An overview of hyperbolic PDEs models on networks

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In this talk we discuss some models in the form of dynamical systems on networks and their relevance in natural sciences. In this context, a network is represented by a metric graph, i.e., a graph with edges identified with intervals. This additional structure allows to consider dynamics occuring along the edges, while the interaction between particular edges is described by suitable transmission condition in the vertices. We will focus mainly on hyperbolic systems, i.e., PDEs of first order both in space and time. Such models describe, for example, blood flow [3], synapses [4] and population dynamics [2]. We will finish with the recent results of A.B. and J. Banasiak [1].

Keywords: hyperbolic systems, networks, semigroups of operators

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