SEISMICITY OF A MECHANICAL MODEL OF EARTHQUAKE: LESSONS FROM ANHARMONIC ELASTIC FORCES

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OUTLINES

The group

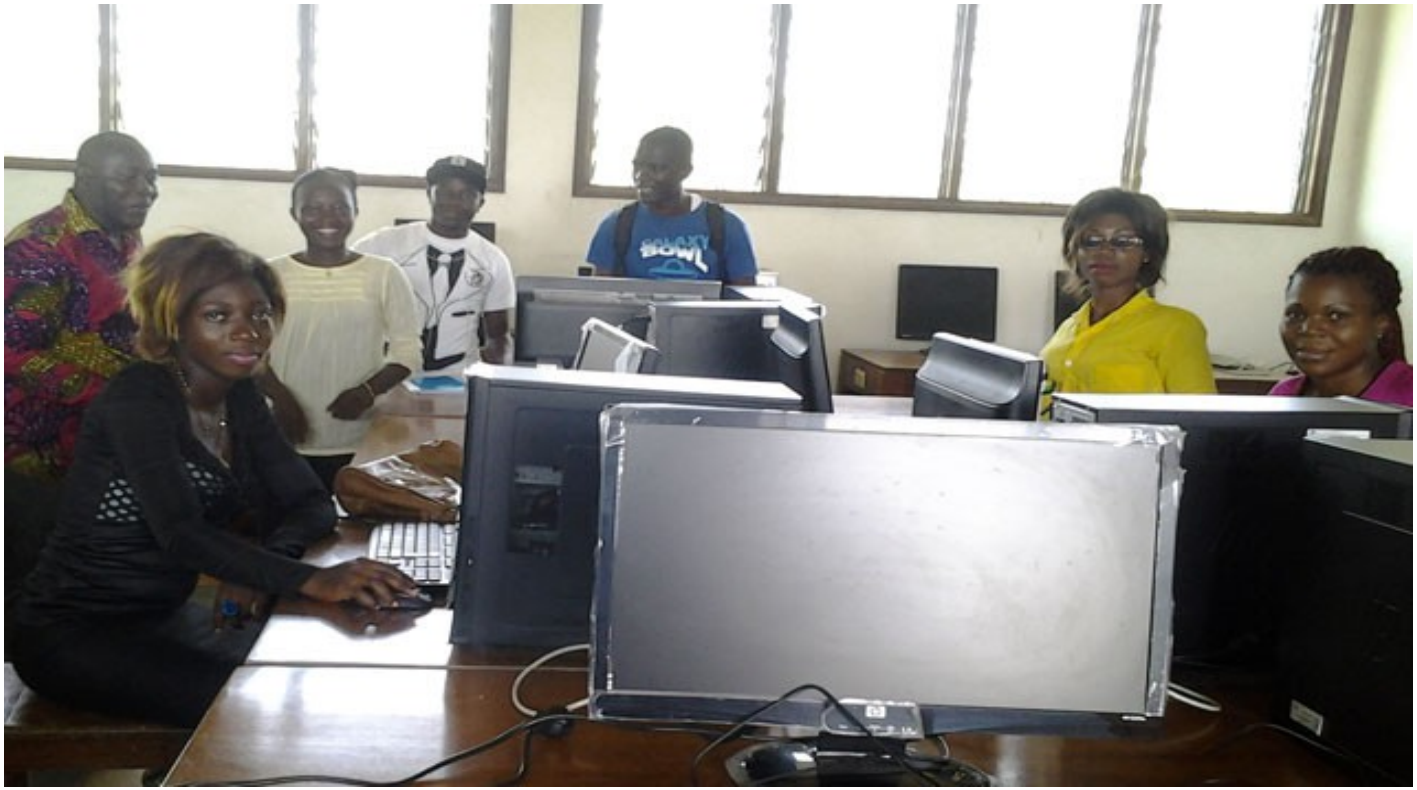
LaRAMaNS is a research team in the Physics Department of the University of Buea. Our current activities focus on Condensed Matter Physics and Nonlinear Sciences, with a wide-range interest in these fields.

Current members of the team include:

- 8 senior members (Pr. Moukam Kakmeni, Dr. Mkam Tchouobiap, Dr. Atoneche Fred, etc.)
- 25 postgraduate students (17 M.Sc. students and 8 Ph.D. students)
- Group's blog: http://laramans.blogspot.com/

The group

Some pictures .....



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Research in LaRAMaNS is theoretical (mainly modelling), with a strong component of numerical simulations. In this respect, the group has a fairly equiped simulation lab (with a 6-Rack HP proliant server + several pcs), from grant of the Alexander von Humboldt (AvH) Foundation (20,000 euros).

The group

Some recent inputs from the team:

- A. N. Nkongho Ayuketang, Alain M. Dikandé and J. Ebobenow: Nonlinear multi-frequency dynamics of wind turbine components with a single-mesh helical gear train, Mathematical and computational applications vol. 23, 12 (2018).
- E. Nji Nde Aboringong and Alain M. Dikandé: Exciton dynamics in amide-I α-helix protein chains with long-range intermolecular interactions, in press in European Physical Journal e (EPJ E, Springer)

The group

- R. D. Dikandé Bitha and Alain M. Dikandé: Elliptic-type soliton combs in optical ring microresonators, Physical Review A 97, 033813, 2018.
- G. Fongang Achu, F. M. Moukam Kakmeni and A. M. Dikandé: Breathing pulses in the damped model for nerves, Physical Review E 97, 012211, 2018.
- R. D. Dikandé Bitha, D. S. Mbieda and Alain M. Dikandé: Trapping and reshaping of low-intensity radiations by soliton trains in gas-filled hollow-core photonic crystal fibers, arXiv:1712.06350, 2017.

The group

- Alain M. Dikandé, J. Voma and B. Z. Essimbi: Continuous-wave to pulse regimes for a family of passively mode-locked lasers with saturable nonlinearity, Journal of Optics (iop) 19, 105504, 2017.
- D. Welakuh Mbangheku and Alain M. Dikandé: Storage and retrieval of time-entangled soliton trains in a three-level atom system coupled to an optical cavity, Optics Communications 403, 27, 2017.

The group

- Defi Jr. Jubgang Fandio and Alain M. Dikandé: Pulse train uniformity and nonlinear dynamics of soliton crystals in mode-locked fiber ring lasers, Journal of the Optical Society of America (JOSA) B34, 2721, 2017.
- Duplex S. Mbieda Petmegni, Alain M. Dikandé and B. Z. Essimbi: Raman self-induced-transparency soliton trains in hollow-core photonic crystals, Applied Physics (Springer) B123, 171, 2017. etc....

What is Earthquake? simple picture

Earthquakes are usually triggered by a sudden slip on a fault, much like what happens when we snap our fingers. Before the snap we push the fingers together and sideways, and because we are pushing them together, friction keeps them from moving to the side. When we push sideways hard enough to overcome the friction, the fingers move suddenly causing the release of energy in the form of sound waves that travels from the hand to our ears.

What is Earthquake? simple picture

An Earthquake occurs the same way: stresses from inside the Earth crust push the two sides of the fault. The friction across the surface of the fault holds the rocks together so they do not slip immediately when pushed sideways. When enough stress has built up within the fault, the rocks slip suddenly thus releasing energy in form of waves travelling through the rocks to cause the shaking felt during Earthquakes.

What is Earthquake? simple picture

Earthquakes happen over an area of the fault called the rupture surface, just as we snap our fingers with the whole area of our fingertip and thumb. However, unlike our fingers, the whole fault plane does not slip at once. The rupture begins at a point on the fault plane called the hypocenter, a point usually deep down on the fault. The epicenter is the point on the surface directly above the hypocenter. The rupture keeps spreading until all the accumulated stress is released.

EARTHQUAKE FAULTS

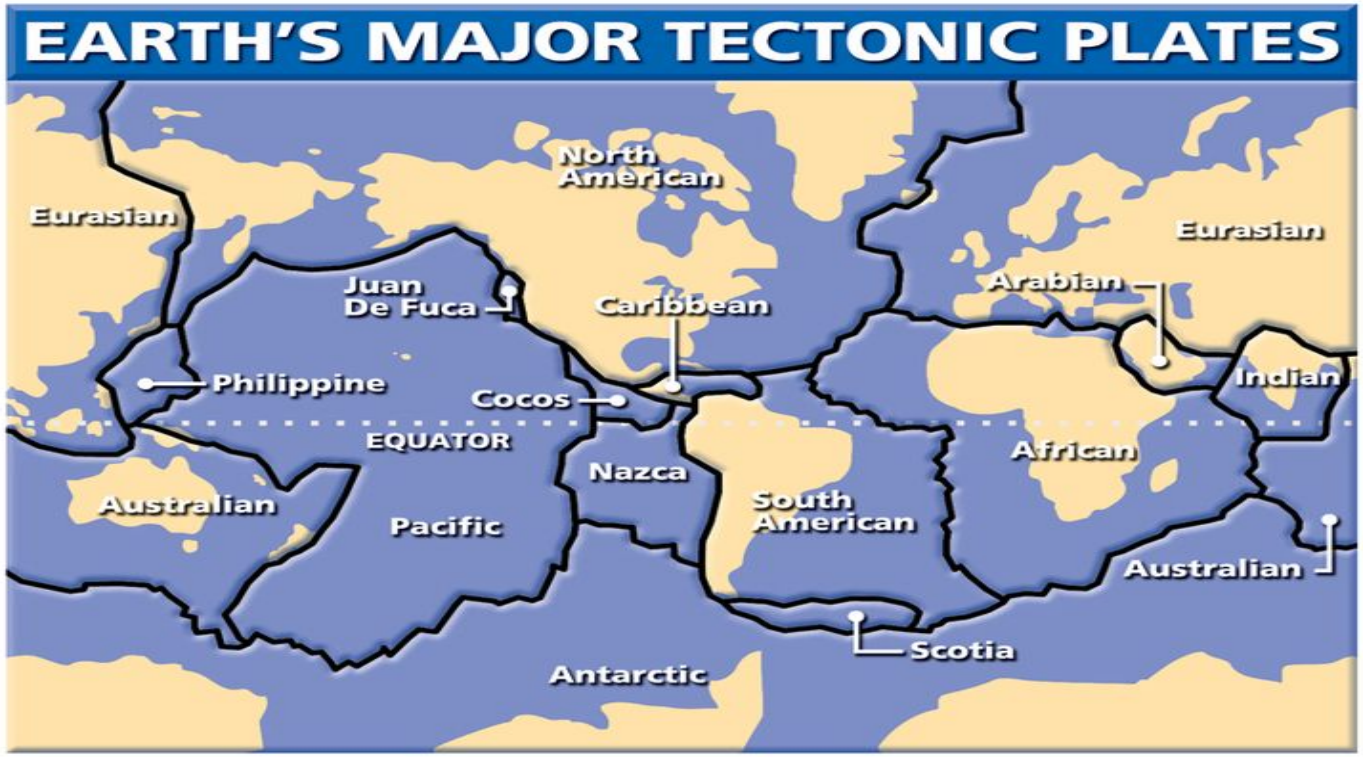
A fault (or fracture on the outer Earth crust) is a thin zone of crushed rock separating blocks of the earth's crust. When an Earthquake occurs on one of these faults, the rock on one side of the fault slips with respect to the other. Faults can be centimeters to thousands of kilometers long. The fault surface can be vertical, horizontal, or at some angle to the surface of the Earth. Faults can extend deep into the Earth and may or may not extend up to the Earth's surface.

EARTHQUAKE FAULTS

Seismic faults: a low-scale (minor) fault...

EARTHQUAKE FAULTS

Earth's major Tectonic plates separated by faults:



PHYSICAL MODEL OF EARTHQUAKE

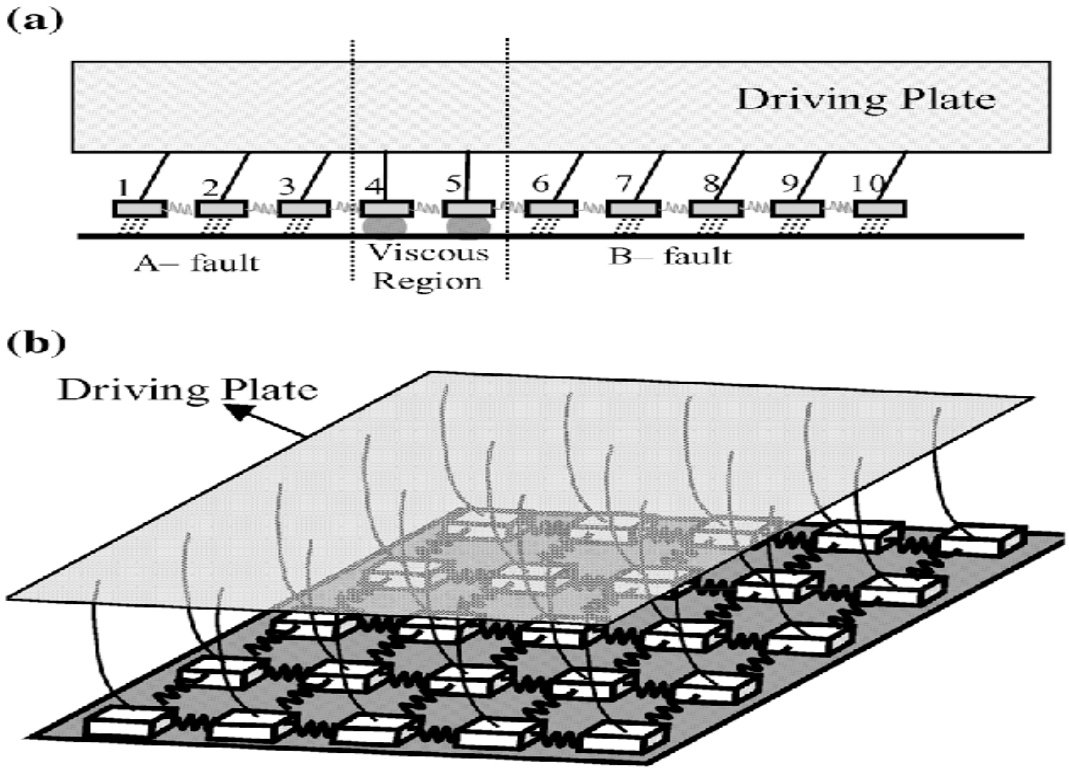
The assumption that Earthquakes are governed mainly by a competition between the stress and friction, leads to the so-called "stick-slip" picture proposed by Burridge and Knopoff [1]. In this picture one considers the fault zone (i.e. the two sides of the fracture) as the contact plane between two tectonic plates [1, 2, 3].

PHYSICAL MODEL OF EARTHQUAKE

The two sides (faces) of the fault consist of rocks, represented as regularly ordered blocks of finite masses, interconnected by springs and interacting with blocks of the other side also via springs.

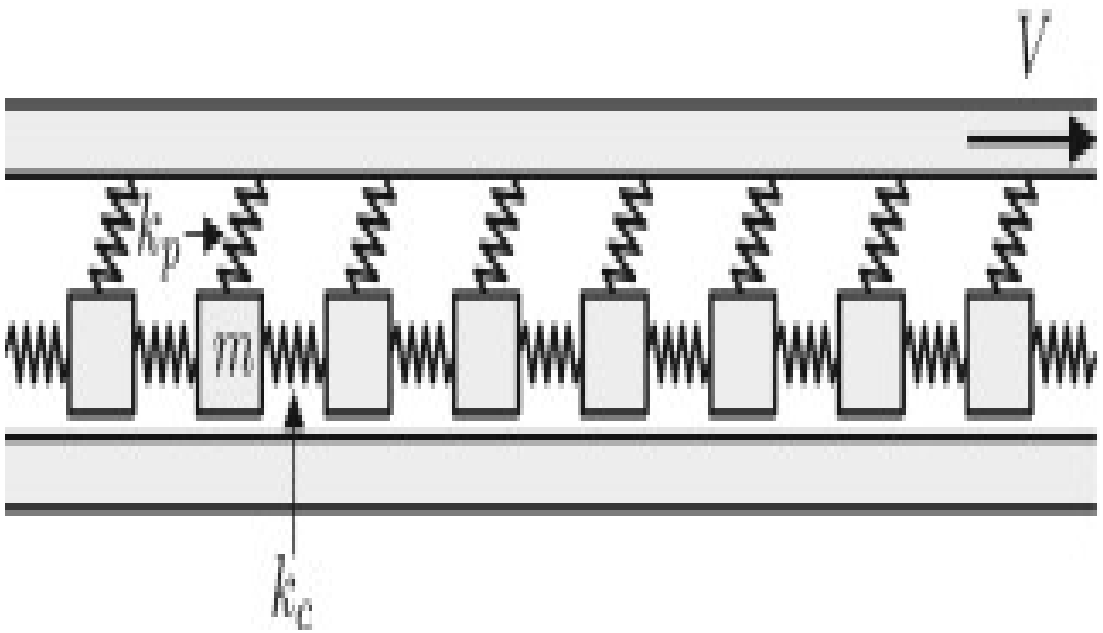
EARTHQUAKE OCCURENCE: MECHANICAL ASPECTS

2D Mechanical model of Earthquake fault:



EARTHQUAKE OCCURENCE: MECHANICAL ASPECTS

Further simplification leads to the following 1D Mechanical model, so-called Burridge-Knopoff model:



MATHEMATICAL MODEL

Equations of motion for block number i:

$$\begin{aligned} mX_{itt} &= -k_c(X_i - X_{i+1}) - k_c(X_i - X_{i-1}) - k_p(X_i - vt) - F(\dot{X}_i) \\ &= k_c(X_{i+1} - 2X_i + X_{i-1}) - k_p(X_i - vt) - F(\dot{X}_i), \end{aligned} \tag{1}$$

where X_i is the relative displacement of the *ith* block with respect to its equilibrium, and v is the velocity of uniform displacement of one plate respective to the other.
We can define a new discrete displacement variable say $U_i = X_i - vt$, such that the above equation can be rewritten:

$$mU_{itt} = k_c(U_{i+1} - 2U_i + U_{i-1}) - k_p U_i - F(\dot{U}_i + v). \tag{2}$$

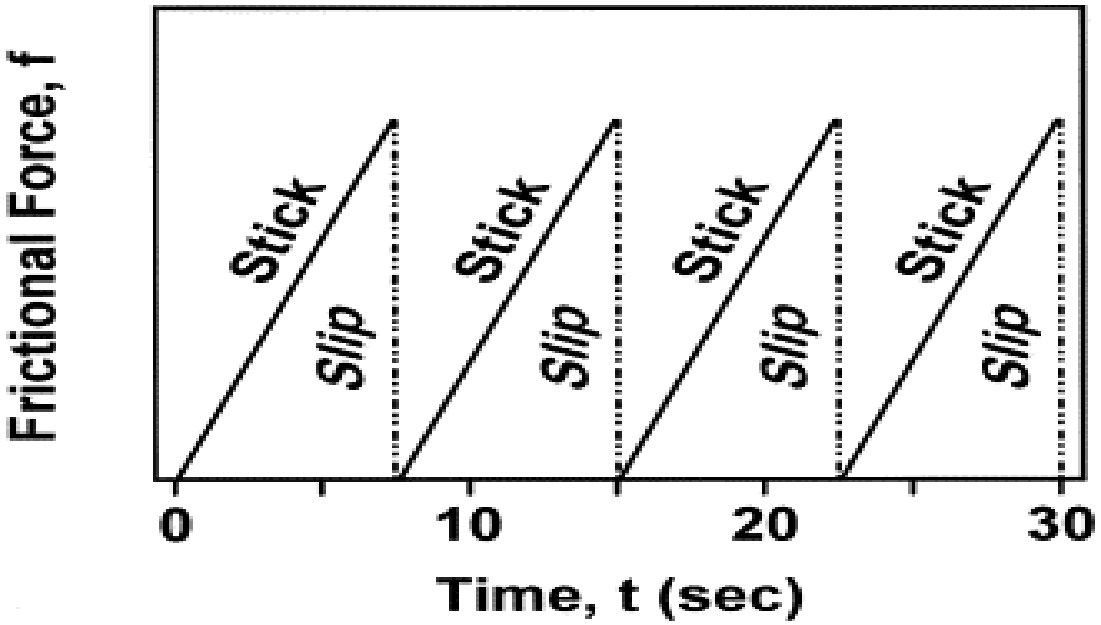
MATHEMATICAL MODEL

What is peculiar to the BK model:

1. the springs are of Hooke's type, hence elastic-only motions are harmonic (i.e. sinusoidal) oscillations.
2. As the stress accumulates, the friction force $F(\dot{U}_i)$ increases to resist the block displacements (stick phase). The friction reaches a finite threshold and then starts decreasing, thus favoring the release of stress (slip phase) consequent upon the slipping of the plates one respective to the other.
   From what precedes, it is very likely that the frictional force $F(\dot{U}_i)$ should be nonlinear.

MATHEMATICAL MODEL

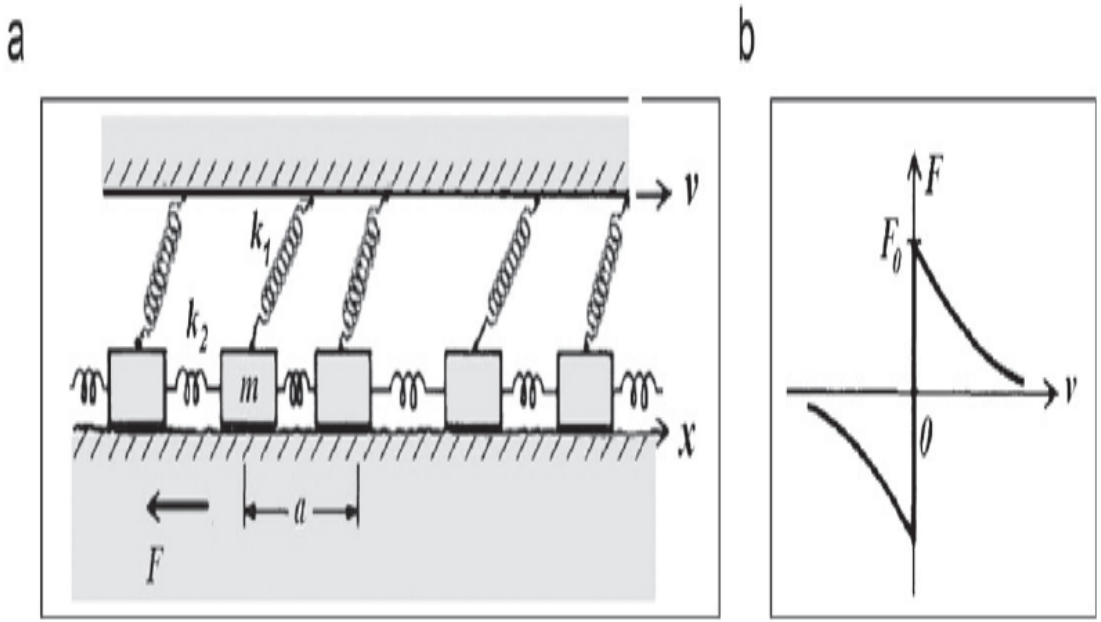
Velocity-weakening frictional force in the BK model: time series



Time variation of the frictional force: stick and slip phases.

MATHEMATICAL MODEL

Velocity-weakening frictional force in the BK model: force versus velocity.



A LITTLE ABOUT LINEAR STABILITY...

Note that the frictional force in the BK model is defined as [2, 3]:

$$F(y) = \frac{f_0 sgn(y)}{1 + \alpha|y|}, \tag{3}$$

where the constant α is inversely proportional to v. The system stability is governed by the nonlinear frictional force (3). The friction exhibits a velocity-weakening force, i.e. there exists a range of velocities for which $dF(\dot{U}_i + v)/dU_i < 0$. In this range of velocities, a linear stability analysis [3] shows that all Fourier modes will grow exponentially at finite rate, so a small disturbance from equilibrium will be magnified during slipping.

MOMENT AND MAGNITUDE OF AN EVENT

Given a nonuniform initial condition, the system is expected to reach a statistically steady state after a few cycle of "displacive" (or large) events.
How do we define an event?
In general an event is associated with a collective (not necessarily homogeneous) slipping motion of a connected set of blocks. The total moment M of an event is by definition, the sum of relative displacements $\delta U_i = U_{i+1} - U_i$ of all the blocks i.e. [2, 3]:

$$M = \sum_{i=1} \delta U_i \tag{4}$$

THEORETICAL IMPLICATIONS...

From the above it appears clearly that having U_i provides key infomation about the Earthquake event. We will therefore focus on solving eq. (2).

NONLINEAR CONTINUUM EQUATION

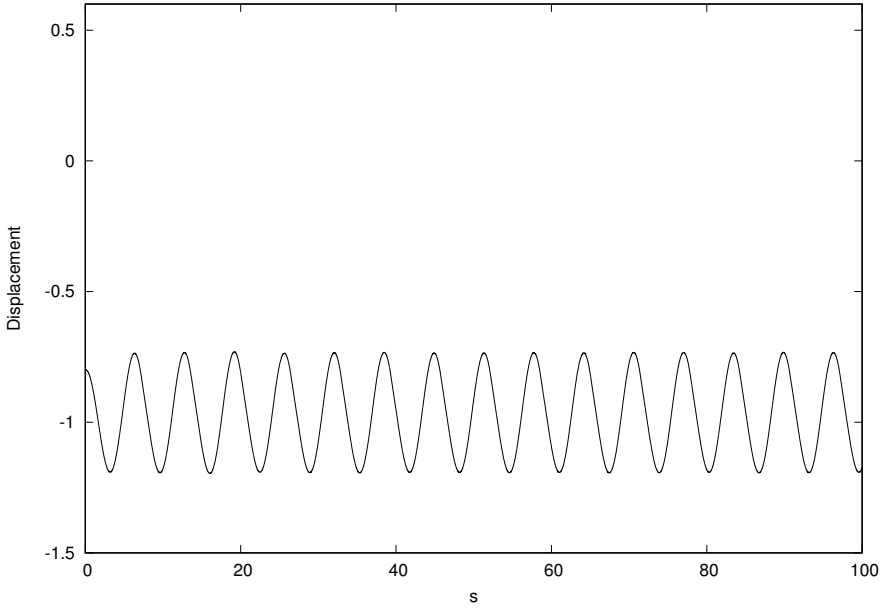
To solve eq. (2), we assume waves are of long wavelength such that the whole fault zone can be regarded as a continuous medium. With this assumption eq. (2) becomes:

$$U_{ss} = \omega_0^2 U + \frac{1}{m} F(U_s + v), \tag{5}$$

with $s = (ia - \vartheta t)\gamma$, $\gamma^{-2} = \vartheta_0^2(1 - \vartheta_0^2/\vartheta^2)$, $\vartheta_0^2 = k_c a^2/m$. Also $\omega_0^2 = k_p/m$ (S-wave characteristic frequency).

SOME DISPLACEMENT AND VELOCITY PROFILES

Displacement profile



Velocity profile

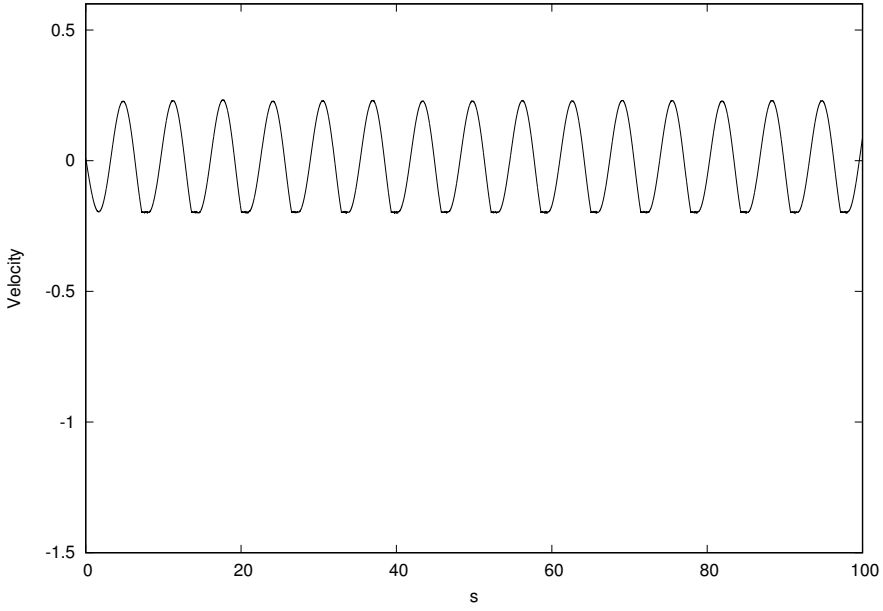


Fig.3: Profiles of U and U_s versus s for $\alpha = 0.05$, $\omega_0^2 \equiv 1$.

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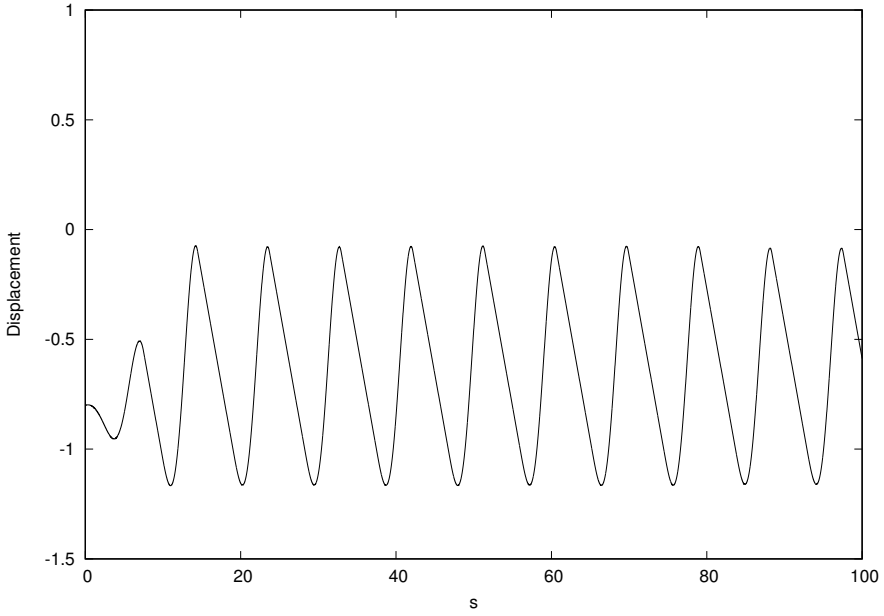
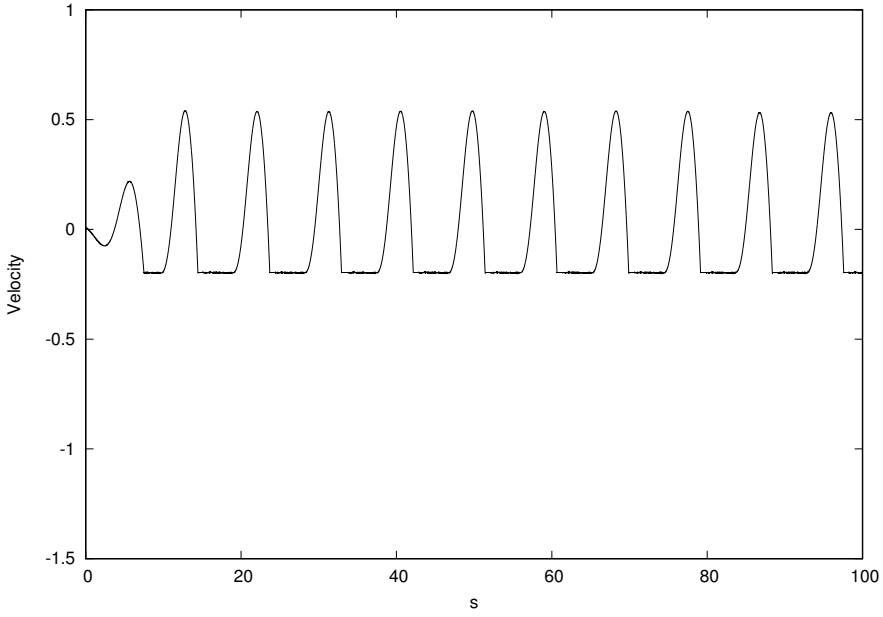


Velocity profile



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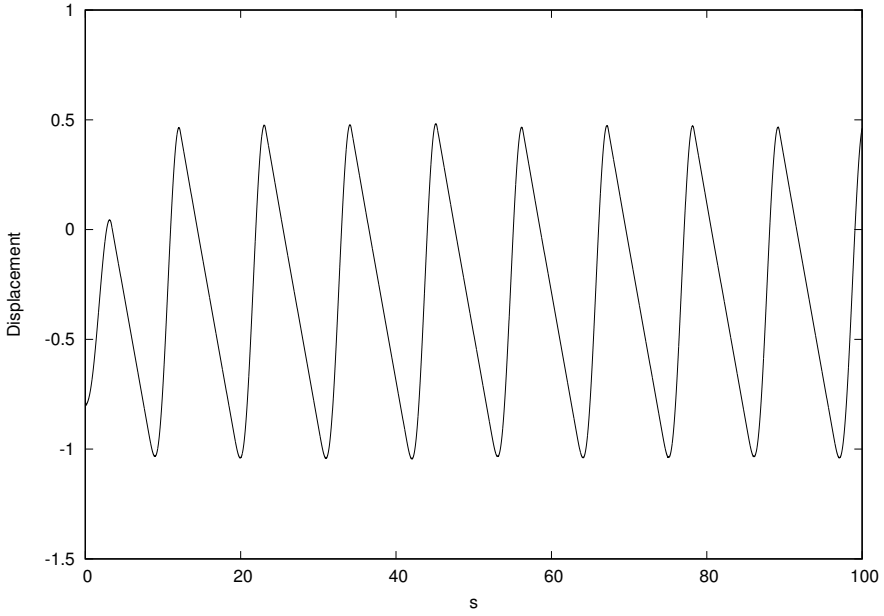


Velocity profile

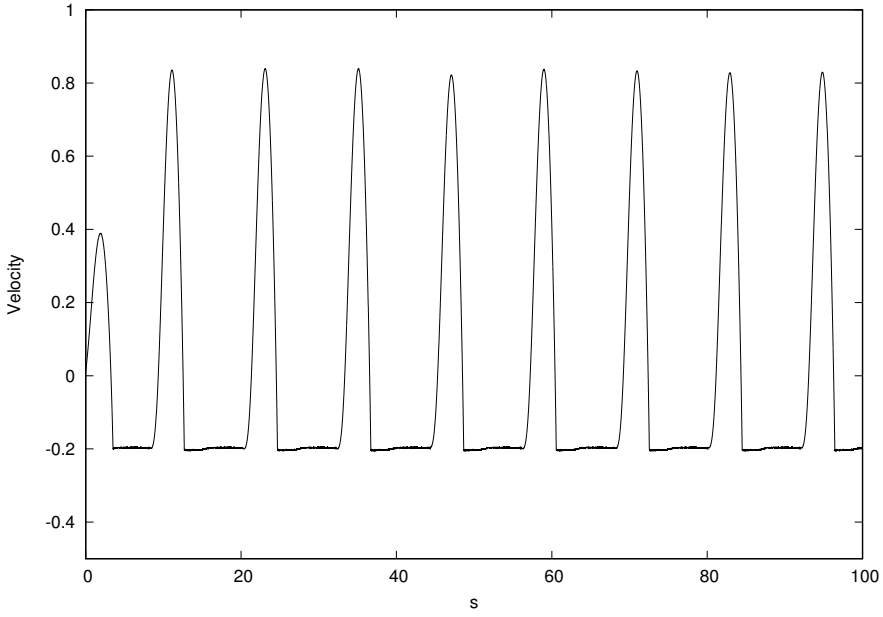


Fig.3: Profiles of U and U_s for $\alpha = 1.5$, $\omega_0^2 \equiv 1$.

WHY ANHARMONIC ELASTIC FORCE?

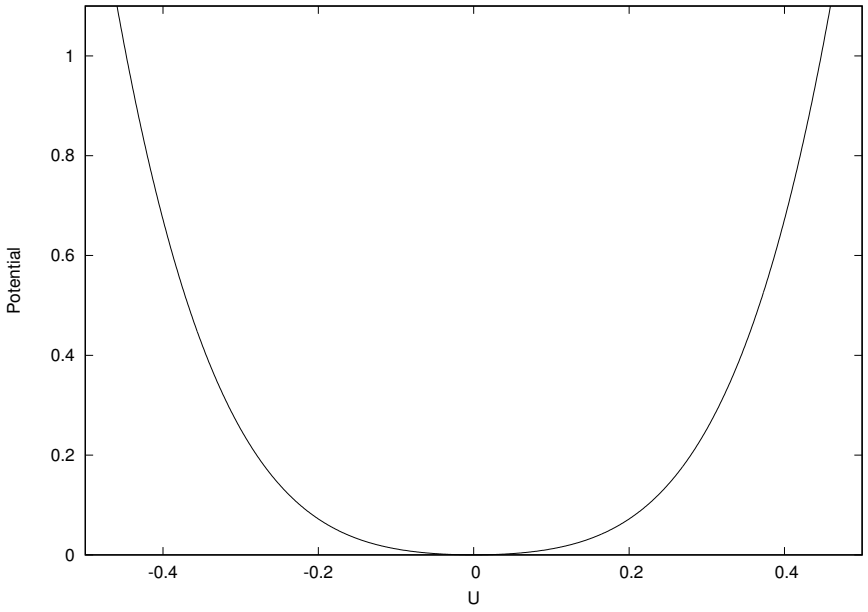
Harmonic force between tectonic plates means in the absence of friction, the plates oscillate "sinuzoidally" one with respect to the other. Such harmonic motions would ve valid regardless of the amplitude of block displacements.
On the contrary, anharmonic force means a stronger interaction between plates. Moreover, with anharmonic force harmonic oscillations will be confined at very small block displaments. Here we shall consider a quartic anharmonic potential providing the anharmonic force i.e.:

$$V(U_i) = \frac{k_c}{2} U_i^2 + \frac{\beta}{4} U_i^4, \tag{6}$$

Symmetry requirements imply that $V(U_i) = V(-U_i)$.

THE ANHARMONIC POTENTIAL AND FORCE

Anharmonic potential



Anharmonic force

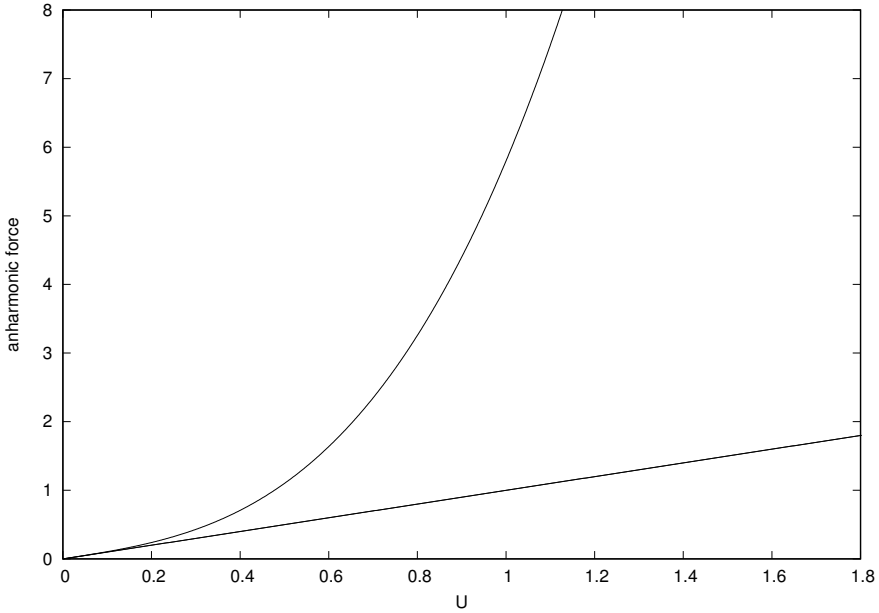


Fig.3: Profiles of U and U_s for $\alpha = 1.5$, $\omega_0^2 \equiv 1$.

DYNAMICS OF BK MODEL WITH ANHARMONIC ELASTIC FORCES

With the anharmonic force, the discrete equation (2) becomes:

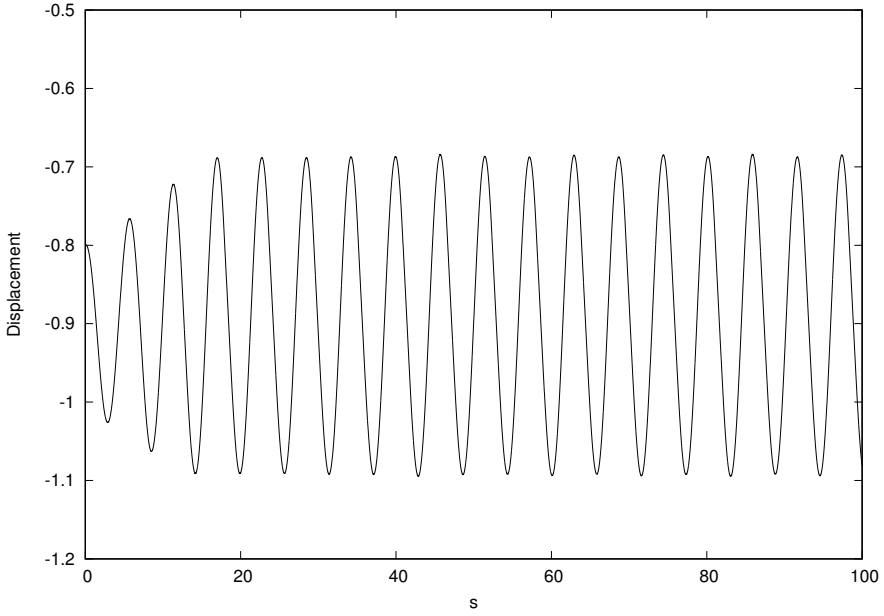
$$mU_{itt} = k_c(U_{i+1} - 2U_i + U_{i-1}) - k_p(1 + \frac{\beta}{k_c}U_i^2)U_i - F(\dot{U}_i + v). \quad (7)$$

The above equation can be seen as the one of a discrete set of harmonic oscillators, with a "displacement-dependent" stiffness $k_2(1 + \beta U_i^2)$. In the continuum limit ($\beta_0 = \beta/\omega_0^2$):

$$U_{ss} = \omega_0^2(1 + \beta_0 U^2)U + \frac{1}{m}F(U_s + v). \quad (8)$$

SOME DISPLACEMENT AND VELOCITY PROFILES

Displacement profile



Velocity profile

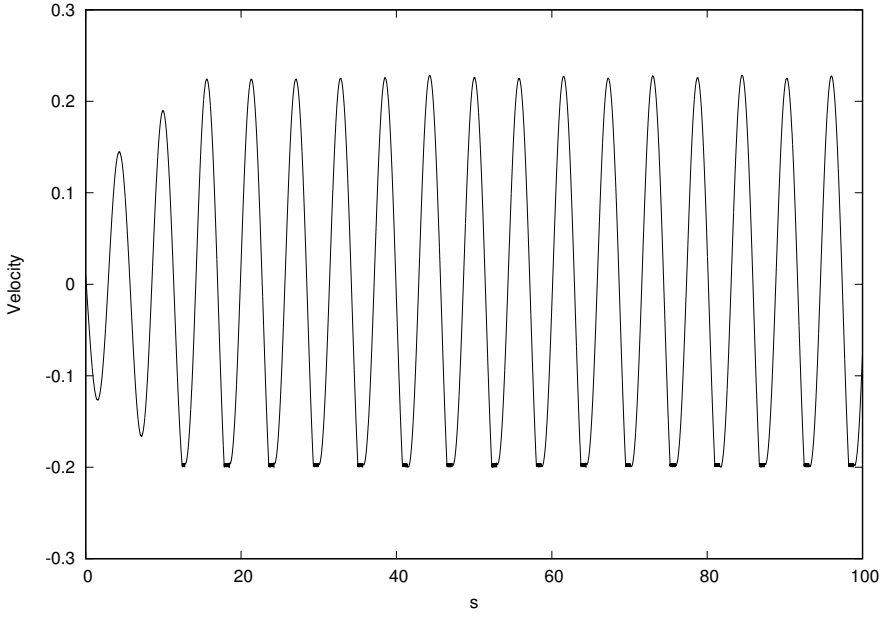


Fig.3: Profiles of U and U_s for $\alpha = 0.05$, $\beta_0 = 0.1$, $\omega_0^2 \equiv 1$.

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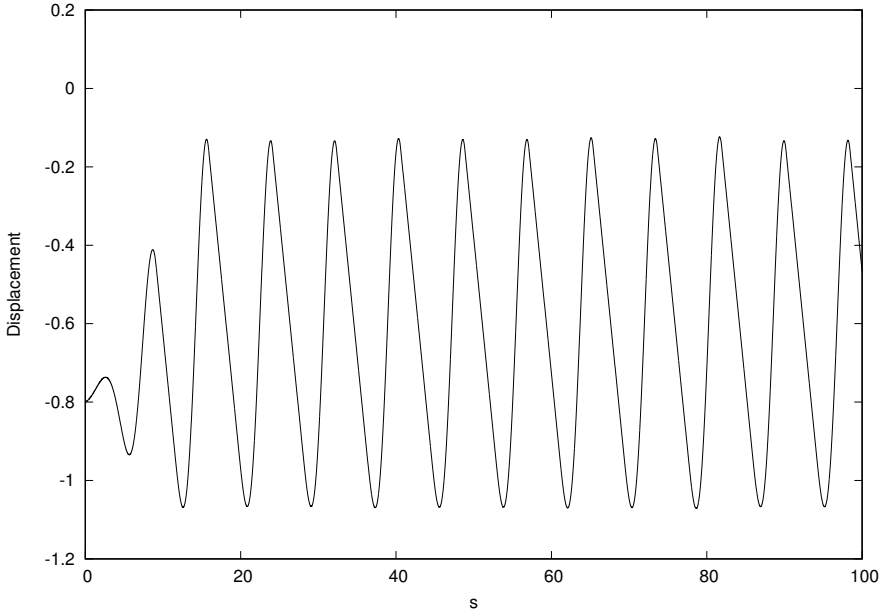
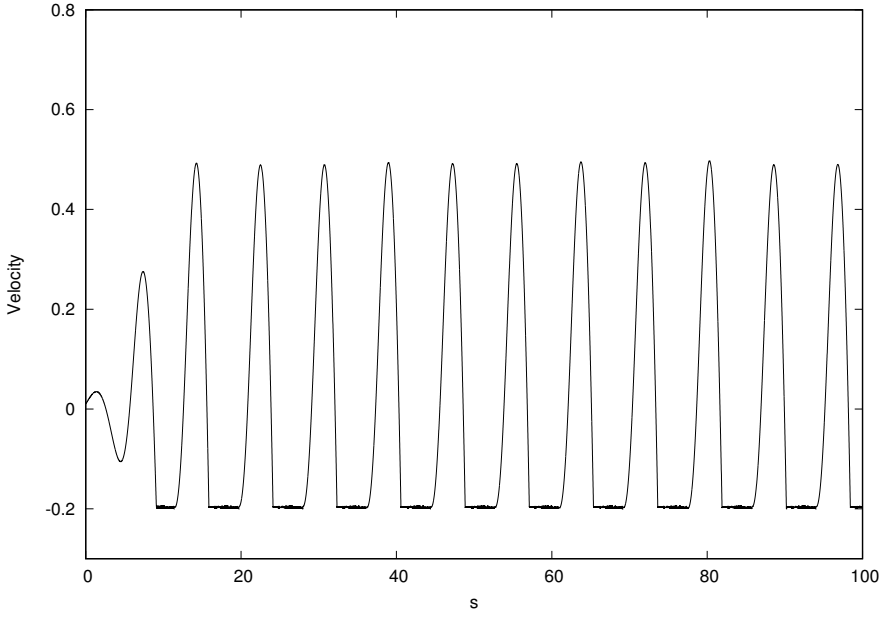


Velocity profile



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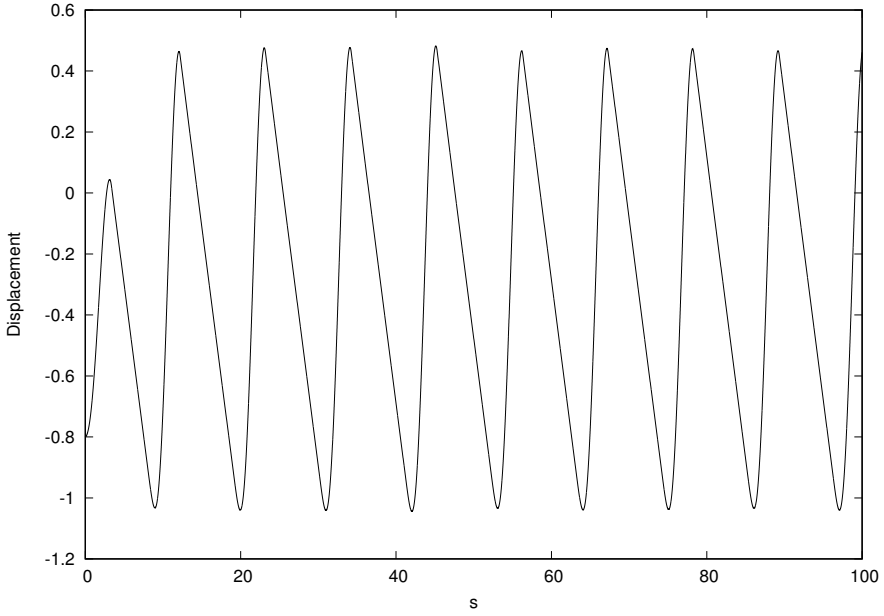

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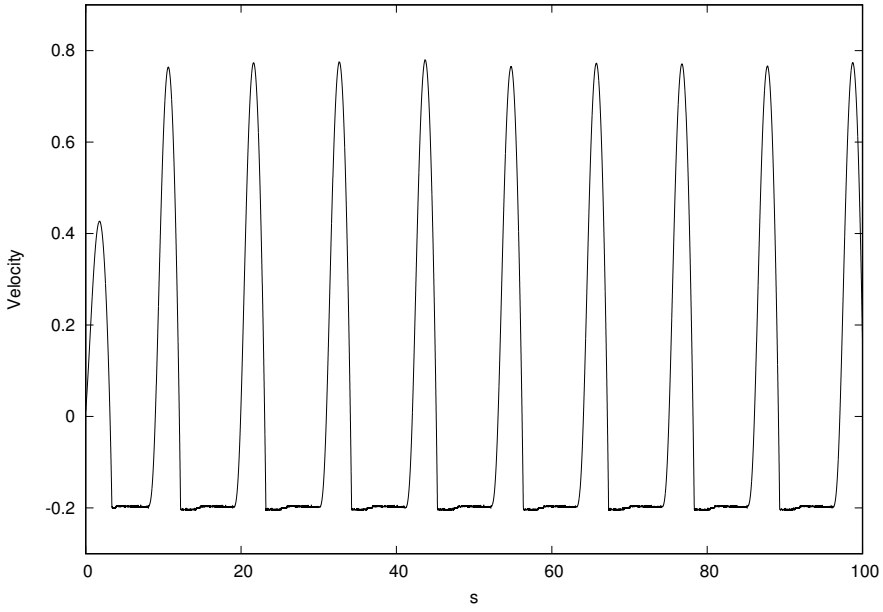

Fig.3: Profiles of U and U_s for $\alpha = 1.5$, $\beta_0 = 0.1$, $\omega_0^2 \equiv 1$.

MAIN LESSONS FROM ANHARMONICITY

- Anharmonic inter-plate interactions give rise to smaller amplitudes of block displacements and velocities, due to a stronger elastic stress resisting the friction.
- The displacement and velocity profiles are always anharmonic, even for small values of the anharmonicity constant β. Therefore, only for very strong friction a rupture between the plates can induce their slip and hence an Earthquake.

MAIN LESSONS FROM ANHARMONICITY

- When the frictional force is zero the anharmonic elastic force will govern the system dynamics. Namely it prevents blocks from oscillating harmonically, instead the blocks will "rattle" as a consequence of a relatively broad bottom of the anharmonic potential in which they can perform large excursion without leaving their equilibrium states.
- From the standpoint of fundamental physics, the consideration of an anharmonic force between tectonic plates enriches the system dynamics with new displacement and velocity patterns with soliton features, including kinks, pulses, and so on.

MAIN LESSONS FROM ANHARMONICITY

- For (arbitrarily) very small values of α, the "anharmonic" BK model can be readily approximated by a discrete set of coupled Duffing oscillators with a constant drive i.e.:

$$mU_{itt} = k_c(U_{i+1} - 2U_i + U_{i-1}) - k_p(1 + \frac{\beta}{k_p} U_i^2) U_i + \alpha_0 |\dot{U}_i| - F(v), \tag{9}$$

where $\alpha_0 = \alpha f_0$.

Some references

[1] R. Burridge and L. Knopoff: "Model and theoretical Seismicity", Bull. Seismol. Soc. Am. vol. 57, p. 341 (1967).

[2] J. M. Carlson and J. S. Langer: "Properties of Earthquakes generated by fault dynamics", Phys. Rev. Lett. vol. 62, p. 2632 (1989).

[3] J. M. Carlson and J. S. Langer: "Mechanical model of an Earthquake fault", Phys. Rev. A vol. 40, p. 6470 (1989).

[4] B. Gutenberg and C. F. Richter: "Sismicity of Earth and related phenomena" (Princeton University Press, 1954)

Some references

[1] F. C. Davison and C. H. Scholz: "Frequency-moment distribution of Earthquakes in the aleutian arc: a test of the characteristic Earthquake model", Bull. Seismol. Soc. Am. vol. 75, p. 1349 (1985)

[2] B. E. Shaw, J. M. Carlson and J. S. Langer: "Patterns of seismic activity preceding Earthquakes", J. Geophys. Res. vol. 97, p. 479 (1992).