Operator Algebras and Quantum Dynamics

University of Pretoria

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12 July

A proposal for the thermodynamics of certain open systems

F. Fidaleo

Department of Mathematics, University of Rome "Tor Vergata"

Motivated by the fact that the (inverse) temperature might be a function of the energy levels in the Planck distribution $n_\varepsilon = \frac{1}{z^{-1}e^{\beta(\varepsilon)\varepsilon}-1}$ for the occupation number n_ε of the level ε for free bosons, we show that it can be naturally achieved by imposing the constraint concerning the conservation of a weighted sum $\sum_\varepsilon f(\varepsilon)\varepsilon n_\varepsilon$, with a fixed positive weight function f, of the contributions of the single energy levels occupation in the Microcanonical Ensemble scheme, obtaining $\beta(\varepsilon) \propto f(\varepsilon)$. This immediately addresses the possibility that also a weighted sum $\sum_\varepsilon g(\varepsilon)n_\varepsilon$ of the particles occupation number is conserved, having as a consequence that the chemical potential might be a function of the energy levels of the system as well. This scheme leads to a thermodynamics of open systems in the following way:

the equilibrium is reached when the entropy function is maximised under the constraints that some weighed sums of occupation of the energy levels and the occupation numbers are conserved.

The standard case of isolated systems corresponds to the weight functions being trivial (i.e. f, g are identically 1).

09:30 OAQD 1

Concerning the theoretical investigation of such open systems, new and unexpected phenomena can appear. Among them, we mention the appearance of the Bose Einstein Condensation both in dimension less than 3 in configuration space, and even in excited levels of the energy spectrum. In addition, this suggests a new approach to the condensation which allows an unified analysis involving also the condensation of q-particles, $-1 \leq q \leq 1$, where $q = \pm 1$ corresponds to the Bose/Fermi alternative. For such q-particles, it is shown that the condensation can occur only if $0 < q \leq 1$, the case 1 corresponding to the standard Bose-Einstein condensation. In this more general approach, completely new and unexpected states exhibiting condensation phenomena naturally occur even in the usual situation of equilibrium thermodynamics involving bosons.
The new approach proposed for the situation of 2^{nd} quantisation of free particles, is based on the theory of the distributions, which might hopefully be extended to more general cases involving nontrivial interaction.

The talk is based on the following papers:

Accardi L., Fidaleo F. *Condensation of Bose and q-particles in equilibrium and non equilibrium thermodynamics*, Rep. Math. Phys. **77** (2016), 153-182.

Fidaleo F., Viaggiu S. *A proposal for the thermodynamics of certain open systems*, Physica A **468** (2017), 677-690.

10:30-11:00 **TEA**

On some ergodic properties for C^*-dynamical systems arising from Yang-Baxter-Hecke quantisation

V. Crismale[1], F. Fidaleo[2], Y. G. Lu[1]

1 Department of Mathematics, University of Bari, Italy
2 Department of Mathematics, University of Rome 2, Italy

11:00
OAQD 2

We firstly give sufficient conditions ensuring the strong ergodic property of unique mixing for C^*-dynamical systems arising from Yang-Baxter-Hecke quantisation.
We discuss whether they can be applied to the monotone and boolean C^*-algebras.
We show that the set of stationary stochastic processes is isomorphic to a segment in both the monotone and boolean situations. Moreover in the talk we will point out the boolean processes enjoy the very strong property of unique mixing with respect to the fixed-point subalgebra and the Monotone ones do not.
Finally, what it happens in Bose and Fermi case will be broadly explained.

Relative weak mixing in W^*-dynamical systems

R.D.V. Duvenhage[1], M.B. King[2]

1 Department of Physics, University of Pretoria, 0002, Pretoria, South Africa
2 Department of Mathematics and Applied Mathematics, University of Pretoria, 0002, Pretoria, South Africa

11:45
OAQD 3

We characterise relative weak mixing in W*-dynamical systems in terms of joinings.

12:30-14:00 **LUNCH**

Dissipative dynamics in Noncommutative Spaces

B Zegarlinski

Imperial College

14:00
OAQD 4

I will review recent progress including properties of norms and entropy functionals and discuss construction and study of dissipative dynamics in noncommutative spaces.

15:00-15:30 **TEA**

15:30
OAQD 5

Entropy and ergodicity in general quantum systems

Open discussion

13 July

Translation invariant Dirichlet forms in the context of locally compact quantum groups

A. Skalski,[1], A.Viselter[2]

1 Institute of Mathematics of the Polish Academy of Sciences, ul. Sniadeckich 8, 00-656 Warszawa, Poland

2 Department of Mathematics, University of Haifa, 31905 Haifa, Israel

09:30 OAQD 6

Since the work of Cipriani on one hand and Goldstein and Lindsay on the other in the 1990s it is known that certain natural class of symmetric Markov semigroups on a von Neumann algebra M equipped with a faithful normal state admits extensions to associated Haagerup L^p-spaces and is characterised via a Dirichlet property of the generating quadratic form on the L^2-space. Recently Cipriani, Franz and Kula studied a special class of such semigroups associated to compact quantum groups. In this talk I will discuss how their results extend to the framework of locally compact quantum groups, where two new important technical features appear: there is no natural 'algebraic' domain for the generator and one needs to work with weights, as opposed to finite states (using the appropriate Dirichlet form result provided by Goldstein and Lindsay). I will also present some applications of Dirichlet forms to the study of geometric properties of quantum groups.

10:30-11:00 **TEA**

Balance between quantum Markov semigroups

11:00 OAQD 7

R. Duvenhage, M. Snyman

Department of Physics, University of Pretoria

Balance between quantum Markov semigroups is introduced as an extension of concepts appearing in the theory of quantum detailed balance. It is then shown how a specific form of detailed balance, standard quantum detailed balance, can be expressed in terms of balance.

Properties of balance between quantum Markov semigroups

11:45 OAQD 8

R. Duvenhage[1], M. Snyman[1]

1 Department of Physics, University of Pretoria, South Africa

We study the notion of balance between quantum Markov semigroups on von Neumann algebras. In particular a characterization of balance is given in terms of unital completely positive maps, and transitivity is discussed.

12:30-14:00 **LUNCH**

Quantum stochastic semigroups

Martin Lindsay
Department of Mathematics and Statistics, Lancaster University, Lancaster LA1 4YF, UK

14:00
OAQD
9

In this talk I shall describe some aspects of a stochastic enrichment of the theory of one-parameter semigroups.

15:00-15:30 **TEA**

Dirichlet forms and dynamical maps for general quantum systems

15:30
OAQD
10

Open discussion

14 July

Quantum Orlicz spaces and Quantum Field Theory

WA Majewski[1,21], LE Labuschagne[2]
1 University of Gdansk
2 North-West University (Potchefstroom)

09:30 OAQD 11

We will show that the formalism based on non-commutative Orlicz spaces is well suited to study Quantum Field Theory. The pair of Orlicz spaces we explicitly use are respectively built on the exponential function (for the description of regular observables) and on an entropic type function (for the corresponding states). They form a dual pair (both for classical and quantum systems). This pair $\langle L^{\cosh -1}, L\log(L+1)\rangle$ has the advantage of being general enough to encompass regular observables, and specific enough for the latter Orlicz space to select states with a well-defined entropy function. We will show that the strategy based on $\langle L^{\cosh -1}, L\log(L+1)\rangle$ can be successfully applied to quantum field theory. To this end we propose regularity conditions which ensure good behavior of field operators as observables in the context of local algebras. It is important to note that to carry out these modifications the basic ingredients of the principle of relativity will be used.

10:30-11:00 **TEA**

Interpolation of quantum Markov maps

LE Labuschagne[2], WA Majewski[1,2]
1 University of Gdansk
2 North-West University (Potchefstroom)

11:00 OAQD 12

We revise the basic theory of quantum Orlicz spaces for type III von Neumann algebras and their significance for describing "good" states and observables. We then proceed to show that for general von Neumann algebras, the imposition of fairly mild restrictions ensure that the action of quantum Markov maps may be extended to a large class of quantum Orlicz spaces. In particular for any von Neumann algebra M, the action of CP quantum Markov semigroups canonically extends to the space $L^{\cosh -1}(M)$ (the natural home for regular observables). This result is achieved through the development of a novel interpolation method.

Isometries on symmetric spaces associated with semi-finite von Neumann algebras

J.J. Conradie[1], P. de Jager[2], [1], R.W. Martin[1]
1 University of Cape Town
2 North West University

11:45 OAQD 13

In the commutative setting, isometries between Banach function spaces can typically be characterized as weighted composition operators. Several non-commutative analogues of these results have been proven, but these have at times been confined to isometries on symmetric spaces associated with trace-finite von Neumann algebras. In this talk we provide a brief overview of the topic and present some new results regarding isometries on symmetric spaces associated with semi-finite von Neumann algebras.

12:30-14:00 **LUNCH**

W^*-correspondence approach to multivariable Schur classes

S ter Horst

North-West University(Potchefstroom)

Classically the Schur class is defined as the closed ball of H^∞ over the unit disc $\mathbb{D}$ of the complex plane. However, there are many different ways of characterizing the Schur class, as the contractive multipliers on H^2, the contractive analytic Toeplitz operators on ℓ^2_+, positivity of the associated De Branges-Rovnyak kernel, and as transfer functions of contractive operator colligations. When the domain $\mathbb{D}$ is replaced by another domain, such as the unit ball $\mathbb{B}_d$ in $\mathbb{C}^d$ or the polydisk $\mathbb{D}^d$, a more intricate theory appears where some of the implication may disappear and others require significantly more complicated proofs. Since the late 1990s Paul Muhly and Baruch Solel developed an operator algebra approach to H^∞ and its associated Schur class based on the notion of a W^*-correspondence, in which some of these (multivariable) extensions as well as many others can be encoded by the appropriate choice of the W^*-correspondence. In this talk we discuss this approach and some special cases.

14:00
OAQD
14

(1) P.S. Muhly and B. Solel, Hardy algebras, W^*-correspondences and interpolation theory, *Math. Annalen* **330** (2004), 353–415.

(2) P.S. Muhly and B. Solel, Schur class operator functions and automorphisms of Hardy algebras, *Documenta Math.* **13** (2008), 365-411.

(3) J.A. Ball, A. Biswas, Q. Fang, and S. ter Horst, Multivariable generalizations of the Schur class: positive kernel characterization and transfer function realization, in: *Recent Advances in Operator Theory and Applications*, pp. 17–79, *Oper. Theory Adv. Appl.* **187**, Birkhäuser, Basel, 2009.

(4) J.A. Ball and S. ter Horst, Multivariable operator-valued Nevanlinna-Pick interpolation: a survey, in: *Operator Algebras, Operator Theory and Applications*, pp. 1–72, *Oper. Theory Adv. Appl.* **195**, Birkhäuser, Basel, 2010.

14:50-15:20 **TEA**

15:20
OAQD
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Operator algebras and Quantum Field Theory

Open discussion

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