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Predicting Multi-Scale Positive and Negative Stock Market Bubbles in a Panel of G7 Countries: The Role of Oil Price Uncertainty

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Abstract

In this paper, as a first step, we use the Multi-Scale Log-Periodic Power Law Singularity Confidence Indicator (MS-LPPLS-CI) approach to detect both positive and negative bubbles in the short-, medium- and long-term in the stock markets of the G7 countries. While detecting major crashes and booms in the seven stock markets over the monthly period of 1973:02 to 2020:05, we also observe similar timing of strong (positive and negative) LPPLS-CIs across the G7, suggesting synchronized boom-bust cycles. Given this, in the second step, we apply dynamic heterogeneous coefficients panel databased regressions to analyze the predictive impact of a model-free robust metric of oil price uncertainty on the bubbles indicators. After controlling for the impacts of output growth, inflation, and monetary policy, we find that oil price uncertainty predicts a decrease in all the time scales and countries of the positive bubbles and increases strongly the medium term for five countries (and weakly the short-term) negative LPPLS-CIs. The aggregate findings continue to hold with the inclusion of investor sentiment indicators. Our results have important implications for both investors and policymakers, as the higher (lower) oil price uncertainty can lead to a crash (recovery) in a bullish (bearish) market.

Keywords: Multi-Scale Bubbles; Oil Price Uncertainty; Panel Data Regressions; G7 Stock Markets

JEL Codes: C22; C32; C33; G15; Q02

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1. Introduction

As pointed out by Bernanke (1983) and Pindyck (1991), investment under uncertainty and real options implies that high oil price uncertainty creates cyclical fluctuations in investment by lowering the firms' incentive for current investment. This, in turn, affects cash flows generated by a firm and the discount rate used to calculate stock prices and, hence, negatively impacts stock prices and/or stock returns (Swaray and Salisu, 2018; Chen and Demirer, 2022). Moreover, since stock prices are the sum of discounted cash flows, including dividends, oil price uncertainty can adversely affect stock prices by decreasing the overall profit that a firm generally uses to pay dividends, with this resulting from the fact that firms need to bear additional costs to avoid risk associated with oil price uncertainty (Demirer et al., 2015). The theoretical prediction that oil price uncertainty negatively drives international stock prices and or/returns via the investment and dividends channels has been widely empirically evaluated (see, for example, Sadorsky (1999), Basher and Sadorsky (2006), Masih et al., (2011), Alsalman (2016), Diaz et al., (2016), Bass (2017), Benavides et al. (2019), Rahman (2021), Balcilar et al. (2022), and Salisu et al. (2022)).

While existing studies tend to agree that oil uncertainty would adversely impact equity prices and/or returns, an associated important question would be how does it affect stock market bubbles, i.e., its boom-bust cycles? Intuitively, if stock prices are accelerating away from their fundamental value, higher (lower) oil uncertainty is likely to lead to a burst of the bubble (further growth) in the market. While the decline in stock prices would continue in the wake of higher oil uncertainty, a rally could be witnessed when oil price uncertainty declines. Moreover, Zhang and Wong (2023) pointed out that oil price uncertainty negatively impacts stock liquidity, which, in turn, is central to the efficient functioning of trade and investor confidence in the financial markets. Naturally, deteriorating (improving) investment confidence following higher (lower) oil uncertainty could also lead to a collapse (recovery) of the stock market (see, Scherbina and Schlusche (2014) for detailed discussions of the theoretical models, based on investor disagreement, feedback trading and biased self-attribution, used to relate investor sentiment or confidence to bubbles). Understandably, with tremendous fluctuations in oil prices witnessed since the Global Financial Crisis, what we propose to investigate in this paper is pertinent from the perspective of not only investors but also policymakers, as bubbles are known to historically not only impact economic activity (Reinhart and Rogoff, 2009; Jordà et al. 2015), but also welfare (Narayan et al., 2016).

Against this backdrop, we aim to analyze the effect of a robust metric of oil uncertainty on stock market bubbles of the G7 countries (i.e., Canada, France, Germany, Italy, Japan, the United Kingdom (UK), and the United States (US)) over the monthly period of 1973:02 to 2020:05 in a panel data setting. The choice of the G7 is not only driven by the availability of data that allows us to cover nearly five decades of extreme movements in the stock markets of these developed economies but also due to the fact that the G7 bloc accounts for nearly two-thirds of global net wealth and nearly half of world output, and hence, dynamics of bubbles in these stock markets are likely to have worldwide spillover effects and impact the sustainability of the global financial system (Das et al., 2019). At the same time, the decision to rely on panel data regressions is motivated by the high degree of synchronization of the indicators of the bubbles of these countries, which we discuss in detail below. But even though we conduct the estimation in a panel setting, we allow for heterogeneous responses of bubbles to oil uncertainty (and other controls) by utilizing the Random Coefficients (RC) approach of Swamy (1970) to derive both overall and country-specific results.

As far as detecting bubbles, we not only use the Log-Periodic Power Law Singularity (LPPLS) model, originally developed by Johansen et al. (1999, 2000) and Sornette (2003), for both positive (upward accelerating price followed by a crash) and negative (downward accelerating price followed by a rally) bubbles, but we also apply the Multi-Scale LPPLS Confidence Indicators (MS-LPPLS-CI) of Demiret et al. (2019) to characterize positive and negative bubbles at different time scales, i.e., short-, medium- and long-term, corresponding to estimation windows associated with trading activities over one to three months, three months to a year, and one year to two years, respectively. Note that the identification of both positive and negative multi-scale bubbles is not possible based on other existing wide array of statistical tests (see, Balcilar et al. (2016), Zhang et al. (2016), and Sornette et al. (2018) for detailed reviews), which points to the suitability and added value of our applied methodology. In fact, we consider this as important because it would allow us to gauge the possible asymmetric effect of oil uncertainty on the equity market bubbles of the G7, given that crash and recovery at different horizons can carry different information for market participants as suggested by the Heterogeneous Market Hypothesis (HMH; Müller et al., 1997). To the best of our knowledge, this is the first paper to analyze the effect of oil uncertainty on six indicators of multi-scale positive and negative bubbles in the G7 countries based on a heterogeneous coefficients panel data model.

The remainder of the paper is organized as follows: Section 2 discusses the data and the basics of

the econometric model. Section 3 presents the empirical findings involving the detection of bubbles, as well as the effects of investor sentiment on the six LPPLS-CIs of bubbles in the panel of G7 countries. Finally, Section 4 concludes the paper.

2. Data and Econometric Model

2.1. Data

We first obtain weekly bubble indicators, derived based on the natural logarithmic values of the daily dividend-price ratio of the seven countries, using the dividend and the stock price index series in their local currencies, obtained from Refinitiv Datastream. Appendix A of the paper outlines the mathematical details of how the MS-LPPLS-CIs are obtained. The generated bubbles indicators cover the weekly period of the first week of January, 1973 to the fourth week of May, 2020. Since our controls, following Caraiani et al. (2023), namely, the macroeconomic variables, besides the indicator of oil uncertainty, are at a monthly frequency, to obtain a monthly value for each of the multi-scale confidence indicator, we take the average for each of the scales weekly values that fall within a given month.

The evolution of the MS-LPPLS-CIs can be used to detect crashes and rallies in real time. To this end, we plot the short-, medium-, and long-term indicators (green, purple, and red lines) while we show the log price-to-dividend ratio as a black line in Figure 1(a). A larger LPPLS-CI value for a particular scale shows that the LPPLS signature is present for many of the fitting windows to which we calibrated the model, making it a more reliable bubble indicator. The key message conveyed by Figure 1(a) is that there are many peaks in the LPPLS-CIs preceding substantial shifts in the log price-to-dividend ratio.

We note that the bubble indicators across the G7 countries, in general, display peaks in the periods corresponding to crashes and recoveries before and around the collapse of the Bretton Woods system in 1973, the “Black Monday” episode in 1987, the Asian Financial Crisis of 1997, the Dot-com bubble burst from 2000 to 2002, the Global Financial Crisis of 2007 to 2008, the European sovereign debt crisis from 2009 to 2012, the “Brexit” in 2016, and to some extent during the COVID-19 episode. In other words, the MS-LPPLS-CIs are capable of providing leading information on all the major episodes of booms and busts witnessed globally from 1973 to 2020.

In general, smaller crashes or rallies can best be recovered using shorter time scales, while longer time scales help to detect larger crashes or rallies, with the short-term LPPLS-CIs preceding the

medium-term ones, and the latter leading the long-run indicators, i.e., maturation of the bubble heading towards instability is present across several distinct time-scales. More importantly, we observe a similar timing of the strong (positive as well as negative) MS-LPPLS-CI values in the cross-section of G7 countries, in line with the intuition that boom and bust cycles of the seven developed equity markets often occur in tandem, motivating the need to use a panel-based approach to analyze the impact of oil price uncertainty on stock market bubbles.

[INSERT FIGURE 1]

Next, we next turn our attention to the main predictor, i.e., oil price uncertainty, depicted in Figure 1(b). One must realize that uncertainty is a latent variable, and needs to be measured. Given this, majority of the studies, mentioned in the introduction, rely on univariate or bivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models applied to the oil price returns to derive metrics of oil price uncertainty to relate to stock price and/or returns. In other words, GARCH-based oil price uncertainty is fully determined by changes in the level of oil price, and as a result, it is impossible to disentangle uncertainty about the oil price and changes in the oil price level (Jo, 2014). Given this, Rahman (2021) proposes a new measure of oil price uncertainty by utilizing Stochastic Volatility (SV) in a Structural Vector Autoregressive (SVAR) model (involving oil and stock prices, and a monetary policy instrument). In this model, oil price uncertainty is the conditional variance of the oil price change forecast error, and thus, it evolves independently of any change in the oil price level.¹ Despite the innovativeness of this approach over GARCH-based models in measuring oil price uncertainty, the metric is not free from the structure of any specific theoretical model. Given these empirical issues in constructing an appropriate metric of oil price uncertainty, Nguyen et al. (2021) have proposed a novel construction of the oil price uncertainty index that is unconditional on a model.² These authors develop a measure of oil price uncertainty as the one-period-ahead forecast error variance of a forecasting regression with SV in the residual terms. The novelty of this construction approach lies in its flexibility in including a large number of additional information that is important in explaining fluctuations in oil prices, namely, exchange rate, oil production, global economic condition and co-movement in the fuel market. In this sense, the index is able to capture uncertainty in oil prices rather

¹ Using this framework, Rahman (2021) provides evidence that increased oil price uncertainty has a negative effect on (real) stock returns of the US.

² The data for the oil uncertainty index can be obtained from the website of Dr. Bao H. Nguyen at: <https://sites.google.com/site/nguyenhoibao/datasets/oil-market-uncertainty?authuser=0>.

than volatility as measured by both GARCH and SV models.

According to Figure 1(b), heightened oil price uncertainty coincides with the first and second oil crises of 1973 and 1979. These events are also associated with substantial positive and negative stock market bubbles across all G7 countries, as evident from Figure 1(a). Following the oil crises of the 1970s, oil uncertainty peaked again during the first half of 1986, coinciding with a prolonged period of positive stock market bubbles across all G7 countries and at short- medium- and long-term time scales. A strong bull market overdue for a correction since 1982, exacerbated by heightened oil uncertainty, culminated in the Black Monday stock market crash of October 1987.

Iraq's invasion of Kuwait in 1990 had significant ramifications for global stock markets, leading to increased uncertainty and bearish sentiment. The invasion led to a sharp spike in oil prices and consequently increased inflation and reduced economic growth, typically negative for stock market performance. As with many geo-political crises, investors pull out riskier assets like equities and move towards safer assets such as gold and government bonds. This effect shows up as positive asset bubbles, most notably in Germany and the US.

The next episode of heightened oil market uncertainty started towards the end of 1998, with substantial positive stock market bubbles across all countries, but most pronounced for the US. The positive bubble indicators for the US remained high up to 2002, reflecting both the impact of the East-Asian crisis and the Dot-Com Bubble on global stock markets.

Our model-free estimate of oil price uncertainty indicates another spike towards the end of 2008, further negatively contributing to the Global Financial Crisis.

As is further evident from Figure 1(b), the COVID-19 pandemic, which emerged at the end of 2019 and became a global health crisis in 2020, had profound effects on the global economy and various industries, with the oil and gas industries most severely impacted through a collapse in demand, storage issues and a price war between major oil producers in the OPEC+ group. The pandemic also accelerated discussions about the future of oil and the potential for a more rapid transition to renewable energy sources – leading to a significant increase of uncertainty in the oil market, driven by not only immediate demand-side shocks but also longer-term considerations about the future of energy consumption. Although most countries in the sample register some positive bubble effects, positive bubbles are most noticeable in the case of the US.

Regarding the macroeconomic control variables included in the analysis, we use month-on-month growth of industrial production, month-on-month Consumer Price Index (CPI)-based inflation rate, and change in the interest rate, with all transformations to the data ensuring stationarity of the variables under consideration, as per the unit root testing approach of Im et al. (2003). As far as the interest rate variable is concerned, we use the three-month money market interest rates, merged with the Shadow Short Rate (SSR) of the individual countries (of course, from 1999 onwards France, Germany, and Italy have the same values), from the time the latter became available. Industrial production, CPI, and the money market interest rates are all sourced from the Main Economic Indicators (MEI) database of the Organization for Economic Co-operation and Development (OECD).³ Specifically speaking, barring the US data, which begins in 1985:11, the SSRs of the remaining six countries are available from 1995:01.⁴

Ultimately, based on data availability and transformations to ensure stationarity, our panel databased regression covers monthly data from 1973:02 to 2020:05. Though the model-free measure of oil price uncertainty starts in 1975:02 and ends in 2020:05, in line with the extant literature on oil uncertainty and stock price, we also use GARCH-based measure of the former in our estimations, which in turn, based on the West Texas Intermediate (WTI) oil price log-returns can be computed from 1973:02, with the data on oil price derived from the Global Financial Data.⁵ This is important for us, as it not only allows us to go two years back in time to relate oil price uncertainty with the bubbles associated with the collapse of the Bretton Woods system but also provides a robustness check.

³ <https://www.oecd.org/sdd/oecdmaineconomicindicatorsmei.htm>.

⁴ The SSRs are derived from the website of Dr. Leo Krippner: <https://www.ljkmfa.com/>. Note that, the SSR estimates used in this paper are derived from the works of Krippner (2013, 2015), due to their coverage involving the G7, besides being considered an improvement over those obtained by Wu and Xia (2016) (for the Euro area, the UK and the US), as discussed in detail by Krippner (2020). The SSR is based on models of the term-structure, which essentially removes the effect that the option to invest in physical currency (at an interest rate of zero) has on yield curves, resulting in a hypothetical “shadow yield curve” that would exist if the physical currency were not available. The “shadow policy rate” generated in this manner, therefore, provides a measure of the monetary policy stance after the actual policy rate reaches zero. The main advantage of the SSR is that it is not constrained by the Zero Lower Bound (ZLB), and thus allows us to combine the data from the ZLB period with that of the non-ZLB era, and in turn to use it as the common metric of monetary policy stance across the conventional and unconventional monetary policy episodes.

⁵ <https://globalfinancialdata.com/>.

2.1. Econometric Framework

To capture the predictive effect of oil price uncertainty on equity market bubbles at various time scales, we specify the following dynamic panel data model:

$$eq_bubble_{i,t}^j = \beta_{0i} + \beta_{1,i}eq_bubble_{i,t-1}^j + \beta_{2,i}opu_{i,t-1} + \beta_{ki}Z_{i,t-1} + \varepsilon_{i,t} \quad (1)$$

where $eq_bubble_{i,t}^j = \{lt_neg_{i,t}, mt_neg_{i,t}, st_neg_{i,t}, lt_pos_{i,t}, mt_pos_{i,t}, st_pos_{i,t}\}, j = 1, 2, \dots, 6$ represents negative and positive equity market bubbles at short, medium and long-run time scales, which correspond to estimation windows associated with trading activities over one to three months, three months to a year, and one year to two years, respectively (see, Appendix A for further details); $opu_{i,t-1}$ is the one-period lagged oil price uncertainty, which involves either the metric developed by Nguyen et al. (2021) or the GARCH-based one; while $Z_{i,t-1}$ is the set of lagged macroeconomic control variables, with $Z'_{i,t-1} = \{ip_growth_{i,t-1}, infl_{i,t-1}, ir_diff_{i,t-1}\}$, comprising industrial production growth, CPI inflation, and changes in interest rates. The β 's in Equation (1) capture the cross-section-specific (country-level) parameters associated with the predictors, which also involves the lagged MS-LPPLS-CIs of the G7 in an attempt to capture the persistence of these indicators. The idiosyncratic error term ($\varepsilon_{i,t}$) is distributed with mean zero and variance $\sigma_{ii,t}I$. The model is estimated using the Random Coefficients (RC) approach, as discussed in Appendix B.

3. Empirical Findings

In this section, the Random Coefficient (Swamy, 1970) estimation results for Equation (1) for all countries combined, as well as the country-specific results of the effect of oil price uncertainty on equity market bubbles, are reported.

We model the lagged impact of oil price uncertainty on equity market bubbles to capture the notion of predictive impact and avoid any possible concerns of endogeneity.⁶ The impact of lagged opu on negative and positive equity market bubbles across the three time scales are presented in Table 1.

[INSERT TABLE 1 HERE]

⁶ Note that, the application of the Hausman (1978) test suggested that oil price uncertainty and the control variables are exogenous to the specification, with complete details of these results available upon request from the authors. Hence, in Table C1 in Appendix C we depict the contemporaneous effects of all the predictors.

From Table 1, it is evident that, in line with the observations made in Figure 1, with relatively longer time scales capturing stronger crashes or rallies, the persistence, which is always statistically significant at the 1% level, increases as we move from the short- to the long-term positive and negative LPPLS-CIs. More importantly, as per our intuition outlined in the introduction, lagged oil price uncertainty exerts a positive and negative impact on the negative and positive MS-LPPLS-CIs, respectively.⁷ While the effect is statistically significant at the 1% level across all the time scales of the positive bubbles, significance (at 10% and 1%, respectively) is observed for the cases of the short- and medium-term negative bubbles.⁸ In sum, lagged higher oil price uncertainty is likely to cause a burst in the stock market relatively more strongly, as depicted by the higher absolute value of the regression coefficient related to oil price uncertainty when it is in its bullish phase, than further collapse in its bearish state. This evidence of asymmetry in terms of the strength and significance of the effect of oil price uncertainty provides us with a strong justification for decomposing bubbles into their positive and negative counterparts. This result is possibly an indication that market agents tend to react less to (oil price uncertainty) news, in particular during deep stock market downturns, as captured by the long-term negative LPPLS-CI, as sentiments are already low (Çepni and Gupta, 2021; Çepni et al., 2023), especially given that they might have foreseen the situation with short- and medium-term indicators leading long-horizon bubbles, as shown in Figure 1.

As far as the effects from the other controls are concerned, they are somewhat sporadic, especially for output growth and changes in interest rates. For instance, in line with economic sense, higher growth in industrial production reduces the negative long-term-LPPLS-CI and increases the positive medium-term-LPPLS-CI in a statistically significant manner. Higher inflation can be considered bad news and is found to significantly increase the negative short- and long-term-MS-LPPLS-CIs, whereas the effect is opposite, in a statistically significant fashion, for all the scales of the positive LPPLS-CIs. Finally, the lagged interest rate tends to increase and reduce, at conventional statistically significant levels, the short-term negative and positive bubbles indicators, respectively, in line with what is expected. But, a weak counter-intuitive negative effect is detected for the negative medium-term LPPLS-CI. All in all, inflation, like oil uncertainty, tends to carry quite a strong influence on the MS-LPPLS-CIs, especially

⁷ These results in terms of sign are also confirmed, along with delayed significant effects, in Figure C1 in the Appendix, via impulse response functions, following an oil price uncertainty shock, identified using Cholesky decomposition, on six Panel VAR (PVAR) models with variables ordered as follows: *opu*, *ip_growth*, *infl*, *ir_diff*, and a specific MS-LPPLS-CI for the G7.

⁸ These findings are consistent for contemporaneous *opu*, as reported in Table C1.

in the context of positive bubbles.

We next turn to country-specific results for the sample of the G7 economies to understand the drivers of the overall results, with the tests of parameter constancy, barring the cases of short-, and long-term positive LPPLS-CIs, suggesting statistically different slope parameters of the predictors, i.e., heterogeneous impacts. Table 2 presents the results for the impact of lagged *opu* on negative and positive equity market bubbles at the short-, medium-, and long-term scales. Consistent with the overall strong results in Table 1 for positive MS-LPPLS-CIs, we find that barring the case of the short-term positive bubbles indicator of Italy, oil price uncertainty tends to predict a decline of the positive MS-LPPLS-CIs in a statistically significant manner in all other twenty instances. Comparatively, as shown in Table 1, the country-specific effects for the negative MS-LPPLS-CIs are weak, with the overall strong effect for the medium-term being driven by France, Germany, Italy, Japan and the UK. Interestingly, while the overall effect is insignificant for the positive long-term-LPPLS-CI, significance is observed for France and Japan, with counter-intuitive insignificant effects observed for the UK and the US (likely to have nullified the aggregate influence). At the same time, while weak predictive content is shown to be carried by lagged oil uncertainty for the short-term negative LPPLS-CI, there is no evidence of significance at the country level under this case. The reason behind this is technical, with the software (STATA, Version 18, in this case) not allowing for producing bootstrapped standard errors for the country-specific parameter estimates, unlike that in the overall case. In fact, if we do not bootstrap the standard errors for the results in Table 1, the response of the short-term negative LPPLS-CI to *opu* is no longer significant at the 10% level. More importantly, the other predictive effects from *opu*, i.e., on all three time scales of the positive bubbles indicators and on the medium-term negative LPPLS-CI, continue to be significant at the 1% level.⁹ In sum, our results tend to confirm that higher values of lagged oil price uncertainty are likely to have a strong negative impact on the positive MS-LPPLS-CIs, i.e., cause a crash across all of the G7 markets and, hence, the aggregate. For the negative medium-term LPPLS-CI involving the whole of the G7, the relatively strong positive influence is due to five out of the seven markets.

[INSERT TABLE 2 HERE]

In order to validate the model-free measure of oil price uncertainty used by us, we present in Table

⁹ All non-bootstrapped results are available from the authors upon request.

3 the overall predictive results for the bubbles of G7 countries obtained now based on the GARCH-based measure of oil returns volatility. As can be seen, compared to Table 1, the lagged effects are weaker, with statistical significance detected only under the medium-term positive and short-term negative LPPLS-CIs, with the sign being counter-intuitive (negative) in the latter case. Clearly, this set of results justifies the need to utilize a robust metric of an otherwise latent variable when drawing appropriate inferences of prediction involving stock market bubbles of the G7 based on oil price uncertainty.

[INSERT TABLE 3 HERE]

Finally, realizing the role of behavioral factors in driving stock market bubbles of the G7 (Pan, 2020; Van Eyden et al. (2023)), we have reported in Table 4 the results from an extended version of the model given by Equation (1), where we have now additionally included lagged standardized seasonally-adjusted survey-based business confidence indicator (*bci*) and consumer confidence indicator (*cci*) as predictors, with both these variables obtained from the MEI database of the OECD. Our results of Table 1, i.e., oil uncertainty tends to negatively impact positive MS-LPPLS-CIs relatively strongly, statistically and economically, than the corresponding positive effects on the negative bubbles indicators continue to hold robustly under the extended model. The fact that the sentiment indicators, barring the negative effect (as higher values depict better sentiments) of the lagged *cci* for the long-term negative bubbles, are hardly significant, is possibly an indication that oil price uncertainty affects the stock markets not only via investment and dividends channels, but also through a behavioral route.

[INSERT TABLE 4 HERE]

In general, we highlight the importance of oil price uncertainty, when measured accurately, in predicting positive bubbles across various time scales and medium-term negative bubbles, in particular for the G7 stock markets.

4. Conclusion

The primary objective of our paper is to analyze the predictive impact of a model-free robust measure of oil price uncertainty on equity market bubbles of the G7 countries. In the first step, we detect positive and negative bubbles in the short-, medium-and long-run for these advanced equity markets using the Multi-Scale Log-Periodic Power Law Singularity Confidence Indicator (MS-LPPLS-CI) approach. Our findings reveal major crashes and booms in the seven stock markets over the monthly period of 1973:02

to 2020:05. We also observe similar timing of strong (positive and negative) LPPLS indicator values across time scales for the G7 countries, suggesting commonality in the boom-bust cycles of these equity markets. In other words, diversification of investor portfolios across these developed stock markets is not a possibility for the market agents over investment horizons during both booms and crashes. In the second step, due to the detected evidence of synchronicity in the bubble indicators across the G7, we use a dynamic panel data-based heterogeneous coefficients regression model to study the overall and country-specific impact of oil price uncertainty. After controlling for the impacts of output growth, inflation, and monetary policy, we find that oil price uncertainty predicts a decrease across all time scales for the positive LPPLS-CI, and reduces the next period negative LPPLS-CIs primarily at the medium- and weakly at the short-term scale for the G7 countries considered all together. At the country level, predictability of the positive MS-LPPLS-CIs by oil price uncertainty is observed for all the G7 stock markets, while the effect on the medium-run negative LPPLS-CIs is recorded for five countries. The aggregate findings continue to hold when we include investor sentiment indicators, but when oil price uncertainty is captured by a conditional measure of volatility, our results weaken considerably and, hence, highlight the need to appropriately model the latent variable of uncertainty in oil price movements.

With oil price uncertainty showing up as having strong negative effects on positive bubbles compared to other traditional macroeconomic and financial indicators, it is recommended that investors and policymakers should be careful when the level of oil uncertainty tends to rise at the time the stock markets are booming, because this could imply an imminent crash. At the same time, when stock prices are facing relatively less severe bearish regimes, higher oil price uncertainty can lead to deep equity market downturns. Accordingly, policymakers should monitor rising oil price uncertainty closely and implement expansionary monetary and fiscal policies to ensure the revival of the equity market (Gupta et al., 2023; André et al., forthcoming; Çepni et al., forthcoming), as directly controlling oil price uncertainty is likely to be difficult due to it being driven by oil market-specific shocks and geopolitical events (Demirer et al., 2020; Qian et al., 2022).

As part of future research, in light of the large literature on the relationship between oil price or returns and stock price or returns (see, Degiannakis et al. (2018), and Smyth and Narayan (2018) for comprehensive reviews), it would be interesting to consider the effect of oil prices on stock market bubbles. But realizing that oil prices are driven by various shocks namely, oil-supply, global economic

activity, oil-specific consumption demand and inventory-demand (Kilian, 2009; Baumeister and Hamilton, 2019), having different directional impacts on stock prices (Kilian and Park, 2009), we will need to decompose oil price movements due to these innovations to detect the impact on bubbles in possibly a time series-based SVAR model, which will also allow us to distinguish between opposing effects of higher oil prices on stock markets for oil-exporting and importing countries (Wang et al., 2013).¹⁰

¹⁰ This line of reasoning is perhaps confirmed by our finding of insignificant predictive impacts of replacing oil price uncertainty in our model in Equation (1) with nominal WTI oil price returns on the MS-LPPLS-CIs, as reported in Table C2 in Appendix C.

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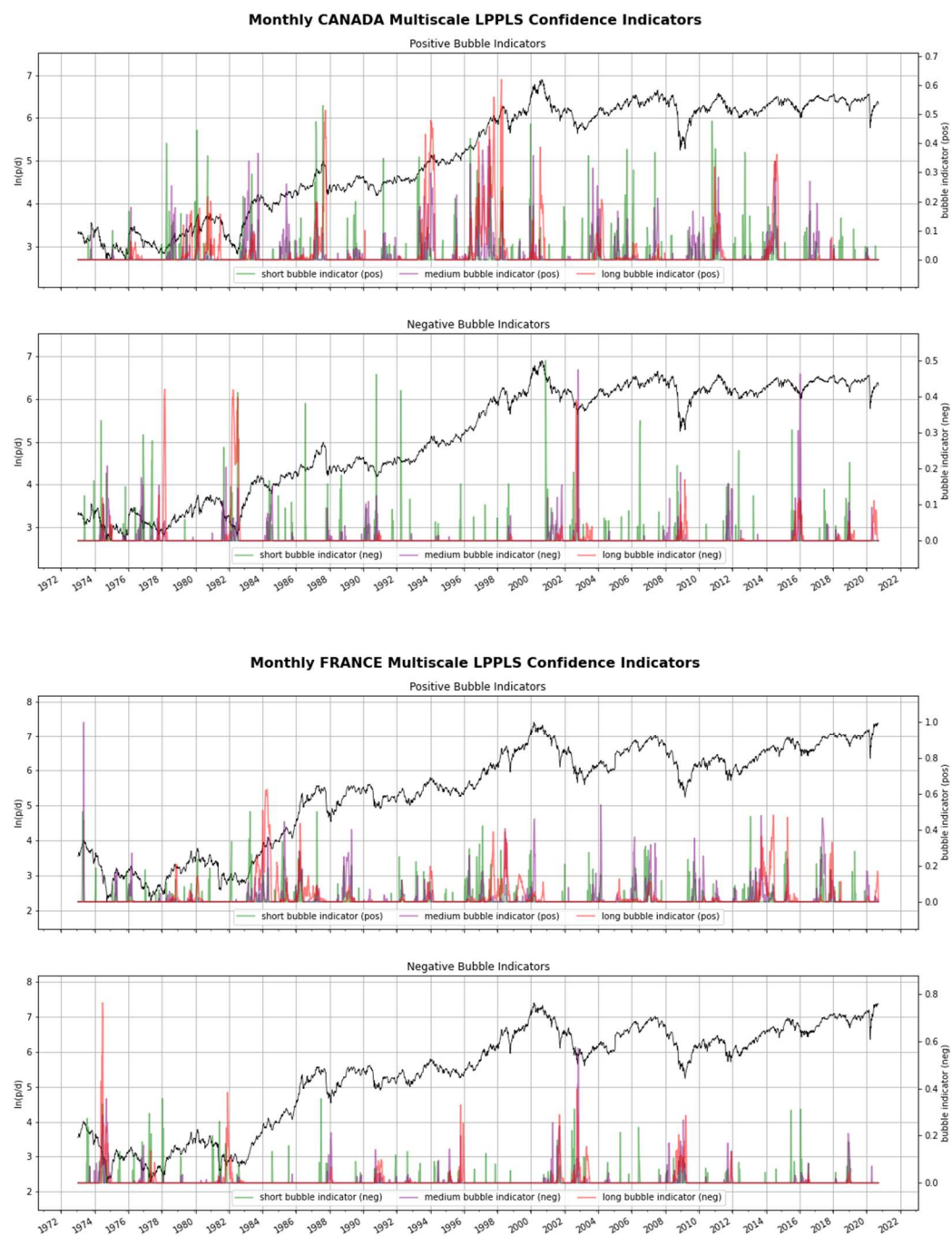
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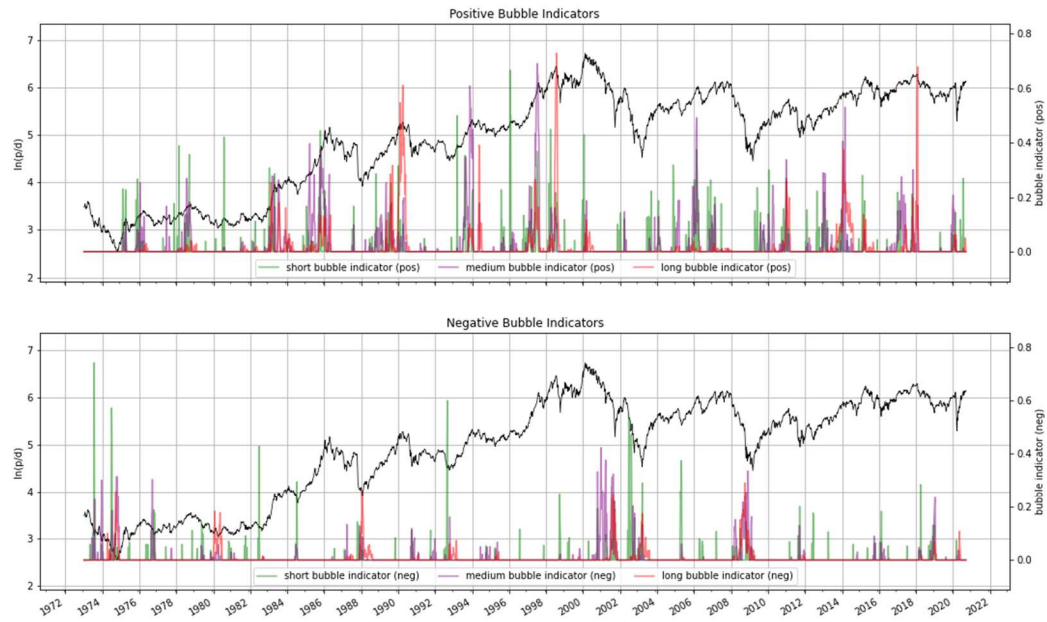
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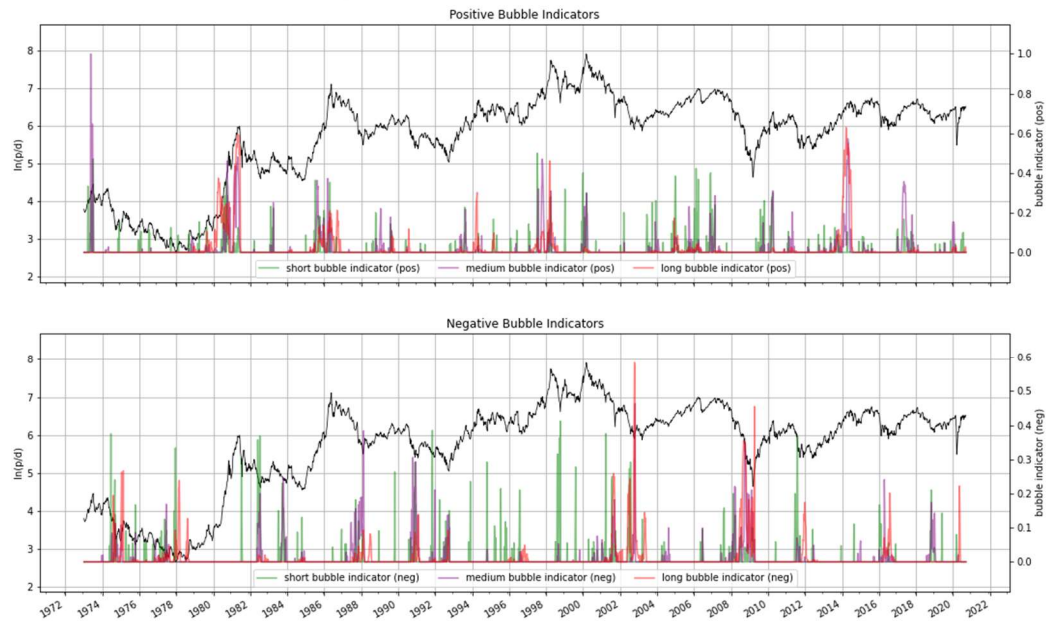
Figure 1. Data Plots
1(a). *Monthly Multi-Scale LPPLS-CIs of the G7 Countries*



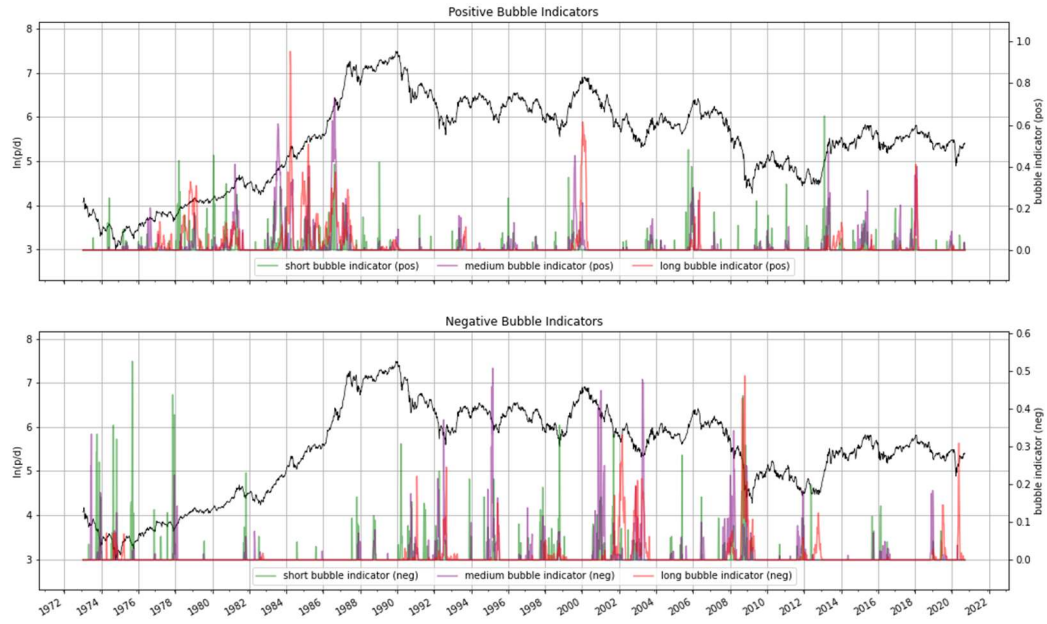
Monthly GERMANY Multiscale LPPLS Confidence Indicators



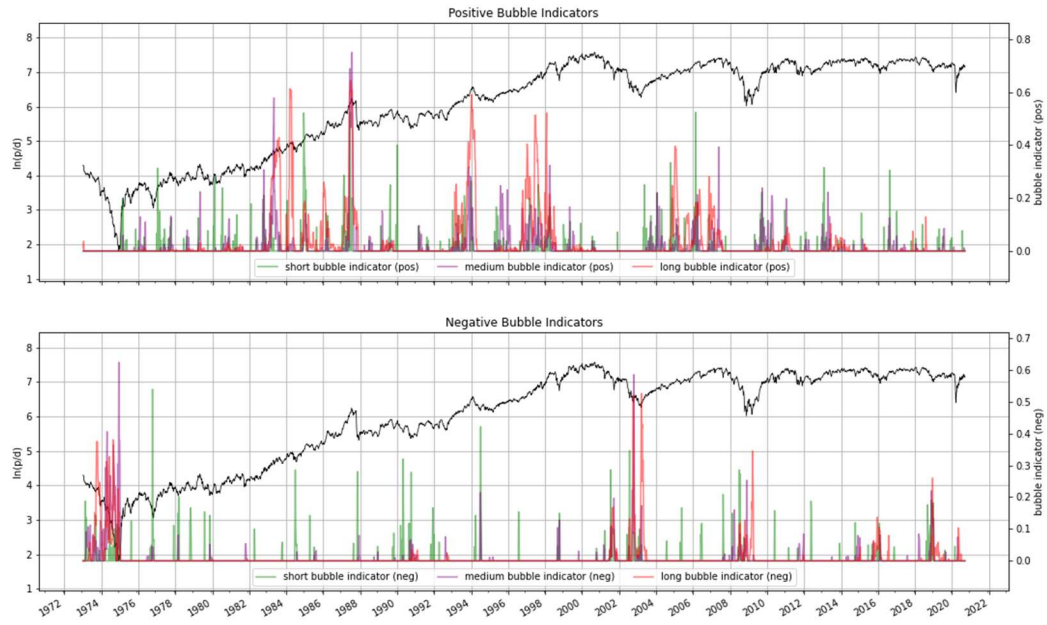
Monthly ITALY Multiscale LPPLS Confidence Indicators

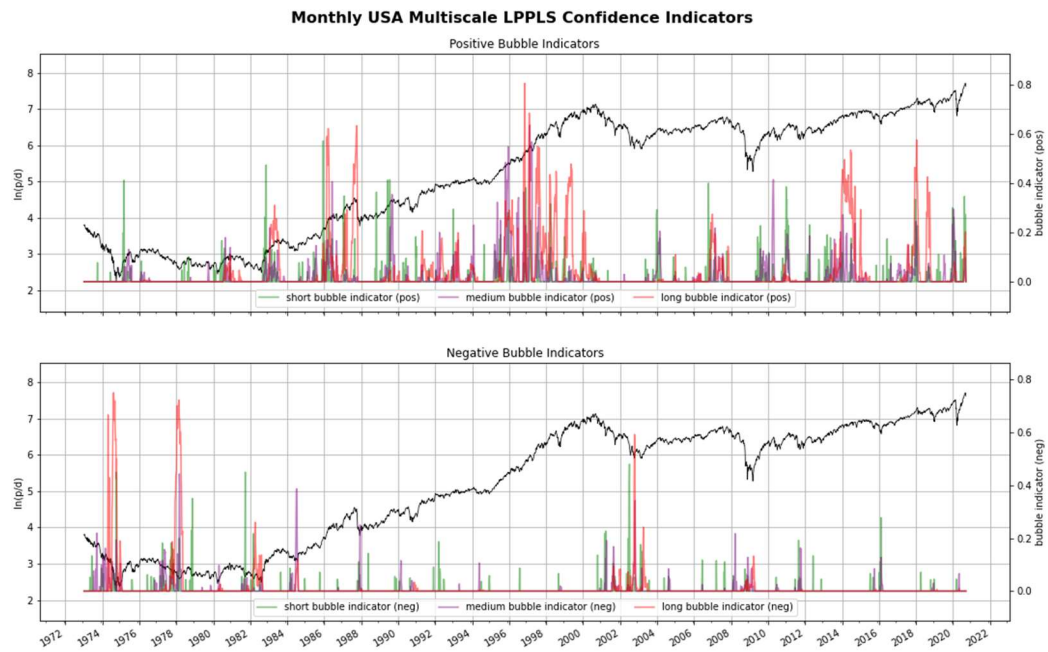


Monthly JAPAN Multiscale LPPLS Confidence Indicators



Monthly UK Multiscale LPPLS Confidence Indicators





1(b). Model-Free Estimate of Oil Price Uncertainty

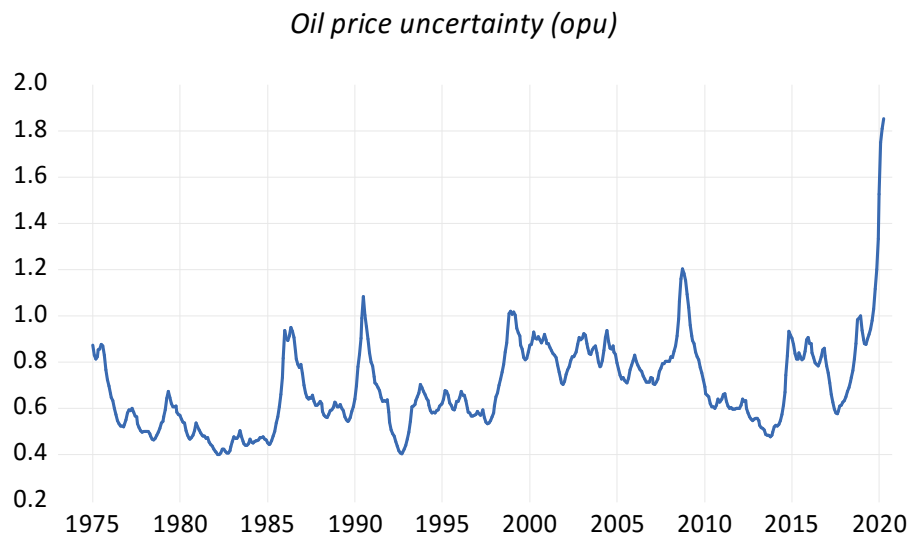


Table 1. Random coefficient estimation predictive results for negative and positive equity bubbles due to a model-free estimate of oil price uncertainty: 1975:02 to 2020:05

	(1)	(2)	(3)	(4)	(5)	(6)
	<i>lt_neg</i>	<i>mt_neg</i>	<i>st_neg</i>	<i>lt_pos</i>	<i>mt_pos</i>	<i>st_pos</i>
<i>l.lt_neg</i>	0.670*** (20.42)					
<i>l.mt_neg</i>		0.416*** (15.00)				
<i>l.st_neg</i>			0.248*** (8.24)			
<i>l.lt_pos</i>				0.775*** (46.76)		
<i>l.mt_pos</i>					0.580*** (17.25)	
<i>l.st_pos</i>						0.320*** (15.34)
<i>l.opu</i>	0.00330 (1.23)	0.00951*** (4.02)	0.00169* (1.70)	-0.0132*** (-7.05)	-0.0126*** (-6.04)	-0.00748*** (-5.30)
<i>l.ip_growth</i>	-0.186*** (-2.97)	0.00925 (0.17)	-0.0530 (-0.96)	0.0507** (2.02)	0.117 (1.46)	-0.0196 (-0.37)
<i>l.infl</i>	0.239 (0.97)	-0.0584 (-0.36)	0.261*** (3.09)	-0.417 (-0.85)	-1.270*** (-3.89)	-0.756*** (-6.68)
<i>l.ir_diff</i>	-0.0000363 (-0.03)	-0.000632 (-1.28)	0.00160*** (2.77)	-0.00194 (-1.28)	-0.00285 (-1.41)	-0.00250* (-1.75)
<i>constant</i>	0.000109 (0.06)	-0.00262 (-1.59)	0.00323*** (4.37)	0.0154*** (6.08)	0.0182*** (8.84)	0.0148*** (12.46)
# observations	3808	3808	3808	3808	3808	3808
# groups	7	7	7	7	7	7
Test for par constancy, χ^2	91.33	60.06	53.88	38.81	76.37	35.10
d.o.f	36	36	36	36	36	36
Prob.	0.0000	0.0072	0.0281	0.3440	0.0001	0.5114

Note: *l* (one-month lag); Oil price uncertainty (*opu*); industrial production growth (*ip_growth*); consumer price index inflation (*infl*); interest rate difference (*ir_diff*); long-term negative bubble (*lt_neg*); medium-term negative bubble (*mt_neg*); short-term negative bubble (*st_neg*); long-term positive bubble (*lt_pos*); medium-term positive bubble (*mt_pos*); short-term positive bubble (*st_pos*) *t*-statistics (based on bootstrapped robust standard errors) in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 2. Random coefficient estimation predictive results for the country-specific impact of a model-free estimate of oil price uncertainty on negative and positive equity market bubbles: 1975:02 to 2020:05

		(1)	(2)	(3)	(4)	(5)	(6)
		<i>lt neg</i>	<i>mt neg</i>	<i>st neg</i>	<i>lt pos</i>	<i>mt pos</i>	<i>st pos</i>
<i>Canada</i>	<i>l.opu</i>	0.0006 (02.12)	0.0034 (0.85)	0.0013 (0.54)	-0.0152*** (-3.06)	-0.0151*** (-2.98)	-0.0096*** (-2.88)
<i>France</i>	<i>l.opu</i>	0.0104** (2.44)	0.0103*** (3.46)	0.0023 (0.99)	-0.0107** (-2.20)	-0.0113** (-2.25)	-0.0073** (-2.18)
<i>Germany</i>	<i>l.opu</i>	0.0031 (0.85)	0.0159*** (3.95)	0.0016 (0.71)	-0.0329*** (-3.22)	-0.0140*** (-2.79)	-0.0082** (-2.54)
<i>Italy</i>	<i>l.opu</i>	0.0072 (1.56)	0.0120*** (4.00)	0.0005 (0.20)	-0.0076*** (-1.65)	-0.0119** (-2.37)	-0.0047 (-1.44)
<i>Japan</i>	<i>l.opu</i>	0.0077* (1.66)	0.0218*** (3.00)	0.0026 (1.10)	-0.0114*** (-2.44)	-0.0092* (-1.83)	-0.0062** (-1.87)
<i>United Kingdom</i>	<i>l.opu</i>	-0.0017 (-0.33)	0.0102*** (2.78)	0.0024 (1.03)	-0.0145*** (-2.96)	-0.0114** (-2..26)	-0.00104*** (-3.16)
<i>United States</i>	<i>l.opu</i>	-0.0041 (-0.83)	0.0018 (0.48)	0.011 (0.51)	-0.0192*** (-3.73)	-0.0155*** (-3.06)	-0.0056* (-1.72)

Note: *l* (one-month lag); Oil price uncertainty (*opu*); long-term negative bubble (*lt neg*); medium-term negative bubble (*mt neg*); short-term negative bubble (*st neg*); long-term positive bubble (*lt pos*); medium-term positive bubble (*mt pos*); short-term positive bubble (*st pos*) *t*-statistics in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3. Random coefficient estimation predictive results for negative and positive equity bubbles due to a conditional volatility estimate of oil price uncertainty: 1973:02 to 2020:05

	(1)	(2)	(3)	(4)	(5)	(6)
	<i>lt_neg</i>	<i>mt_neg</i>	<i>st_neg</i>	<i>lt_pos</i>	<i>mt_pos</i>	<i>st_pos</i>
<i>l.lt_neg</i>	0.658*** (21.04)					
<i>l.mt_neg</i>		0.464*** (14.23)				
<i>l.st_neg</i>			0.244*** (8.37)			
<i>l.lt_pos</i>				0.778*** (46.81)		
<i>l.mt_pos</i>					0.581*** (17.26)	
<i>l.st_pos</i>						0.319*** (16.57)
<i>l.GARCH_opu</i>	0.0000020 (1.14)	0.00000013 (0.15)	-0.0000012** (-2.11)	-0.0000025 (-1.40)	-0.0000058*** (-4.00)	-0.00000030 (-0.13)
<i>l.ip_growth</i>	-0.214*** (-2.79)	-0.0376 (-0.57)	-0.0658 (-1.15)	0.154*** (2.63)	0.184** (2.44)	0.0543 (1.55)
<i>l.infl</i>	0.315 (1.06)	0.183 (0.85)	0.369*** (4.57)	-0.255 (-0.63)	-0.966*** (-3.36)	-0.539*** (-5.63)
<i>l.ir_diff</i>	0.00132 (0.95)	0.000345 (0.90)	0.00224*** (5.49)	-0.00175 (-1.43)	-0.00228 (-1.49)	-0.00277** (-2.32)
<i>constant</i>	0.00238*** (4.04)	0.00400*** (6.88)	0.00469*** (15.76)	0.00590*** (4.58)	0.00959*** (10.11)	0.00940*** (22.20)
# observations	3997	3997	3997	3997	3997	3997
# groups	7	7	7	7	7	7
Test for par constancy, χ^2	92.92	69.05	51.01	40.95	76.11	38.81
d.o.f	36	36	36	36	36	36
Prob.	0.0000	0.0008	0.0499	0.2621	0.0001	0.3440

Note: *l* (one-month lag); GARCH model-based oil price uncertainty (*GARCH_opu*); industrial production growth (*ip_growth*); consumer price index inflation (*infl*); interest rate difference (*ir_diff*); long-term negative bubble (*lt_neg*); medium-term negative bubble (*mt_neg*); short-term negative bubble (*st_neg*); long-term positive bubble (*lt_pos*); medium-term positive bubble (*mt_pos*); short-term positive bubble (*st_pos*) *t*-statistics (based on bootstrapped robust standard errors) in parentheses; ** $p < 0.05$, *** $p < 0.01$.

Table 4. Random Coefficient estimation predictive results for negative and positive equity bubbles due to a model-free estimate of oil price uncertainty with investor sentiment indicators: 1975:02 to 2020:05

	(1) <i>lt neg</i>	(2) <i>mt neg</i>	(3) <i>st neg</i>	(4) <i>lt pos</i>	(5) <i>mt pos</i>	(6) <i>st pos</i>
<i>l.lt_neg</i>	0.614*** (10.38)					
<i>l.mt_neg</i>		0.406*** (11.51)				
<i>l.st_neg</i>			0.243*** (7.08)			
<i>l.lt_pos</i>				0.769*** (42.92)		
<i>l.mt_pos</i>					0.569*** (15.71)	
<i>l.st_pos</i>						0.308*** (11.11)
<i>l.opu</i>	0.00492 (1.44)	0.0109*** (5.65)	0.00230** (2.03)	-0.0140*** (-3.81)	-0.0124*** (-3.56)	-0.00729*** (-2.81)
<i>l.ip_growth</i>	-0.127** (-2.09)	0.0382 (0.75)	-0.0223 (-0.55)	0.0112 (0.31)	0.0715 (0.84)	-0.0310 (-0.46)
<i>l.infl</i>	0.196 (1.09)	-0.0967 (-0.29)	0.360*** (4.12)	-0.284 (-0.58)	-1.081*** (-3.13)	-0.949*** (-5.52)
<i>l.ir_diff</i>	-0.000122 (-0.08)	-0.00186** (-2.10)	0.00106** (2.08)	-0.00163 (-0.88)	-0.000374 (-0.14)	-0.000492 (-0.19)
<i>l.cci</i>	-0.00113* (-1.89)	-0.000467 (-1.33)	-0.000131 (-0.37)	0.000551 (0.70)	0.000873 (1.19)	-0.00000175 (-0.00)
<i>l.bci</i>	0.000185 (0.71)	0.000392 (1.59)	0.0000964 (0.31)	0.000216 (0.37)	-0.000727 (-1.14)	-0.000215 (-0.45)
<i>constant</i>	0.0942** (2.00)	0.00393 (0.16)	0.00605 (0.31)	-0.0611 (-1.08)	0.00332 (0.10)	0.0367 (1.36)
<i># observations</i>	3377	3377	3377	3377	3377	3377
<i># groups</i>	7	7	7	7	7	7
<i>Test for par constancy, χ^2</i>	139.33	65.54	67.28	55.70	84.07	56.31
<i>d.o.f</i>	48	48	48	48	48	48
<i>Prob.</i>	0.0000	0.0469	0.0345	0.2077	0.0010	0.1920

Note: *l* (one-month lag); Oil price uncertainty (*opu*); industrial production growth (*ip_growth*); consumer price

index inflation (*infl*); interest rate difference (*ir_diff*); consumer confidence indicator (*cci*), business confidence indicator (*bci*); long-term negative bubble (*lt_neg*); medium-term negative bubble (*mt_neg*); short-term negative bubble (*st_neg*); long-term positive bubble (*lt_pos*); medium-term positive bubble (*mt_pos*); short-term positive bubble (*st_pos*) *t*-statistics (based on bootstrapped robust standard errors) in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Appendix A: Estimating the Multi-Scale Log-Periodic Power Law Singularity (LPPLS) Model

Filimonov and Sornette (2013) developed a stable and robust calibration scheme for the following LPPLS model given by:

$$\ln E[p(t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t)^m - \phi) \quad (\text{A1})$$

where parameter t_c represents the critical time (the date of the termination of the bubble); A is the expected log value of the observed time series, i.e., the stock price-dividend ratio, at time t_c ; B is the amplitude of the power law acceleration; and C is the relative magnitude of the log-periodic oscillations. The exponent of the power law growth is given by m , while ω and ϕ represent the frequency of the log-periodic oscillations and a phase shift parameter, respectively

We make use of this stable and robust calibration scheme, and following Filimonov and Sornette (2013), reformulate Equation (A1) to reduce the complexity of the calibration process by eliminating the nonlinear parameter ϕ and expanding the linear parameter C to $C_1 = C \cos \phi$ and $C_2 = C \sin \phi$.

The new formulation can be written as

$$\ln E[p(t)] = A + B(f) + C_1(g) + C_2(h) \quad (\text{A2})$$

where

$$f = (t_c - t)^m$$

$$g = (t_c - t)^m \cos[\omega \ln(t_c - t)]$$

$$h = (t_c - t)^m \sin[\omega \ln(t_c - t)]$$

To estimate the three nonlinear parameters: $\{t_c, m, \omega\}$, and 4 linear parameters: $\{A, B, C_1, C_2\}$, we fit Equation (A2) to the log of the price-dividend ratio. This is done by using L^2 norm to obtain the following sum of squared residuals:

$$F(t_c, m, \omega, A, B, C_1, C_2) = \sum_{i=1}^N [\ln p(\tau_i) - A - B(f_i) - C_1(g_i) - C_2(h_i)]^2 \quad (\text{A3})$$

Since the estimation of the three nonlinear parameters depends on the four linear parameters, we obtain the following cost function:

$$F(t_c, m, \omega) = \min_{A, B, C_1, C_2} F(t_c, m, \omega, A, B, C_1, C_2) = F(t_c, m, \omega, \hat{A}, \hat{B}, \hat{C}_1, \hat{C}_2) \quad (\text{A4})$$

Solving the optimization problem allows for the estimation of the four linear parameters:

$$\{\hat{A}, \hat{B}, \hat{C}_1, \hat{C}_2\} = \arg \min_{A, B, C_1, C_2} F(t_c, m, \omega, A, B, C_1, C_2) \quad (\text{A5})$$

which can be done analytically by solving the following matrix equation:

$$\begin{pmatrix} N & \sum f_i & \sum g_i & \sum h_i \\ \sum f_i & \sum f_i^2 & \sum f_i g_i & \sum f_i h_i \\ \sum g_i & \sum f_i g_i & \sum g_i^2 & \sum g_i h_i \\ \sum h_i & \sum f_i h_i & \sum g_i h_i & \sum h_i^2 \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{B} \\ \hat{C}_1 \\ \hat{C}_2 \end{pmatrix} = \begin{pmatrix} \sum \ln p_i \\ \sum f_i \ln p_i \\ \sum g_i \ln p_i \\ \sum h_i \ln p_i \end{pmatrix} \quad (\text{A6})$$

Next, the three nonlinear parameters can be determined by solving the following nonlinear optimization problem:

$$\{\hat{t}_c, \hat{m}, \hat{\omega}\} = \arg \min_{t_c, m, \omega} F(t_c, m, \omega) \quad (\text{A7})$$

We use the Sequential Least Squares Programming (SLSQP) search algorithm (Kraft, 1988) to find the best estimation of the three nonlinear parameters $\{t_c, m, \omega\}$.

The LPPLS confidence indicator, introduced by Sornette et al. (2015), is used to measure the sensitivity of bubble patterns in each country's log price-dividend ratio time series. The larger the LPPLS confidence indicator (CI), the more reliable the LPPLS bubble pattern and vice versa. It is calculated by calibrating the LPPLS model to shrinking time windows by shifting the initial observation t_1 forward in time toward the final observation t_2 with a step dt . For each LPPLS model fit, the estimated parameters are filtered against established thresholds, and the qualified fits are taken as a fraction of the total number of positive or negative fits. A positive fit has estimated $B < 0$, and a negative fit has estimated $B > 0$.

Following the work of Demirer et al. (2019), we incorporate bubbles of varying multiple time scales into this analysis. We sample the time series in steps of 5 trading days. We create the nested windows $[t_1, t_2]$ and iterate through each window in steps of 2 trading days. In this manner, we obtain a weekly resolution from which we construct the following indicators:

- Short-term bubble: A number $\in [0,1]$ which denotes the fraction of qualified fits for estimation

windows of length $dt := t_2 - t_1 \in [30:90]$ trading days per t_2 . This indicator comprises of $(90 - 30)/2 = 30$ fits.

- Medium-term bubble: A number $\in [0,1]$ which denotes the fraction of qualified fits for estimation windows of length $dt := t_2 - t_1 \in [30:90]$ trading days per t_2 . This indicator comprises of $(300 - 90)/2 = 105$ fits.
- Long-term bubble: A number $\in [0, 1]$ which denotes the fraction of qualified fits for estimation windows of length $dt := t_2 - t_1 \in [30:90]$ trading days per t_2 . This indicator comprises of $(745 - 300)/2 = 223$ fits.
- Filter conditions: After calibrating the model, the following filter conditions are applied to determine which fits are qualified:

$$m \in [0.01, 0.99]$$

$$\omega \in [2, 15]$$

$$t_c \in [\max(t_2 - 60, t_2 - 0.5(t_2 - t_1)), \min(252, t_2 + 0.5(t_2 - t_1))]$$

$$O > 2.5$$

$$D > 0.5$$

where, $O = \frac{\omega}{2\pi} \ln \left(\frac{t_c - t_1}{t_c - t_2} \right)$ is the number of oscillations, and $D = \frac{m|B|}{\omega|C|}$ captures the damping parameter required to ensure that the crash hazard rate, $h(t)$, is non-negative.

Appendix B: Random Coefficients (RC) Estimation

Traditional fixed- and random-effects models incorporate panel-specific heterogeneity by including a set of nuisance parameters that provide each panel with its own constant term. However, in these models all panels share common slope parameters – a restriction that is often less desirable as changes in independent variables may exert a heterogeneous impact on the dependent variable in question. Random-coefficients (RC) models (Swamy, 1970) are more general, allowing each panel to have a vector of randomly drawn slopes from a distribution common to all panels. According to Poi (2003), the implementation of the RC estimator ensures the best linear unbiased predictors of the panel-specific draws from this distribution.

Consider a general random-coefficients model, with y being the dependent variable and X being the predictor, of the form:

$$y_i = X_i\beta_i + \varepsilon_i \quad (\text{B1})$$

In the case of RC, each panel-specific β_i is related to an underlying common parameter vector β :

$$\beta_i = \beta + v_i \quad (\text{B2})$$

where $E\{v_i\} = 0$, $E\{v_i v_i'\} = \Sigma$, $E\{v_i v_j'\} = 0$ for $j \neq i$, and $E\{v_i \varepsilon_j'\} = 0$ for all i and j . Equations (B1) and (B2) may be combined to get:

$$\begin{aligned} y_i &= X_i(\beta + v_i) + \varepsilon_i \\ &= X_i\beta + u_i \end{aligned}$$

with $u_i \equiv X_i v_i + \varepsilon_i$. Furthermore:

$$\begin{aligned} E\{u_i u_i'\} &= E\{(X_i v_i + \varepsilon_i)(X_i v_i + \varepsilon_i)'\} \\ &= X_i \Sigma X_i' + \sigma_{ii} I \\ &\equiv \Pi_i \end{aligned}$$

The P panels can be represented in stack format:

$$y = X\beta + u \quad (\text{B3})$$

where:

$$\Pi \equiv E\{u_i u_i'\} = \begin{bmatrix} \Pi_1 & 0 & \cdots & 0 \\ 0 & \Pi_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Pi_p \end{bmatrix}$$

Estimating the parameters in Equation (B2) is a standard problem, which can be solved with generalized least squares (GLS):

$$\begin{aligned} \hat{\beta} &= (X' \Pi^{-1} X)^{-1} X' \Pi^{-1} y \\ &= \left(\sum_i X_i' \Pi_i^{-1} X_i \right)^{-1} \sum_i X_i' \Pi_i^{-1} y_i \\ &= \sum_i W_i b_i \end{aligned} \tag{B4}$$

where W_i is the Generalized Least Squares (GLS) weight and $b_i = (X_i' X_i)^{-1} X_i' y$. The resulting $\hat{\beta}$ for the overall (national) result is, therefore, a weighted average of the state-specific OLS estimates. For more details on $\hat{\beta}$ variance specification and GLS weight, refer to Poi (2003).

To obtain the state-specific $\hat{\beta}_i$ vectors, Judge et al. (1985) suggest that if attention is restricted to the class of estimators $\{\beta_i^*\}$ for which $E\{\beta_i^* | \beta_i\} = \beta_i$, then the state-specific OLS estimator b_i is appropriate. Following Green's (1997) suggested method of obtaining the variance of $\hat{\beta}_i$, it follows that $\hat{\beta}$ is both consistent and efficient, and although inefficient, b_i is also a consistent estimator of β .

Poi (2003) also suggests a test to determine whether the country-specific β_i s are significantly different from one another. The null hypothesis is stated as:

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_p \tag{B5}$$

and the test statistic is defined as:

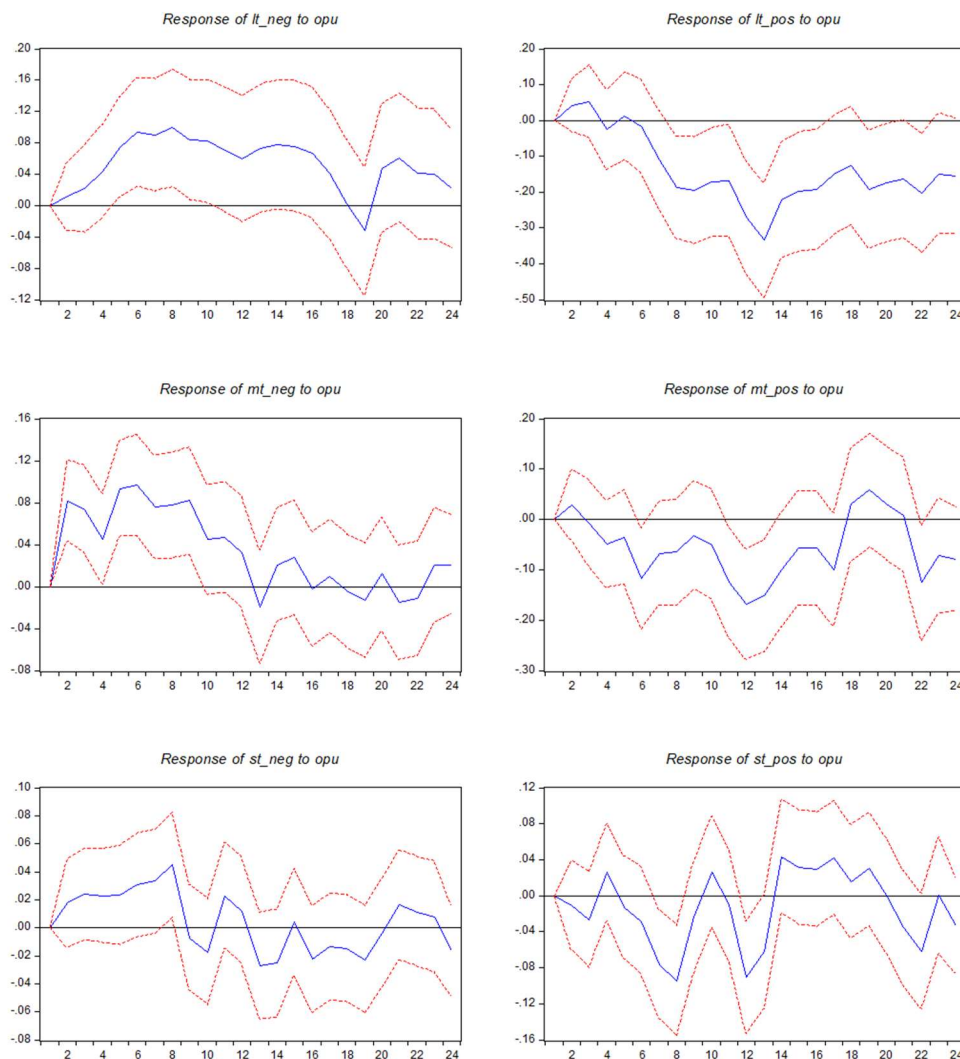
$$T \equiv \sum_{t=1}^P (b_i - \beta^\dagger)' \{ \hat{\sigma}_{ii}^{-1} (X_i X_i) \} (b_i - \beta^\dagger) \tag{B6}$$

where $\beta^\dagger = \{ \sum_{t=1}^P \hat{\sigma}_{ii}^{-1} (X_i X_i) \}^{-1} \sum_{t=1}^P \hat{\sigma}_{ii}^{-1} (X_i X_i) b_i$.

The test statistic T is distributed as χ^2 with $k(P - 1)$ degrees of freedom.

Appendix C: Additional Results

Figure C1. Impulse response functions from a panel vector autoregressive model for negative and positive equity bubbles due to a one unit shock to the model-free estimate of oil price uncertainty identified using Cholesky decomposition: 1975:02 to 2020:05



Note: The PVAR models of the G7 comprise of the variables in the following order: Oil price uncertainty (opu); industrial production growth (ip_growth); consumer price index inflation ($infl$); interest rate difference (ir_diff); long-term negative bubble (lt_neg) or medium-term negative bubble (mt_neg) or short-term negative bubble (st_neg) or long-term positive bubble (lt_pos) or medium-term positive bubble (mt_pos) or short-term positive bubble (st_pos), with the blue line showing the mean responses to a one unit shock to opu , along with the 95% confidence bands (red dotted lines),

Table C1. Random coefficient estimation of contemporaneous effects for negative and positive equity bubbles due to a model-free estimate of oil price uncertainty: 1975:02 to 2020:05

	(1)	(2)	(3)	(4)	(5)	(6)
	<i>lt_neg</i>	<i>mt_neg</i>	<i>st_neg</i>	<i>lt_pos</i>	<i>mt_pos</i>	<i>st_pos</i>
<i>l.lt_neg</i>	0.678*** (20.54)					
<i>l.mt_neg</i>		0.413*** (14.79)				
<i>l.st_neg</i>			0.248*** (8.42)			
<i>l.lt_pos</i>				0.775*** (46.19)		
<i>l.mt_pos</i>					0.582*** (17.55)	
<i>l.st_pos</i>						0.323*** (16.82)
<i>oilunc</i>	0.00549 (1.46)	0.00847*** (4.08)	0.00246* (1.84)	-0.0140*** (-7.05)	-0.0106*** (-7.25)	-0.00514*** (-3.69)
<i>ip_growth</i>	-0.244** (-2.42)	-0.164*** (-8.29)	0.0436 (1.61)	-0.0445 (-0.42)	0.122*** (3.26)	0.141* (1.89)
<i>infl</i>	0.418** (2.16)	-0.285*** (-4.19)	0.0369 (0.36)	-0.861* (-1.86)	-0.696*** (-3.35)	-0.0167 (-0.09)
<i>ir_diff</i>	0.000924 (0.91)	-0.000903 (-0.73)	0.000470 (0.64)	0.000316 (0.32)	-0.000104 (-0.06)	-0.00355*** (-3.09)
<i>constant</i>	-0.00155 (-0.61)	-0.00161 (-1.21)	0.00290*** (3.08)	0.0164*** (6.39)	0.0162*** (11.16)	0.0123*** (11.29)
<i># observations</i>	3808	3808	3808	3808	3808	3808
<i># groups</i>	7	7	7	7	7	7
<i>Test for par constancy, χ^2</i>	112.70	60.96	54.60	44..60	66.14	35.38
<i>d.o.f</i>	36	36	36	36	36	36
<i>Prob.</i>	0.0000	0.0058	0.0241	0.1538	0.0016	0.4978

Note: *l* (one-month lag); Oil price uncertainty (*opu*); industrial production growth (*ip_growth*); consumer price index inflation (*infl*); interest rate difference (*ir_diff*); long-term negative bubble (*lt_neg*); medium-term negative bubble (*mt_neg*); short-term negative bubble (*st_neg*); long-term positive bubble (*lt_pos*); medium-term positive bubble (*mt_pos*); short-term positive bubble (*st_pos*) *t*-statistics (based on bootstrapped robust standard errors) in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table C2. Random coefficient estimation predictive results for negative and positive equity bubbles due to nominal oil price returns: 1973:02 to 2020:05

	(1)	(2)	(3)	(4)	(5)	(6)
	<i>lt_neg</i>	<i>mt_neg</i>	<i>st_neg</i>	<i>lt_pos</i>	<i>mt_pos</i>	<i>st_pos</i>
<i>l.lt_neg</i>	0.658*** (21.21)					
<i>l.mt_neg</i>		0.463*** (13.80)				
<i>l.st_neg</i>			0.244*** (8.41)			
<i>l.lt_pos</i>				0.779*** (47.09)		
<i>l.mt_pos</i>					0.582*** (17.40)	
<i>l.st_pos</i>						0.321*** (16.47)
<i>l.oil_returns</i>	-0.000106 (-1.62)	-0.0000317 (-1.09)	-0.00000734 (-0.28)	-0.0000668 (-1.49)	-0.00000533 (-0.12)	-0.0000428 (-0.76)
<i>l.ip_growth</i>	-0.179** (-2.08)	-0.0234 (-0.37)	-0.0655 (-1.04)	0.172*** (3.16)	0.167** (2.40)	0.0565 (1.58)
<i>l.cpi_growth</i>	0.371 (1.19)	0.226 (1.05)	0.387*** (4.53)	-0.178 (-0.42)	-0.951*** (-2.94)	-0.522*** (-3.85)
<i>l.ir_diff</i>	0.00151 (1.13)	0.000351 (0.93)	0.00224*** (5.63)	-0.00156 (-1.31)	-0.00213 (-1.44)	-0.00265** (-2.24)
<i>constant</i>	0.00257*** (4.40)	0.00398*** (7.44)	0.00454*** (13.25)	0.00555*** (4.71)	0.00893*** (9.84)	0.00934*** (26.01)
# observations	3997	3997	3997	3997	3997	3997
# groups	7	7	7	7	7	7
Test for par constancy, χ^2	102.76	70.23	533.72	40.66	76.26	35.94
d.o.f	36	36	36	36	36	36
Prob.	0.0000	0.0006	0.0290	0.2727	0.0001	0.4716

Note: *l* (one-month lag); Oil price returns (*oil_returns*); industrial production growth (*ip_growth*); consumer price index inflation (*infl*); interest rate difference (*ir_diff*); long-term negative bubble (*lt_neg*); medium-term negative bubble (*mt_neg*); short-term negative bubble (*st_neg*); long-term positive bubble (*lt_pos*); medium-term positive bubble (*mt_pos*); short-term positive bubble (*st_pos*) *t*-statistics (based on bootstrapped robust standard errors) in parentheses; ** $p < 0.05$, *** $p < 0.01$.