University of Pretoria
Department of Economics Working Paper Series

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Working Paper: 2022-56
November 2022

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Investor Sentiment and Multi-Scale Positive and Negative Stock Market Bubbles in a Panel of G7 Countries

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Abstract

In this paper, firstly, we use the Log-Periodic Power Law Singularity Multi-Scale Confidence Indicator (LPPLS-CI) approach to detect both positive and negative bubbles in the short-, medium- and long-term of the stock market indices of the G7 countries. After detecting major crashes and booms in the seven stock markets over the monthly period of 1973:02 to 2020:09, we observe similar timing of strong (positive and negative) LPPLS-CIs across the G7 countries, suggesting synchronized extreme movements in these stock markets. Given this, secondly, we apply heterogeneous coefficients panel data-based regressions to analyze the impact of investor sentiment, proxied by business and consumer confidence indicators, on the indicators of bubbles of the G7. After controlling for the impacts of output growth, inflation, monetary policy, stock market volatility, and growth in trading volumes, we find that investor sentiment increases the positive and reduces the negative LPPLS-CIs, primarily at the medium- and long-term scales for the G7 considered all together, with the result being driven by at least five of the seven countries. Our results have important implications for both investors and policymakers, as the collapse (improvement) of investor sentiment can lead to a crash (recovery) in a bull (bear) market.

Keywords: Multi-Scale Bubbles and Crashes; Investor Sentiment; Business and consumer confidence; Panel Regressions; G7 Stock Markets

JEL Codes: C22; C32; G41

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1. Introduction

Many market participants are emotional and reactionary, and thus tend to make overly optimistic or pessimistic judgments and choices. Following the seminal contributions of Baker and Wurgler (2006, 2007), which underlines the importance of investor sentiment for movements in the US stock market, many studies (see for example, Bathia and Bredin (2013), Bathia et al. (2016), Jawadi et al., (2018), Rahman and Shamsuddin (2019)) have emerged highlighting the driving role of investor sentiment for stock market returns of the G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom (UK), and the United States (US)).

While existing studies agree that market sentiment can drive movements in stock market indices, an associated important question to ask would be how does it impact stock market bubbles? The theoretical models of Barberis et al. (1998) and Daniel et al. (1998) suggests that reversal of investor sentiment could be associated with the bursting of equity market bubbles. Given this, the only available study that tends to lend empirical support to the above-mentioned theoretical proposition, is the work of Pan (2020). It examines the relationship between US stock market bubbles and consumer confidence indexes, acting as proxies for investor sentiment, and indicates that investor sentiment positively and in a statistically significant manner affects the probability of stock bubble occurrences.

In this paper, we aim to extend the work of Pan (2020) to an international context by going beyond the US context, and considering six other advanced equity markets comprising the G7 bloc. Specifically, we analyze the impact of the metrics of business or consumer confidence on equity market bubbles of these countries over the monthly period of 1973:02 to 2020:09 in a panel data setting. The choice of the G7 is not only driven by the availability of data that allows us to cover nearly 5 decades of extreme movements in the stock markets of these developed economies, but also due to the fact that the G7 bloc accounts for nearly two-thirds of global net wealth and nearly half of world output, and hence, dynamics of bubbles in these stock markets are likely to have a worldwide spillover effects and impact the sustainability of the global financial system (Das et al., 2019). The decision to rely on panel data regressions, itis motivated by the high degree of synchronization of the indicators of the bubbles, which we discuss it in detail below, with strong evidence of connectedness in terms of investor sentiment (and speculation) within these markets also being reported in the works of Plakandaras et al. (2020), Demirer et al. (2021), and Tiwari et al. (2021). But even though we conduct the estimation in a panel setting, we allow for heterogeneous responses of bubbles to investor sentiment (and other controls) by utilizing the Random Coefficients (RC) approach of Swamy (1970) to derive both overall and country-specific results.

As far as detecting bubbles, we not only use the Log-Periodic Power Law Singularity (LPPLS) model, originally developed by Johansen et al. (1999, 2000) and Sornette (2003), for both positive (upward accelerating price followed by a crash) and negative (downward accelerating price followed by a rally) bubbles, but we also apply the multi-scale LPPLS Confidence Indicators (LPPLS-CI) of Demirer et al. (2019) to characterize positive and negative bubbles at different time scales, i.e., short-, medium- and long-term, corresponding to estimation windows associated with trading activities over one to three months, three months to a year, and one year to two years, respectively. Note that the identification of both positive and negative multi-scale bubbles is not possible based on other existing wide array of statistical tests (see, Balcilar et al. (2016), Zhang et al. (2016), and Sornette et al. (2018) for detailed reviews), which points to the suitability and added value of our applied methodology. In fact, we consider this as important because it would allow us to gauge the possible asymmetric effect of investor sentiment on the equity market bubbles of the G7, given that crash and recovery at different horizons can carry different information for market participants as suggested by the Heterogeneous Market Hypothesis (HMH; Müller et al., 1997). In this regard, it must be pointed out that the study of Pan (2020) only dealt with positive bubbles and did not involve distinction across time-scales, which makes our analysis more comprehensive not only by considering six advanced equity markets other than the US within the G7 bloc but at the US equity market context too. To the best of our knowledge, this is the first paper to analyze the effect of investor sentiment, as captured by business and consumer confidence measures, on six indicators of multi-scale positive and negative bubbles in the G7 countries based on a heterogeneous coefficients panel data model.

Our results show major crashes and booms in the G7 stock markets over the monthly sample period of 1973:02 - 2020:09. The impact of investor sentiment on bubble indicators is asymmetric, increasing the positive and reducing the negative bubbles mainly at the medium- and long-term scales, which points to the importance of the behavioural indicators of investors for the boom and bust cycles in G7 equity markets.

The remainder of the paper is organized as follows: Section 2 discusses the data and the basics of the econometric model. Section 3 presents the empirical findings involving the detection of bubbles, as well as the effects of investor sentiment on the six LPPLS-CIs of bubbles in the panel of G7 countries.

Finally, Section 4 concludes the paper.

2. Data and Econometric Model

2.1. Data

We first obtain weekly bubble indicators, with them derived based on the natural logarithmic values of the daily dividend-price ratio of the seven countries, using the dividend and the stock price index series, in their local currencies, obtained from Refinitiv Datastream. Appendix A of the paper outlines the mathematical details of how the multi-scale LPPLS CIs are obtained. The generated bubbles indicators cover the weekly period of the 1st week of (7th) January, 1973 to the 2nd week of (13th) September, 2020. Since, our controls, following Pan (2020) and Caraiani et al. (forthcoming), namely, the macroeconomic variables trading volume and (realized) volatility, besides the indicators of investor sentiment, are at monthly frequency, to obtain a monthly value for each multi-scale confidence indicators, we take the average for each of the scales weekly values that fall within a given month. Regarding the macroeconomic control variables, we use month-on-month growth of industrial production, month-month Consumer Price Index (CPI)-based inflation rate, and change in the interest rate, with all transformations to the data ensuring stationarity of the variables under consideration. As far as the interest rate variable is concerned, we use the three-month money market interest rates, merged with the Shadow Short Rate (SSR) of the individual countries (of course from 1999 onwards France, Germany, and Italy have the same values), from the time the latter became available. Data Industrial production, CPI, and the money market interest rates are all sourced from the Main Economic Indicators (MEI) database of the Organization for Economic Co-operation and Development (OECD).[1] Specifically, barring the US data, which begins in 1985:11, the SSRs of the remaining six countries are available from 1995:01. The SSRs are derived from the website of Dr. Leo Krippner.[2]

[1] https://www.oecd.org/sdd/oecdmaineconomicindicatorsmei.htm.

[2] https://www.ljkmfa.com/. Note that, the SSR estimates used in this paper are derived from the works of Krippner (2013, 2015), due to their coverage involving the G7, besides being considered an improvement over those obtained by Wu and Xia (2016) (for the Euro area, the UK and the US), as discussed in detail by Krippner (2020). The SSR is based on models of the term-structure, which essentially removes the effect that the option to invest in physical currency (at an interest rate of zero) has on yield curves, resulting in a hypothetical shadow yield curve" that would exist if the physical currency were not available. The “shadow policy rate” generated in this manner, therefore, provides a measure of the monetary policy stance after the actual policy rate reaches zero. The main advantage of the SSR is that it is not constrained by the Zero Lower Bound (ZLB), and thus allows us to combine the data from the ZLB period with that of the non-ZLB era, and in turn to use it as the common metric of monetary policy stance across the conventional and unconventional monetary policy episodes.

To capture volatility, we use the measure of realized volatility of Andersen and Bollerslev (1998), whereby we take the sum of squared of daily log-returns over a month. The trading volume is obtained from Refinitiv Datastream, and we take its month-on-month growth rates to ensure stationarity.

Finally, our main predictor, investor sentiment, is measured using the OECD standardized seasonally-adjusted survey-based Consumer Confidence Indicator (CCI) and Business Confidence Indicator (BCI),[3] with both being amplitude adjusted and having a long-term average of 100. The BCI and CCI are also obtained from the MEI of the OECD. The BCI provides leading information, based upon opinion surveys on developments in production, orders, and stocks of finished goods in the industry sector. Numbers above 100 suggest an increased confidence in near future business performance, and numbers below 100 indicate pessimism towards future performance. The CCI provides a leading indication of households' consumption and savings, based upon answers regarding their expected financial situation, their sentiment about the general economic situation, unemployment, and capability of savings. An indicator above 100 signals a boost in consumers' confidence towards the future economic situation, as a consequence of which they are less prone to save, and more inclined to spend money on major purchases in the next 12 months. Values below 100 indicate a pessimistic attitude towards future developments in the economy, possibly resulting in a tendency to save more and consume less.

Ultimately, based on data availability and transformations to ensure stationarity, our panel data-based regression covers monthly data from 1973:02 to 2020:09, and is an unbalanced panel, due to lack of data for trading volume and the investor sentiment indicators for some countries over the entire sample period.

[3] Traditionally, in the literature, two approaches have been followed to measure the latent investor sentiment (see, Zhou (2018) for a detailed discussion). The first relies on various market-based measures (for example, trading volume, closed-end fund discount, initial public offering (IPO) first-day returns, IPO volume, option implied volatilities (VIX), or mutual fund flows) as proxies for investor sentiment. The second comprises survey-based indexes (such as, AAII Investor Sentiment Survey, University of Michigan Consumer Sentiment Index just as our CCI and BCI, the UBS/GALLUP Index for Investor Optimism, or investment newsletters). We utilize the second approach (i.e. survey-based indexes) due to free availability of their data, besides being comparable as derived from the same source, and also follow Pan (2020) in this regard, who concludes that such survey-based indexes are "good proxies for investor sentiment".

2.1. Econometric Framework

To capture the effect of investor sentiment on equity market bubbles at various time scales, we specify the following panel data model:

$$eq\_bubble_{i,t}^{j} = \beta_{0i} + \beta_{1,i} is_{i,t} + \beta_{ki} Z_{i,t} + \varepsilon_{i,t} \tag{1}$$

where $eq\_bubble_{i,t}^{j} = \left\{ lt_{neg_{i,t}}, mt_{neg_{i,t}}, st_{neg_{i,t,}} lt_{pos_{i,t}}, mt_{pos_{i,t}}, st_{pos_{i,t}} \right\}, j = 1,2,..,6$ represents negative and positive equity market bubbles at short, medium and long-run time scales, which correspond to estimation windows associated with trading activities over one to three months, three months to a year, and one year to two years, respectively (see, Appendix A for further details); is_{it} is the investor sentiment indicator, which involves either the $bci_{i,t}$ or the $cci_{i,t}$ capturing business and consumer confidence indicators, respectively; while Z_{it} is a set of control variables, with

$Z_{i,t}' = \left\{ ip_{grow\ \ i,t}, cpi_{growth_{i,t}}, ir_{diff_{i,t}}, rv_{it}, tv_{growth_{i,t}} \right\}$, comprising industrial production growth, CPI inflation growth, changes in interest rates, realized volatility, and total volume growth. The β's in Eq. (1) capture the cross-section-specific (country-level) parameters, and the idiosyncratic error term ($\varepsilon_{i,t}$) is distributed with mean zero and variance $\sigma_{ii,t} I$. The model is estimated using the Random Coefficients (RC) approach, as discussed in detail in Appendix B of the paper.

3. Empirical Findings

We start by discussing each scale of the Multi-Scale LPPLS-CI values for G7 countries, and then the impact of investor sentiment measures on these indicators based on the panel data regression.

3.1. Identification of Bubbles in the G7 Countries

The short, medium and long-term indicators are displayed in different colors (green, purple and red, respectively) and the log price-to-dividend ratio is displayed in black in Figure 1. Higher LPPLS-CI values from a corresponding scale indicate the LPPLS signature is present for many of the fitting windows to which the model was calibrated.

[INSERT FIGURE 1]

We see four strong positive long-term LPPLS-CI values. The first value is observed in Canada, France, Germany, Italy, the UK and the US from 1973 to 1974. This strong indicator value preceded one of the worst global market downturns since the "Great Depression" lasting from 1973:01 through 1974:12. This crash came on the heels of the collapse of the Bretton Woods system, and the dollar devaluation from the Smithsonian Agreement. The second positive long-term LPPLS-CI value is strong, preceding "Black Monday" in 1987:10 in Canada, Japan, the UK and the US. A third positive value is observed for Canada, the UK and the US, as well as to some extent for Germany, can be made during the Asian Financial Crisis of 1997. The fourth value involves a clustering of highly positive LPPLS-CI values leading up to the Dot-com bubble burst over 2000:03 to 2002:10, especially for Canada, France, Italy, the UK and the US; but immediately following the crash, we see strong negative LPPLS-CI values, which in turn, signal booms in these countries. Regarding the negative LPPLS-CI values, while they not as many as positive LPPLS-CIs, there are strong and exist for all G7 constituents, except the US following the GFC, suggesting faster stock market recoveries in the remaining six countries.

In general, for the medium-term we observe pronounced LPPLS-CI values (positive and negative) at points where we detected the same for the long-term indicators. In addition, strong positive medium-term LPPLS-CI values are formed before strong long-term LPPLS-CI values leading up to the GFC. The short-term, LPPLS-CI produces the most signals. It can also be inferred from Figure 1 that the smallest crashes/booms are signaled from this short-term scale, possibly due to it picking up idiosyncratic signals. However, we still can see small corrections immediately following a strong short-term LPPLS-CI value. It is also interesting to notice, just as with the medium-term indicators preceding the long-term indicators, that short-term indicators tend to lead medium-term ones, in the context of the major bubble dates identified by the medium- and long-run indicators discussed above. This adds support to the finding from Demirer et al., (2019) that the maturation of the bubble towards instability is present across several distinct time-scales.

Note that, besides the crises episodes discussed above, these indicators in general also show spikes associated with crashes and recoveries before and around the European sovereign debt crisis over 2009 to 2012, the "Brexit" in 2016, and to some extent the COVID-19 outbreak as well, especially for the US involving the positive bubble indicator.

As we observe similar timing of strong (positive and negative) LPPLS-CI values across the G7, i.e., synchronized boom and bust cycles of the seven developed equity markets, this in turn motivates the use of a panel-based approach to analyze the impact of investor sentiments on stock market bubbles.

3.2. Effect of Investor Sentiment on Bubbles

In this section, the Random Coefficient (Swamy, 1970) estimation results for Equation (1) for all countries combined, as well as the country-specific results of the effect of investor sentiment on equity market bubbles, are reported.

We model the contemporaneous impact of investor sentiment on equity market bubbles, given that application of the Hausman (1978) test for endogeneity suggests that business and consumer confidence and control variables are exogenous to the specification, with complete details of these results available upon request from the authors. The impact of *bci* and *cci* on negative equity market bubbles across the three time-scales is presented in Table 1, while the same for the multi-scale positive bubble indicators is reported in Table 2.

[INSERT TABLES 1 AND 2 HERE]

From Table 1, it is evident that both *bci* and *cci* exert a negative and statistically significant impact on negative equity market bubbles, primarily in the medium and long-term scales. The impact of *bci* and *cci* on short-term negative equity market bubbles is also negative, but this impact is not statistically significant. Furthermore, when considering the impact of the two investor sentiment indicators on positive equity market bubbles, we note that the impact of both business and consumer sentiments is positive and statistically significant on the positive equity market bubbles, but again it is restricted to the medium and long-term. The impact of *bci* and *cci* on positive equity market bubbles is positive in the short-term, but not statistically significant.

Intuitively, these findings make sense, when we recall that, a positive bubble indicator signals rapid growth in the stock markets before the crash, while the negative bubble indicator captures the recovery following a decline. Specifically, we find that, higher values of investor sentiments tend to increase the positive LPPLS-CI, while the same reduces the corresponding negative indicators. This is understandable as strong investor sentiments cause the market to grow fast before the crash, and in the same vein when the market is declining, the rebound occurs quicker. Even though Pan (2020) does not identify negative bubbles, our evidence is in line with the findings of the author that investor sentiments enhance the likelihood of the occurrence of (positive) stock market bubbles. Furthermore, with the long- and medium-term scales based on larger calibration time-period compared to the short-run LPPLS-CI, the former two scales tend to be relatively less idiosyncratic, as outlined in the preceding sub-section. With the behavioral variables significantly impacting the long- and medium-term LPPLS-CIs, the evidence suggests that investor sentiment is associated with deeper equity market crashes and recoveries − thus, making investor sentiment an important driver of the boom-bust cycles in the G7 equity markets. Interestingly, the *bci* has a stronger impact than the *cci* for the medium-term bubble indicators, at (least at) the 5% level of significance, while the reverse is true for the long-term bubble indicators. With the medium-term LPPLS-CIs leading the long-run indicators, the importance of business-related sentiment becomes comparatively more important, with consumer confidence making these effects stronger in the long-term. Finally, in general, the absolute values of the coefficients of the investor sentiment variables reveal a stronger effect on the positive bubble indicators compared to the negative ones. This implies that, higher investor sentiment can indeed instigate recovery when markets are down, but when markets are booming, the crash effect becomes more powerful.[4]

As far as the effects from the other controls, besides sporadic impact from output growth, inflation, and interest rate changes, we detect strong associations for realized volatility and growth in trading volume. In line with Pan (2020), particularly realized volatility negatively (positively) impacts the positive (negative) LPPLS-CI indicator, while trading volume growth has the reverse effect on the generation of bubbles.

[INSERT TABLES 3 AND 4 HERE]

We next turn to country-specific results for the sample of the G7 economies to understand the drivers of the overall results. Table 3 presents the results for the impact of *bci* and *cci* on negative equity market bubbles at the short-, medium-, and long-term scales, while Table 4 reports the results of the impact of those two alternative metrics of investor sentiment on the positive equity market bubbles indicators across the three time-scales.

For the negative LPPLS-CI, we observe negative and significant effects from the *bci* for France,

[4] Since the bubble indicators are originally at daily frequency, and a measure of daily global economic sentiment is available namely, Societe Generale (SG) Global Sentiment Index (SGGSI; https://sg-global-sentiment.com/) from 11th March of 2002, we utilized the extracted first Principal Component (PC) for each of the six bubble indicators across the G7, and then estimated Ordinary Least Squares (OLS) regressions relating the six PCs with the SGGSI, which was detrended linearly to make it stationary. We found that, short- and medium-term PCs of the negative LPPLS-CIs were negatively impacted by the SGGSI in a statistically significant manner (with coefficients -0.277 and -0.350 of at 1% levels), and PC of the short-term positive indicators were positively driven by SGGSI in a statistically significant fashion (with a coefficient of 2.065 at 1% level). In essence, investor sentiment positively impact positive bubbles and reduces the negative ones. Further details are available upon request from the authors.

Japan at the long-term scale, Italy at both medium- and long-term, and the US at the medium-term. Interestingly, the UK shows a counter-intuitive positive impact from *bci* at the long run. As far as *cci* is concerned under negative bubbles, the main impact is from France, Germany, Japan, the UK and the US, with effects at the long- and short-term, long- and medium-term, all the three time-scales, long-term, and long-and medium-term, respectively. In other words, in line with the overall result, the most significant impact of *bci* and *cci* is observed for the long- and medium-term scales, though some effects are also observed at the shortterm under the latter for France and Japan. In sum, 5 countries (except Canada, and Germany or Italy), out of the G7 bloc are affected by *bci* and *cci* (respectively).

For the positive LPPLS-CI, *bci* impacts the long-term scale only of Canada and Germany, but both medium- and long-run indicators of France, Japan, the UK and the US. As far as *cci* is concerned, a significant effect is captured for Canada for the long-term scale only, at medium- and long-term for Germany and the UK, and at all the scales for Japan (just as under the negative bubbles indicator). For France, a positive and significant effect from the *cci* is registered under the long-term scale, but a contradictory negative impact is detected in the short-term. Again, as with the negative bubbles case, *bci* and *cci* tend to affect the medium- and long-term scales, shaping the overall impact of the G7 countries for the positive bubbles too. Overall, *bci* drives positive bubbles in 6 (except Italy) countries, while *cci* does so in 5 (except Italy and the US) countries of the G7.

To put our findings for the US into comparison with those of Pan (2020), we find that consumer confidence has a significantly negative impact on long- and medium-term negative equity market bubbles, but no impact is detected for the positive bubbles from the *cci*. However, business confidence has a pronounced positive impact on long and medium-term positive equity market bubbles – a finding we cannot compare to Pan (2020), as the author only concentrated on alternative measures of *cci*. Despite this discussion, it is worth noting that a one-to-one correspondence between our findings and that of Pan (2020) is not possible due to different methods of detecting bubbles, the sample period, the underlying data, and model employed.[5]

[5] With a measure of daily economic sentiment available for the US dating back to 1st January of 1980, as developed by Shapiro et al. (2020), we ran OLS regressions to capture the effect of this metric of economic sentiment on the corresponding six daily LPPLS-CIs (of the US). We found that, the medium- and long-term negative indicators were statistically significantly affected in a negative manner (with coefficients of -0.020 and -0.008 at 1% level), while the medium-term positive LPPLS-CI was affected positively in a statistically significant way (with a coefficient of 0.002 at the 1% level). But the long-term indicator was found to be negatively affected, with the coefficient (-0.013) being significant at the 1% level. As far as the PCs in Footnote 4 is concerned, this sentiment indicator negatively impacted

In general, the majority of country-specific results, albeit with some degree of heterogeneity, tend to confirm the overall findings that investor sentiment drives the medium and long-term scales of the LPPLS-CIs, with relatively stronger (absolute) effects for the positive bubbles compared to the negative ones.

4. Conclusion

The primary objective of our paper is to analyze the impact of investor sentiment, as captured by business and consumer confidence indicators on equity market bubbles of the G7 countries. In the first step, we detect positive and negative bubbles in the short-, medium-and long-run for the G7 equity markets using the Multi-Scale Confidence Indicator approach. Our findings reveal major crashes and booms in the seven stock markets over the monthly period of 1973:02 to 2020:09. We also observe similar timing of strong (positive and negative) LPPLS indicator values across the G7 countries, suggesting commonality in the boom-bust cycles of these equity markets. In other words, diversification of investor portfolios across advanced equity markets is not a possibility for the market agents across investment horizons during both booms and crashes. In the second step, due to of the detected evidence of synchronicity in the bubble indicators across the G7, we use a panel data-based regression characterized by heterogeneous response to investor sentiment to study the overall and country-specific impact of business and consumer confidence indictors. After controlling for the impacts of output growth, inflation, monetary policy, stock market volatility, and growth in trading volumes, we find that the behavioral variables increase the positive and reduces the negative LPPLS-CIs primarily at the medium- and long-term scales for the G7 countries considered all together. Notably, the significant effects on the relatively longer time-scales is an important finding, as the medium- and long-run LPPLS-CIs are observed to be highly reliable when it comes to detecting severe crashes and strong recoveries on the stock markets. At the country-level, while there is some minor degree of heterogeneity, we find that, barring Canada under negative bubbles, at least one metric of sentiment, associated with businesses or consumers, strongly predicts crashes and/or recoveries in all of cases

the PCs of the medium- and long-term negative LPPLS-CIs (with coefficients of -2.182 and -0.876), with the effect being statistically significant at the 1% level, while the corresponding effects on the PCs of the short-, medium- and long-term positive LPPLS-CIs were positive (0.415), positive (0.249), and negative (-1.135) with statistical significance holding at 1%, 5% and 1% respectively. In general, the relationship between investor sentiment and positive bubbles is positive, while, it is negatively related to negative bubbles. Further details of these results are available upon request from the authors.

considered.

With investor sentiment showing up as having strong positive effects on positive bubbles, compared to other traditional macroeconomic and financial, it is recommended that investors and policymakers should be careful when the level of investor sentiment tends to peak at the time the stock markets are booming, because this could imply an imminent market crash. At the same time, when stock prices are declining, then higher investor sentiment can help revive the market quickly. Accordingly, policymakers should implement policies that keep investor sentiment in check during bullish-regimes of the G7 equity markets, but boost the same when a bearish-phase is underway. With contractionary and expansionary monetary policies known to impact stock markets and investor sentiment in similar direction (Çepni and Gupta, 2021; Çepni et al., 2021), the role of the state-contingent interest rate decisions of the central banks becomes of paramount importance. Having said this, in spite of the high-degree of similar movements in the bubble indicators, due to the underlying heterogeneous impact of investor sentiment, the policy authorities should be designing country-specific monetary policy responses.

As part of future research, it would be interesting to extend our analysis to emerging markets, contingent on the availability of consistent data on measures of investor sentiment. In addition, sentiment of the monetary policy committees about the state of the macroeconomy and the financial system can also be investigated in driving stock market bubbles, given the recent evidence provided by Gardner et al. (2022) that the sentiment conveyed by the Federal Open Market Committee (FOMC) statements has a significant effect on the US stock market.[6]

[6] In fact, using the event-based FOMC sentiment data of this study starting in 2000:02 (and available for meeting dates), OLS estimation suggested that perceptions about inflation, output and the labor market had a positive and significant effect (with coefficients of 0.063, 0.188, and 0.111 at 10%, 10% and 1% levels respectively) on the positive long-term LPPLS-CI of the US, while, the latter two negatively impacted the negative medium-term LPPLS-CI in a statistically significant fashion (with coefficients of -0.019 and -0.016 at the 5% level in both cases). Interestingly, FOMC's sentiment about the monetary and financial conditions were not found to carry significant impacts. Further details of these results are available upon request from the authors.

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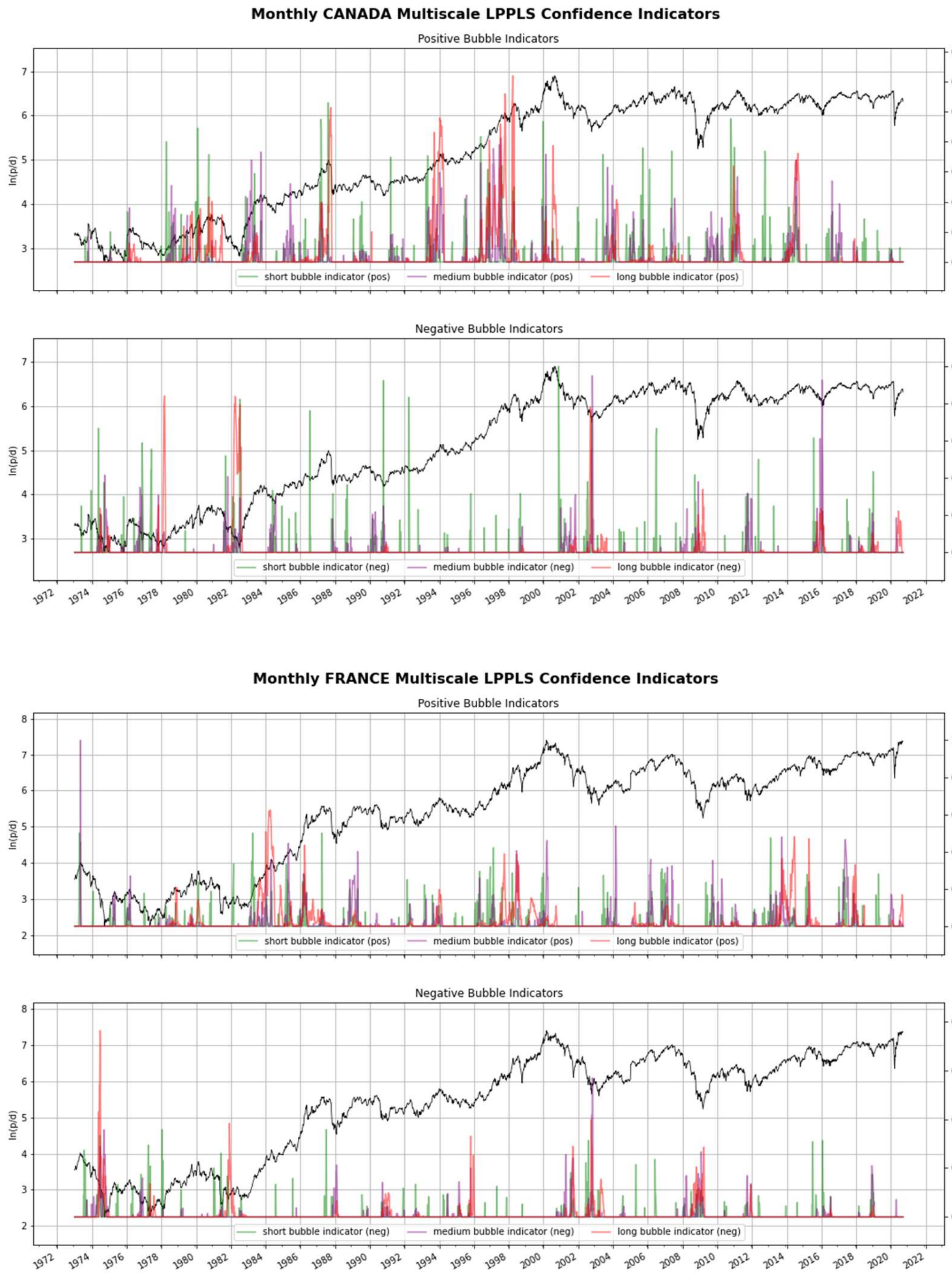
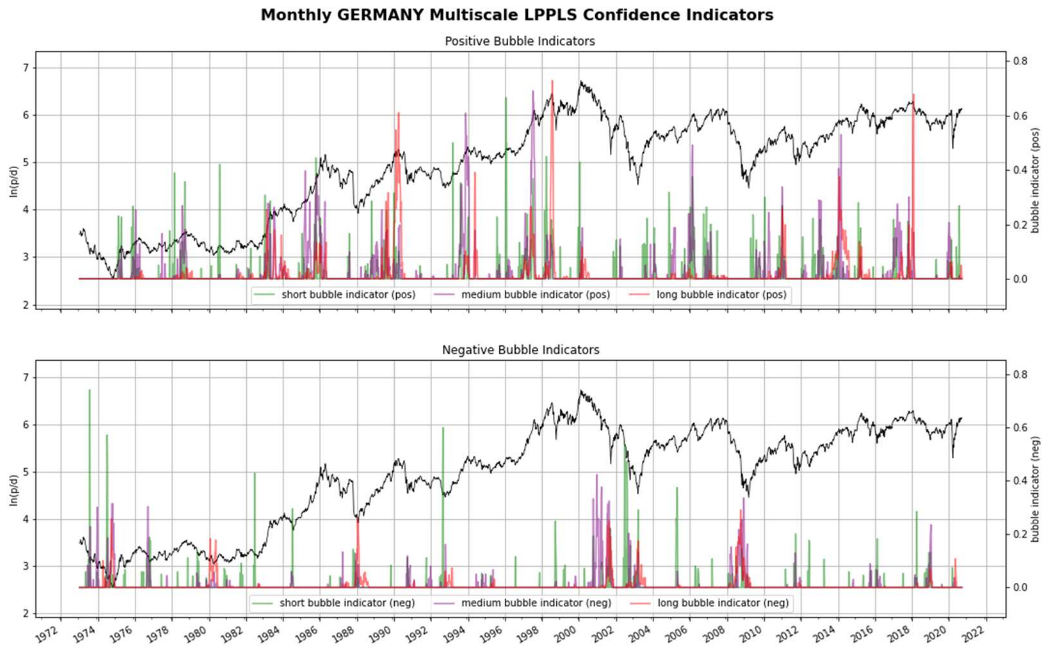
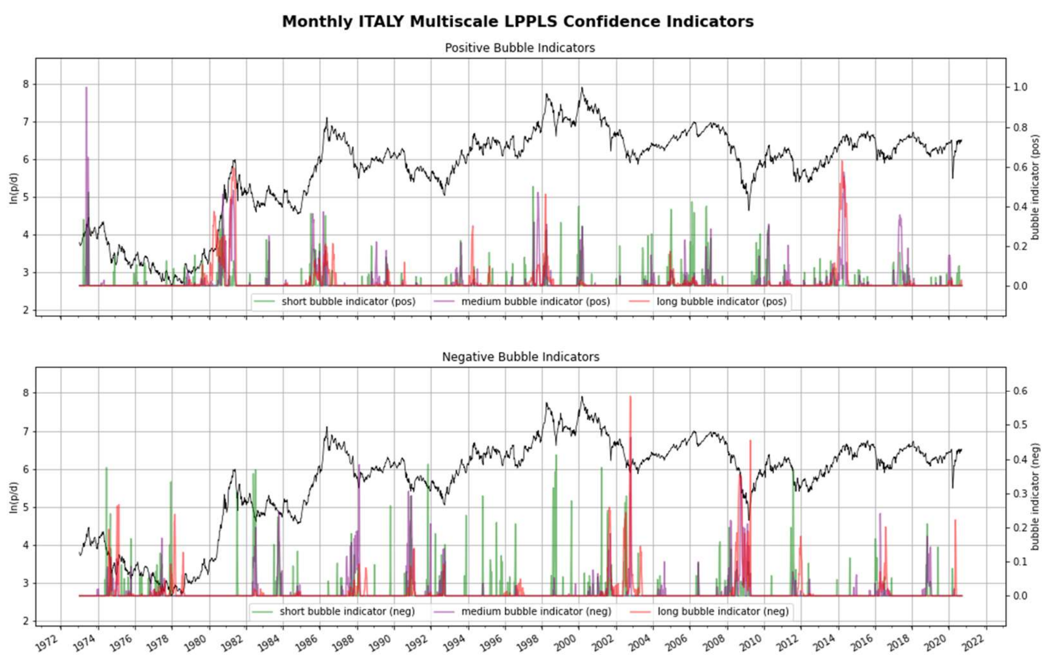
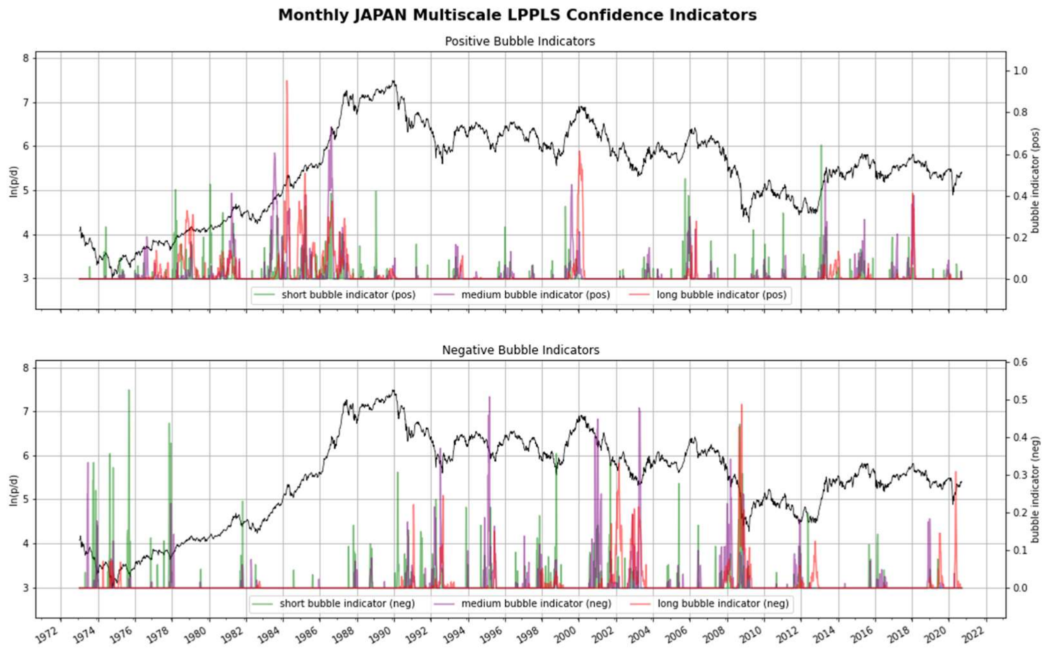
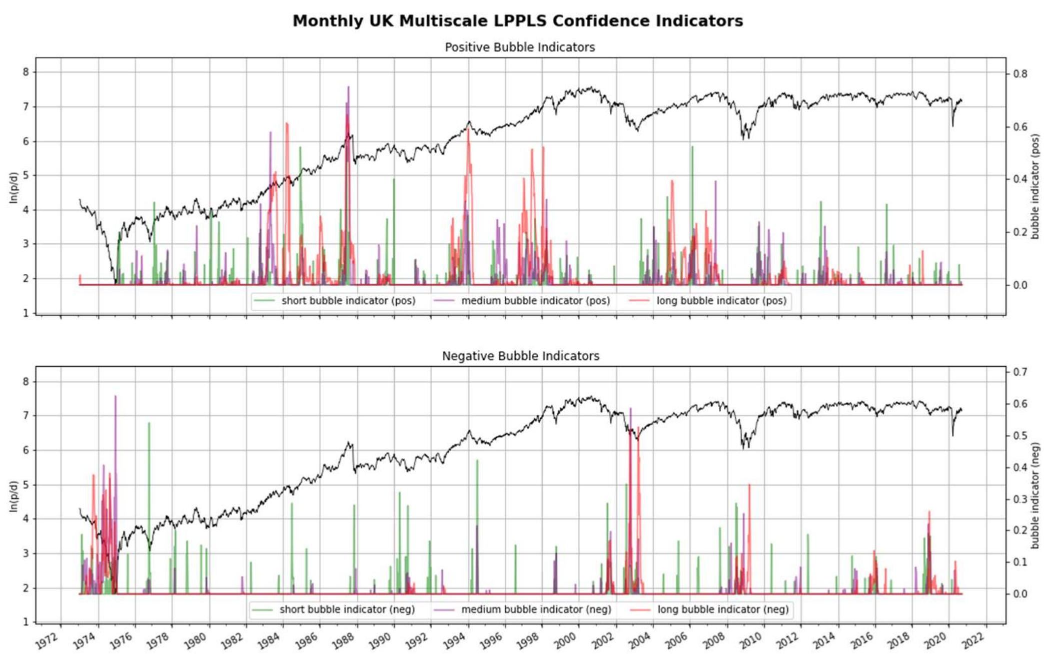
Figure 1. Monthly Multi-Scale LPPLS-CIs of the G7 Countries

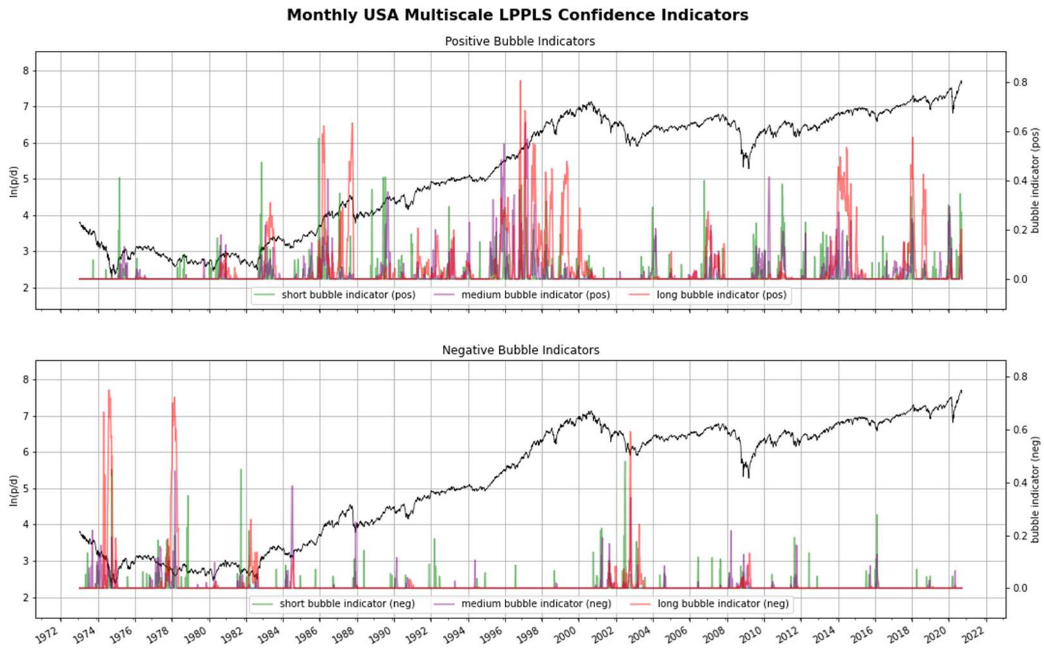



Monthly GERMANY Multiscale LPPLS Confidence Indicators
Positive Bubble Indicators
Negative Bubble Indicators
ln(p/d)
bubble indicator (pos)
bubble indicator (neg)
short bubble indicator (pos)
medium bubble indicator (pos)
long bubble indicator (pos)
short bubble indicator (neg)
medium bubble indicator (neg)
long bubble indicator (neg)


Monthly ITALY Multiscale LPPLS Confidence Indicators
Positive Bubble Indicators
Negative Bubble Indicators
ln(p/d)
bubble indicator (pos)
bubble indicator (neg)
short bubble indicator (pos)
medium bubble indicator (pos)
long bubble indicator (pos)
short bubble indicator (neg)
medium bubble indicator (neg)
long bubble indicator (neg)


Monthly JAPAN Multiscale LPPLS Confidence Indicators
Positive Bubble Indicators
ln(p/d)
bubble indicator (pos)
short bubble indicator (pos)
medium bubble indicator (pos)
long bubble indicator (pos)
Negative Bubble Indicators
bubble indicator (neg)
short bubble indicator (neg)
medium bubble indicator (neg)
long bubble indicator (neg)


Monthly UK Multiscale LPPLS Confidence Indicators
Positive Bubble Indicators
ln(p/d)
bubble indicator (pos)
short bubble indicator (pos)
medium bubble indicator (pos)
long bubble indicator (pos)
Negative Bubble Indicators
bubble indicator (neg)
short bubble indicator (neg)
medium bubble indicator (neg)
long bubble indicator (neg)



Monthly USA Multiscale LPPLS Confidence Indicators
Positive Bubble Indicators
ln(p/d)
bubble indicator (pos)
short bubble indicator (pos)
medium bubble indicator (pos)
long bubble indicator (pos)
Negative Bubble Indicators
ln(p/d)
bubble indicator (neg)
short bubble indicator (neg)
medium bubble indicator (neg)
long bubble indicator (neg)

Table 1. Random Coefficient (RC) estimation results for negative equity bubbles: 1973:02 to 2020:09

	(1) lt_{neg}	(2) mt_{neg}	(3) st_{neg}	(4) lt_{neg}	(5) mt_{neg}	(6) st_{neg}
bci	-0.00215*	-0.000600**	0.000207			
	(-1.69)	(-1.98)	(0.74)			
cci				-0.00344**	-0.00115*	-0.000325
				(-2.20)	(-1.76)	(-1.10)
ip_{growth}	-0.174	-0.104	0.0774**	-0.123	-0.131	0.0817***
	(-1.30)	(-1.09)	(2.36)	(-1.12)	(-1.64)	(2.74)
cpi_{growth}	-0.457	-0.315	0.115	-0.734**	-0.329	0.161
	(-1.36)	(-0.69)	(0.57)	(-2.31)	(-0.82)	(0.90)
ir_{diff}	-0.000430	-0.00517	-0.00200	0.000655	-0.00287	-0.00125
	(-0.31)	(-1.07)	(-1.30)	(0.38)	(-0.84)	(-0.96)
rv	1.759***	1.694***	0.829***	1.756**	1.672***	0.785***
	(2.68)	(3.42)	(3.71)	(2.48)	(3.30)	(3.54)
tv_{growth}	-0.00395	-0.0190**	0.00477	-0.00145	-0.0158**	0.00821**
	(-0.67)	(-2.52)	(1.29)	(-0.30)	(-2.21)	(2.04)
constant	0.218*	0.0624**	-0.0180	0.346**	0.117*	0.0355
	(1.70)	(2.06)	(-0.65)	(2.21)	(1.78)	(1.23)
#observations	1720	1720	1720	1873	1873	1873
# grroups	7	7	7	7	7	7
Test for par constancy χ^2	400.22	174.52	75.96	452.64	193.74	81.03
d.o.f	42	42	42	42	42	42
Prob.	0.0000	0.000	0.0010	0.0000	0.0000	0.0003

Note: Business confidence indicator (*bci*); consumer confidence indicator (*cci*); industrial production growth (ip_{growth}); consumer price index growth (cpi_{growth}); interest rate difference (ir_{diff}); realized volatility (*rv*); total volume growth (tv_{growth}); long-term negative bubble (lt_{neg}); medium-term negative bubble (mt_{neg}); short-term negative bubble (st_{neg}); *t*-statistics (based on bootstrapped robust standard errors) in parentheses; * p<0.10, ** p<0.05, *** p<0.01.

Table 2. RC estimation results for positive equity bubbles: 1973:02 to 2020:09

	(1) lt_{pos}	(2) mt_{pos}	(3) st_{pos}	(4) lt_{pos}	(5) mt_{pos}	(6) st_{pos}
bci	0.0132*	0.00281***	0.000163			
	(1.79)	(3.49)	(0.26)			
cci				0.00369**	0.00152*	-0.000310
				(2.16)	(1.89)	(-0.30)
ip_{growth}	-0.0799	0.0533	0.0394	0.452	0.153	0.0741
	(-0.59)	(0.87)	(0.51)	(1.06)	(1.14)	(0.65)
cpi_{growth}	-1.293	0.216	1.070	-1.583	0.00218	0.929
	(-1.08)	(0.21)	(1.15)	(-1.01)	(0.00)	(0.98)
ir_{diff}	0.00816	0.0153**	0.00216	0.0142	0.0150**	0.000248
	(1.20)	(2.13)	(0.49)	(1.49)	(2.09)	(0.06)
rv	-0.0954	-0.339**	-0.650***	-0.125	-0.386**	-0.642***
	(-0.38)	(-2.24)	(-4.10)	(-0.65)	(-2.30)	(-4.91)
tv_{growth}	-0.00185	0.00525	0.0194**	-0.00308	0.00129	0.0190**
	(-0.25)	(0.99)	(2.47)	(-0.35)	(0.24)	(2.55)
constant	-1.297*	-0.264***	-0.00176	-0.343**	-0.134*	0.0461
	(-1.77)	(-3.28)	(-0.03)	(-1.98)	(-1.65)	(0.44)
# observations	1720	1720	1720	1873	1873	1873
# groups	7	7	7	7	7	7
Test for par constancy χ^2	109.59	51.95	50.57	122.03	59.65	62.30
d.o.f	42	42	42	42	42	42
Prob.	0.0000	0.1397	0.1711	0.0000	0.0377	0.0225

Note: Business confidence indicator (*bci*); consumer confidence indicator (*cci*); industrial production growth (ip_{growth}); consumer price index growth (cpi_{growth}); interest rate difference (ir_{diff}); realized volatility (*rv*); total volume growth (tv_{growth}); long-term positive bubble (lt_{pos}); medium-term positive bubble (mt_{pos}); short-term positive bubble (st_{pos}); *t*-statistics (based on bootstrapped robust standard errors) in parentheses; * $p<0.10$, ** $p<0.05$, *** $p<0.01$.

Table 3. RC estimation results: Country-specific impact of sentiment on negative equity market bubbles: 1973:02 to 2020:09

	Investor Sentiment	lt_{neg}	mt_{neg}	st_{neg}
Canada	*bci*	-0.00164	-0.00037	0.00079
		(-1.39)	(-0.57)	(1.27)
	cci	-0.00068	-0.00138	-0.00052
		(-0.74)	(-1.49)	(-1.05)
France	*bci*	-0.00276**	-0.00041	-0.00029
		(-2.31)	(-0.65)	(-0.58)
	cci	-0.00708***	-0.00042	-0.00100*
		(-3.71)	(-0.37)	(-1.89)
Germany	*bci*	-0.00135	0.00053	0.00053
		(-1.15)	(0.84)	(0.82)
	cci	-0.00360***	-0.00234**	-0.00018
		(-3.02)	(-2.28)	(-0.33)
Italy	*bci*	-0.00346**	-0.00141**	0.00051
		(-2.34)	(-2.45)	(0.83)
	cci	-0.00011	0.00111	0.00100
		(-0.09)	(1.25)	(2.07)
Japan	*bci*	-0.00807***	-0.00089	0.00044
		(4.37)	(-1.34)	(0.69)
	cci	-0.01074***	-0.00273**	-0.00103**
		(-6.74)	(-2.33)	(-2.00)
UK	*bci*	0.00259**	-0.00040	0.00027
		(2.24)	(-0.72)	(0.55)
	cci	-0.00193*	0.00030	-0.00023
		(-1.79)	(0.47)	(-0.53)
US	*bci*	-0.00004	-0.00134**	-0.00062
		(-0.71)	(-2.43)	(-0.98)
	cci	-0.00007**	-0.00228***	-0.00035
		(-2.23)	(-3.16)	(-0.79)

Note: Business confidence indicator (*bci*); consumer confidence indicator (*cci*); long-term negative bubble (lt_{neg}); medium-term negative bubble (mt_{neg}); short-term negative bubble (st_{neg}); *t*-statistics (based on bootstrapped robust standard errors) in parentheses; * p<0.10, ** p<0.05, *** p<0.01.

Table 4. RC estimation results: Country-specific impact of sentiment on positive equity market bubbles: 1973:02 to 2020:09

	Investor Sentiment	lt_{pos}	mt_{pos}	st_{pos}
Canada	*bci*	0.00640***	0.00042	0.00127
		(2.71)	(0.30)	(1.19)
	cci	0.00454*	0.00037	0.00128
		(1.67)	(0.32)	(0.97)
France	*bci*	0.00463**	0.00300*	-0.00062
		(2.22)	(1.91)	(-0.65)
	cci	0.00600**	0.00156	-0.00432***
		(2.14)	(1.23)	(-2.81)
Germany	*bci*	0.00774***	0.00184	-0.00077
		(2.98)	(1.12)	(-0.72)
	cci	0.00934***	0.00220*	-0.00177
		(4.16)	(1.68)	(-1.34)
Italy	*bci*	0.00237	0.00383***	0.00096
		(0.93)	(2.66)	(1.09)
	cci	-0.00191	-0.00084	0.00080
		(-0.99)	(-0.74)	(0.78)
Japan	*bci*	0.00513***	0.00262*	-0.00149
		(2.62)	(1.79)	(-1.40)
	cci	0.00320**	0.00374***	0.00250**
		(2.14)	(3.16)	(1.97)
UK	*bci*	0.00700***	0.00295**	0.00070
		(2.87)	(2.22)	(0.91)
	cci	0.00560**	0.00196*	0.00036
		(2.55)	(1.72)	(0.45)
US	*bci*	0.05665***	0.00489***	0.00101
		(5.29)	(2.92)	(0.94)
	cci	0.00047	0.00175	-0.00071
		(0.15)	(1.41)	(-0.42)

Note: Business confidence indicator (*bci*); consumer confidence indicator (*cci*); long-term positive bubble (lt_{pos}); medium-term positive bubble (mt_{pos}); short-term positive bubble (st_{pos}); *t*-statistics (based on bootstrapped robust standard errors) in parentheses; * p<0.10, ** p<0.05, *** p<0.01.

Appendix A: Estimating the Multi-Scale e Log-Periodic Power Law Singularity (LPPLS) Model

Given the LPPLS model as follows, we use the stable and robust calibration scheme developed by Filimonov and Sornette (2013):

$$\ln E[p(t)] = A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t)^m - \phi) \tag{A1}$$

The parameter t_c represents the critical time (the date of the termination of the bubble). A is the expected log value of the observed time-series, i.e., the stock price-dividend ratio, at time t_c. B is the amplitude of the power law acceleration. C is the relative magnitude of the log-periodic oscillations. The exponent of the power law growth is given by m. The frequency of the log-periodic oscillations is given by ω and ϕ represents a phase shift parameter.

Following Filimonov and Sornette (2013), equation (A1) is reformulated so as to reduce the complexity of the calibration process by eliminating the nonlinear parameter ϕ and expanding the linear parameter C to be $C_1 = C \ \cos\phi$ and $C_2 \ = \ C \ \cos\phi$.

The new formulation can be written as

$$\ln E[p(t)] = A + B(f) + C_1(g) + C_2(h) \tag{A2}$$

where

$$f = (t_c - t)^m$$

$$g = (t_c - t)^m \cos[\omega \ln(t_C - t)]$$

$$h = (t_c - t)^m \sin[\omega \ln(t_c - t)]$$

To estimate the 3 nonlinear parameters: $\{t_c, m, \omega\}$, and 4 linear parameters: $\{A, B, C_1, C_2\}$, we fit equation (A2) to the log of the price-dividend ratio. This is done by using L^2 norm to obtain the following sum of squared residuals:

$$F(t_c, m, \omega, A, B, C_1, C_2) = \sum_{i=1}^{N} \left[\ln p(\tau_i) - A - B(f_i) - C_1(g_i) - C_{2(h_i)}\right]^2 \tag{A3}$$

Since the estimation of the 3 nonlinear parameters depend on the four linear parameters, we have the following cost function:

$$F(t_c, m, \omega) = \min_{A,B,C_1,C_2} F(t_c, m, \omega, A, B, C_1, C_2) = F\left(t_c, m, \omega, \hat{A}, \hat{B}, \hat{C}_1, \hat{C}_2\right) \tag{A4}$$

The 4 linear parameters are estimated by solving the optimization problem:

$$\{\hat{A}, \hat{B}, \hat{C}_1, \hat{C}_2\} = \arg \min_{A,B,C_1,C_2} F(t_c, m, \omega, A, B, C_1, C_2) \tag{A5}$$

which can be done analytically by solving the following matrix equation:

$$\begin{pmatrix} N & \sum f_i & \sum g_i & \sum h_i \\ \sum f_i & \sum f_i^2 & \sum f_i g_i & \sum f_i h_i \\ \sum g_i & \sum f_i g_i & \sum g_i^2 & \sum g_i h_i \\ \sum h_i & \sum f_i h_i & \sum g_i h_i & \sum h_i^2 \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{B} \\ \hat{C}_1 \\ \hat{C}_2 \end{pmatrix} = \begin{pmatrix} \sum \ln p_i \\ \sum \mathrm{f_i} \ln p_i \\ \sum \mathrm{g_i} \ln p_i \\ \sum \mathrm{h_i} \ln p_i \end{pmatrix} \tag{A6}$$

Next, the 3 nonlinear parameters can be determined by solving the following nonlinear optimization problem:

$$\{\hat{t}_c, \widehat{m}, \widehat{\omega}\} = \arg \min_{t_c,m,\omega} F(t_c, m, \omega) \tag{A7}$$

We use the Sequential Least Squares Programming (SLSQP) search algorithm (Kraft, 1988) to find the best estimation of the three nonlinear parameters $\{t_c, m, \omega\}$.

The LPPLS confidence indicator, introduced by Sornette et al. (2015), issued to measure the sensitivity of bubble patterns in the log price-dividend ratio time series of each country. The larger the LPPLS confidence indicator (CI), the more reliable the LPPLS bubble pattern and vice versa. It is calculated by calibrating the LPPLS model to shrinking time windows by shifting the initial observation t_1 forward in time towards the final observation t_2 with a step dt. For each LPPLS model fit, the estimated parameters are filtered against established thresholds and the qualified fits are taken as a fraction of the total number of positive or negative fits. A positive fit has estimated $B < 0$ and a negative fit has estimated $B > 0$.

Following the work of Demirer et al. (2019), we incorporate bubbles of varying multiple time-scales into this analysis. We sample the time series in steps of 5 trading days. We create the nested windows $[t_1, t_2]$ and iterate through each window in steps of 2 trading days. In this manner, we obtain a weekly resolution from which we construct the following indicators:

- Short-term bubble: A number $\in [0,1]$ which denotes the fraction of qualified fits for estimation windows of length $dt := t_2 - t_1 \in [30:90]$ trading days per t_2. This indicator is comprised of $(90 - 30)/2 = 30$ fits.
- Medium-term bubble: A number $\in [0,1]$ which denotes the fraction of qualified fits for estimation windows of length $dt := t_2 - t_1 \in [30:90]$ trading days per t_2. This indicator is comprised of $(300 - 90)/2 = 105$ fits.
- Long-term bubble: A number $\in$ [0, 1] which denotes the fraction of qualified fits for estimation windows of length $dt := t_2 - t_1 \in [30:90]$ trading days per t_2. This indicator is comprised of $(745 - 300)/2 = 223$ fits.

Filter Conditions: After calibrating the model, the following filter conditions are applied to determine which fits are qualified:

$$m \in [0.01, 0.99]$$

$$\omega \in [2, 15]$$

$$t_c \in [max(t_2 - 60, t_2 - 0.5(t_2 - t_1)), min(252, t_2 + 0.5(t_2 - t_1))]$$

$$O > 2.5$$

$$D > 0.5$$

where

$$O = \frac{\omega}{2\pi} \ln\left(\frac{t_c - t_1}{t_c - t_2}\right)$$

$$D = \frac{m|B|}{\omega|C|}$$

Appendix B: Random Coefficients (RC) Estimation

Fixed- and random-effects models incorporate panel-specific heterogeneity by including a set of nuisance parameters that provide each panel with its own constant term. However, all panels share common slope parameters, which is undesirable in the current context. Random-coefficients (RC) models (Swamy, 1970) are more general, allowing each panel to have its vector of randomly drawn slopes from a distribution common to all panels. The implementation of the estimator ensures the best linear unbiased predictors of the panel-specific draws from said distribution (Poi, 2003).

Consider a general random-coefficients model, with y being the dependent variable and X being the predictor, of the form:

$$y_i = X_i\beta_i + \varepsilon_i \tag{B1}$$

In the case of RC, each panel specific β_i is related to an underlying common parameter vector β:

$$\beta_i = \beta + v_i \tag{B2}$$

where $E\{v_i\} = 0, E\{v_i v_i'\} = \Sigma, E\{v_i v_j'\} = 0$ for $j \neq i$, and $E\{v_i \epsilon_j'\} = 0$ for all i and j. We may combine equations (B1) and (B2) to get:

$$y_i = X_i(\beta + v_i) + \varepsilon_i$$

$$= X_i\beta + u_i$$

with $u_i \equiv X_i v_i + \varepsilon_i$. Furthermore:

$$E\{u_i u_i'\} = E\{(X_i v_i + \varepsilon_i)(X_i v_i + \varepsilon_i)'\}$$

$$= X_i \Sigma X_i' + \sigma_{ii} I$$

$$\equiv \Pi_i$$

We can stack the P panels:

$$y = X\beta + u \tag{B3}$$

where:

$$\Pi \equiv E\{u_i u_i'\} = \begin{bmatrix} \Pi_1 & 0 & \cdots & 0 \\ 0 & \Pi_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Pi_P \end{bmatrix}$$

Estimating the parameters in equation (B2) is a standard problem, which can be solved with generalized least squares (GLS):

$$\begin{aligned} \hat{\beta} &= (X'\Pi^{-1}X)^{-1}X'\Pi^{-1}y \\ &= \left(\sum_i X_i'\Pi_i^{-1}X_i\right)^{-1}\sum_i X_i'\Pi_i^{-1}y_i \\ &= \sum_i W_i b_i \end{aligned} \tag{B4}$$

with W_i the Generalized Least Squares (GLS) weight and $b_i = (X_i'X_i)^{-1}X_i'y$. The resulting $\hat{\beta}$ for the overall (national) result is therefore a weighted average of the state-specific OLS estimates. For more details on GLS weight and $\hat{\beta}$ variance specification, the reader can refer to Poi (2003).

To obtain the state-specific $\widehat{\beta_i}$ vectors, Judge et al. (1985) suggest that if attention is restricted to the class of estimators $\{\beta_i^*\}$ for which $E\{\beta_i^*|\beta_i\} = \beta_i$, then the state-specific OLS estimator b_i is appropriate. Following Green's (1997) suggested method of obtaining the variance of $\widehat{\beta_i}$, it follows that $\hat{\beta}$ is both consistent and efficient; and although inefficient, b_i is also a consistent estimator of β.

Poi (2003) also suggests a test to determine whether the panel-specific β_is are significantly different from one another. The null hypothesis is stated as:

$$H_0: \beta_1 = \beta_2 = \cdots = \beta_P \tag{B5}$$

and the test statistic is defined as:

$$T \equiv \sum_{t=1}^{P} \left(b_i - \beta^{\dagger}\right)'\{\hat{\sigma}_{ii}^{-1}(X_i X_i)\}\left(b_i - \beta^{\dagger}\right) \tag{B6}$$

where $\beta^{\dagger} = \left\{\sum_{t=1}^{P} \hat{\sigma}_{ii}^{-1}(X_i X_i)\right\}^{-1} \sum_{t=1}^{P} \hat{\sigma}_{ii}^{-1}(X_i X_i) b_i$.

The test statistic T is distributed as χ^2 with $k(P-1)$ degrees of freedom.