life and strain-life techniques converge, since effects of plasticity negligible.

- Mostly the case in automotive applications, although controversial.
Fundamental theory and methodologies

Methodologies:
- Remote parameter analysis (RPA)
- Time domain modal superposition
- Random vibration analysis
- Covariance methodology
- Fatigue Damage Response Spectrum
- Direct Integration
## Fundamental theory and methodologies

### Comparison

<table>
<thead>
<tr>
<th>Type</th>
<th>Load input</th>
<th>Stress analysis</th>
<th>Fatigue analysis</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remote Parameter Analysis</td>
<td>Quasi-static, time domain</td>
<td>Strain-gauge measurement</td>
<td>Static FEA</td>
<td>Rainflow counting + various methods of fatigue-life analysis</td>
<td>Can use remote measured strain-gauge data, economic FEA</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Not suitable for complex dynamic response, rainflow on each stress point</td>
</tr>
<tr>
<td>Covariance method</td>
<td>Quasi-static, frequency</td>
<td>Measured /simulated</td>
<td>Static FEA</td>
<td>Dirlik formula + various methods of fatigue-life analysis</td>
<td>Takes account of complex dynamic response, economic FEA</td>
</tr>
<tr>
<td></td>
<td>domain</td>
<td>input forces</td>
<td></td>
<td></td>
<td>Requires stationary random input data, Dirlik formula approximations</td>
</tr>
<tr>
<td>Random vibration</td>
<td>Dynamic, frequency domain</td>
<td>Measured /simulated</td>
<td>Eigenvalue FEA</td>
<td>Dirlik formula + various methods of fatigue-life analysis</td>
<td>Takes account of complex dynamic response, economic FEA</td>
</tr>
<tr>
<td></td>
<td>domain</td>
<td>input forces</td>
<td></td>
<td></td>
<td>Requires stationary random input data, forces must be measured, Dirlik formula approximations</td>
</tr>
<tr>
<td>Fatigue-domain response</td>
<td>Dynamic, fatigue/ frequency</td>
<td>Measured accelerations</td>
<td>Eigenvalue FEA</td>
<td>Ho3M cycle counting + Stress Life</td>
<td>Takes account of complex dynamic response, economic FEA</td>
</tr>
<tr>
<td>spectrum</td>
<td>domain</td>
<td></td>
<td></td>
<td></td>
<td>Requires stationary random input data</td>
</tr>
<tr>
<td>Time-domain modal superposition</td>
<td>Dynamic, time or frequency</td>
<td>Measured /simulated</td>
<td>Eigenvalue FEA</td>
<td>Dirlik formula + various fatigue life analysis methods</td>
<td>Takes account of complex dynamic response, economic FEA</td>
</tr>
<tr>
<td></td>
<td>domain</td>
<td>input forces</td>
<td></td>
<td></td>
<td>Forces must be measured, Dirlik formula approximations</td>
</tr>
<tr>
<td>Direct integration with large mass, relative inertia, Lagrange multipliers</td>
<td>Dynamic, time domain</td>
<td>Measured accelerations</td>
<td>Dynamic FEA</td>
<td>Rainflow counting + various methods of fatigue-life analysis</td>
<td>Takes account of complex and transient dynamic response, accelerations may be measured</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Expensive FEA</td>
</tr>
</tbody>
</table>
Thesis structure

- Measurements, surveys and simulation
- Design and testing requirements
- Fatigue assessment and correlation
- Formalisation
- Conclusion
Case studies: Fuel tanker

- Dual purpose, aluminium fuel tanker development.
- Challenges: box shaped, aluminium, lightweight design.
- Comprehensive fatigue design, unique application requiring definition of design loading.
Case studies: Fuel tanker

- **Measurements**
  - Strain gauges placed to measure nominal stresses in as many areas required to reasonably characterise stress response of the structure.
  - Measurements included typical 300 kilometre trip with liquid load, as well as a return dry-load trip.

- **Calculation of fatigue design loads**
  - Development of **Fatigue Equivalent Static Load (FESL)** methodology.
  - Based on stress-life method, requiring only static finite element analysis, achieving simplified definition of fatigue loads for design purposes.
  - Numerical equivalent of constant amplitude or block loading fatigue tests.
Case studies: Fuel tanker

- Assumption: only vertical g-loading.
- Use a single strain gauge channel that measured bending stresses on front trailer chassis beam

FESL calculation:

- Measured stress-time history of chosen channel cycle-counted to yield spectrum of stress ranges $\Delta\sigma_i$ and number of counted cycles $n_i$.
- Relative fatigue damage calculation ($b=-0.333$, $S_f$ arbitrary):

$$\text{Damage} = \sum \frac{n_i}{N_i} = \sum \frac{n_i}{\left(\frac{\Delta\sigma_i}{S_f}\right)^{1/b}}$$
Case studies: Fuel tanker

Calculate equivalent bending stress range which would, when repeated arbitrary $n_e$ (chosen to be 2 million to allow for direct comparison with welding SN curve category) times, cause same damage to beam to what would be caused during total life (2 million km) of vehicle, made out of repetitions of measured trip ($n_i$ multiplied with 2 million over 600):

$$\sum \frac{n_i}{\left(\frac{\Delta\sigma_i}{S_f}\right)^{1/b}} = \frac{n_e}{\left(\frac{\Delta\sigma_e}{S_f}\right)^{1/b}}$$

$$\Delta\sigma_e = \left(\sum \frac{\Delta\sigma_i^{m} n_i}{n_e}\right)^{1/m}$$
Case studies: Fuel tanker

- Bending stress $\sigma_{1g}$, caused by 1 g (unit) vertical inertial loading at the strain gauge position, is then calculated using static finite element analysis.

- Fatigue equivalent static loading (FESL) calculated as follows:

$$\text{FESL} = \frac{\Delta \sigma_e}{\sigma_{1g}} = 0.62\, \text{g}$$
Case studies: Fuel tanker

- Fatigue life prediction
  - FESL applied to finite element model in a static analysis.
  - Stresses calculated interpreted as stress ranges, which would be repeated 2 million times during life of 1 million kilometres.
  - Fatigue life at each critical position calculated, using appropriate SN-curve relevant to detail at each position.
Case studies: Fuel tanker

- Design code correlation
  - $20 \text{ MPa}/0.62\text{g} = 32 \text{ MPa/g}$
  - $56 \text{ MPa}/2\text{g} = 28 \text{ MPa/g}$

<table>
<thead>
<tr>
<th></th>
<th>SABS 1398</th>
<th>SABS 1518</th>
<th>BS 8118 Static</th>
<th>BS 8118 Fatigue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal</td>
<td>2 g</td>
<td>0.75 g (2 g)</td>
<td>1 g</td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td>2 g</td>
<td>1.7 g</td>
<td>2.26 g</td>
<td>0.62 g</td>
</tr>
<tr>
<td>Lateral</td>
<td>1 g</td>
<td>0.4 g</td>
<td>0.53 g</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Separate</td>
<td>Combined</td>
<td>Combined</td>
<td></td>
</tr>
<tr>
<td>Allowable stress plates</td>
<td>62 MPa</td>
<td>77.5 MPa (232.5 MPa)</td>
<td>181 MPa for raw material</td>
<td>Typ. &gt; 35 MPa for raw material</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>188 MPa for weld material</td>
<td>Typ &gt; 20 MPa for weld material</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>115 MPa for HAZ material</td>
<td>(depending on classification)</td>
</tr>
<tr>
<td>Allowable stress extru.</td>
<td>56 MPa</td>
<td>70 MPa</td>
<td>185 MPa for raw material</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>146 MPa for weld material</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>112 MPa for HAZ material</td>
<td></td>
</tr>
</tbody>
</table>
Case studies: Fuel tanker

- Correlation with field failures
  - Majority of the fleet has to date exceeded required life without failures, thereby substantiating the FESL process.
  - Failures occurred on vehicles fitted with underslung axles, introduced due to availability problems of original design overslung axles.
  - Engineering change procedure failed to highlight large access hole for airbag introduced on inside of lower flange of chassis beam in critical stress area.
  - Cracks experienced originated from this hole (after typically 800 000 km) and in some cases propagated into web to almost sever the beam.
Case studies: Fuel tanker

- Correlation with field failures

  - Calculation using FEA results with access hole results in life prediction of 1.2 million km.
Case studies: Fuel tanker

- Formalisation

  FESL methodology in framework

  Case study in generalised process
Case studies: Load Haul Dumper

- LHD employed in underground mines to load blasted rock at stope face.
- Harsh road conditions, high dynamic loads induced during loading & dumping.
- Fatigue problems prompted life extension project.
Case studies: Load Haul Dumper

- Measurements
Case studies: Load Haul Dumper

- Finite element analysis
  - Model A where bucket is empty, boom is resting stops, inertial loading applied to simulate empty travelling.
  - Model B where bucket is full, boom is resting, inertial loading applied to simulate full travelling.
  - Model C where boom is lifted, loading applied on boom to simulate effect of forces on bucket during loading or off-loading.
Case studies: Load Haul Dumper

FESL calculation (three models)

\[
\left( \frac{2 \times 10^6}{\text{FESL}_A \sigma_{A_j}} \right)^{-3} + \left( \frac{2 \times 10^6}{\text{FESL}_B \sigma_{B_j}} \right)^{-3} + \left( \frac{2 \times 10^6}{\text{FESL}_C \sigma_{C_j}} \right)^{-3} = \text{TD}_j
\]

![Diagram showing empty vehicle and full vehicle.](image)

\[= \quad + \quad + \quad \text{Bucket load} \]

10,000 hours

- Empty vehicle
- Full vehicle
- Bucket load

Bucket load

1.85 x rated bucket load

2 million

4.2g

1.1g

2 million
Case studies: Load Haul Dumper

- Correlation with field failures
Case studies: ISO tank container

- Transport of dangerous liquids by ship, rail and truck.
- Harsh dynamical loading conditions: shunting impacts, handling by cranes, forklifts, loading in storms when stacked 8 high in ship holds, fatigue loads induced by rough roads.

- Design/testing loads of codes are static, accounting for dynamic & fatigue effects through safety factors.
- Field failures resulting from normal fatigue loading still often experienced.
- Project to determine loading from extensive measurements.
Case studies: ISO tank container

Measurements

- Amplifier
- Signal conditioning
- A/D
- RAM
- Processor Algorithms Memory
- Computer for extraction
- Battery
- Strain gauges
- Accelerometers
- Data Logger

RAM
Amplifier
Signal conditioning
A/D
Computer for extraction
Battery
Accelerometers
Data Logger
Strain gauges
Case studies: ISO tank container

- Measurements
Case studies: ISO tank container

- Dynamic simulation
Case studies: ISO tank container

- Dynamic FEA
Case studies: ISO tank container

- FESL calculation (multi-axial)

\[
\text{FESL} = \begin{cases}
\Delta g_{e,\text{vert}} \\
\Delta g_{e,\text{long}} \\
\Delta g_{e,\text{lat}} \\
\Delta g_{e,\text{pitch}}
\end{cases}
= \begin{bmatrix}
\sigma_{\text{vert},\text{ch}1} & \sigma_{\text{long},\text{ch}1} & \sigma_{\text{lat},\text{ch}1} & \sigma_{\text{pitch},\text{ch}1} \\
\sigma_{\text{vert},\text{ch}2} & \sigma_{\text{long},\text{ch}2} & \sigma_{\text{lat},\text{ch}2} & \sigma_{\text{pitch},\text{ch}2} \\
\sigma_{\text{vert},\text{ch}3} & \sigma_{\text{long},\text{ch}3} & \sigma_{\text{lat},\text{ch}3} & \sigma_{\text{pitch},\text{ch}3} \\
\sigma_{\text{vert},\text{ch}4} & \sigma_{\text{long},\text{ch}4} & \sigma_{\text{lat},\text{ch}4} & \sigma_{\text{pitch},\text{ch}4}
\end{bmatrix}^{-1}\begin{bmatrix}
\Delta \sigma_{e,\text{ch}1} \\
\Delta \sigma_{e,\text{ch}2} \\
\Delta \sigma_{e,\text{ch}3} \\
\Delta \sigma_{e,\text{ch}4}
\end{bmatrix}
\]
Case studies: ISO tank container

- Formalisation

  Generalised process
Case studies: Ladle Transport Vehicle

- Ladle Transport Vehicle (LTV) for Aluminium Smelter plant.
- Articulated arrangement, with U-shaped trailer.
- Determine input loads during typical operation to allow fatigue durability assessment.
Case studies: Ladle Transport Vehicle

Measurements

- Strain-gauge transducers applied to LTV structure.
- Channels 3 and 4, near the front on top of left and right chassis beams, sensitive to vertical bending stresses; channels 5 and 6, placed to rear of left and right chassis beams, sensitive to vertical bending stresses; channels 7 and 8, measuring bending stress on left and right vertical pillars, were used in analyses.
- Measurements taken during typical operation of prototype vehicle in smelter plant.
Case studies: Ladle Transport Vehicle

- Measurements
  - De-coupling of vertical, lateral channels
Case studies: Ladle Transport Vehicle

- Frequency analysis
  - Pillar gauges exhibited significant response at 4.7 Hz, corresponding to a natural mode (twisting of pillar with lid swinging), determined by performing an eigenvalue analysis.
Case studies: Ladle Transport Vehicle

- Hybrid methodology
  - To augment the Remote Parameter Analysis transfer matrices with modal stresses for the excited mode, calculated by the eigenvalue FEA analysis.
Case studies: Ladle Transport Vehicle

- Dynamic loads calculation

\[
[K_{\text{decoupled}}] = \begin{bmatrix}
\frac{2}{\sigma_{ch7,1gvert}} \left(\sigma_{ch3,1gvert} + \sigma_{ch4,1gvert}\right) & \frac{2}{\sigma_{ch3,1glat}} \left(\sigma_{ch3,1glat} + \sigma_{ch4,1glat}\right) & \frac{2}{\sigma_{ch3,modal}} \left(\sigma_{ch3,modal} + \sigma_{ch4,modal}\right) \\
\frac{2}{\sigma_{ch7,1gvert}} \left(\sigma_{ch3,1gvert} - \sigma_{ch4,1gvert}\right) & \frac{2}{\sigma_{ch3,1glat}} \left(\sigma_{ch3,1glat} - \sigma_{ch4,1glat}\right) & \frac{2}{\sigma_{ch3,modal}} \left(\sigma_{ch3,modal} - \sigma_{ch4,modal}\right) \\
\frac{2}{\sigma_{ch7,1glat}} & \frac{2}{\sigma_{ch3,modal}} & \frac{2}{\sigma_{ch7,modal}}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{Vert}_g(t) \\
\text{Lat}_g(t) \\
\text{Modal}(t)
\end{bmatrix} = [K_{\text{decoupled}}]^{-1} \begin{bmatrix}
\frac{2}{\sigma_{ch3,meas}(t)} \left(\sigma_{ch3,meas}(t) + \sigma_{ch4,meas}(t)\right) \\
\frac{2}{\sigma_{ch3,meas}(t)} \left(\sigma_{ch3,meas}(t) - \sigma_{ch4,meas}(t)\right) \\
\frac{2}{\sigma_{ch7,meas}(t)}
\end{bmatrix}
\]
Case studies: Ladle Transport Vehicle

- Dynamic loads thus calculated used as inputs into critical position / load transfer matrix to calculate dynamic stress histories at all critical positions (including at all strain-gauge positions).
- Good correlation found between calculated and measured stress at redundant strain-gauge positions.
- Fatigue calculations, using rainflow-cycle counting and stress-life criteria, completed the exercise.
Case studies: Minibus

- Testing of new motor vehicle models involves accelerated simulation of operational conditions on test routes, test tracks, or in structural testing laboratory.
- Definition of operational conditions presents a major challenge.
- Extreme South African conditions not included in original usage profiles, necessitating adaptation of designs, therefore requiring qualification testing according to optimal durability requirements.
Case studies: Minibus

- **Measurements**
  - Minibus vehicle instrumented with strain gauges on each torsion bar forming part of front suspension, to capture vertical road-induced loading.
  - Measured data organized in different files categorizing a certain category of road. Several files existed for each category of road.
  - Strategy was to drive in areas typically used by taxis, whilst subjectively classifying road categories. The vehicle was driven at aggressive speeds, considered safe by the driver.
Case studies: Minibus

▲ Fatigue processing

- Cycle counting, stress-life method and Miner damage accumulation law employed to calculate the relative damage for each measurement file, divided by distance, to obtain a damage/km for each terrain type.

- Results for files in each category were averaged to yield an average relative damage/km for each category.

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Average relative damage per kilometre</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Highway</td>
<td>$1.81 \times 10^{-4}$</td>
</tr>
<tr>
<td>2</td>
<td>Secondary tar</td>
<td>$7.38 \times 10^{-4}$</td>
</tr>
<tr>
<td>3</td>
<td>Smooth gravel</td>
<td>$2.53 \times 10^{-3}$</td>
</tr>
<tr>
<td>4</td>
<td>Rough gravel</td>
<td>$3.16 \times 10^{-3}$</td>
</tr>
<tr>
<td>5</td>
<td>Very rough</td>
<td>$3.78 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
Case studies: Minibus

- Survey
  - Customer questionnaire survey performed, obtaining data in terms of percentages driven on each category road, as well as distances travelled per time period.
  - Combination of measurement and survey data is used to define two parameters for each questionnaire participant, namely, a fatigue damage per distance and a distance per time period.

\[
\text{Dam/km} = \sum_{\text{category}=1}^{5} \text{average dam/ km}_{\text{category}} \times \frac{\text{percentage}_{\text{category}}}{100}
\]
Case studies: Minibus

- Statistical processing
  - Probability density functions are fitted to this data, thereby defining a two-parameter usage profile.
Case studies: Minibus

- Two parameter statistical usage profile

\[ f(x_1, x_2) = \frac{1}{x_1 x_2 \sigma_{y_1} \sigma_{y_2}} 2\pi \sqrt{1-\rho^2} e^z \]

with \[ z = \frac{1}{2(1-\rho^2)} \left[ \left( \frac{\ln x_1 - \mu_{y_1}}{\sigma_{y_1}} \right)^2 - 2\rho \left( \frac{\ln x_1 - \mu_{y_1}}{\sigma_{y_1}} \right) \left( \frac{\ln x_2 - \mu_{y_2}}{\sigma_{y_2}} \right) + \left( \frac{\ln x_2 - \mu_{y_2}}{\sigma_{y_2}} \right)^2 \right] \]

\[ \rho = \frac{\sigma_{y_2}}{\sigma_{y_1} \sigma_{y_2}} \]

\( y_1 = \ln x_1 \), being normally distributed (mean = \( \mu_{y_1} \), variance = \( \sigma_{y_1}^2 \))

\( y_2 = \ln x_2 \), being normally distributed (mean = \( \mu_{y_2} \), variance = \( \sigma_{y_2}^2 \))
Case studies: Minibus

- Durability test requirement
Case studies: Minibus

- Laboratory testing: reproduce failure on cross member (dam$_f$)
Case studies: Minibus

- Failure predictions
  - $\text{dam}_f/\text{km} \times \text{km/day} = \text{dam}_f/\text{days on road} = \text{constant}_{\text{month}}$

---

**Actual vs predicted failures**

- **Distance to failure [thousand km]**
- **Number of failures**

- **Graph showing actual vs predicted failures**

- **Legend**
  - Predicted
  - Actual
Case studies: Minibus

- Monte Carlo Simulation with variable component strength give similar results.
- Reasonable to assume that effect on field failure distributions of variance in component strength, small in comparison to variance in input loading.
- Automotive manufacturers expend significant effort into controlling variation in fabrication quality.
Case studies: Minibus

- Usage profile defined by 5 parameters:
  - Mean of y1 (μy1)
  - Mean of y2 (μy2)
  - Standard deviation of y1 (σy1)
  - Standard deviation of y2 (σy2)
  - Correlation coefficient (ρ)

- Bounds may be estimated (e.g. x2 between 100 and 30 000 km/month).

- Monte Carlo simulation used to generate failures for chosen set of parameter values.
Case studies: Minibus

- **Reversed process** developed to determine usage profile from failure data
Case studies: Minibus

- Binned and normalised failure values, when multiplied by an unknown scale factor (K), would approximately fall on PDF surface.
- Estimating usage profile parameters from failure data therefore reduces to curve fitting the PDF, divided by K, onto the set of binned and normalised failure values.
Case studies: Minibus

Mathematically, the problem is defined as follows:
Solve non-linear curve-fitting problem in the least-squares sense, that is, given input data xdata and output ydata, find coefficients P that "best-fit" the equation $F(P, xdata)$, i.e.;

$$
\min_x \frac{1}{2} \left\| F(P, xdata) - ydata \right\|_2^2 = \frac{1}{2} \sum_i (F(P, xdata_i) - ydata_i)^2
$$

where,

xdata = coordinates (ln(D/km), ln(km/month)) of the bins
ydata = the normalised failure values of the bins
F = (f(x_1, x_2) defined by Eq. 5) / K
P = unknown parameters ($\mu_y$, $\mu_2$, $\sigma_1$, $\sigma_2$, $\rho$, $K$)
Case studies: Minibus

- Curve fitting result for 8% failures
Case studies: Minibus

- Generalised process
Conclusion

Contributions:

- Fatigue Equivalent Static Load methodology (multi-axial).
- Hybrid Remote Parameter / Modal Superposition methodology.
- Two Parameter analytical approach for statistical usage profile, as well as inversed approach to estimate usage profile from failure data.
- Formalisations: Framework and generalised process.
- Correlation with actual field performance.
Conclusion

Future work:

- Development of FESL methodology that takes account of resonant dynamics.
- Employ methodologies to update design codes for various applications.
- Investigate possible contribution of using strain-life and multi-axial fatigue methods to increase accuracy.
Sherpas and sponsor

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