GEOMETRIC OPTIMISATION OF CONJUGATE COOLING CHANNELS WITH DIFFERENT CROSS-SECTIONAL SHAPES

By

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Outline

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- Background
- Motivation/Application
- Aims/Objective
- Objective functions
- Methodology
- Work done
- Results/Graphs
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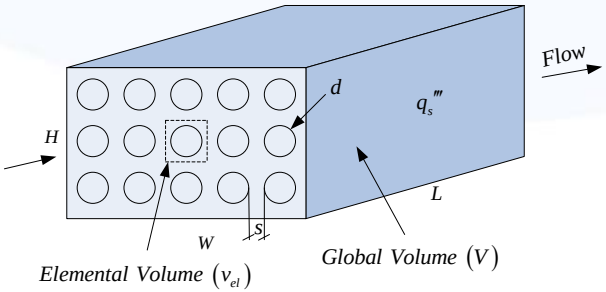
- Future work

Introduction

Heat generating devices and Thermal management

- Heat generating devices, such as high power electronic equipment and heat exchangers are widely applicable in engineering fields e.g electronic chip cooling, power and energy sectors.
- Heat generation can cause overheating problems and thermal stresses and may leads to system failure.
- Cooling of heat generating device critical challenge to thermal design engineers and researchers.
- Heat generating devices are designed in such a way as to optimise the structural geometry by packing and arranging array of cooling channels into given and available volume constraint without exceeding the allowable temperature limit specified by the manufacturers.
- This translates into the maximisation of heat transfer density or the minimisation of overall global thermal resistance, which is a measure of the thermal performance of the cooling devices.

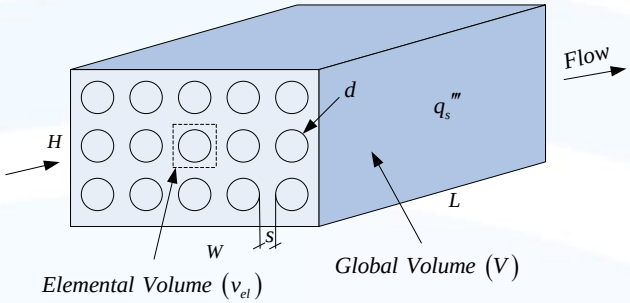


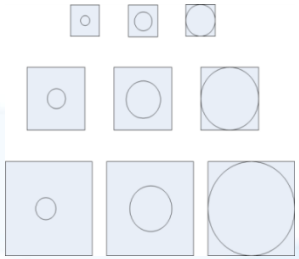
Introduction

Modern Heat Transfer: Geometry and Shape Optimisation

Channel geometric design affects the thermal performance of Heat transfer

Geometry optimization of various shapes and sizes





Convective heat transfer

Characteristic length scale

Geometry

w, h, L, d_h

Heat transfer Performance

$$Nu = \frac{h\,\xi}{k}$$

Shape

Conductive heat transfer

Optimsation parameters

Fig 1. The Nusselt Number (*Nu*) , a measure of heat transfer performance

Backgrounds: Constructal Theory and Design

- Bejan and Sciubba (1992),considered the optimization spacing of board to board of an array of parallel plate that can be fitted in a fixed volume in an electronic cooling system

- Muzychka (2005), analytical optimisation the geometry of circular and non-circular cooling channels.

- Ordonez (2004), Numerically, conducted a two-dimensional heat transfer analysis in a heat-generated volume with cylindrical cooling channels and air as the working fluid.

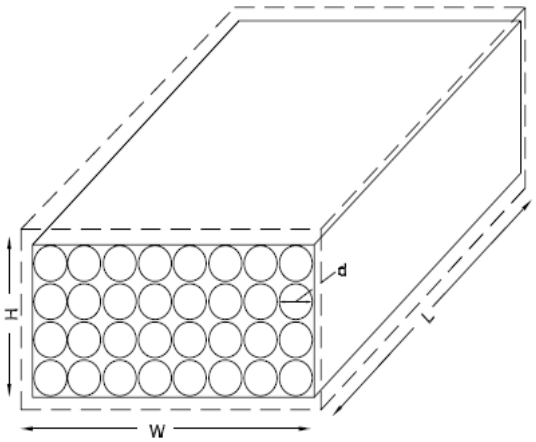
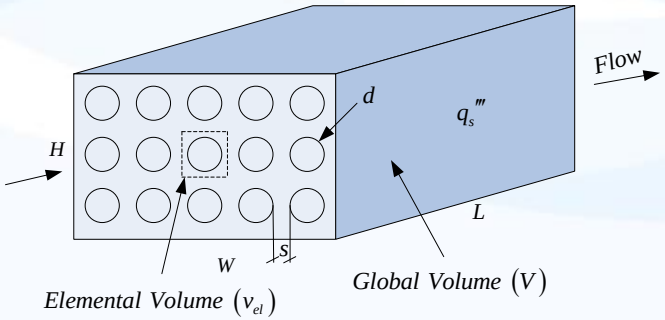


Fig. 1. Convectively cooled finite volume.

$$\frac{d_{opt}}{L} \approx 4.683 Be^{-1/4}$$

$$Q^* \le \frac{Q''' L^2}{k(T_s - T_i)} = C_3 Be^{1/2}$$

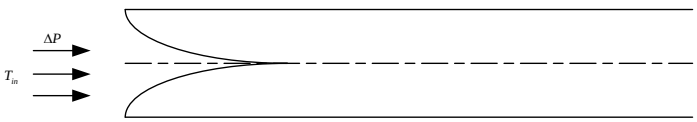
$$n \cong \frac{HW}{d^2}$$



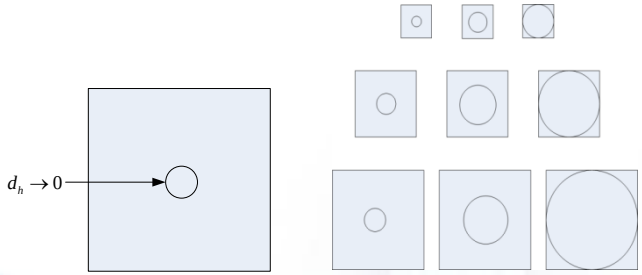
$$n \cong \frac{HW}{(d+s)^2}$$

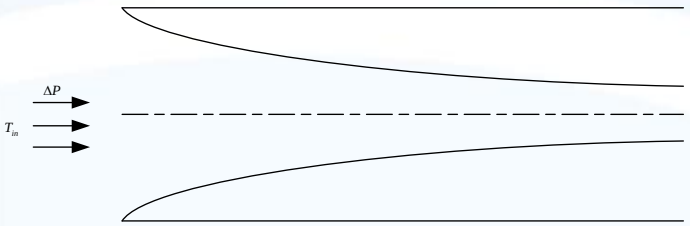
Fig 2. Convectively conducting volume with cooling channels

Method of intersection of asymptotes $\left(0 \le d_h \le \infty\right)$

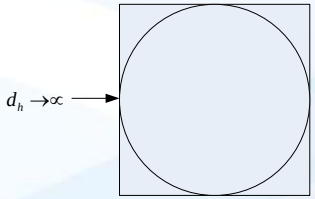


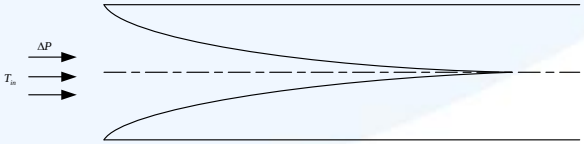
When the channel characteristic dimension scale is small and sufficiently slender, D → 0, D << L



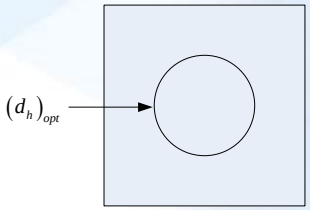


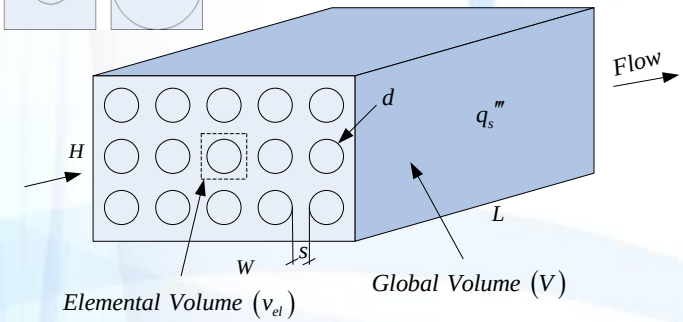
When the channel cross-sectional area is large, D → ∞





When the channel cross-sectional area is at optimum





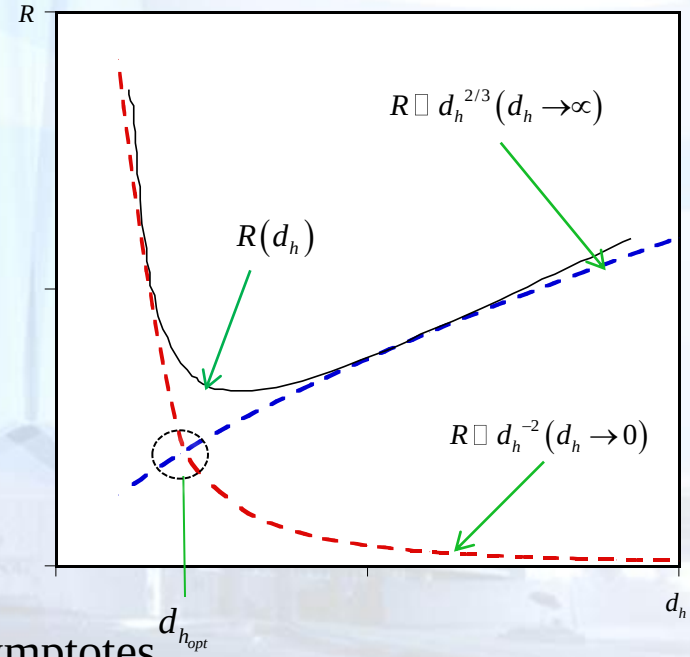


Fig 3. Method of intersection of asymptotes

Motivations/Applications

- The advent of high density components has required investigation of innovative techniques for removing heat from these devices

- Better and optimal performance
- Cost minimization

Applications

- Electronic cooling
- Compact heat exchanger,
- Automotive
- Nuclear power

Aims/Objectives

- **Aim :** To carry out theoretical and numerical optimization studies in conjugate heat transfer in cooling channels with different cross-sections and under varoius conditions

- **Objectives** : To minimise the dimensionless maximal excess of temperature or global thermal resistance

The objectives will be conducted in two phases:

- Analytical (Theory) Analysis

- Numerical Analysis

- Optimisation process by suitable mathematical algorithm

Research activities/work done

Part 1 : Optimisation of Conjugate Heat Transfer In Cooling Channels with Internal Heat Generation for Different Cross-sectional Shapes

Part 2 : Optimisation of Laminar-forced Convection Heat Transfer Through a Vascularised Solid with Cooling Channels

Part 3 : Effect of flow orientation on forced convective heat transfer in cooling channel with internal heat generation

PART 1

Optimisation of Conjugate Heat Transfer In Cooling Channels with Internal Heat Generation for Different Cross-sectional Shapes

Numerical Modelling: Problem under consideration

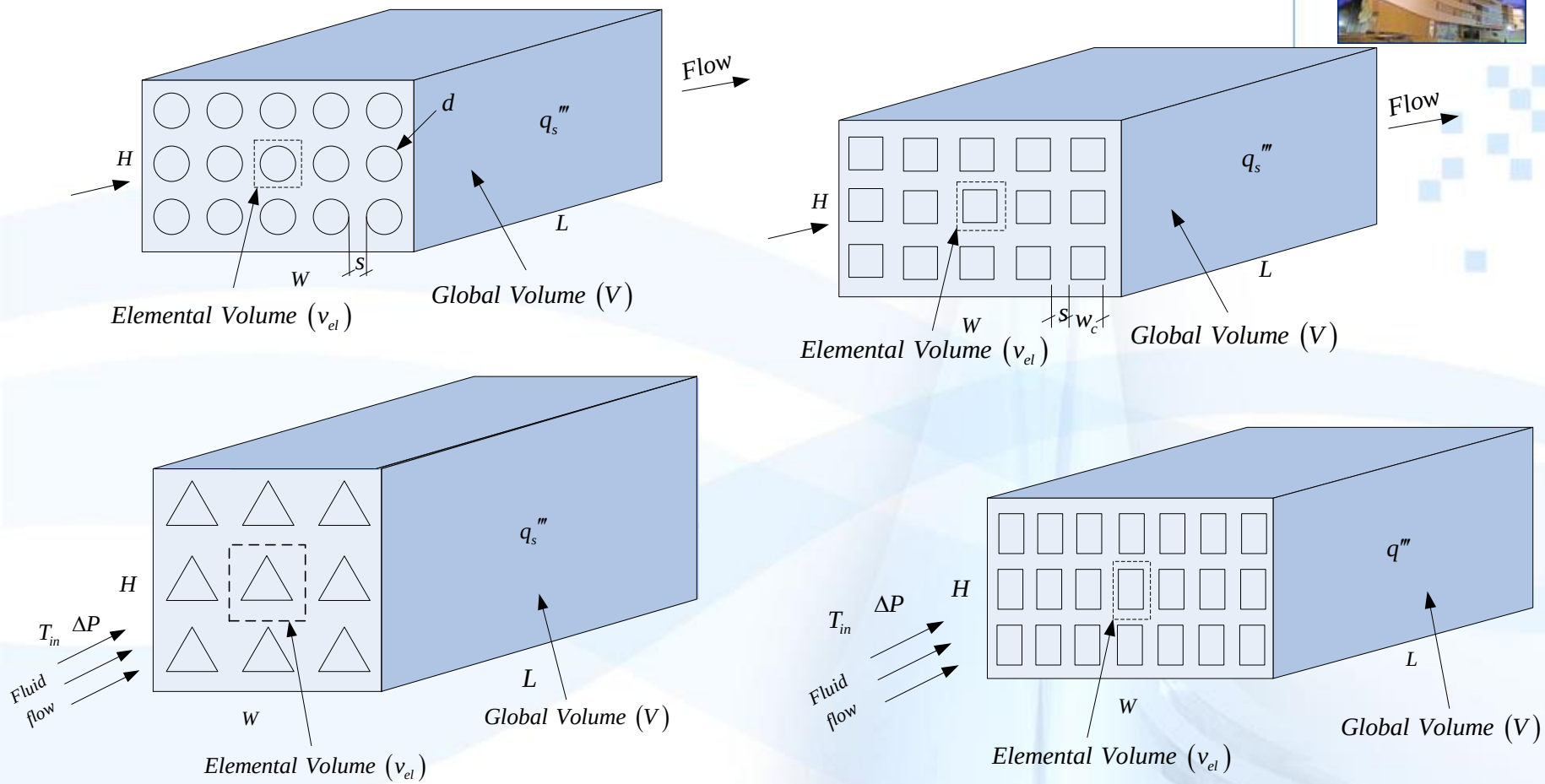


Fig 4. Three-dimensional parallel channels with different cross section across a slab with internal heat generation and forced flow.

Problem under consideration

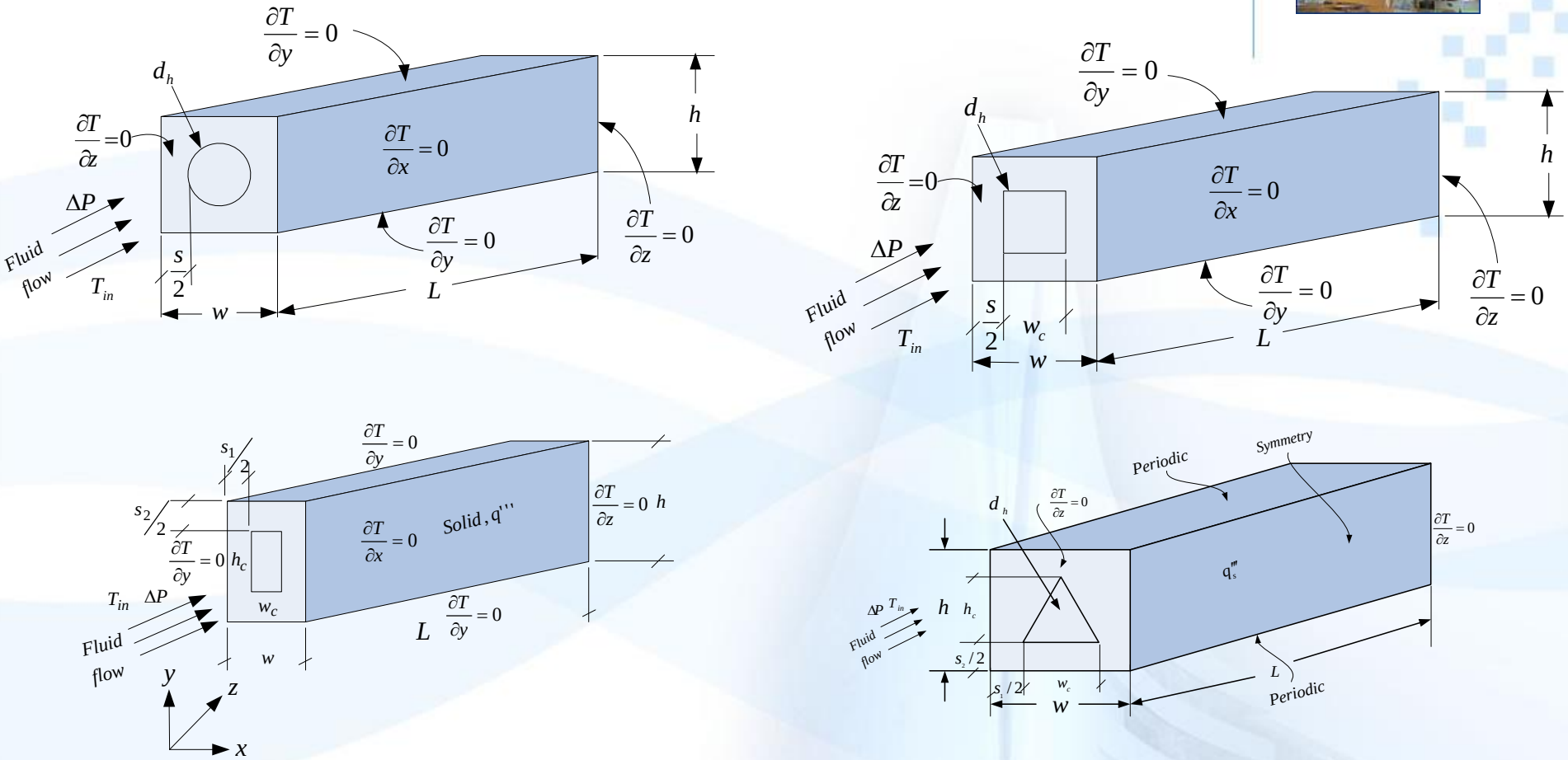


Fig 5 The three dimensional computational domain Elemental volume with cooling channels

Objective functions and Assumptions

The objective is the minimisation of the global thermal resistance

$$R_{\min} = \frac{k_f \left(T_{\max} - T_{in}\right)_{\min}}{q'' L^2} \qquad R_{\min} = f\left(d_{h_{opt}}, \quad v_{el_{opt}}, \quad \left(T_{\max}\right)_{\min}\right) \qquad \text{(P1.1)}$$

Assumptions

- Fluid flow and heat transfer :
- steady-steady state condition
- three dimensional.
- single phase
- Laminar
- Newtonian fluid with constant properties (Water)
- Micro-scale cooling channels

Numerical Modelling /Analysis/Optimisation

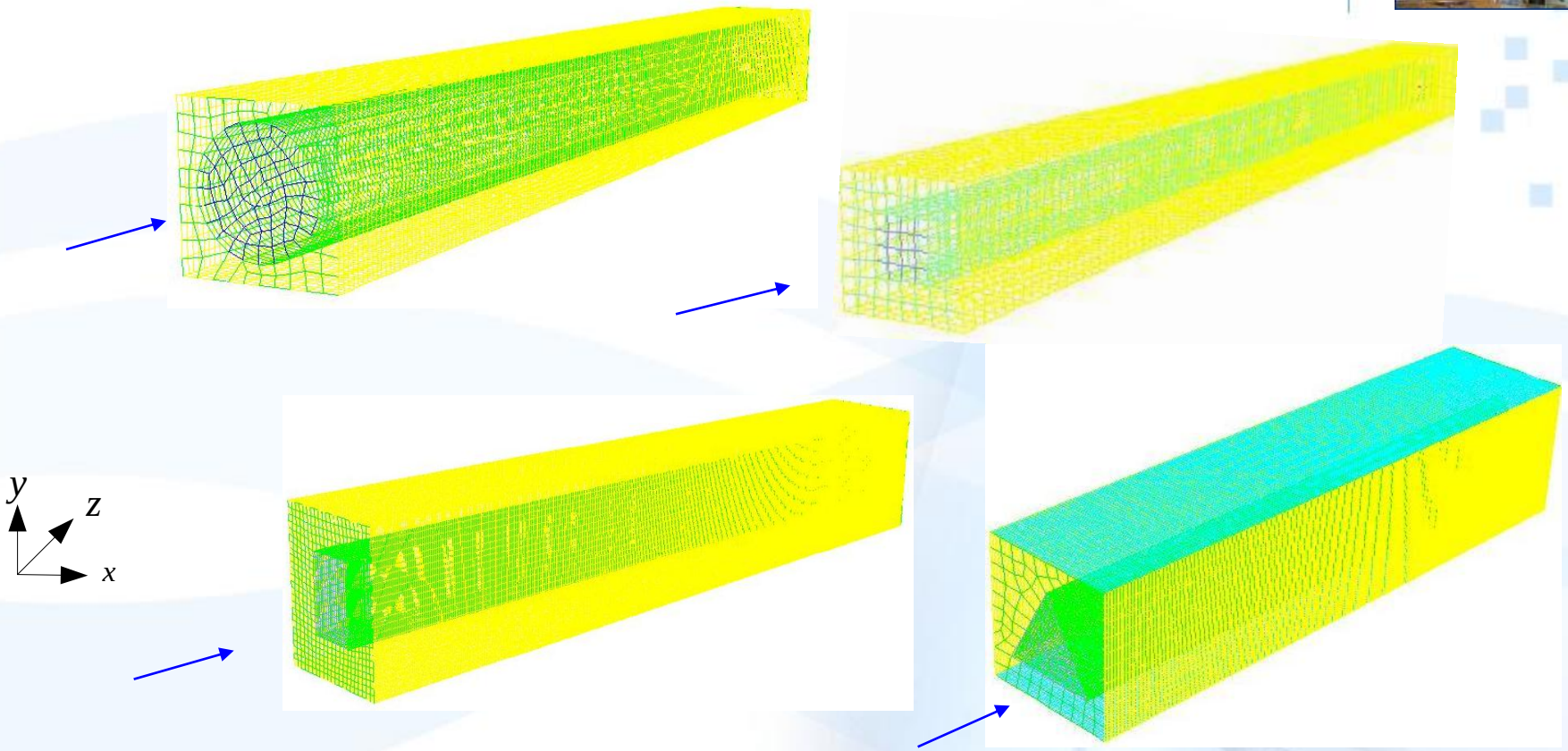


Fig 6: The discretised 3-D computational domain

Numerical Modelling/Analysis

Govering Equations

$$\nabla \cdot \vec{u} = 0 \qquad (P1.2)$$

$$\rho\left(\vec{u} \cdot \nabla \vec{u}\right) = -\nabla P + \mu \nabla^2 \vec{u} \qquad (P1.3)$$

$$\rho_f C_{Pf} \left(\vec{u} \cdot \nabla T\right) = k_f \nabla^2 T \qquad (P1.4)$$

Energy equation for a solid region is given as:

$$k_s \nabla^2 T = 0 \qquad (P1.5)$$

Numerical Modelling/Analysis

- **Boundary Conditions**
  - Unit cell using symmetry
  - Internal heat generation

The continuity of the heat flux at the interface between the solid and the liquid is given as

$$k_s \left.\frac{\partial T}{\partial n}\right| = k_f \left.\frac{\partial T}{\partial n}\right| \qquad \text{(P1.6)}$$

A no-slip boundary condition is specified at the wall of the channel,

$$\vec{u} = 0 \qquad \text{(P1.7)}$$

At the inlet ($x = 0$) $\quad u_x = u_y = 0, \quad T = T_{in}, \quad P_{in} = \frac{Be\alpha\mu}{L^2} + P_{out}$ (P1.8)

At the outlet ($x = L$), zero normal stress

$$P_{out} = 1\ atm \qquad \text{(P1.9)}$$

At the solid boundaries

$$\Delta T = 0 \qquad \text{(P1.10)}$$

Numerical Modelling/Analysis

Summary of Boundary Conditions

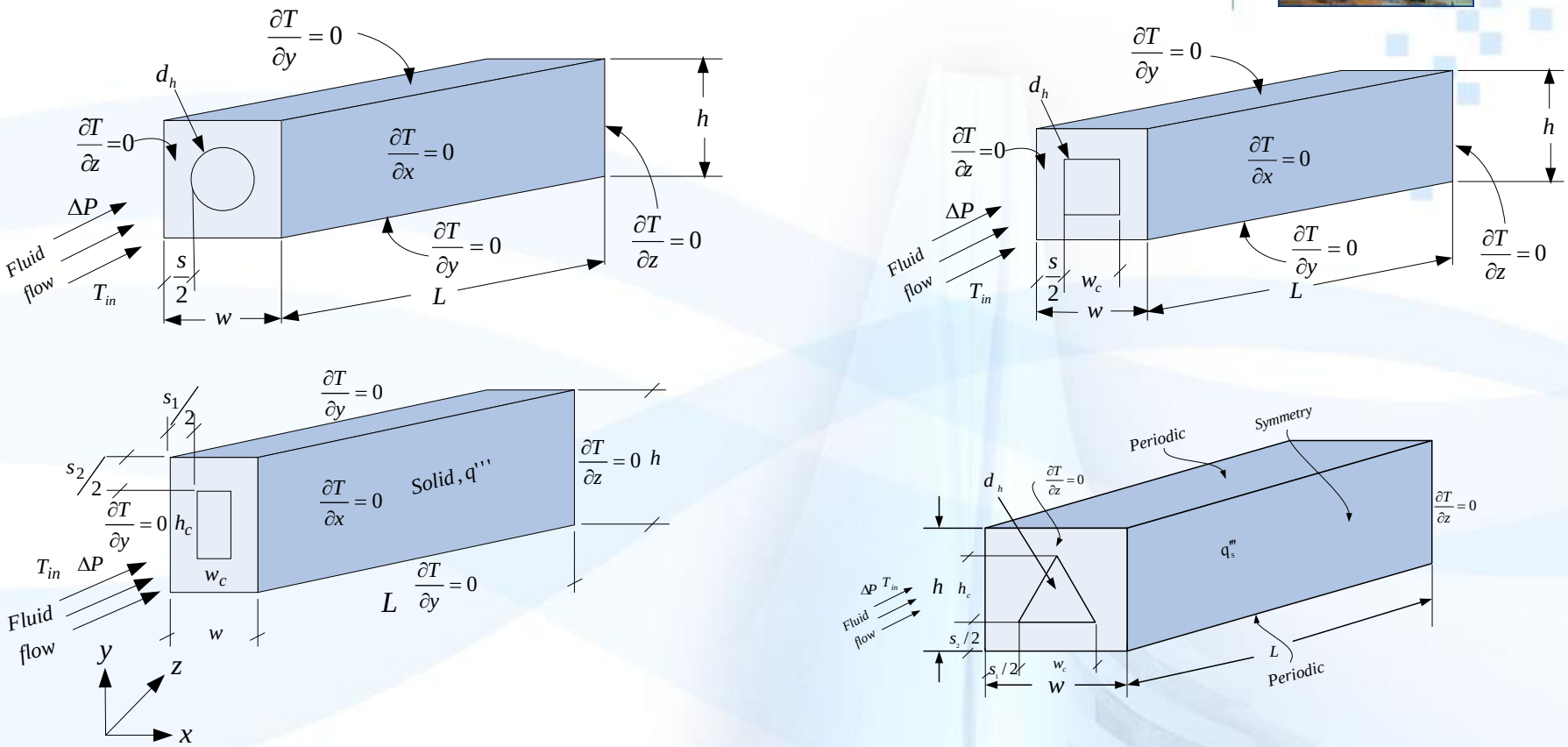


Fig 7 The boundary conditions of the three dimensional Computational domain of the cooling channel

Optimisation Constraints

An elemental volume constraint is considered to compose of elemental cooling channel of hydraulic diameter

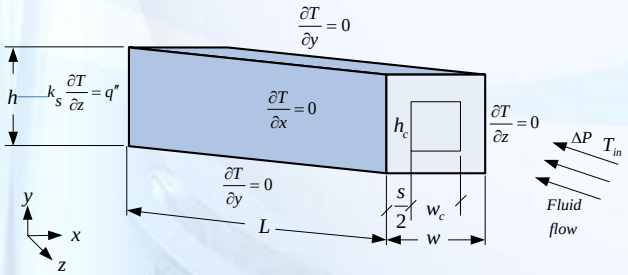
$$v_{el} = w^2 L, \qquad w = d_h + s, \qquad \text{(P1.11)}$$

The number of channels in the structure arrangement can be defined as:

$$N = \frac{HW}{hw} \qquad \text{(P1.12)}$$

The void fraction or porosity of the unit structure can be defined as:

$$\phi = \frac{v_c}{v_{el}} \qquad \text{(P1.13)}$$



The constraint ranges are:

$$0.1 \le \phi \le 0.2, \quad 50\mu m \le w \le 500\mu m, \quad 0 \le d_h \le w, \quad 0 \le s \le w \qquad \text{(P1.14)}$$

Numerical analysis/ Grid independent tests

The numerical solution of the continuity, momentum and energy Equations alongside with boundary conditions was obtained by using a three dimensional commercial package FLUENT™ that employs a finite volume method.

The solution is said to be converged when the normalized residual of the mass and momentum equations falls below 10^{-6} and that of the energy equation is less than 10^{-10}.

Grid independent tests for several mesh refinement were carried out to ensure the accuracy of the numerical results.

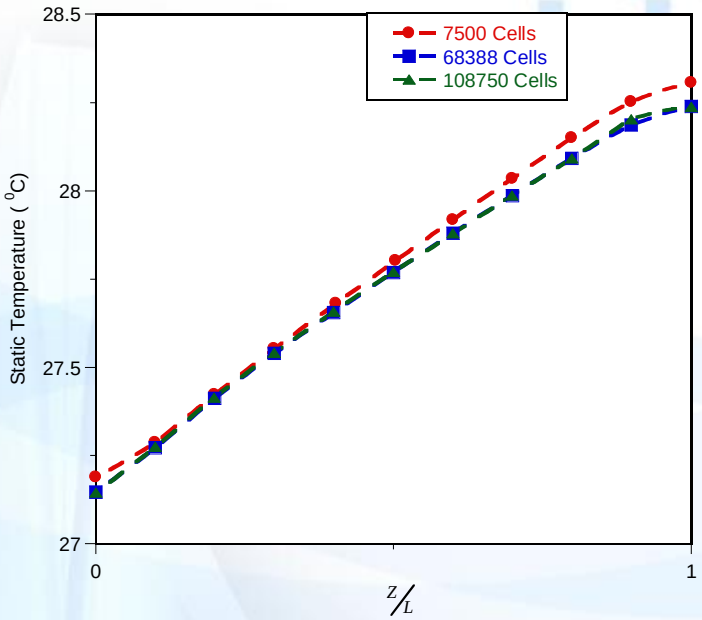


Fig 8 : Grid independent tests

The convergence criterion for the overall thermal resistance as the quantity monitored

$$\gamma = \frac{\left|(T_{\max})_i - (T_{\max})_{i-1}\right|}{\left|(T_{\max})_i\right|} \leq 0.01 \qquad (P1.15)$$

Numerical Results Findings /Graphs

CASE STUDY 1: Cylindrical and square cooling channel embedded in high-conducting solid

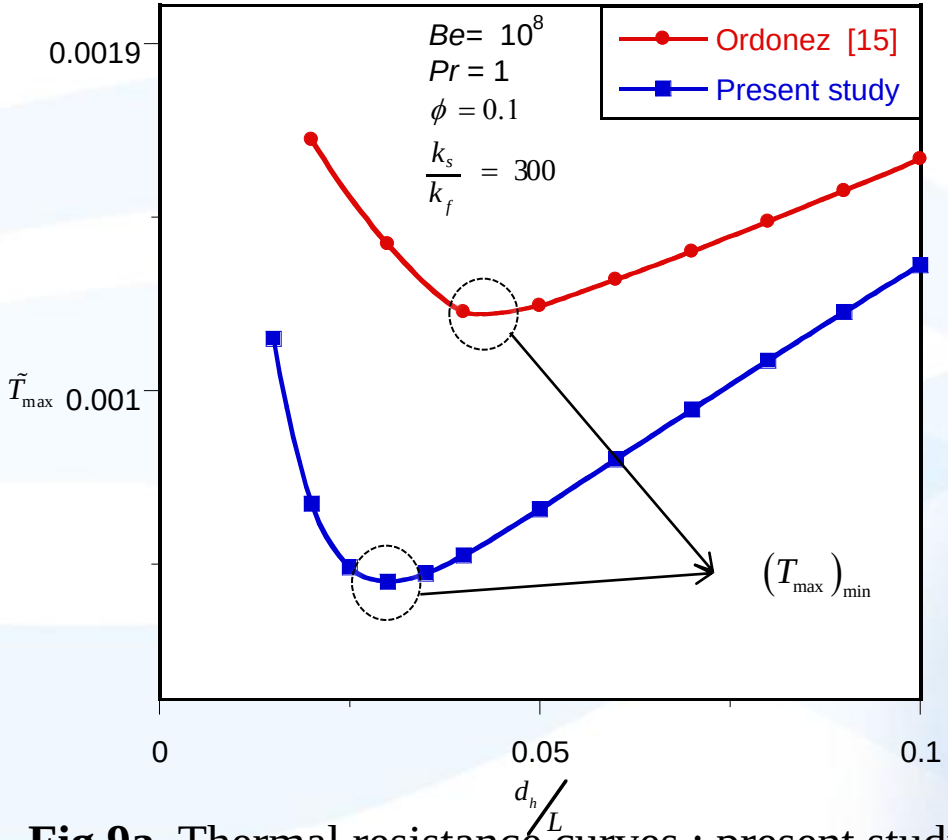


Fig 9a. Thermal resistance curves : present study and ordoenez

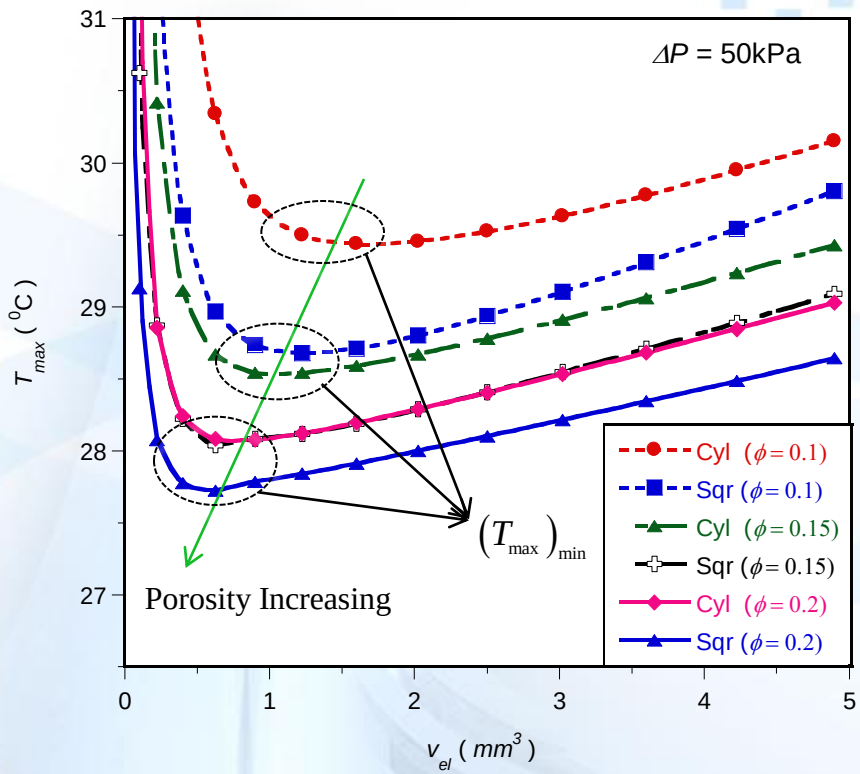
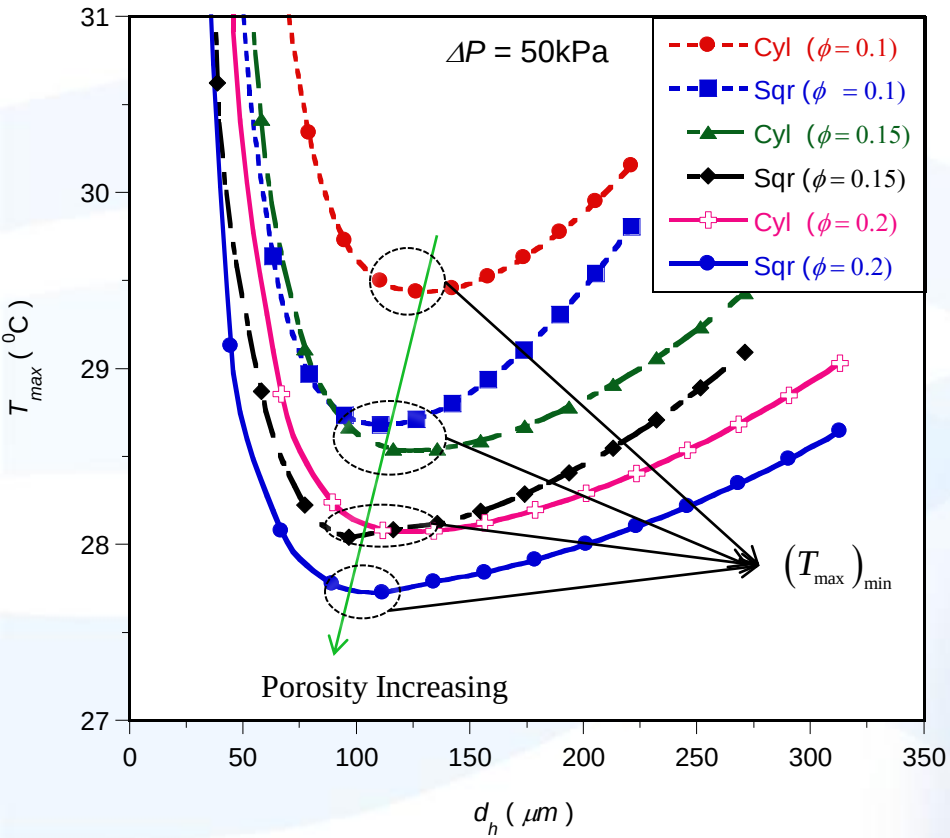


Fig 9b Effect of optimised elemental volume on the peak temperature

Numerical Results Findings /Graphs

CASE STUDY 1: Cylindrical and square cooling channel embedded in high-conducting solid



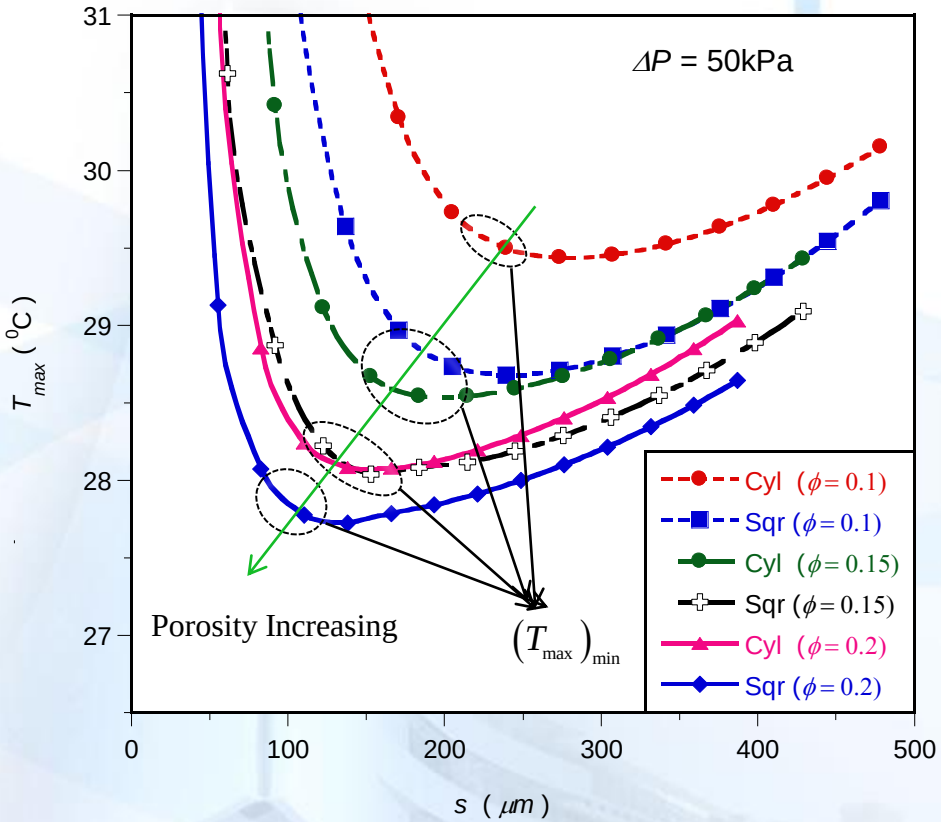
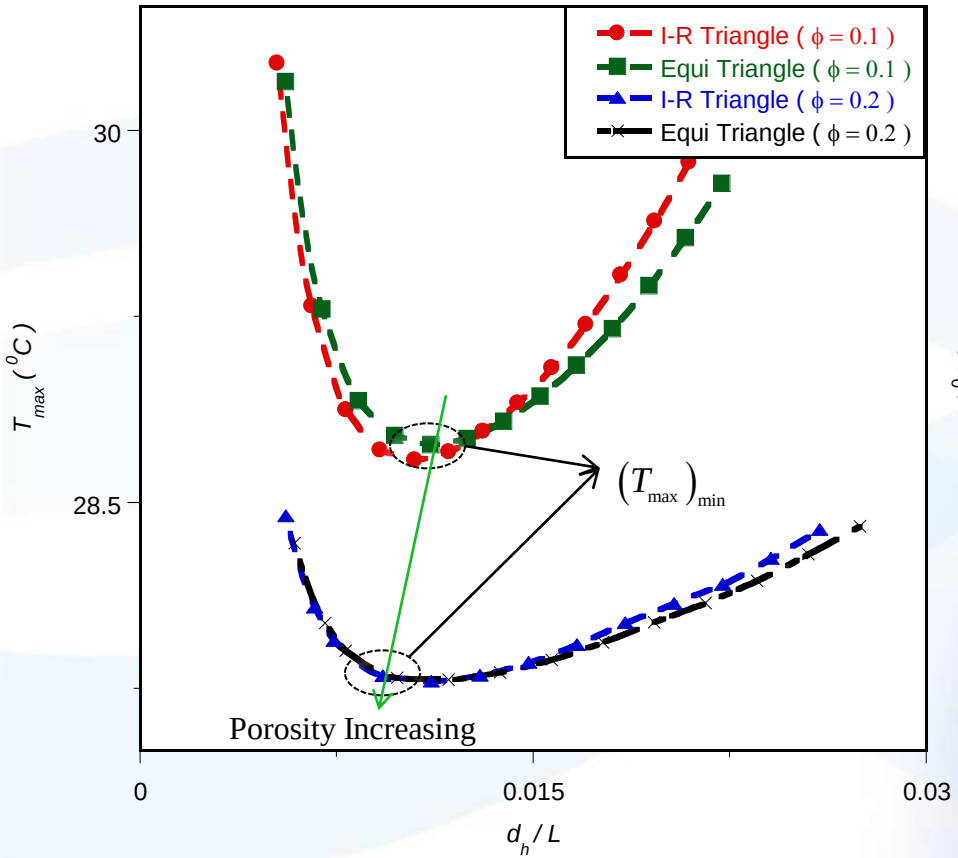


Fig 10 Effect of optimised hydraulic diameter and spacing on the peak temperature

Numerical Results Findings /Graphs

CASE STUDY 2: Truagular cooling channel embedded in high-conducting solid



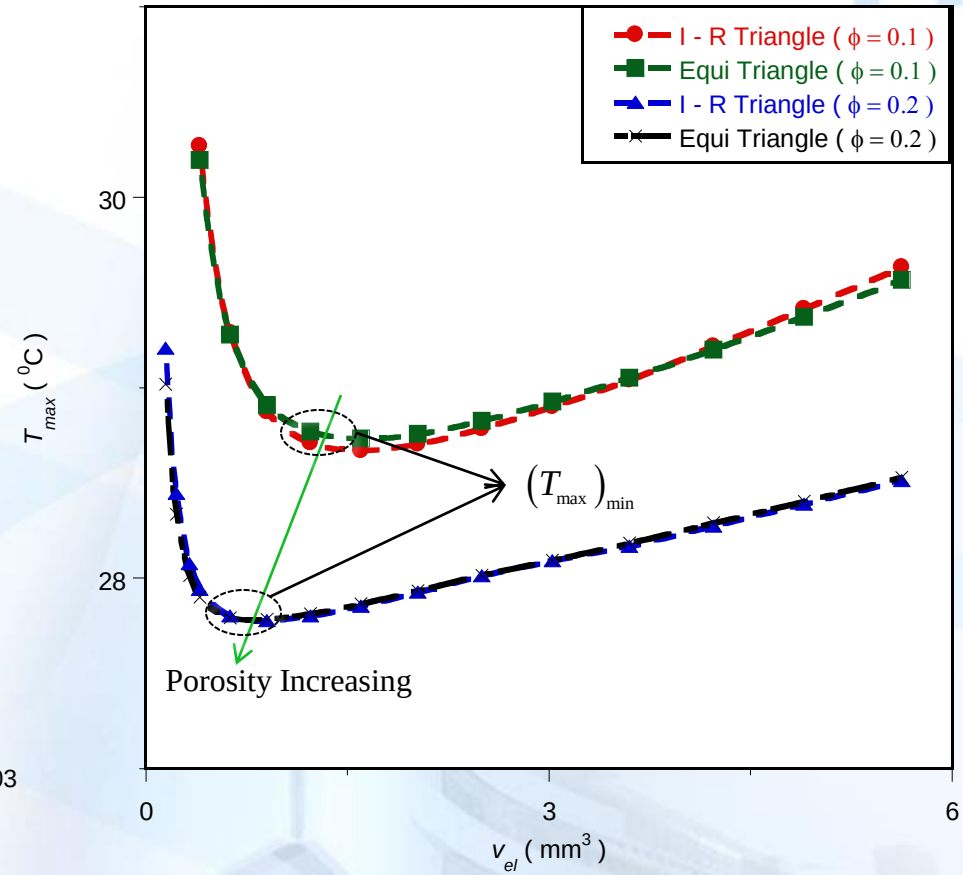
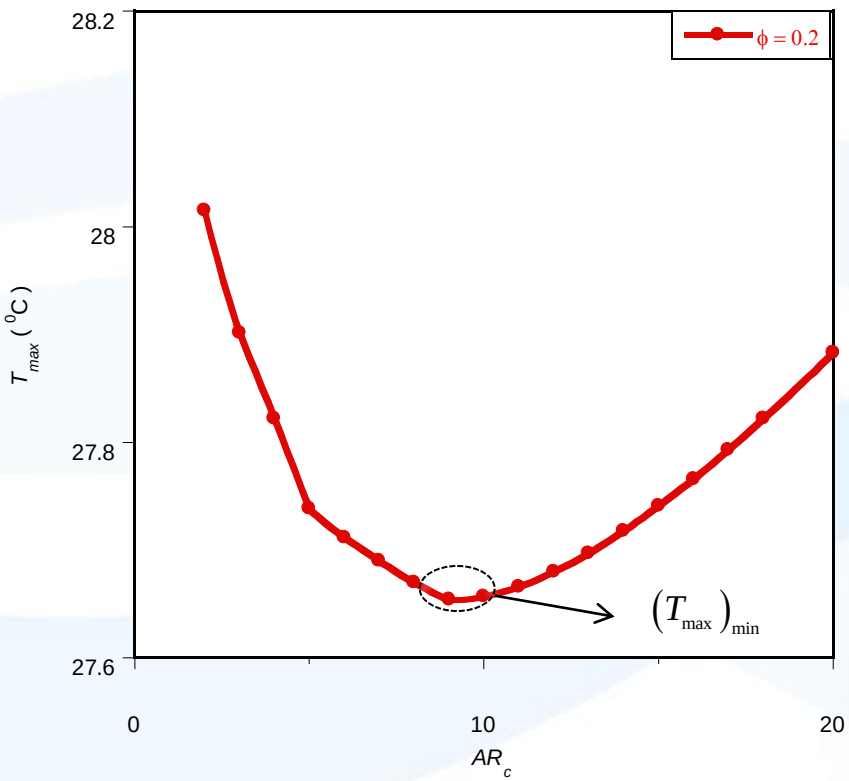


Fig. 11 Effect of optimised hydraulic diameter and elemental volume on the peak temperature

Numerical Results Findings /Graphs

CASE STUDY 3: Rectangular cooling channel embedded in high-conducting solid



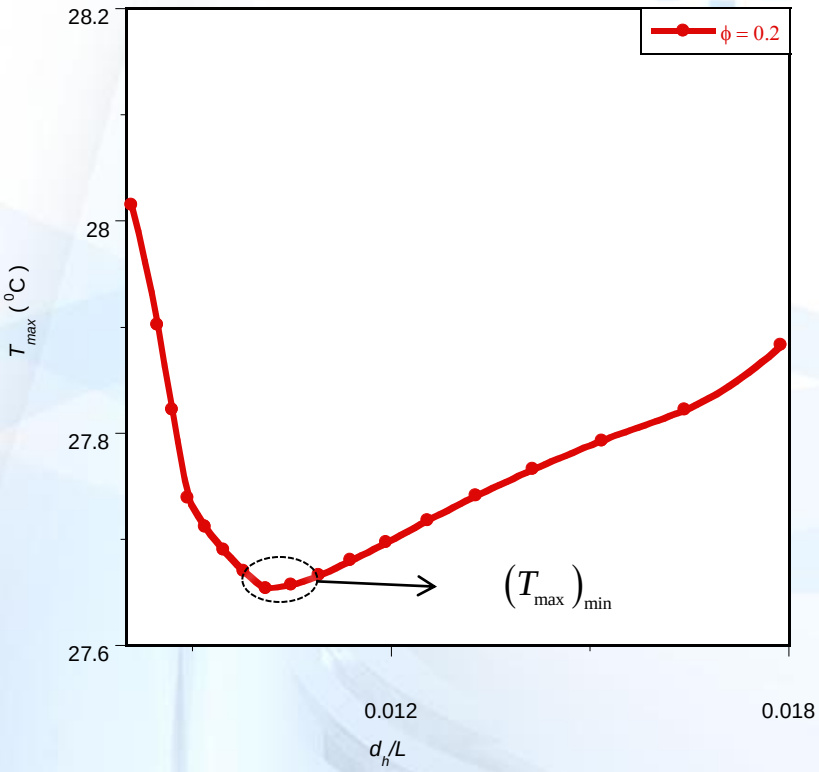


Fig. 12. Effect of optimised aspect ratio and hydraulic diameter on the peak temperature

Mathematical Optimisation:

DYNAMIC-Q Algorithm (by Prof. Snyman)

- **Standard optimization problem**

$$\min_{x} f(x); \left[x_1, \ldots x_2, \ldots . x_i \ldots X_n, \right]^T, \; x_i \in \square^n \qquad (\text{P1.16})$$

Subject to

$$g_j(x) \le 0, \; j = 1, 2, \ldots . p \qquad (\text{P1.17})$$

$$h_k(x) = 0, k = 1, 2, \ldots . q \qquad (\text{P1.18})$$

To search for the:

$$d_{hopt} \qquad (0 \le d_h \le \infty)$$

$$s_{opt} \qquad (0 \le s \le \infty)$$

$$R_{\min} = f\left(d_{hopt}, s_{opt}, \phi_{opt}\right)$$

- **Constraints**

Porosity

$$0.1 \le \left[\phi = \frac{v_c}{v_{el}} \approx \left(\frac{d_h}{w} \right)^2 \right] \le 0.2 \qquad (\text{P1.19})$$

Mathematical Optimisation

- **DYNAMIC-Q Algorithm** Very robust

Gradient based method

- Penalty function technique
- Approximation of numerical functions by spherical quadratic function
- Forward differencing for gradient approximations.
- Automation of the process

$$\tilde{f}(x) = f\left(x^{(l)}\right) + \nabla^T f\left(x^{(l)}\right)\left(x - x^{(l)}\right) + \frac{1}{2}\left(x - x^{(l)}\right)^T A\left(x - x^{(l)}\right)$$

$$\tilde{g}_i(x) = g_i\left(x^{(l)}\right) + \nabla^T g_i\left(x^{(l)}\right)\left(x - x^{(l)}\right) + \frac{1}{2}\left(x - x^{(l)}\right)^T B_i^{(l)}\left(x - x^{(l)}\right), \qquad i = 1, \ldots. p$$

$$\tilde{h}_j(x) = h_j\left(x^{(l)}\right) + \nabla^T h_j\left(x^{(l)}\right)\left(x - x^{(l)}\right) + \frac{1}{2}\left(x - x^{(l)}\right)^T C_j^{(l)}\left(x - x^{(l)}\right), \qquad j = 1, \ldots. q$$

(P1.20)

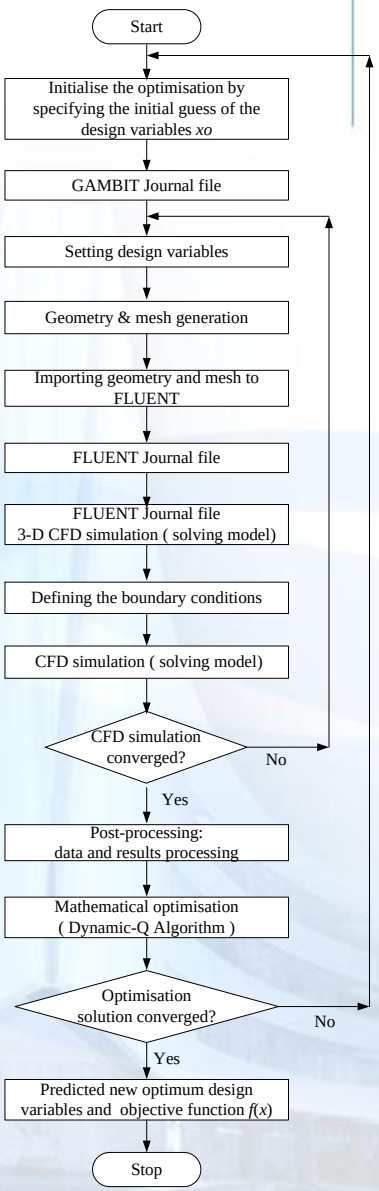


Fig. 13. flow chart of numerical simulation

Numerical Results Findings /Graphs

Cylindrical, square , triangular and rectangular cooling channel embedded in high-conducting solid

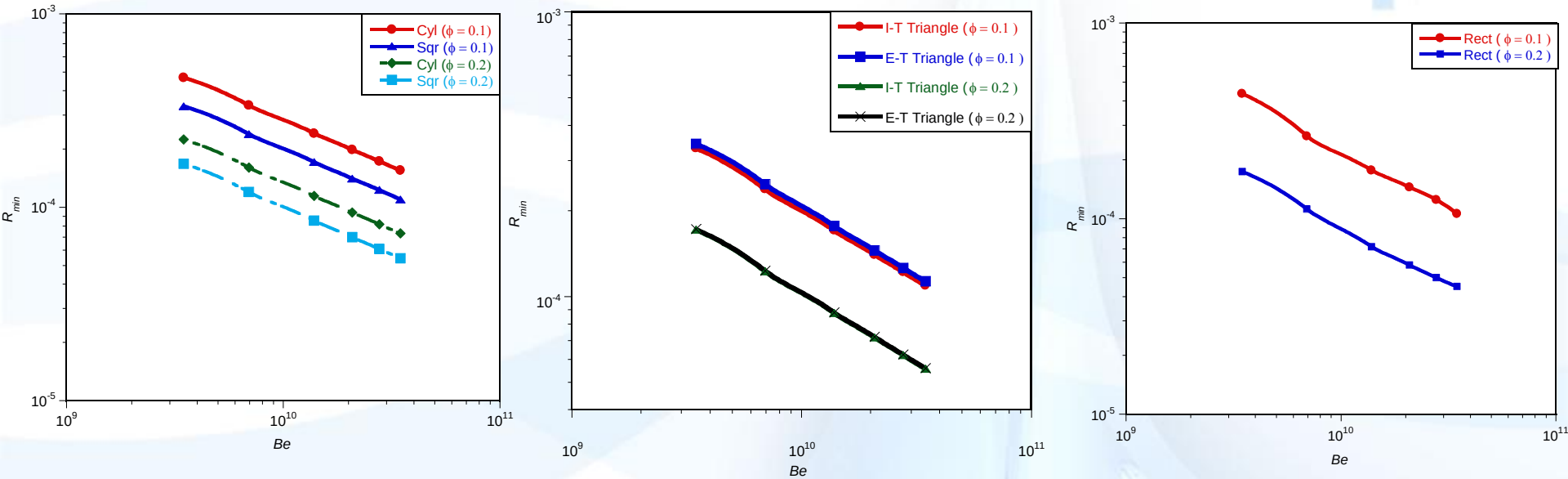


Fig. 14 : Effect of dimensionless pressure difference on the minimised dimensionless global thermal resistance

Numerical Results Findings /Graphs

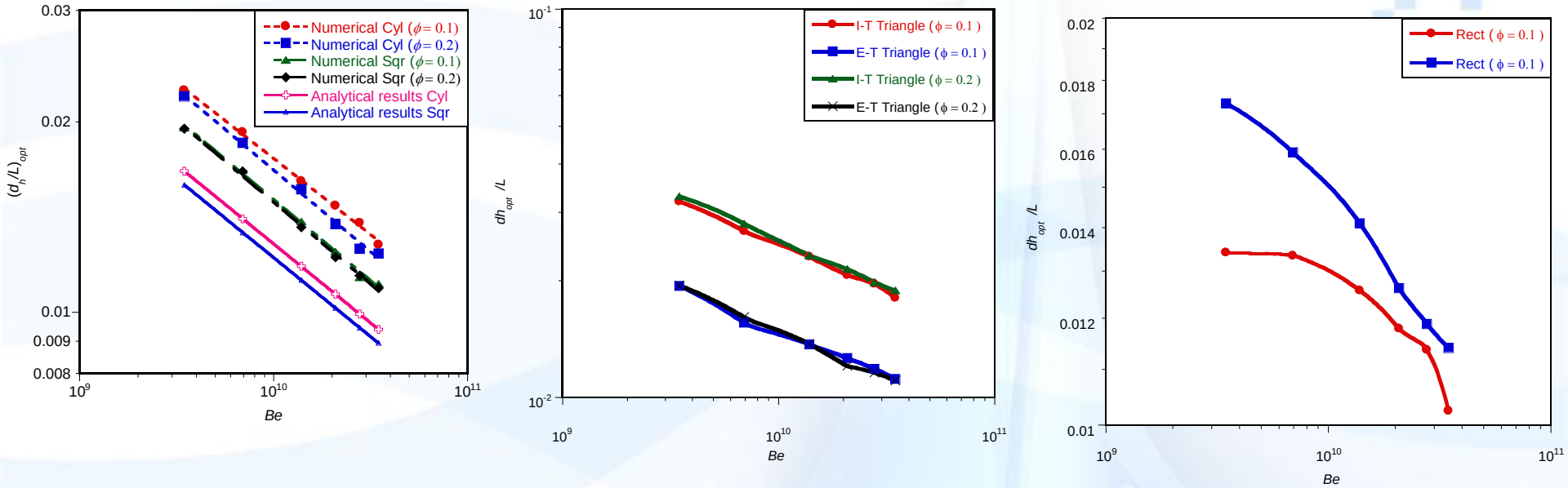
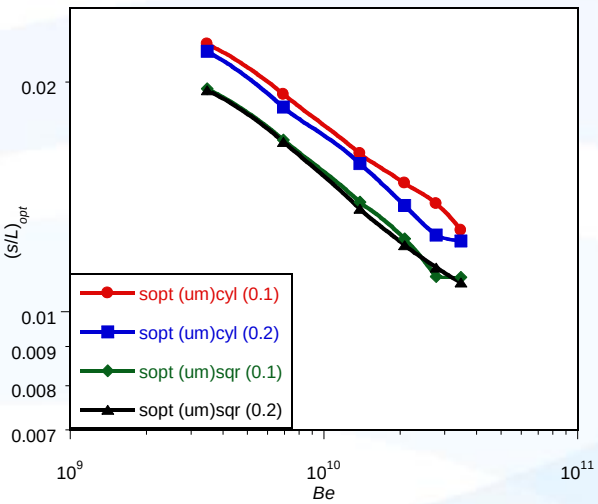
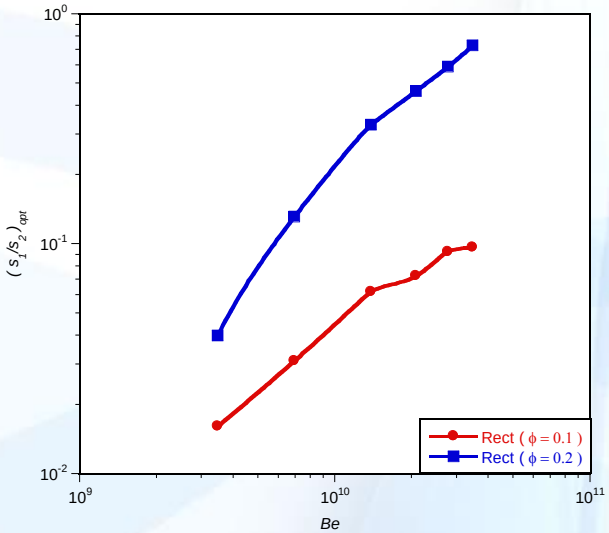


Fig. 15 Effect of dimensionless pressure difference on optimised dimensionless hydraulic diameter

Numerical Results Findings /Graphs





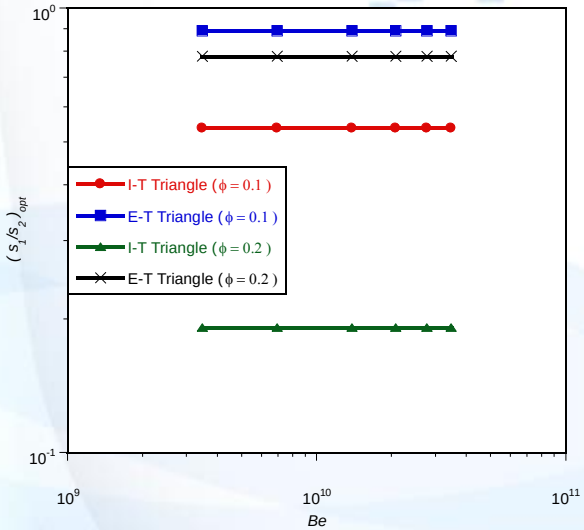


Fig. 16 Effect of dimensionless pressure difference on optimised dimensionless spacing

Analytical Solution

Method of intersection of asymptotes for conjugate channels with internal heat generation

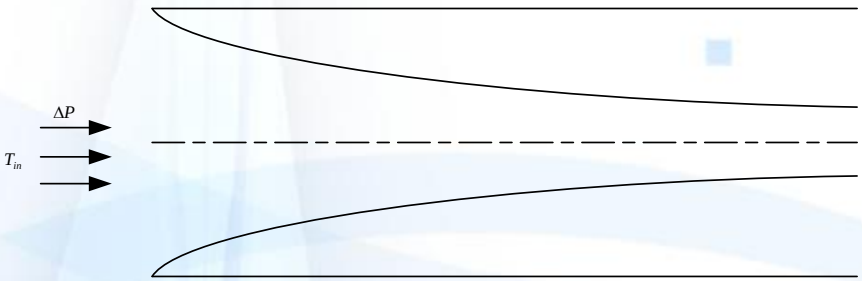
EXTREME LIMIT 1: SMALL CHANNEL

When the channel characteristic dimension scale is small and sufficiently slender, D → 0, D << L

$$R = \left[\frac{k_f\left(T_{\max} - T_{in}\right)}{q'''L^2}\right] \cong 4P_{o_{d_h}}\beta\left(\frac{d_h}{L}\right)^{-2} Be^{-1} \qquad \text{(P1.21)}$$

The hydraulic diameter becomes smaller, the global thermal resistance increases.

EXTREME LIMIT 2: LARGE CHANNEL



When the channel cross-sectional area is large, D → ∞

$$R = \left[\frac{k_f\left(T_{\max} - T_{in}\right)}{q'''L^2}\right] \cong 0.7643\beta\left(\frac{d_h}{L}\right)^{2/3} Be^{-1/3} \qquad \text{(P1.22)}$$

The hydraulic diameter becomes lager, the global thermal resistance increases.

The geometric optimisation in terms of channel diameter could be achieved by combining Eqs. (P1.21) and (P1.22) using the intersection of asymptotes method as shown in

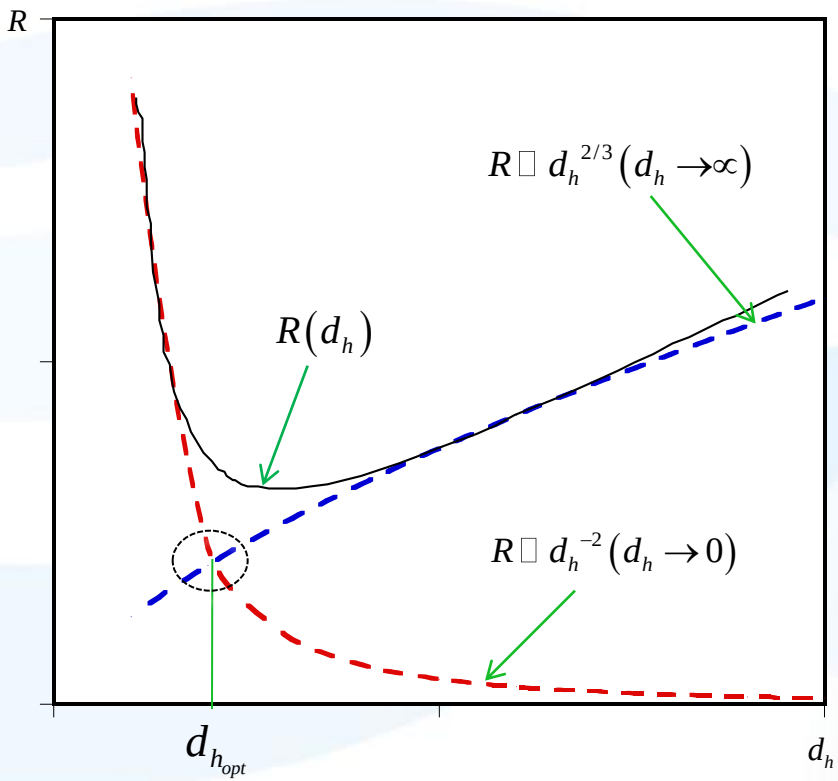
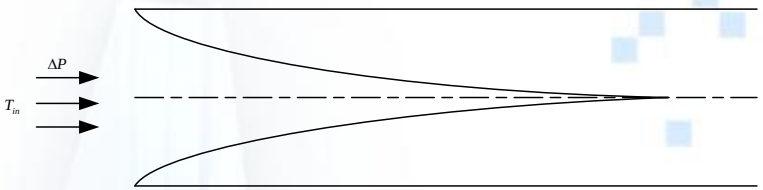


Fig. 17: Intersection of asymptotes method



When the channel cross-sectional area is at optimum

$$\left(\frac{d_h}{L}\right)_{opt} \approx 1.8602 P_{o_{d_h}}^{3/8} Be^{-1/4} \qquad \text{(P1.23)}$$

$$\left(\frac{s}{L}\right)_{opt} \approx 1.8602 \left[(1+\beta)^{1/2} - 1\right] P_{o_{d_h}}^{3/8} Be^{-1/4} \qquad \text{(P1.24)}$$

$$R_{\min} = 1.156 \beta P_{o_{d_h}}^{1/4} Be^{-1/2} \qquad \text{(P1.25)}$$

Comparison of the Theoretical Method and Numerical Optimisation

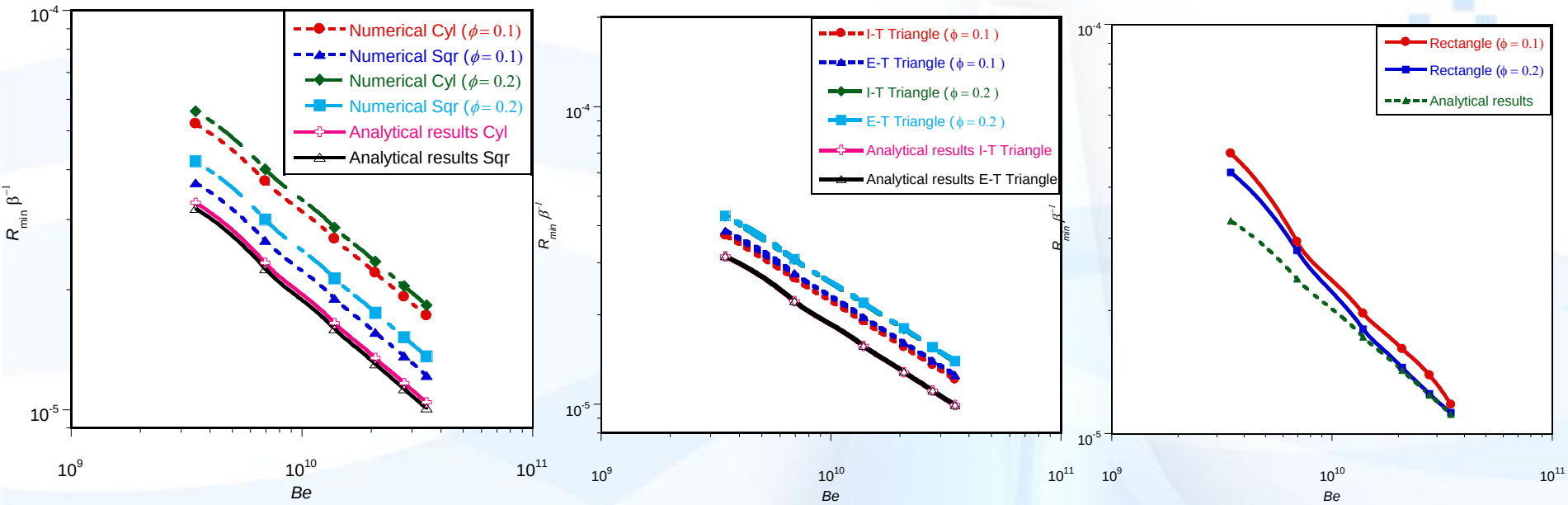


Fig. 18 : Correlation of the numerical and analytical solutions for the minimised global thermal resistance

Comparison of the Theoretical Method and Numerical Optimisation

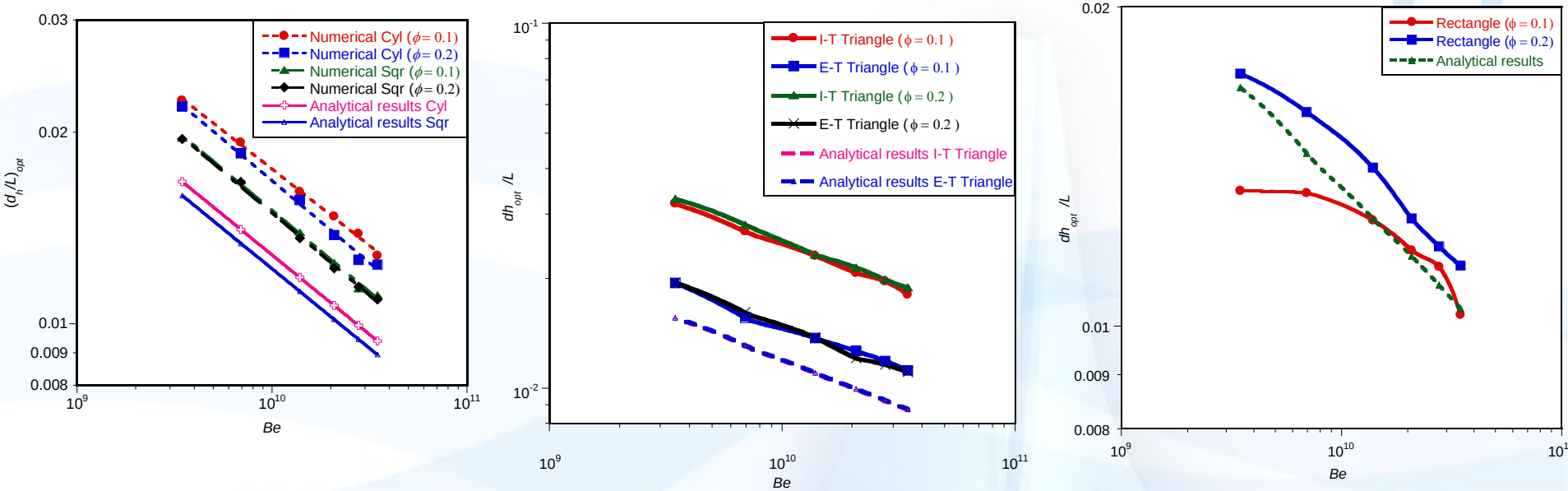
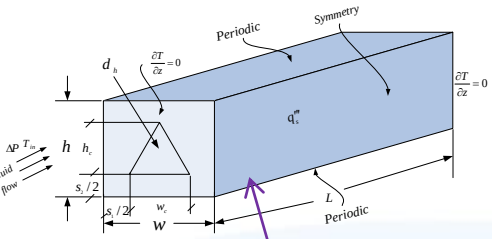
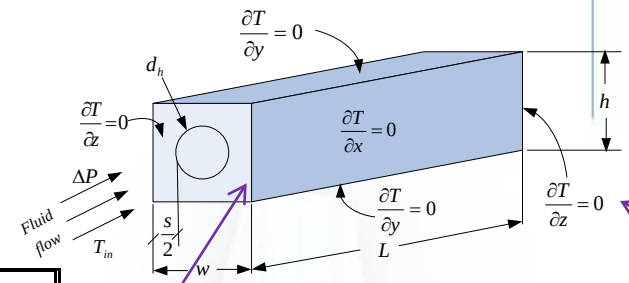


Fig. 19: Correlation between the numerical and analytical solutions for the optimised hydraulic diameter

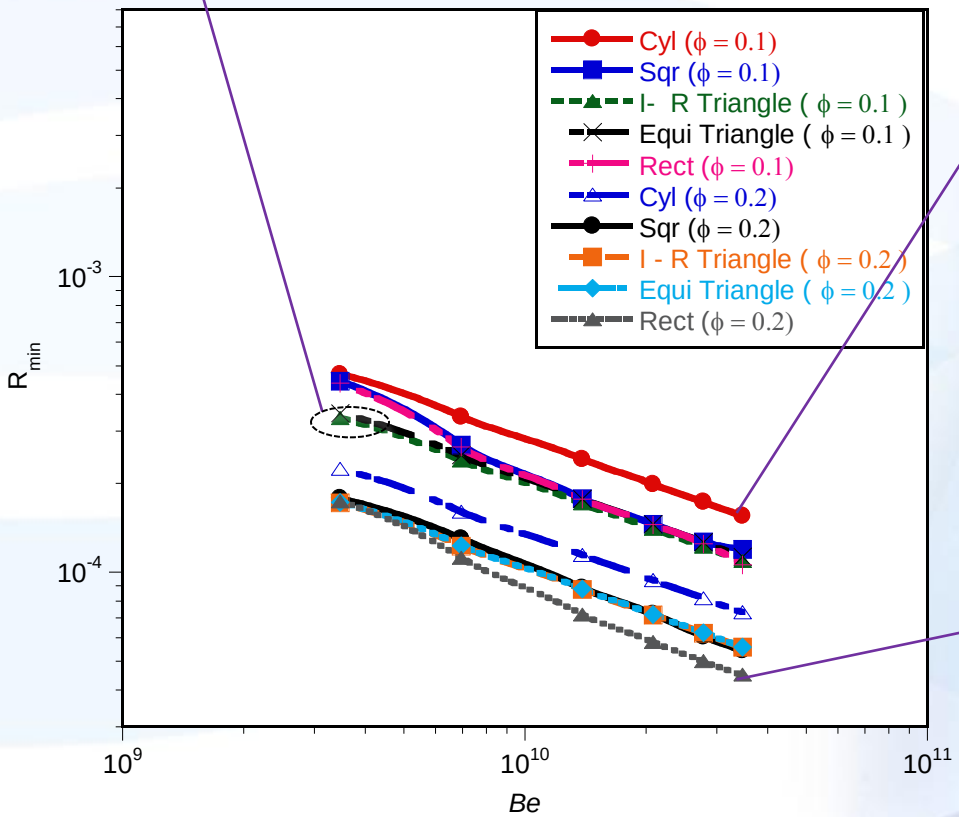
Comparison of the thermal performance of the cooling channels shapes studied



Best



Poor



It was clearly observed that the cooling effect was best achieved at a higher aspect ratio of rectangular channels. However the optimal design scheme could well lead to a design that would be impractical at very high channel aspect ratios, due to the channel being too thin to be manufactured

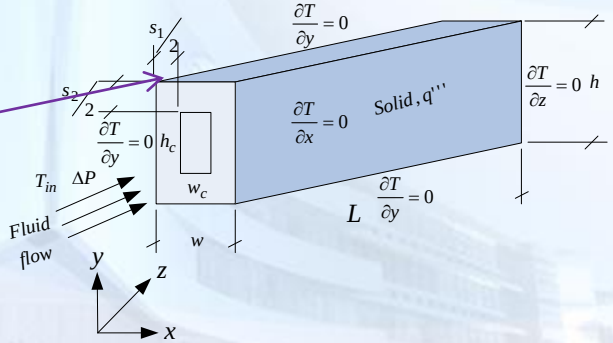


Fig. 20: Comparison of the thermal performance of the cooling channels shapes studied

Numerical Results : Temperature distribution

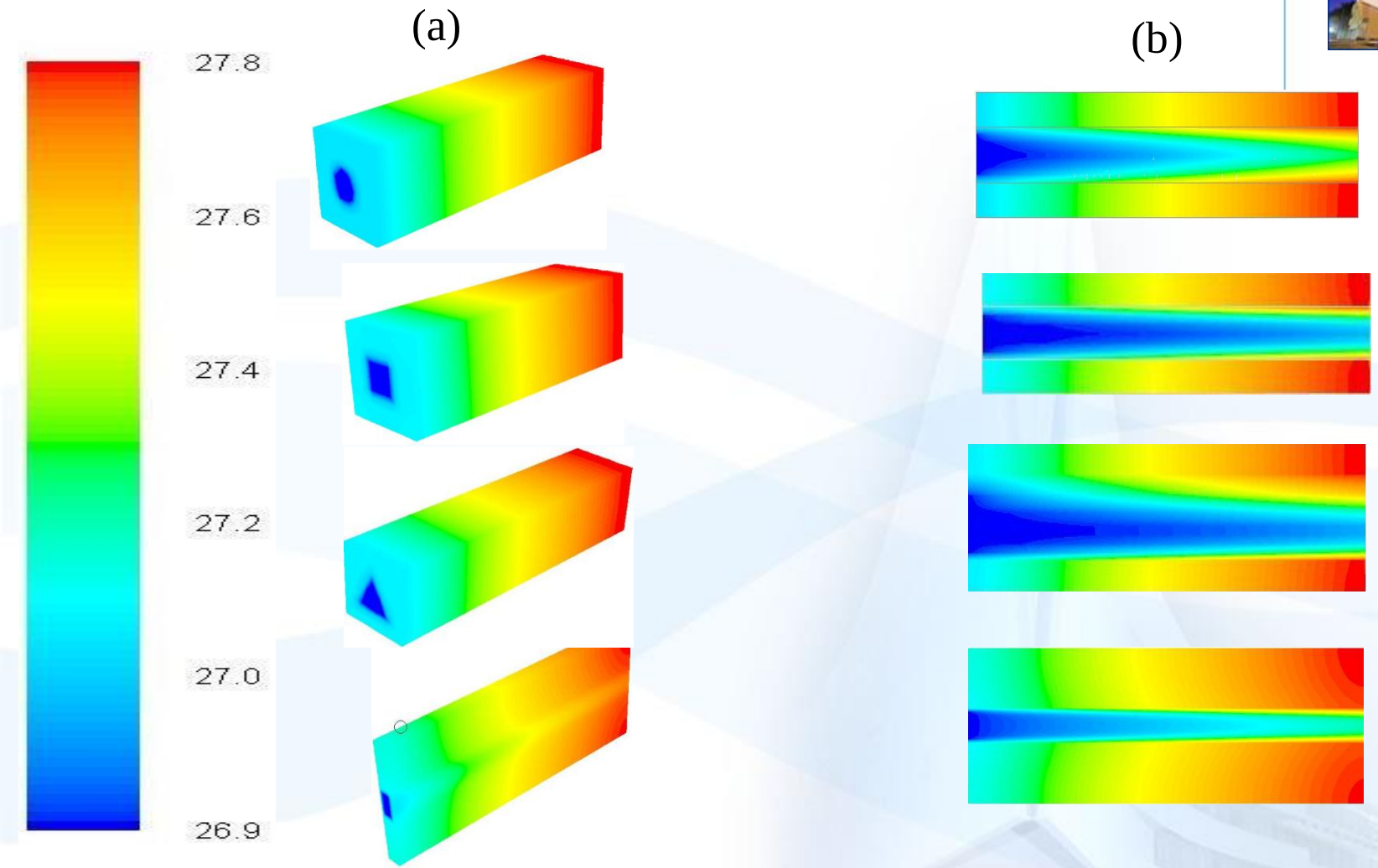


Fig.21. Temperature distribution on (a) the elemental volume and (b) cooling fluid and inner wall

PART 2

Optimisation of Laminar-forced Convection Heat Transfer Through A Vascularised Solid with Cooling Channels

Introduction

- Material with the property of self-healing and self-cooling is becoming more promising in heat transfer analysis.

- The development of vascularisation of the material indicates flow architectures that conduct and circulate fluids at every point within the solid body.

- This solid body (slab) may be performing or experiencing mechanical functions such as mechanical loading, sensing and morphing.

- This self-cooling ability of the vascularised material to bathe at every point of a solid body gave birth to the name “smart material”.

- a solid body of fixed global volume, which is heated with uniform heat flux on the right side; the body is cooled by forcing a single-phase cooling fluid (water) from the left side into the parallel cooling channels

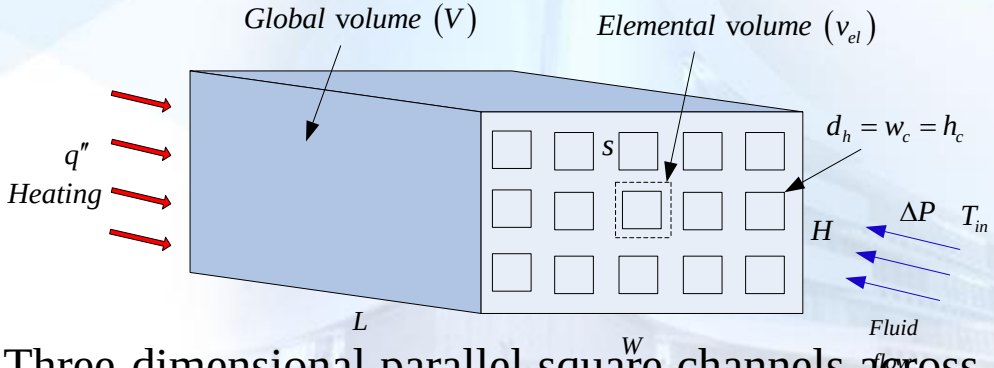


Fig 22. Three-dimensional parallel square channels across a slab with heat flux from one side and forced flow from the other side.

Objective functions and Assumptions

The objective is the minimisation of the global thermal resistance

$$R_{\min} = \frac{k_f \left(T_{\max} - T_{in}\right)_{\min}}{q'' L} \qquad R_{\min} = f\left(d_{h_{opt}}, \quad v_{el_{opt}}, \quad \left(T_{\max}\right)_{\min}\right) \qquad \text{(P2.1)}$$

Assumptions as in part 1

Numerical Modelling /Analysis

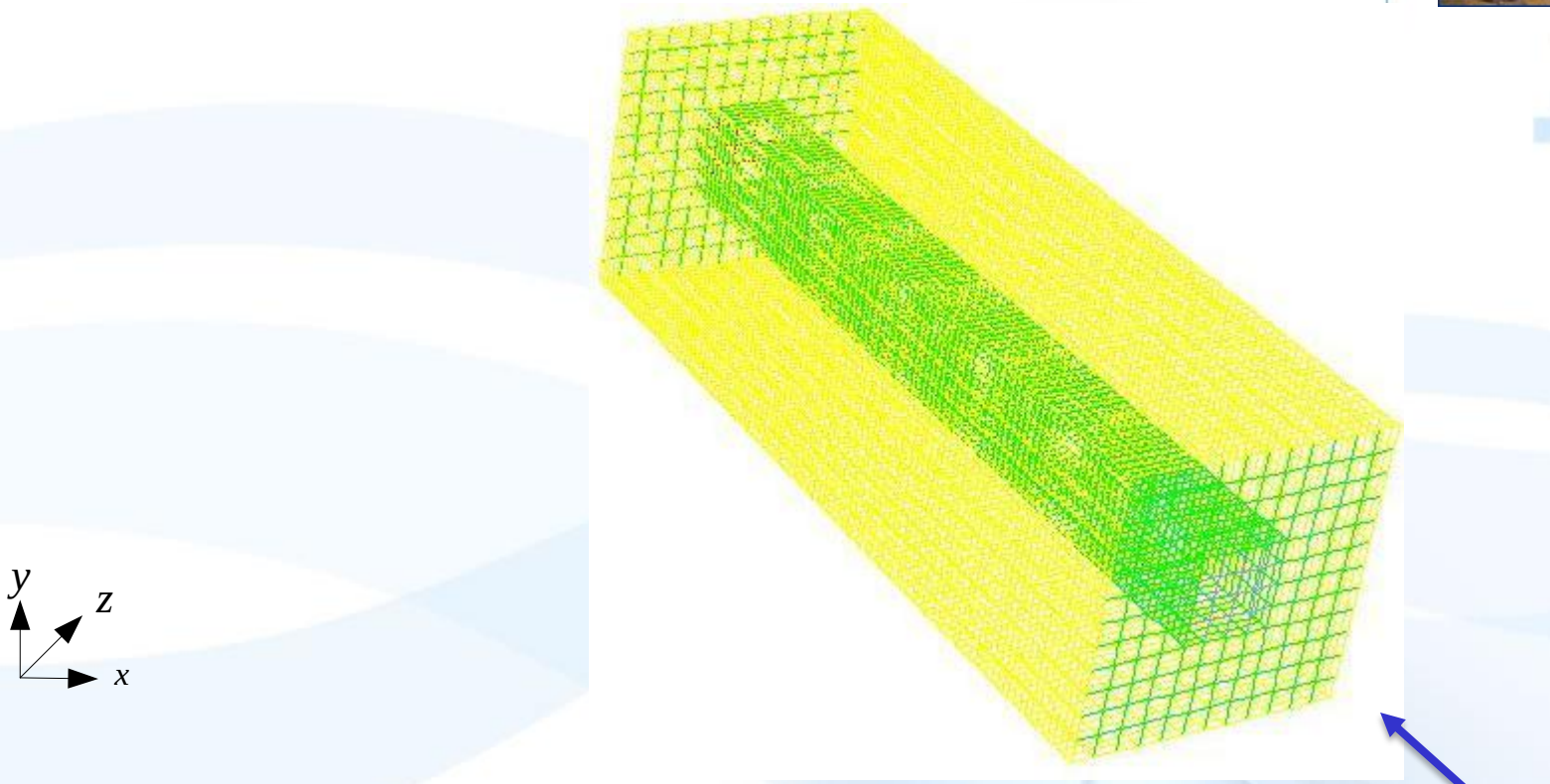


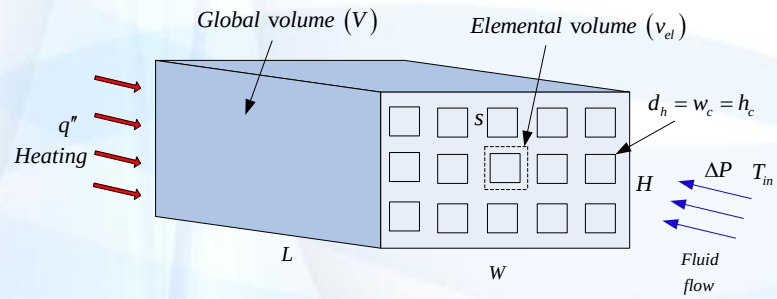
Fig. 23: The discretised 3-D computational domain

Numerical Modelling/Analysis

Governing Equations and BCs as in Part 1 except

Energy equation for a solid region is given as:

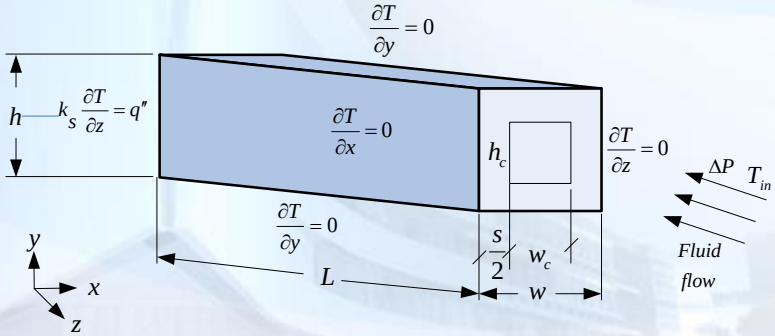
$$k_s \nabla^2 T = 0 \qquad \text{(P2.2)}$$



- **Boundary Conditions**

  - Unit cell using symmetry
  - Heat flux input at the left side

$$k_s \frac{\partial T}{\partial z} = q'' \qquad \text{(P2.3)}$$



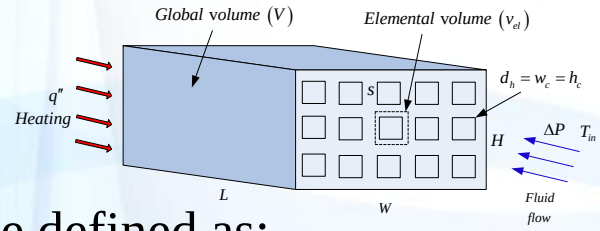
Optimisation Constraints

An elemental volume constraint is considered to compose of elemental cooling channel of hydraulic diameter

$$v_{el} = w^2 L, \quad h = w, \quad w = d_h + s, \qquad \text{(P2.3)}$$

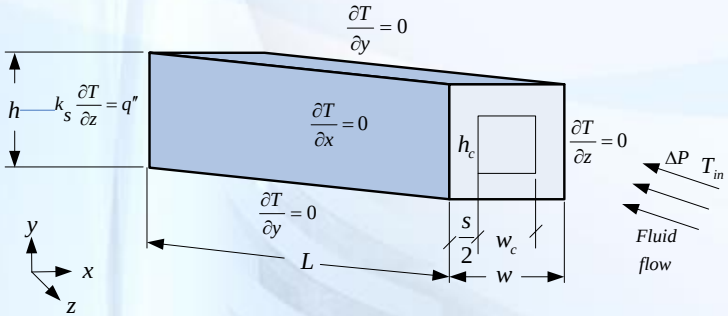
The number of channels in the structure arrangement can be defined as:

$$N = \frac{HW}{\left(d_h + s\right)^2} \qquad \text{(P2.4)}$$



The void fraction or porosity of the unit structure can be defined as:

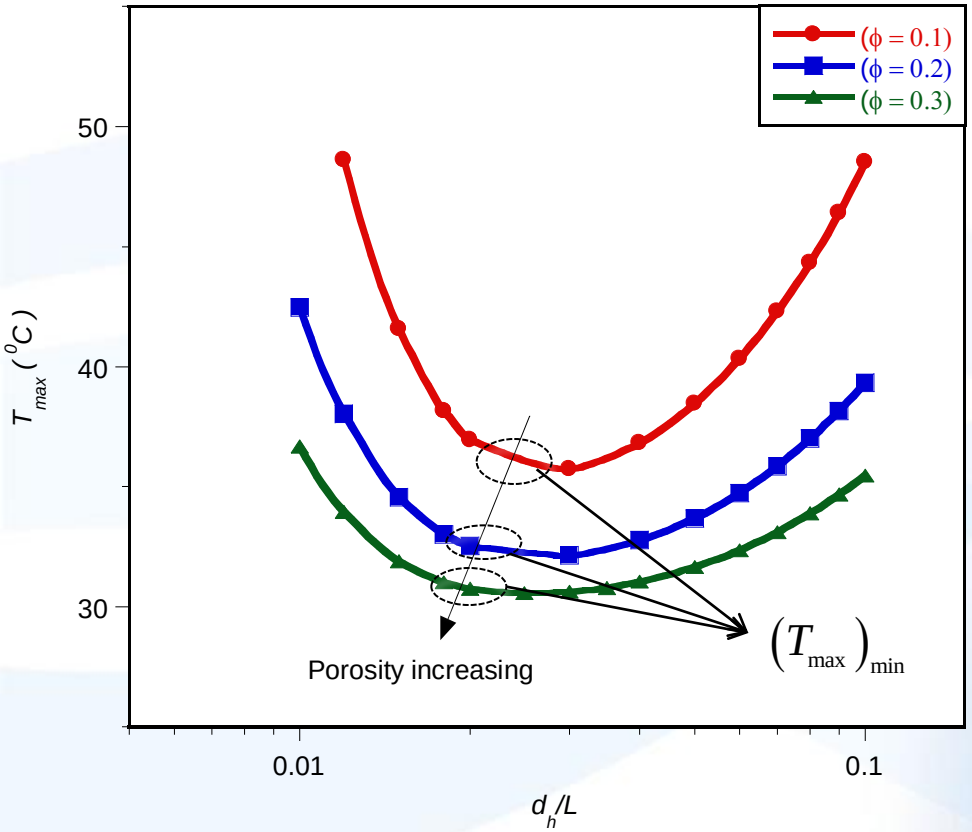
$$\phi = \frac{v_c}{v_{el}} = \left(\frac{d_h}{w}\right)^2 \qquad \text{(P2.5)}$$



The constraint ranges are:

$$0.1 \le \phi \le 0.2, \quad 0.02L \le w \le 0.5L, \quad 0 \le d_h \le w, \quad 0 \le s \le w \qquad \text{(P2.6)}$$

Numerical Results Findings /Graphs



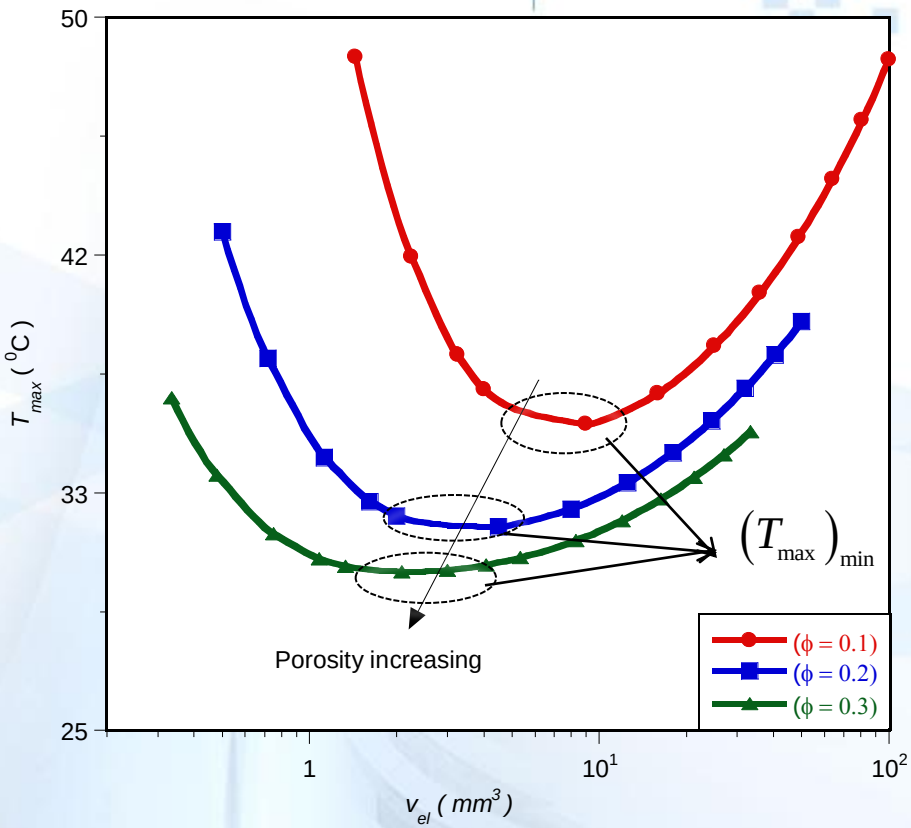
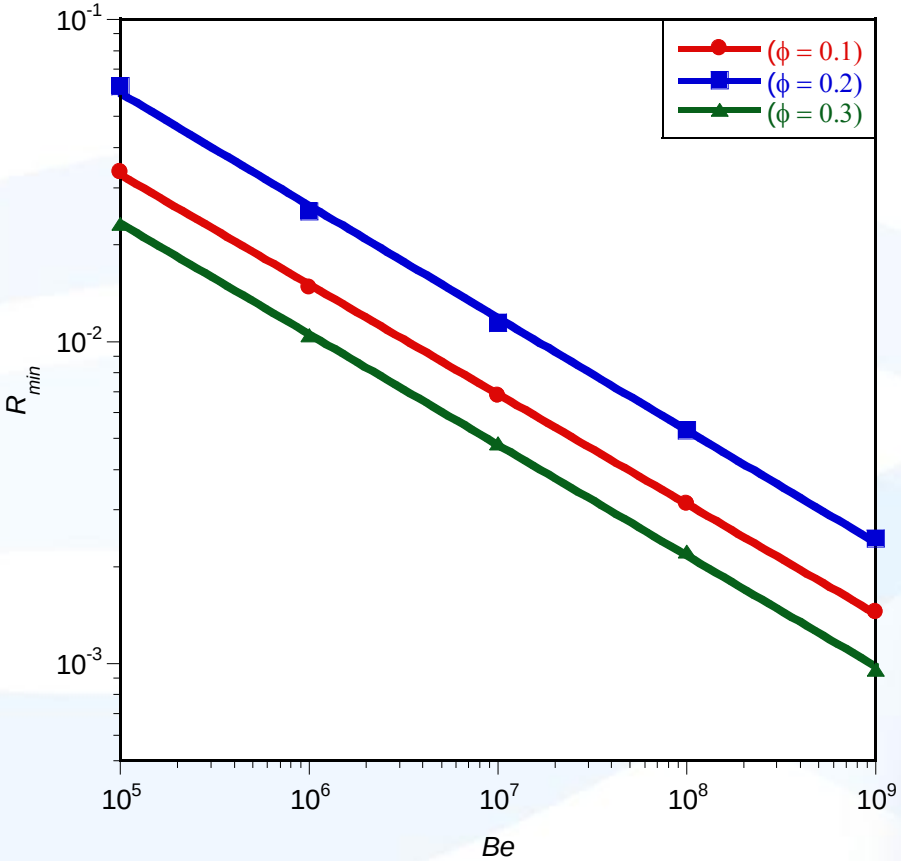


Fig. 24. Effect of optimised dimensionless hydraulic diameter and elemental volume on the peak temperature

Optimisation Results

DYNAMIC-Q Algorithm



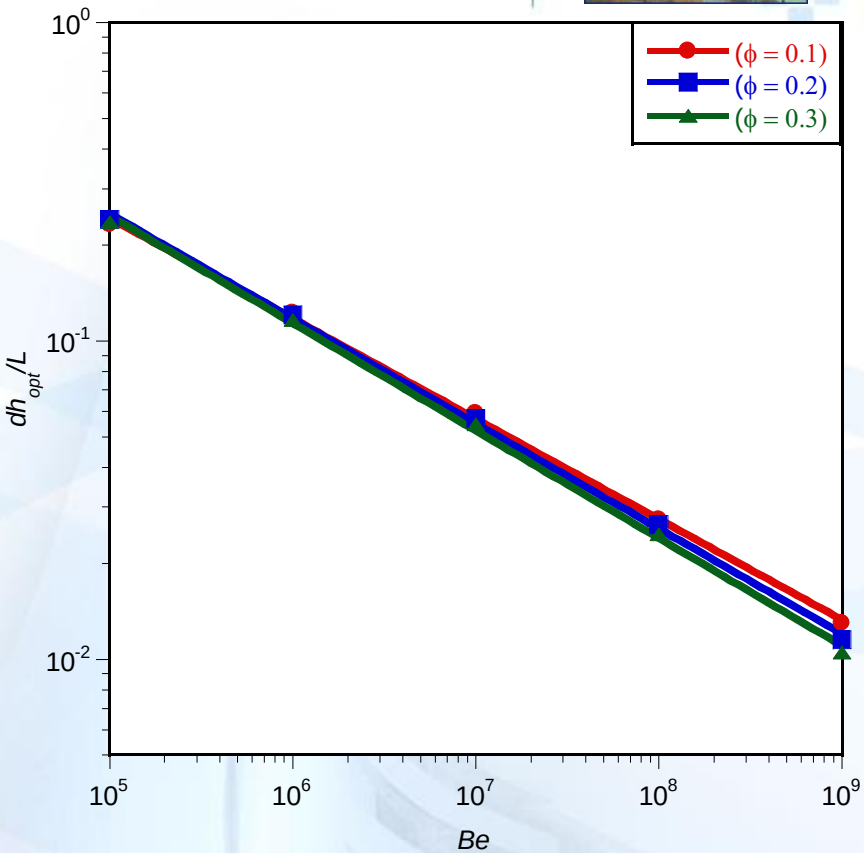
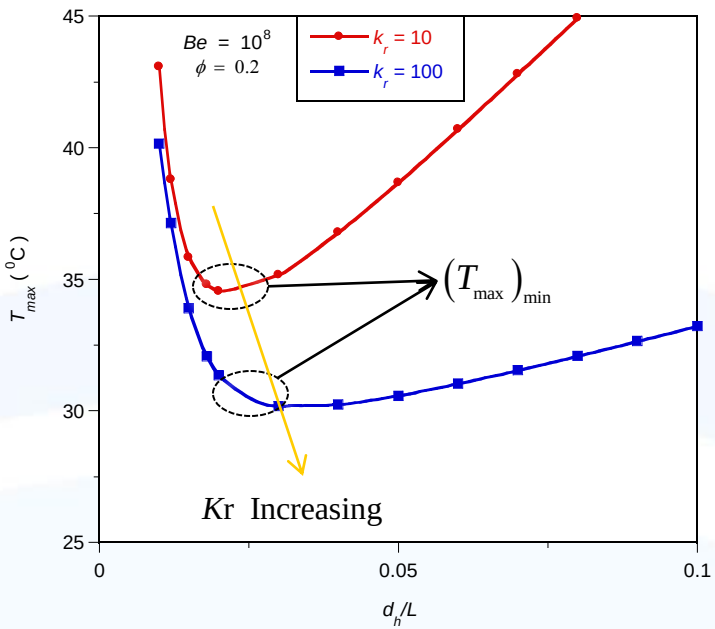
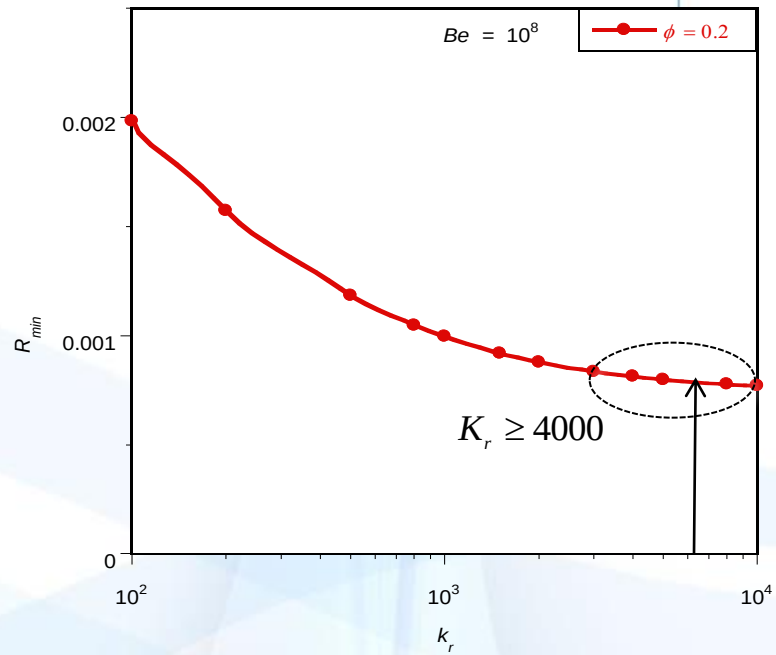
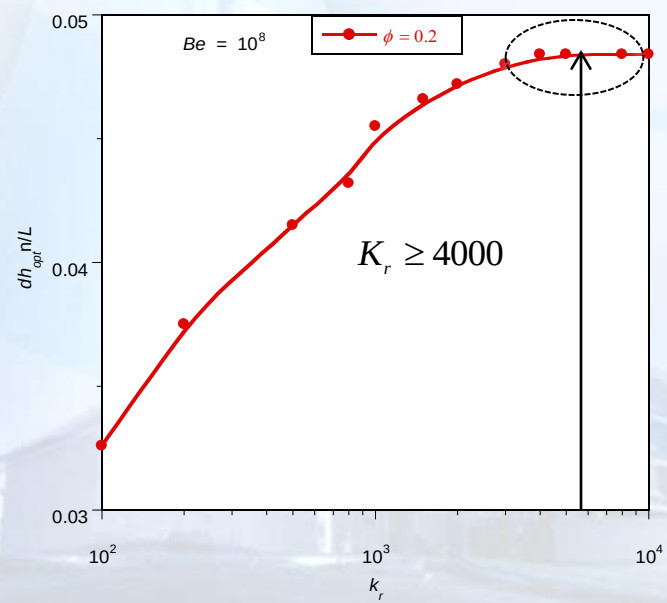


Fig. 25 Effect of dimensionless pressure difference on the dimensionless thermal resistance and the optimised hydraulic diameter

Numerical analysis/results : *Effect of material properties*







At higher thermal conductivity ratio, the thermal conductivity has negligible effect on minimised thermal resistance and optimised hydraulic diameter.

Fig. 26. Effect of material properties on optimised geometry minimised and thermal resistance

Optimisation Results

Effect of material properties Kr on thermal resistance and optimised geometry

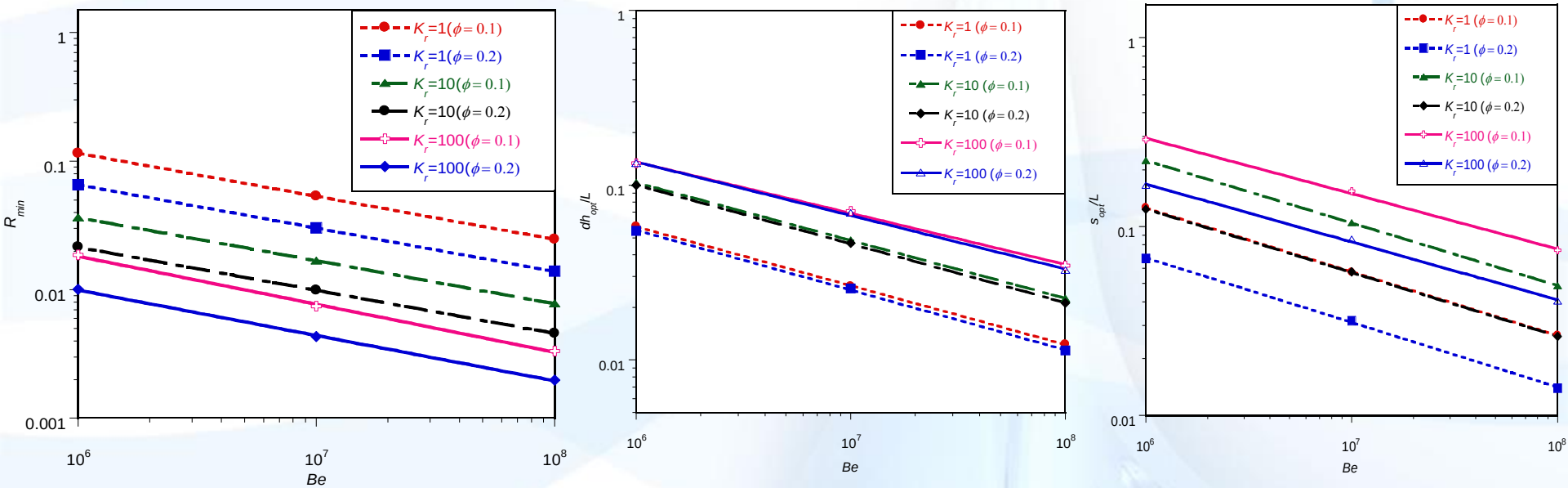


Fig. 27 Effect of material properties Kr on thermal resistance and optimised geometry

Temperature Profile

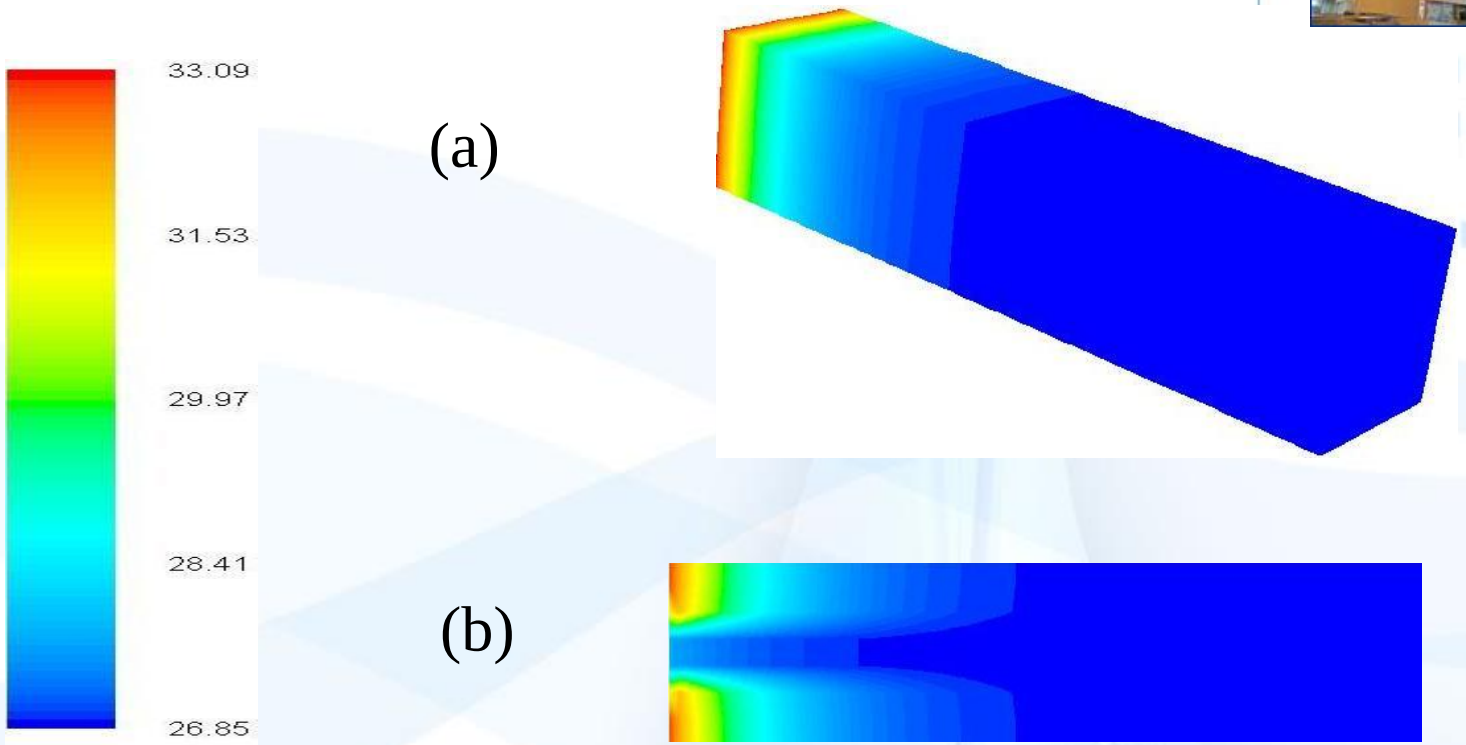
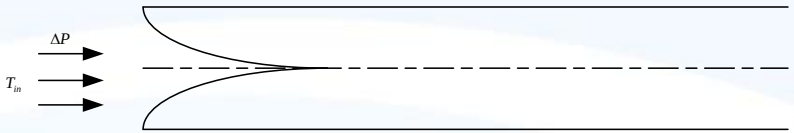


Fig 28. Temperature distribution on (a) the elemental volume and (b) cooling fluid and inner wall.

Analytical Solution

EXTREME LIMIT 1: SMALL CHANNEL

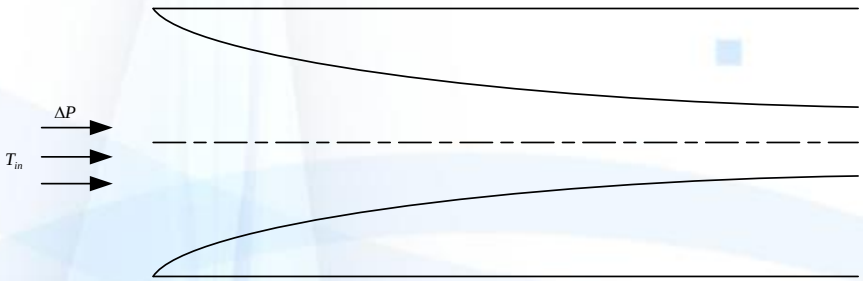


When the channel characteristic dimension scale is small and sufficiently slender, D → 0, D << L

$$R = \frac{k_f\left(T_{\max} - T_{\text{in}}\right)}{q''L} \cong \frac{32}{\phi}\left(\frac{d_h}{L}\right)^{-2} Be^{-1} \qquad (\text{P2.7})$$

As the hydraulic diameter becomes smaller, the global thermal resistance increases.

EXTREME LIMIT 2: LARGE CHANNEL



When the channel cross-sectional area is large, D → ∞

$$R = \frac{k_f\left(T_{\max} - T_{\text{in}}\right)}{q''L} \cong 0.75 k_r^{-1} \phi^{-1/2} \frac{d_h}{L}, \qquad (\text{P2.8})$$

As the hydraulic diameter becomes larger, the global thermal resistance increases.

Analytical Solution

The geometric optimisation in terms of channel diameter could be achieved by combining Eqs. (P2.7) and (P2.8) using the Intersection of asymptotes method as shown in Fig. 9.

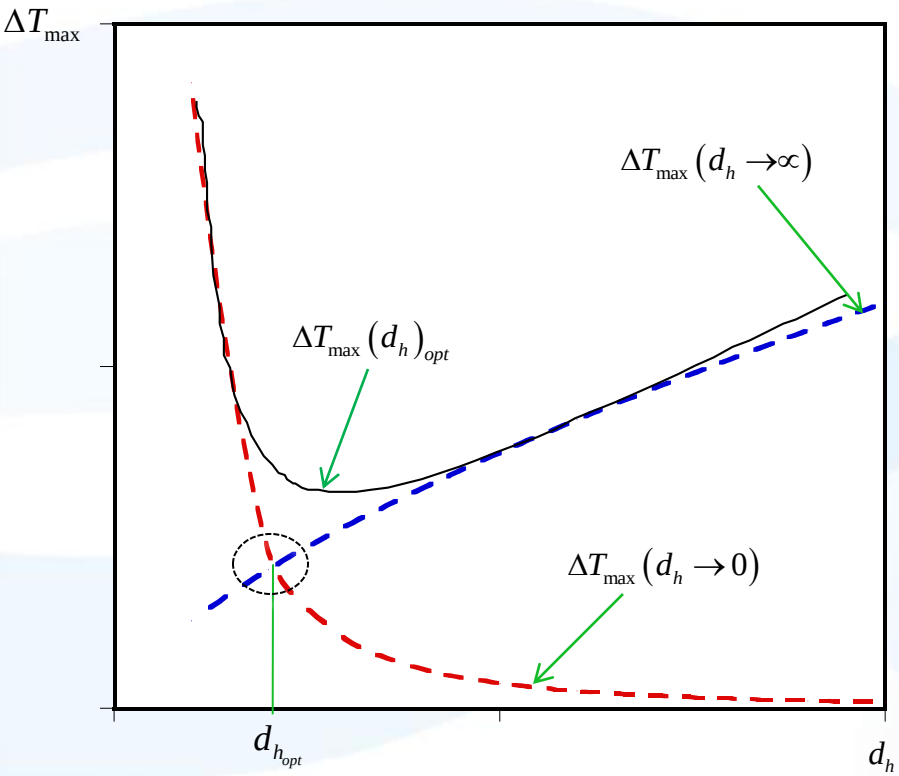
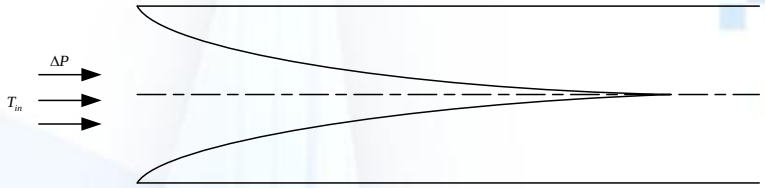


Fig. 29: Intersection of asymptotes method



When the channel cross-sectional area is at optimum

$$\frac{d_{h_{opt}}}{L} = 3.494\phi^{-1/6} k_r^{1/3} Be^{-1/3} \qquad (P2.9)$$

$$R_{min} = \frac{k_f (T_{max} - T_{in})_{min}}{q''L} \cong 2.62 (k_r \phi)^{-2/3} Be^{-1/3}, \qquad (P2.10)$$

Optimisation Results

Effect of material properties Kr on optimised geometry

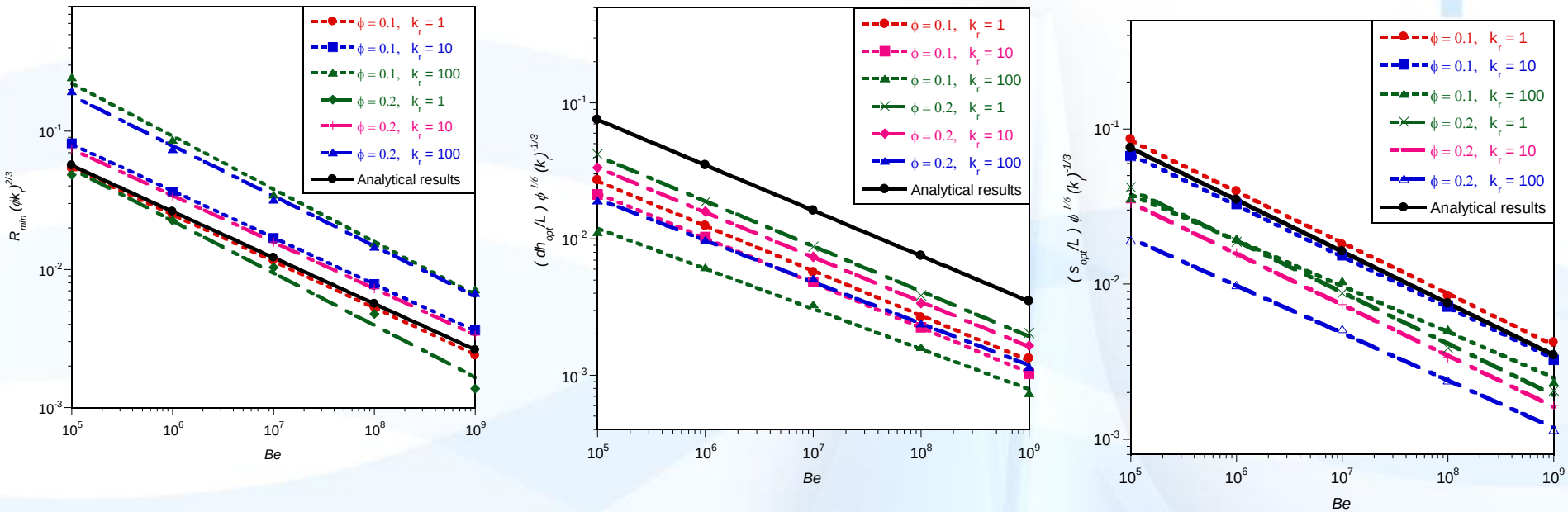
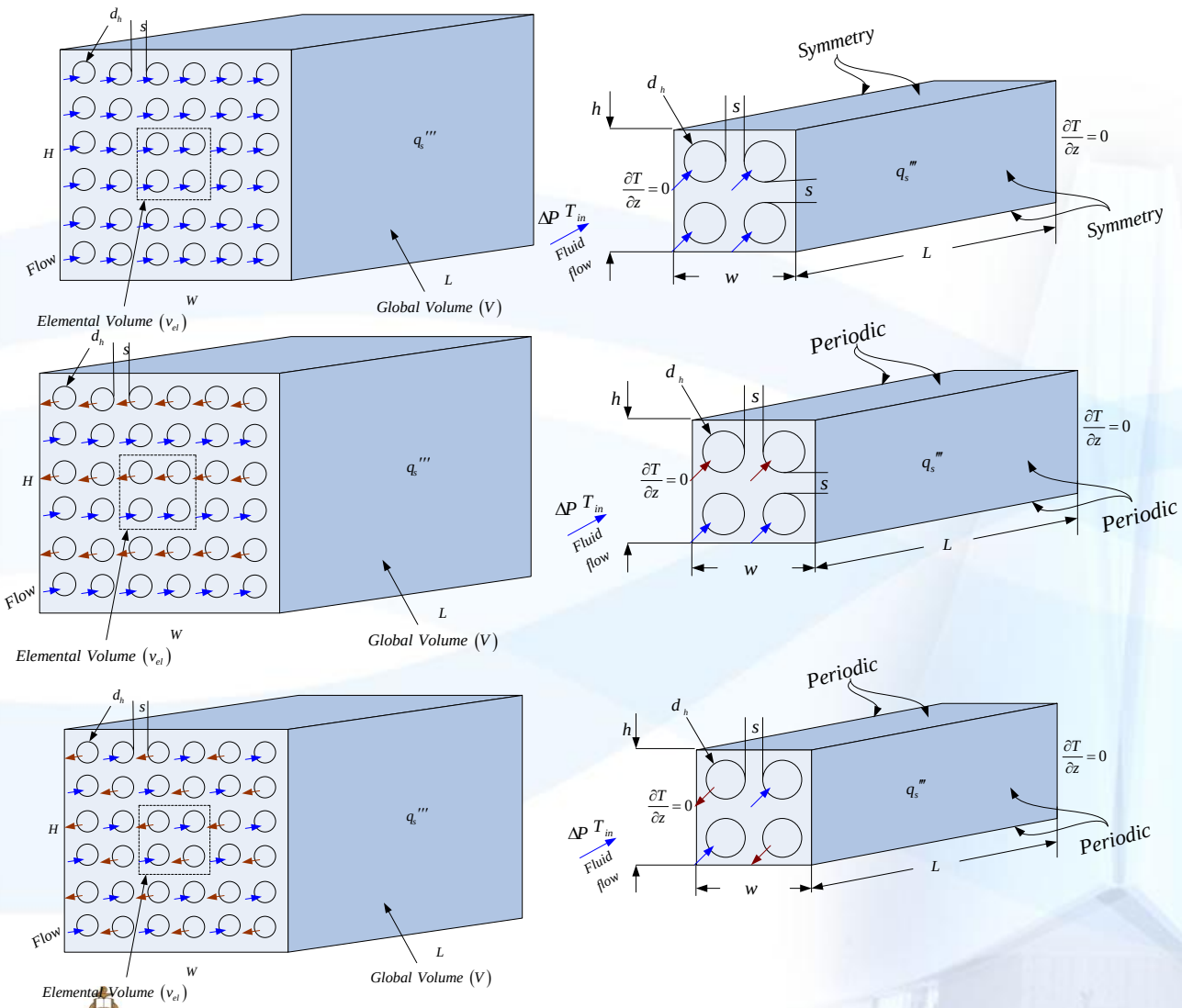


Fig. 30 : Correlation of the numerical and analytical solutions for the minimised global thermal resistance

PART 3

Effect of flow orientation on forced convective heat transfer in cooling channel with internal heat generation

Introduction : Numerical Anaysis



The array of channels with parallel flow refer as PF-1.

The array of channels in which flow of the every other row channel is in counter direction to one another refer as CF-2 .

The every flow in the array of channels is in counter direction to one another, refers as CF-3

Fig. 31 : Three dimensional parallel circular of PF-1, CF-2 and CF-3

Objective functions and Assumptions

The objective is the minimisation of the global thermal resistance

$$R_{\min} = \frac{k_f \left(T_{\max} - T_{in}\right)_{\min}}{q''' L^2} \qquad \text{(P3.1)}$$

$$R_{\min} = f\left(d_{h_{opt}},\ s_{opt},\ v_{el_{opt}},\ T_{\max_{\min}},\ \ \textit{flow orientation}\right)$$

Assumptions as in part 1

Numerical Modelling /Analysis

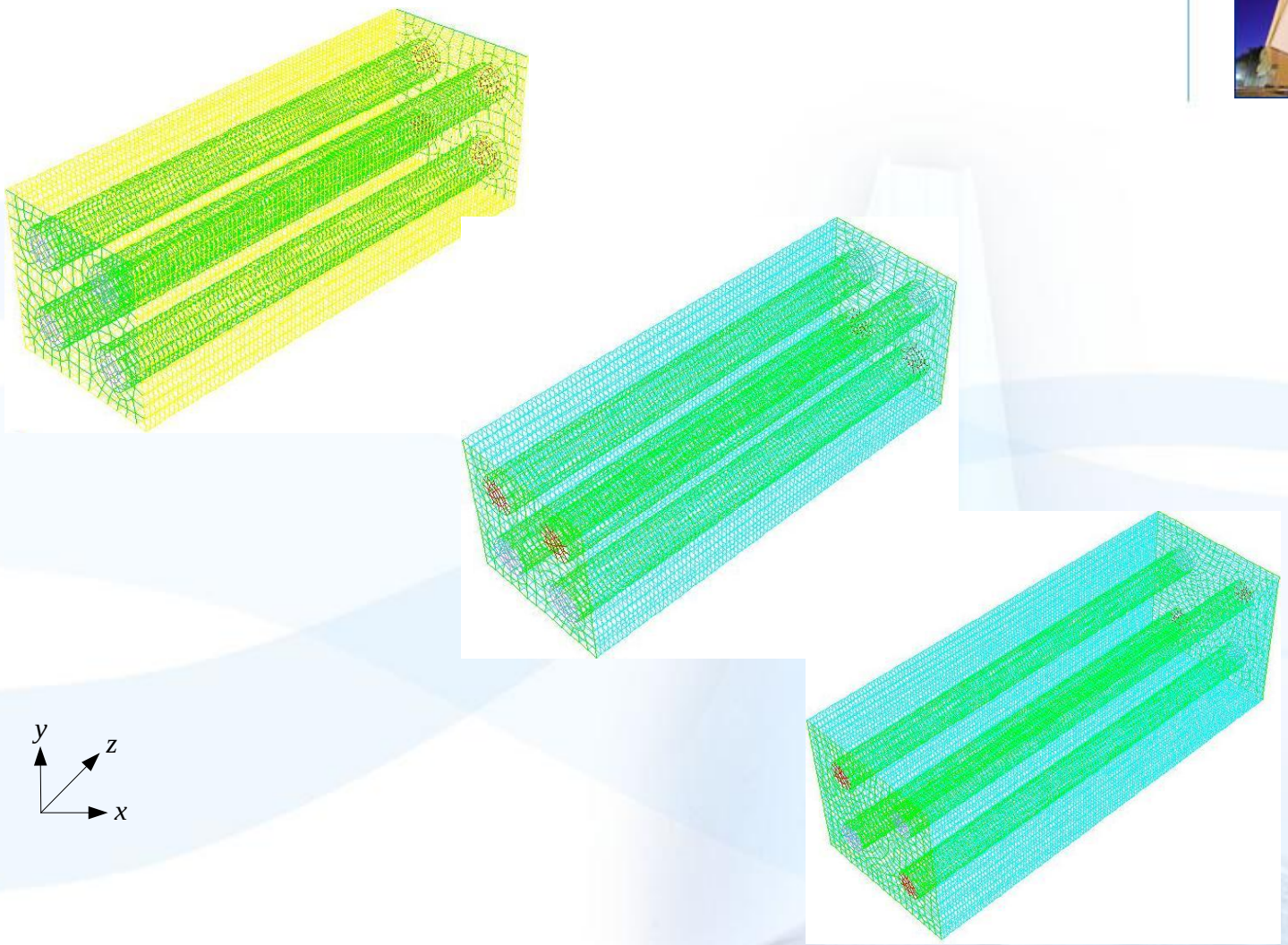


Fig 32. The discretised 3-D computational domain

Numerical Modelling/Analysis

Governing Equations as in Part 1

Boundary Conditions

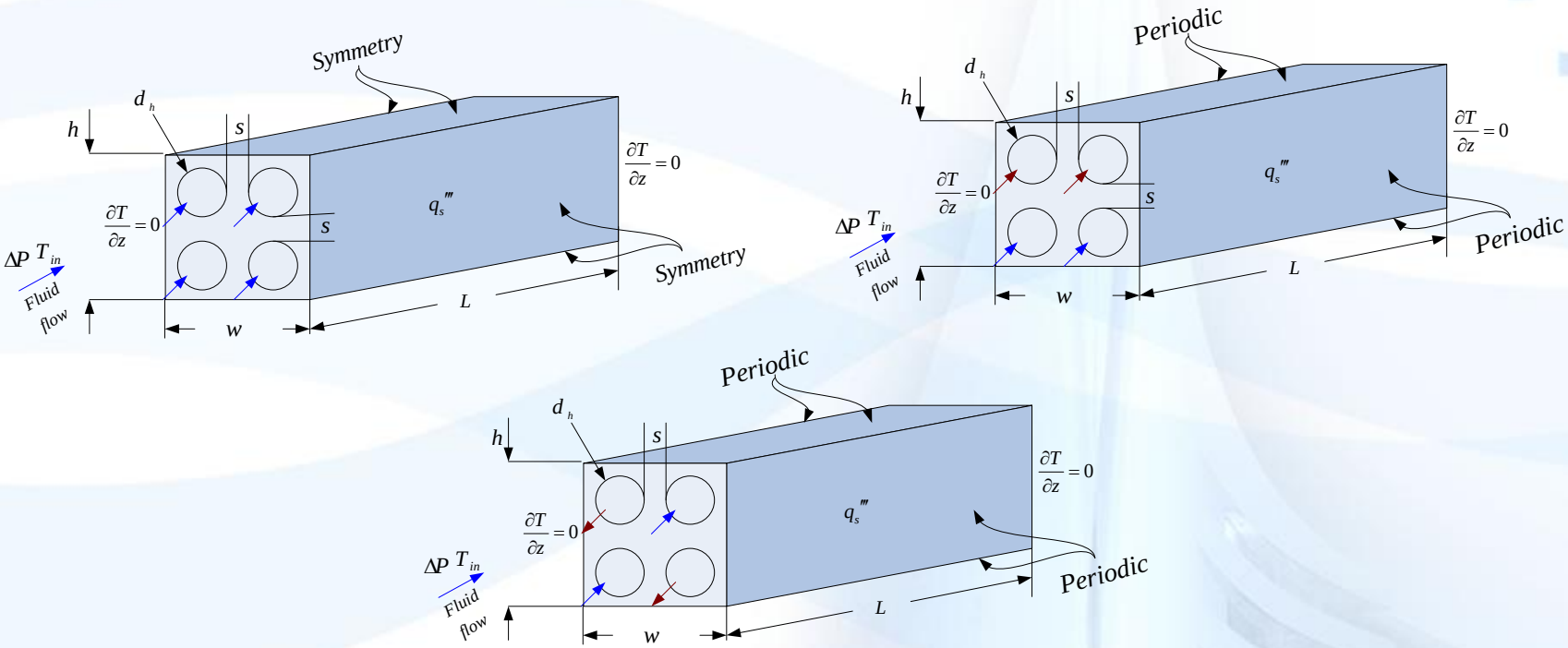


Fig. 33 The boundary conditions of the three dimensional Computational domain of the cooling channel

Optimisation Constraints

An elemental volume constraint is considered to compose of elemental cooling channel of hydraulic diameter

$$v_{el} = w^2 L, \quad h = w, \quad w = d_h + s, \qquad \text{(P3.2)}$$

For a fixed length of the channel, the cross-sectional area of the structure is

$$A_s = HW \qquad \text{(P3.3)}$$

The number of channels in the structure arrangement can be defined as:

$$N = \frac{HW}{hw} \qquad \text{(P3.4)}$$

The void fraction or porosity of the unit structure can be defined as:

$$\phi = 4\frac{v_c}{v_{el}} \qquad v_c = \frac{\pi}{4} d_h^{\,2} L \qquad \text{(P3.5)}$$

The constraint ranges are:

$$0.125\text{mm}^3 \le v_{el} \le 20\text{mm}^3, \quad 0.1 \le \phi \le 0.3, \quad 0 \le w \le L, \quad 0 \le d_h \le w, \quad 0 \le s \le w \qquad \text{(P3.6)}$$

Numerical Results

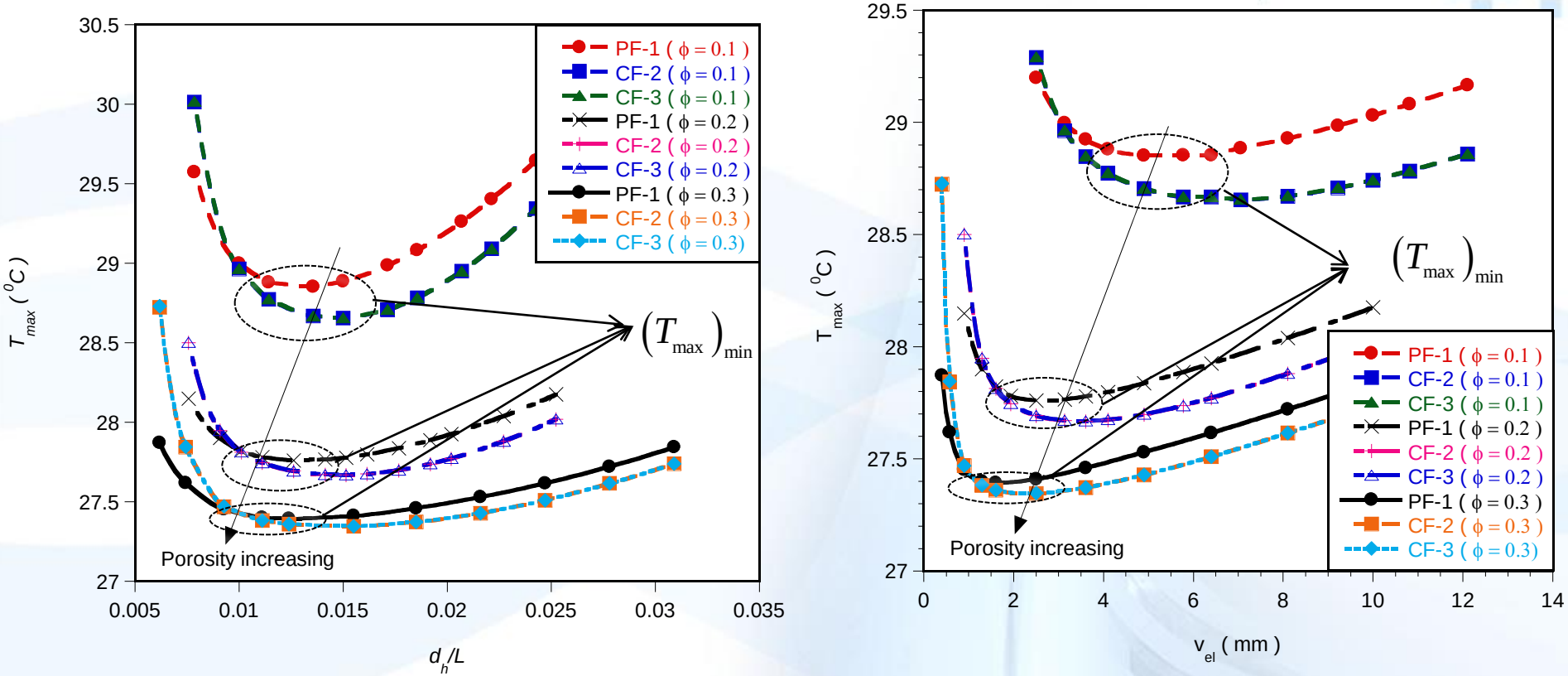


Fig. 34 Effect of optimised dimensionless hydraulic diameter and elemental volume on the peak temperature

Numerical Results

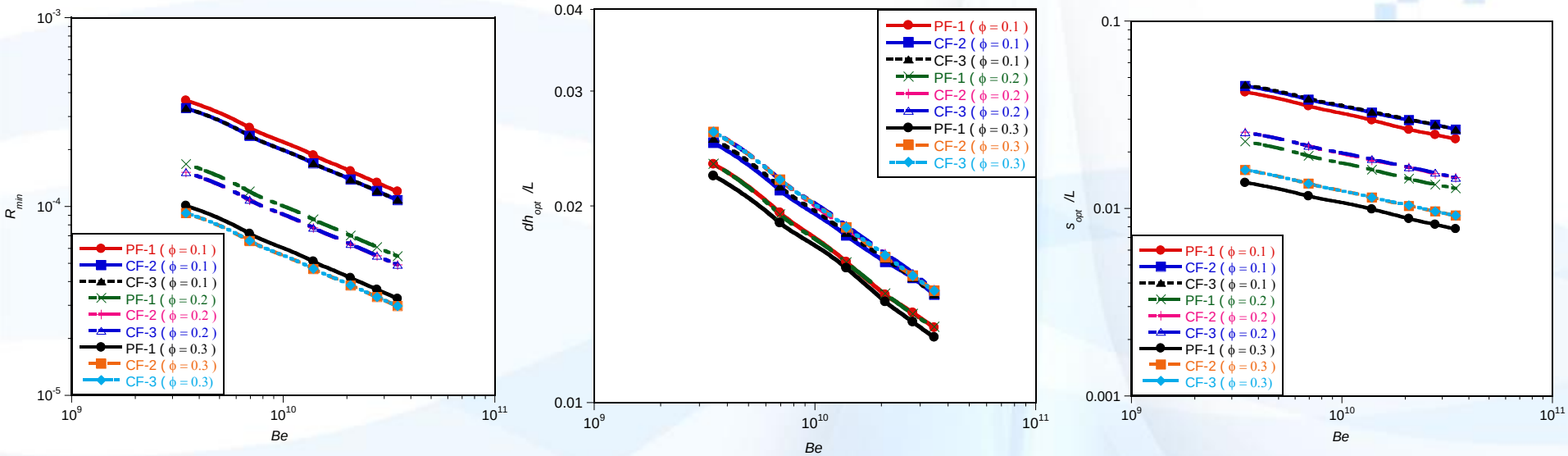


Fig. 35 Effect of dimensionless pressure difference on the dimensionless thermal resistance and the optimised geometries

Conclusion : Part 1

It is all about size and shapes

- Size and shapes significant have effect on the thermal performance of heat-generating devices.
- The global thermal resistance is a function of applied dimensionless pressure difference number (pumping power) and the channel configurations.
- Existence of unique optimal design variables for a given applied dimensionless pressure number for each configuration studied.
- Therefore, thermal designers can pick an optimal solution according to the applied dimensionless pressure difference number (*Be*) available to drive the fluid or thermal resistance required.
- The cooling effect was best achieved at a higher aspect ratio of rectangular channels. The performance of the cylindrical channel was poorer than that of any other channels, it was a more viable option and more often used in industry due to the ease of manufacturability and packaging.
- The optimal channel spacing ratio (s_1/s_2) remains unchanged and insensitive to the performance of the system regardless of the pressure difference number for the two triangular configurations
- The optimal design scheme could well lead to a design that would be impractical at very high channel aspect ratios, due to the channel being too thin to be manufactured

Conclusion : Part 2

- This part studied the numerical optimisation of geometric structures of square cooling channels of vascularised material with the localised self-cooling property subject to heat flux on one side in such a way that the **peak temperature is minimised** at every point in the solid body.

- There is existence of unique optimal design variables (geometries) for a given applied dimensionless pressure number for fixed porosity.

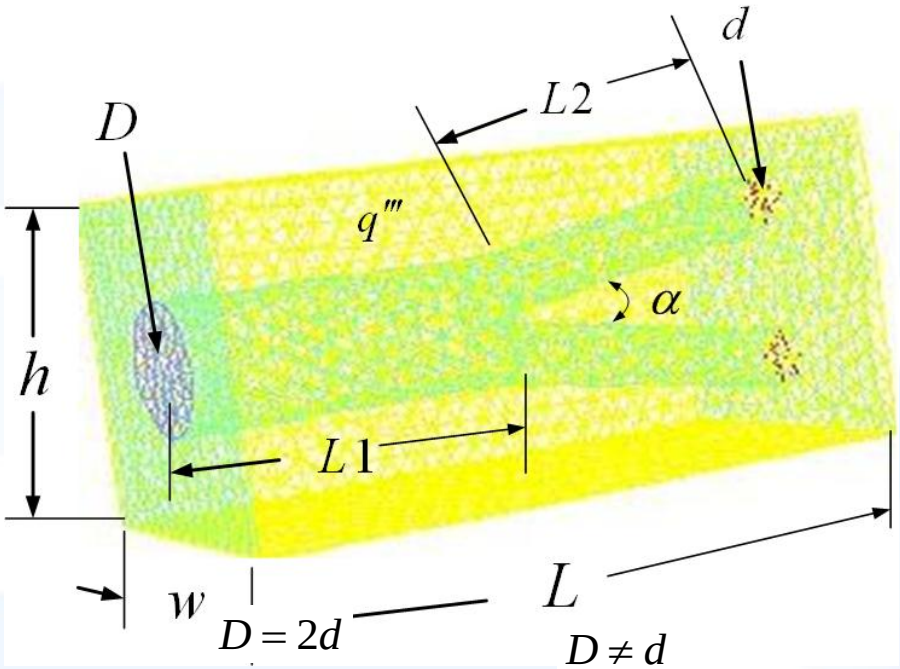
- Minimized thermal resistance decreases with increasing *Kr and Be.* That is the material property and driving force have great influence on the performance of the cooling channel. Therefore, when designing the cooling structure of vascularised material, the internal and external geometries of the structure, material properties and pump power requirements are very important parameters to be considered in achieving efficient optimal designs for the best performance.

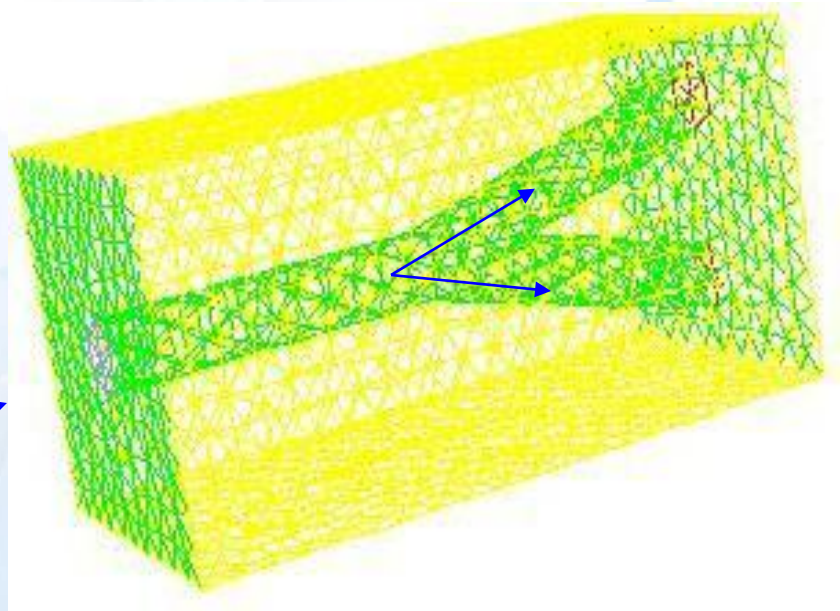
Conclusion : Part 3

- The results also show that the flow orientation has a strong influence on the convective heat transfer.
- For specified applied dimensionless pressure difference and porosity, CF-2 and CF-3 orientations perform better than the PF-1 orientation.
- Therefore, when designing the cooling structure of heat exchange equipment, the internal and external geometries of the structure, flow orientation and the pump power requirements are very important parameters to be considered in achieving efficient and optimal designs for the best performance.
- The thermal designers can pick an optimal solution according to the applied dimensionless pressure difference number (*Be*) available to drive the fluid or thermal resistance required.

Recommendation for Future work

- **Circular Y-shape cooling channel**



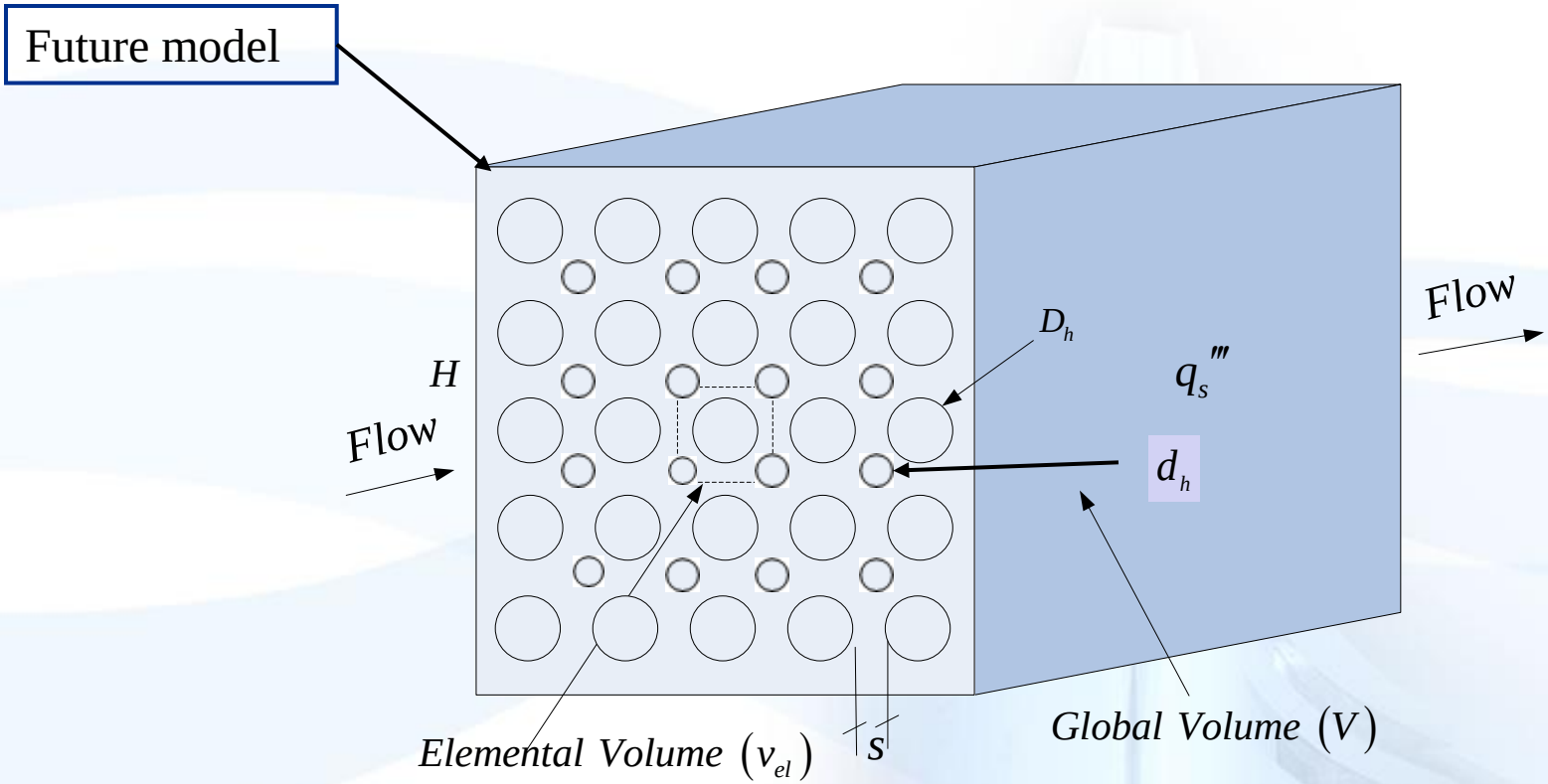


$$\phi = \frac{v_c}{v_{el}}$$

$$R_{\min} = f\left(D_{hopt}, d_{hopt}, L1_{opt}, L2_{opt}, w_{opt}, \phi_{opt}, \alpha_{opt}\right)$$

Recommendation for Future work

- **Multi-scale design of compact cooling channels**



Recommendation for Future work

- Investigation of the effect of pin fins of any shape transversely arranged along the flow channel of the configurations on the temperature distribution and dimensionless pressure difference characteristics with the global objective of minimising thermal resistance and improving thermal performance.

- Turbulent and transient fluid flow.

- Effect of temperature-dependent of the thermo-physical properties of fluid on the minimised thermal resistance.

$$R_{\min} = f(\mu(T), \quad \rho(T), \quad k(T))$$

sensitive to temperature changes due to relatively large variation of working fluid properties at high heat flux and low Reynolds number (*Re*).

List of Publications from the Research

The following articles, book chapter and conference papers were produced during this research.

1. **O.T. Olakoyejo**, T. Bello-Ochende and J.P Meyer, "Mathematical optimisation of laminar forced convection heat transfer through a vascularised solid with square channels", *International Journal of Heat and Mass Transfer*, Vol. 55, pp. 2402-2411, 2012. **(Published)**

2. **O.T. Olakoyejo**, T. Bello-Ochende and J.P Meyer; "Constructal conjugate cooling channels with internal heat generation", *International Journal of Heat and Mass Transfer*, Vol. 55, pp. 4385-4396, 2012. **(Published)**

3. T. Bello-Ochende, **O.T. Olakoyejo** and J.P Meyer, Chapter 11, "Constructal Design of Rectangular Conjugate Channels" **Published in the book, "*Constructal Law and the Unifying Principle of Design*",** L.A.O Rocha, S. Lorente and A. Bejan, eds., pp. 177-194, Springer Publishers, New York, 2012. **(Published)**

4. J.P Meyer**, O.T. Olakoyejo**, and T. Bello-Ochende; "Constructal optimisation of conjugate triangular cooling channels with internal heat generation", *International communication of Heat and Mass Transfer*, Vol. 39, pp. 1093 - 1100, 2012. **(Published)**.

List of Publications from the Research

5. T. Bello-Ochende, **O.T. Olakoyejo**, and J.P Meyer; "Constructal flow orientation in conjugate cooling channels with internal heat generation", *International Journal of Heat and Mass Transfer*, Vol. 57, pp. 241 - 249, 2013. **(Published)**.

6. **O.T. Olakoyejo**, T. Bello-Ochende and J.P Meyer, "Optimisation of circular cooling channels with internal heat generation", *Proceedings of the 7th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics*, Antalya, Turkey, pp. 1345-1350, 19-21 July 2010. **(Presented)**

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List of Publications from the Research

9. **O.T. Olakoyejo**, T. Bello-Ochende and J.P. Meyer, "Constructal optimisation of rectangular conjugate cooling channels for minimum thermal resistance*", Proceedings of the Constructal Law Conference*, 01-02 December, 2011, Porto Alegre, Universidade Federal do Rio Grande do Sul, Brazil. **(Presented)**

10. **O.T. Olakoyejo**, T. Bello-Ochende and J.P Meyer, "Optimisation of conjugate triangular cooling channels with internal heat generation", *9th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics, Malta*, 16 -18 July, 2012. **(Presented)**

9. **O.T. Olakoyejo**, T. Bello-Ochende and J.P Meyer, "Flow orientation in conjugate cooling channels with internal heat generation", *9th International Conference on Heat Transfer, Fluid Mechanics and Thermodynamics, Malta*, July 16 – 18, 2012. **(Presented).**

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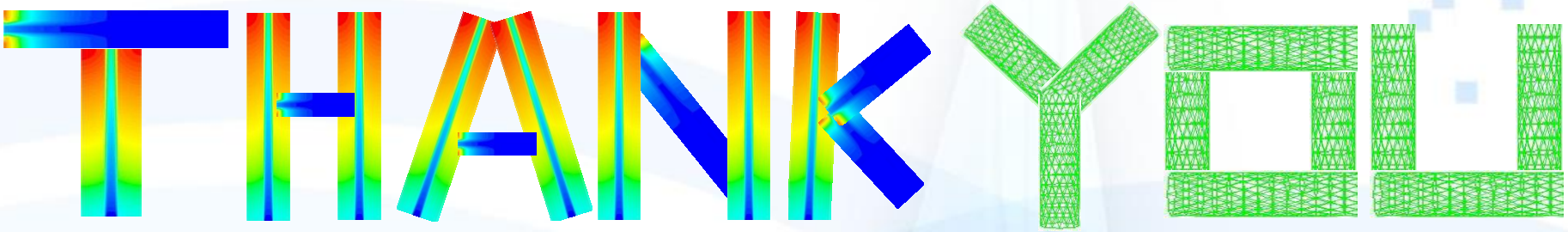
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