First Passage Time & Peak Size Probability Distributions for a Complex Epidemic Model
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The potential of zoonotic diseases to cross species physical boundaries makes them a persistent threat to global public health. Of all the zoonoses, the bubonic plague serves as a historical and modern model. In this work, we investigate a bubonic plague stochastic model using a continuous-time Markov chain (CTMC) model to study the disease dynamics in rats and humans. Using a Galton-Watson multi-type (GWbp) branching process, we have derived an analytical expression for disease extinction probability at the beginning of the epidemic. There is a variation in disease extinction probability calculated via the branching process and numerical simulations, which is the consequence of the discrete assumption of an infected flea instead of being considered in terms of an infected rat in the numerical simulation. An analytical expression for the distribution of first passage time (FPT) to spillover is also obtained in this work using the probability generating function (PGF) technique, and it agrees well with the FPT distribution obtained numerically. Furthermore, the effect of the rat population on the FPT of spillover is also shown, which suggests that as the rat population KR increases, the likelihood of spillover within time t decreases, and it increases as KR decreases. Additionally, we have derived an expression for the mean and variance of the first passage time to spillover, and we incorporate the impact of various parameters on the mean first passage time. Lastly, we perform numerical simulation to estimate the peak size of each infected class and the related time to attain peak infection.

Keywords: Zoonosis; Bubonic plague; Stochastic model; Reducible multi-type branching process; FPT to spillover; Peak size and peak time distribution

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