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Elie Bouri
Lebanese American University
Afees A. Salisu
Centre for Econometrics & Applied Research and University of Pretoria
Rangan Gupta
University of Pretoria
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Department of Economics
University of Pretoria
0002, Pretoria
South Africa
Tel: +27 12 420 2413

Bitcoin Prices and the Realized Volatility of US Sectoral Stock Returns

Elie Bouri*, Afees A. Salisu**, and Rangan Gupta***

Abstract

Recent research suggests stronger ties between Bitcoin and US stock markets. In this paper, we examine the predictive power of Bitcoin prices for the realized volatility of the US stock market index and its various sectoral indices. Using data over the period 22 November 2017 and 30 December 2021, we conduct in-sample and out-of-sample analyses over multiple forecast horizons and evidence that Bitcoin prices contain significant predictive power for the volatility of US stocks. Specifically, an inverse relationship exists between Bitcoin prices and the realized volatility of US stock sector indices. The model that includes Bitcoin prices consistent outperforms the benchmark historical average model, irrespective of the various stock sectors and multiple of forecast horizons. The use of Bitcoin prices as a predictor yields higher economic gains. These findings highlight the power and utility of observing Bitcoin prices when forecasting the realized volatility of US stock sectors, which matter to practitioners, and academics, and policymakers.

Keywords: Bitcoin prices; S&P 500 index; US stock sector indices; realized volatility prediction; economic gains.

* Corresponding Author. Adnan Kassar School of Business, Lebanese American University, Beirut, Lebanon; Email address: elie.elbouri@lau.edu.lb.

** Centre for Econometrics & Applied Research, Ibadan, Nigeria and Department of Economics, University of Pretoria, Private Bag X20, Hatfield 0028, South Africa. Email: adebare1@yahoo.com.

*** Department of Economics, University of Pretoria, Private Bag X20, Hatfield 0028, South Africa; Email: rangan.gupta@up.ac.za.

1. Introduction

Bitcoin has emerged in 2009 as a decentralized cryptocurrency detached from the global financial system and driven by unique factors sprouting around its innovative blockchain technology and attractiveness (Kristoufek, 2015). The hedging property of Bitcoin, especially for the general stock market, is well recognized (see, Bouri et al., 2017; Baur et al., 208; Corbet et al., 2018; Shahzad et al., 2020). However, recent market dynamics of the Bitcoin market in the post-pandemic period show somewhat a less detached and a more synchronized market with the US stock market, as represented by the S&P 500[1]. As indicated by the International Monetary Fund (IMF), the closer connections between Bitcoin and US stocks raise "the risk of contagion across financial markets…the correlation coefficient of their daily moves was just 0.01, but that measure jumped to 0.36 for 2020–21 as the assets moved more in lockstep, rising together or falling together"[2]. A recent academic study by Kumar et al. (2022) point to a stronger relationship between Bitcoin and US stock markets, arising from its progress to become much closer to mainstream finance, investment community, and Exchange Traded Funds. Other studies conducted around the pandemic period show comparable findings. For example, Kristoufek (2020) challenges the role of Bitcoin as a safe haven. Conlon and McGee (2020) argue that "the S&P 500 and Bitcoin move in lockstep, resulting in increased downside risk for an investor with an allocation to Bitcoin"[3].

Fewer studies exist on the relationship between Bitcoin and selected US sectoral returns (see Symitsi and Chalvatzis, 2018; Bouri et al., 2020a, b), although the US stock market comprises heterogonous stocks belonging to 11 various sector indices. Interestingly, many US investors engage in sector rotation strategies based on the variation in market conditions and economic cycles. The consensus, although embryonic, suggests that the magnitude and sign of relationship between Bitcoin and US stocks is somewhat sector dependent (see, Bouri et al., 2020), which points to the utility of conducting an analysis at the sectoral level of US stock indices. However, besides stock returns, what matters to traders, investors and policymakers is stock volatility. In fact, many practitioners are keen to understand the exogenous variables capable of predicting stock volatility, given its important role in forecasting models, option pricing, volatility trading

[1] https://www.bloomberg.com/news/articles/2021-12-03/bitcoin-s-correlation-with-stocks-grows-as-risk-appetite-drops
[2] https://blogs.imf.org/2022/01/11/crypto-prices-move-more-in-sync-with-stocks-posing-new-risks/
[3] On a linked front, Kwon (2020) shows that the tail behavior of Bitcoin is associated with that of the S&P 500 index (see Kwon, 2020).

strategies, and portfolio allocation and risk management. Therefore, it would be informative to provide a comprehensive evidence on whether Bitcoin returns contain valuable information useful to predict stock volatility at the aggregate and sectoral levels. However, it is not clear whether Bitcoin returns has a predictive power on the volatility of the US stock market index and its various sector indices. Addressing this question is timely and relevant given that many retail and institutional stock investors in the US consider Bitcoin as an investment or trading venue. Furthermore, policymakers are now looking for the possibility to build on the cryptocurrency universe to launch a digital US dollar or a central bank digital currency.

Motivated by evidence of stronger ties between Bitcoin and US stock markets in the past two years and the lack of empirical evidence on the predictive power of Bitcoin prices for the volatility of US stock indices, especially at the sectoral level, the aim of this paper is to examine the ability of Bitcoin prices to predict the realized volatility of the S&P 500 composite index and its 11 sector indices. Methodologically, we apply the model of Westerlund and Narayan (2012, 2015), which accounts for key salient data features such as endogeneity, persistence, and conditional heteroscedasticity. We do in-sample and out-of-sample analyses over multiple forecast horizons. To enrich further the predictive capability of the applied model, our analysis allows for possible structural breaks within the model framework (see, Salisu et al. 2019a).

Our main findings indicate an inverse relationship between Bitcoin prices and the realized volatility of US stock indices. The predictive model that accounts for Bitcoin prices and salient features of data outperforms the benchmark historical average model, irrespective of the various stock sectors and multiple forecast horizons. Notably, incorporating Bitcoin prices as a predictor leads to higher economic gains in a large proportion of US stock sector indices. Therefore, it is important and useful to closely watch Bitcoin prices when forecasting the realized volatility of US stock sectors.

Our method and findings contribute to three lines of research. The first line concerns the literature on the linkages between Bitcoin and US stock markets, which seem to indicate mixed findings. For example, some previous studies show a weak relationship and thereby hedging and safe haven implications for asset allocation and risk management (Bouri et al., 2017; Baur et al., 208; Corbet et al., 2018), whereas others find a stronger relationship after the pandemic jeopardizing the hedging ability of Bitcoin (see, Conlon and McGee, 2020; Kristoufek, 2020; Kumar et al., 2022). On a related front, significant volatility linkages between the aggregate stock market returns and the Bitcoin market are documented in Elsayed et al. (2022), Jiang et al. (2022), Maghyereh and Abdoh (2022). Furthermore, most of previous studies try to explain Bitcoin volatility (see, among others, Walther et al., 2019; D'Amato et al., 2022; Sapkota, 2022; Wang et al., 2022), whereas our paper has a different scope by focusing on Bitcoin price ability to predict stock volatility. The second line of research concerns Bitcoin and sectoral stock indices. For example, at the sectoral level of data, Bitcoin and US stock sector returns seem detached or weakly related, which has hedging inferences (Bouri et al., 2020). Other academic studies look into specific sectors such as technology (Bouri et al., 2020b) or technology and energy stocks (see, Symitsi and Chalvatzis). In fact, Symitsi and Chalvatzis (2018) use GARCH-based models and show evidence of significant return and volatility linkages between Bitcoin and energy and technology companies. They indicate evidence of return and volatility spillovers from the technology sector index to Bitcoin prices. Rathi (2022) provides evidence on the impact of the cryptocurrency market on the semiconductor industry. Recent press articles point to the ability of Bitcoin to predict the dynamics of US stock markets index and especially technology stocks. They argue that "Investors are fleeing riskier assets from tech stocks to cryptocurrencies as the Federal Reserve weighs whether to launch a U.S. digital currency"[4]. The third line of research is related to the association between Bitcoin and US stock indices, which can be subject to structural breaks arising from disturbing and irregular events (see, Ciaian et al., 2016; Salisu et al., 2019c). Consequently, we account for structural breaks in the predictive models for the volatility of US sector indices and thereafter we evaluate both the in-sample and out-of-sample predictive contents of bitcoin prices as well as other salient features of the series. Finally, we offer some utility metrics of gauging the benefits of observing bitcoin prices when valuing the stock market risk.

2. Methodology

Here, we specify a Westerlund and Narayan (2012, 2015) [WN]-type distributed lag model for the analysis of the nexus between the realized volatility of the US sector stocks and Bitcoin prices. The WN-type predictive model simultaneously accounts for some salient data features typical of financial series such as endogeneity, persistence and conditional heteroscedasticity. In addition to the theoretical results of Westerlund and Narayan (2012, 2015) validating the consideration of these features in predictability analyses, several empirical studies involving this methodology have

[4] https://www.washingtonpost.com/business/2022/01/22/crypto-crash-bitcoin-fed/

also reported the same outcome (see, Narayan and Bannigidadmath, 2015; Narayan and Gupta, 2015; Phan et al., 2015; Bannigidadmath and Narayan 2016; Devpura et al., 2018; Salisu et al. 2019a, 2019b, 2019c, 2019d, 2021; among others). By way of further enhancing the predictive capability of the applied model, we account for possible structural breaks within the model framework by including break dummies obtained using the Bai and Perron (2003) test which allows for up to five breaks. Accounting for significant structural breaks improves predictability outcomes (see Salisu et al. 2019a, 2019b, 2019c; among others).

We therefore define our predictive model as follows:

$$RV_t = \alpha + \beta btc_{t-1} + \gamma\left(btc_t - \rho btc_{t-1}\right) + \sum_{i=1}^{5} \delta_i brk_{it} + \varepsilon_t \qquad (1)$$

where RV_t is a 20-day annualized realized volatility from the corresponding US sector stock returns computed at period t; btc_t is the log-transformed Bitcoin price at time t; brk_{it} is the i^{th} break dummy; α is the intercept; β is the coefficient associated with our predictor variable of interest, which gives the stance of predictability, or otherwise; the term $\left[\gamma\left(btc_t - \rho btc_{t-1}\right)\right]$ is a persistence-adjustment term that is introduced to simultaneously resolve the inherent persistence effect and endogeneity bias that may have been occasioned by model mis-specification; δ_i is the coefficient associated with the break dummy; while ε_t is a zero mean idiosyncratic error term. Note that the break dates are obtained using the Bai-Perron (2003) multiple breakpoint test, after regressing each US stock returns' realized volatility on a one-period lag of the log-transformed Bitcoin price series, and allowing for up to a maximum of five breaks. The underlying predictability test has the null hypothesis $\left[H_0 : \beta = 0\right]$ against a mutually exclusive alternative, $\left[H_a : \beta \neq 0\right]$, where a rejection (non-rejection) of the null hypothesis implies the predictability (no predictability) of Bitcoin price for realized volatility of US sector stock. Given that our data is of a daily (high) frequency, cum the possibility of exhibiting conditional heteroscedasticity, we weight equation (1) with the standard deviation of the residuals obtained from a $GARCH\left(1,1\right)$ model as a way to account for possible conditional heteroscedasticity effect, and estimate the resulting equation with Ordinary Least Squares to obtain the Feasible Quasi Generalized Least Squares estimates.

A follow up to the predictability stance, is the out-of-sample forecast evaluation of our WN-type predictive model relative to a benchmark historical average model that does not take cognisance of the predictive information inherent in the Bitcoin price series. We therefore subject the forecast of our predictive model to statistical evaluations using the conventional Root Mean Square Error and Clark & West (2007), as well as economic significance. Drawing from extant studies (see Narayan and Gupta, 2015; 2019a, 2019b, 2019c; among others) that have shown the in-sensitivity of estimation outcomes to the choice of data split, the 75:25 data split option is considered for in-sample estimation or predictability and out-of-sample forecast evaluation, respectively. On the out-of-sample period, we consider 30-, 60- and 120-days ahead forecast horizons under a rolling window framework that allows for some time-variation.

On the Clark and West (CW) test that compares the predictive accuracy of two competing models, we are able to formally determine the statistical significance of the observed difference between the forecast errors of the contending models. The CW framework is defined as follows:

$$\hat{f}_{t+k} = \left(r_{t+k} - \hat{r}_{1t,t+k}\right)^2 - \left[\left(r_{t+k} - \hat{r}_{2t,t+k}\right)^2 - \left(\hat{r}_{1t,t+k} - \hat{r}_{2t,t+k}\right)^2\right] \tag{2}$$

where k denotes the forecast period, $\left(r_{t+k} - \hat{r}_{1t,t+k}\right)^2$ and $\left(r_{t+k} - \hat{r}_{2t,t+k}\right)^2$ are respectively the squared errors for the restricted and the unrestricted models, and $\left(\hat{r}_{1t,t+k} - \hat{r}_{2t,t+k}\right)^2$ is the adjusted squared errors introduced by the CW test to correct for any noise associated with the larger model's forecast. Thus, the sample average of $\hat{f}_{t+k}$ can be expressed as $MSE_1 - \left(MSE_2 - \text{adj.}\right)$, and each term is computed as $MSE_1 = P^{-1}\sum\left(r_{t+k} - \hat{r}_{1t,t+k}\right)^2$, $MSE_2 = P^{-1}\sum\left(r_{t+k} - \hat{r}_{2t,t+k}\right)^2$, and adj.$=P^{-1}\sum\left(\hat{r}_{1t,t+k} - \hat{r}_{2t,t+k}\right)^2$, where P denotes the number of predictions used in computing these averages. The equality of forecast performances between the restricted and unrestricted models is tested by regressing the $\hat{f}_{t+k}$ on a constant and drawing inference based on the resulting t-statistic of the constant. Given the null hypothesis of equality of MSEs, the criteria for rejection is based on whether the resulting t-statistics is greater than +1.282 (for a one-sided 0.10 test) or +1.645 (for a one-sided 0.05 test).

3. Data Description

The data comprises daily Bitcoin prices and 20-day annualized realized volatility of the US S&P 500 composite index returns and each of its 11 sector stock (Composite, Consumer Discretionary, Consumer Staples, Energy, Financials, Health Care, Industrials, Information Technology, Materials, Real Estate, Telecommunication Services and Utilities) returns. The log returns of the S&P 500 composite index and each of its 11 sector stock indices are computed as the logarithmic differences between two consecutive daily prices. Then, they are used to compute realized volatility based on the sum of squared returns using 20 days rolling window and subsequently annualized using 252 trading days in a year.[5] All data are collected from DataStream, spanning a period between 22 November 2017 and 30 December 2021. Bitcoin prices are against the US dollar according to Bitstamp, one of the oldest and well-established Bitcoin exchange.

We present, in Table 1, a detailed summary of the data characteristics with respect to the measure of location, spread and shape, as well as some preliminary results on the presence of conditional heteroscedasticity, first and higher order autocorrelation and persistence effects. The log-transformed Bitcoin price series over the period considered averages 9.43 with standard deviation of 0.83 and is found to be positively skewed and platykurtic (exhibiting kurtosis value that is below that of the normal distribution). On the realized volatility of the US sector stock returns, we find the average to range between 0.61 and 1.32, which corresponds to Consumer Staples and Energy, respectively. All the realized volatilities are positively skewed and exhibit heavier than normal tails (leptokurtic feature), which are joint indicators of the non-normality of the realized volatility. We find evidence of ARCH effect (except in the case of Energy), first and higher order autocorrelation effects at all specified lags, as well as persistence effects. The foregoing suggests that the model that would be most appropriate for assessing the nexus between the realized volatility and Bitcoin prices would be one that adequately accounts for most of the observed features of the data. Our WN-type predictive model framework is well suited.

We display the co-movement of US stock returns' realized volatilities and Bitcoin prices graphically in Figure 1. From the figure, we can observe the paired series to exhibit negative relationship, with observable peaks in stock returns' realized volatilities being matched with troughs in Bitcoin prices. We also observe a prominent jump and the highest peak in the stock returns' realized volatilities being observed at March 27, 2020; a period that coincides with the period following the announcement of the COVID-19 as a pandemic. This suggest that there could

[5] For technical details on how the daily RV is computed, see https://www.realvol.com/VolFormula.htm.

be evidence of structural shift in the co-movement of the paired series; however, we test for its presence more formally using the Bai-Perron (2003) test (see last panel of Table 1). We observe that there are at least three significant break dates across the sector stocks realized volatilities, with one of the dates falling between 27 February 2020 and 3 March 2020.

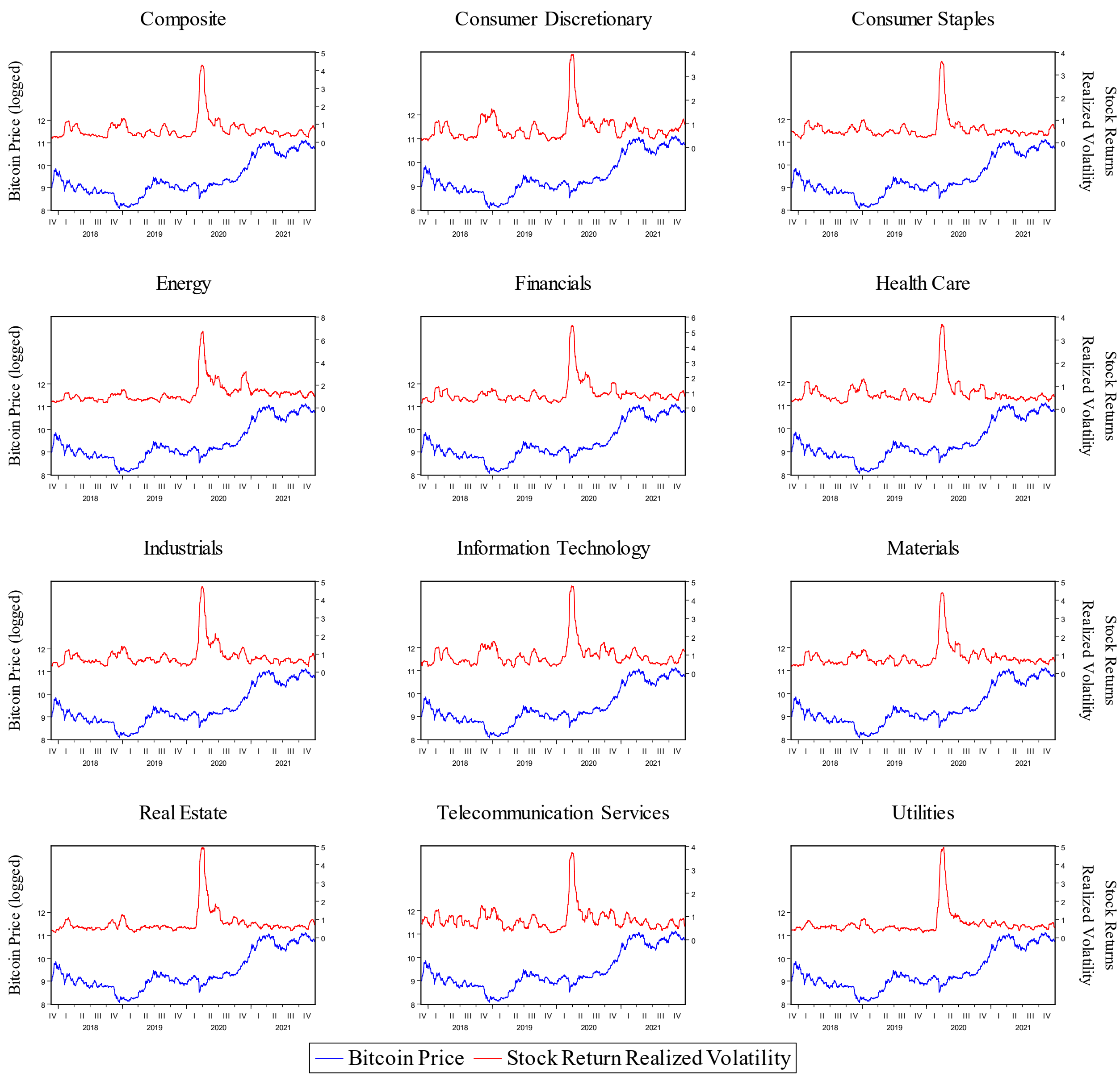

Figure 1: Co-movement between the Realized Volatility of US Stocks and Bitcoin Prices

Table 1: Summary Statistics and Preliminary Analysis

	Bitcoin	Composite	Consumer discretionary	Consumer staples	Energy	Financials	Health Care	Industrials	Info. Technology	Materials	Real Estate	Telecom. Services	Utilities
							Summary Statistics						
Mean	9.43	0.71	0.83	0.61	1.32	0.96	0.71	0.87	0.98	0.89	0.81	0.86	0.76
Deviation	0.83	0.59	0.54	0.47	0.92	0.74	0.5	0.63	0.65	0.57	0.67	0.49	0.65
Skewness	0.65	3.9	3.24	4.52	3.26	3.87	3.83	3.83	3.5	3.85	4.37	3.51	4.69
Kurtosis	2.24	21.66	17.27	26.65	16.4	20.9	20.73	20.76	19.09	20.75	24.32	19.47	26.84
							Conditional Heteroscedasticity Effects						
ARCH (1)	13.88***	45.89***	3.33*	143.91***	0.05	51.71***	15.44***	8.84***	88.24***	2.58	6.05**	71.64***	99.52***
ARCH (5)	29.08***	25.21***	26.72***	43.06***	1.67	23.08***	10.13***	25.61***	21.96***	13.31***	5.60***	21.73***	63.52***
ARCH (10)	19.99***	13.99***	15.91***	29.09***	1.53	11.98***	7.58***	16.36***	11.85***	8.25***	2.96***	11.10***	40.67***
							First and Higher Order Autocorrelation						
$Q(1)$	2.15	85.92***	24.35***	109.94***	7.59***	82.48***	49.02***	50.95***	87.94***	49.63***	71.94***	50.73***	94.94***
$Q(5)$	28.01***	385.58***	222.12***	478.94***	96.922***	312.26***	225.05***	274.15***	266.14***	195.50***	319.54***	183.19***	518.90***
$Q(10)$	47.00***	566.07***	324.58***	682.49***	170.96***	456.86***	381.44***	445.09***	379.3***	306.76***	441.44***	243.13***	791.43***
$Q^2(1)$	13.77***	44.25***	3.34*	127.53***	0.05	49.59***	15.31***	8.82***	81.96***	2.59	6.05**	67.51***	91.54***
$Q^2(5)$	204.07***	156.55***	120.86***	298.51***	8.74	172.29***	58.78***	164.59***	127.29***	80.12***	31.20***	131.32***	408.02***
$Q^2(10)$	419.33***	196.16***	139.09***	430.91***	16.81*	204.49***	106.18***	252.16***	147.59***	108.23***	35.41***	139.23***	563.21***
Persistence	0.999***	0.994***	0.992***	0.994***	0.992***	0.994***	0.993***	0.994***	0.993***	0.993***	0.994***	0.991***	0.995***
							Significant Break Dates (Bai-Perron, 2003)						
Break_1	-	02/01/2019	02/01/2019	07/05/2018	01/23/2019	01/31/2019	02/01/2019	02/01/2019	02/01/2019	02/01/2019	01/23/2019	07/11/2018	02/11/2019
Break_2	-	02/27/2020	02/27/2020	02/21/2019	03/06/2020	03/02/2020	02/25/2020	03/03/2020	02/25/2020	02/27/2020	02/28/2020	02/28/2019	03/02/2020
Break_3	-	10/08/2020	10/08/2020	02/27/2020	10/21/2020	10/12/2020	10/08/2020	10/13/2020	10/08/2020	10/08/2020	10/23/2020	03/02/2020	10/12/2020
Break_4	-	05/21/2021	05/21/2021	10/08/2020	-	-	05/21/2021	-	05/21/2021	05/20/2021	-	10/12/2020	-

Note: The summaries are done for 1,072 observation points, where ARCH(#), Q(#) and Q(#) represent the tests for presence of conditional heteroscedasticity, first and higher order serial correlations, respectively; and statistical significance implying that the tested feature is present in the series, up to the specified lag #. The ***, ** and * denote statistical significance at 1%, 5% and 10% levels, respectively. The break dates are determined using the Bai-Perron (2003) multiple break point test allowing for a maximum of five lags in a regression of each sector stock returns' volatility on a one period lag of log-transformed Bitcoin prices.

4. Results

4.1. Predictability and Forecast Evaluation

We present the results of the predictability (see PANEL A of Table 2) and forecast evaluation (see Panels B and C of Table 2) of our proposed predictive model in comparison with the benchmark model. We only report the estimated coefficient associated with the Bitcoin prices given our interest to show its predictive value for the realized volatility of US stocks and since the other components of the model are used to adjust for the observed salient data features, their interpretation would be redundant. The full data sample is used for the predictability analyses, while we adopt a 75:25 data split for the forecast evaluation, wherein the out-of-sample forecast horizons are drawn from the remaining 25% of the full data after using the first 75% to estimate the parameters.

On the in-sample predictability, we find the existence of an inverse relationship between the realized volatility of each of the US sectors and Bitcoin prices, given that the estimated parameter is significantly negative across the considered US sector stocks. The realized volatilities of the US sector stocks respond negatively to Bitcoin price movements. This formally validates the observation on the graphical presentation of their co-movement, where there are observable stances of peak-trough matches between US sector stock returns' realized volatility and Bitcoin prices. Imperatively, the level of uncertainty/risk associated with each of the US sector stocks decreases with rising Bitcoin prices and increases with declining Bitcoin prices. This nexus can be viewed from a risk-return trade-off perspective where higher returns are associated with higher risks (see French et al, 1987; Bali and Peng, 2006; Beaert et al., 2007; Chiang and Zhang, 2017; among others). Our predictability results suggest that higher prices of Bitcoin will increase its trading (and volatility) which by implication will lower stock trading as well as its volatility. Put differently, lower prices of Bitcoin will stimulate investments in conventional stocks. The improved stock trading resulting from lower Bitcoin prices will raise the level of volatility of the former. In other words, there is somewhat of a hedging relationship between the two assets and we further explore this in a later section titled "Economic Significance" where we provide possible utility gains derivable by a profit maximizing investor in the stock market from observing Bitcoin prices. On the whole, the significantly negative nexus between Bitcoin prices and the realized volatility of US stocks is consistent across the various sectors covered in our analysis.

The out-of-sample forecast evaluation using the relative root mean square error is presented in PANEL B of table 2. The relative root mean square error is the ratio of our WN-type predictive model to the benchmark historical average model, where a value less than unity indicates that the forecast errors of our predictive model is smaller than that of the benchmark model. The relative RMSE is adopted here for ease of interpretation and comparison. From the presented results, we find our predictive model for the realized volatility of each of the US sector stocks to yield more precise forecasts than the benchmark historical average model both in the in-sample and across the specified out-of-sample forecast horizons, since the observed relative RMSE is less than one. The achievement is consistent across the US sector stocks and across forecast horizons, an indication that the results emanating from our predictive model are not sensitive to the sector stocks and out-of-sample forecast horizons.

In the same vein, we consider a more formal pairwise comparison tool – the Clark and West test. For our predictive model to be adjudged the preferred model when compared to the benchmark historical average model, the estimated statistics must be positive and significant. From the result in PANEL C of Table 2, the Clark and West (2007) results further reveal the statistically significant outperformance of our predictive model that accounts for the salient data features, such as endogeneity, persistence, conditional heteroscedasticity and structural breaks, over the benchmark historical average model that neglects these features. We find significantly positive coefficients across the specified periods, and as such we ascertain that these outperformances are sustained regardless of the sample period or forecast horizon.

Table 2: Predictability and Forecast Evaluation

Sectors	PANEL A	PANEL B				PANEL C			
	Parameter Estimate	Relative RMSE				Clark and West (2007)			
		In Sample	Out-of-Sample			In Sample	Out-of-Sample		
			$h = 30$	$h = 60$	$h = 120$		$h = 30$	$h = 60$	$h = 120$
Composite	-0.2168*** [0.0037]	0.9030	0.9026	0.9029	0.9009	0.1944*** [0.0123]	0.1885*** [0.0119]	0.1823*** [0.0115]	0.1739*** [0.0108]
Consumer discretionary	-0.2021*** [0.0004]	0.8571	0.8566	0.8668	0.8774	0.1974*** [0.0161]	0.1949*** [0.0155]	0.1823*** [0.0152]	0.1650*** [0.0101]
Consumer staples	-0.3673*** [0.0040]	0.9289	0.9267	0.9261	0.9240	0.0611*** [0.0041]	0.0608*** [0.0040]	0.0598*** [0.0038]	0.0588*** [0.0036]
Energy	-0.6597*** [0.0033]	0.6856	0.7139	0.7262	0.7691	1.2653*** [0.0937]	1.2321*** [0.0906]	1.2057*** [0.0876]	1.1267*** [0.0825]
Financials	-0.3433*** [0.0029]	0.7650	0.7662	0.7725	0.7744	0.5095*** [0.0471]	0.4947*** [0.0455]	0.4791*** [0.0440]	0.4640*** [0.0412]
Health Care	-0.3106*** [0.0036]	0.9066	0.9031	0.9035	0.8970	0.1616*** [0.0174]	0.1628*** [0.0168]	0.0890*** [0.0054]	0.0962*** [0.0052]
Industrials	-0.3990*** [0.0041]	0.7695	0.7686	0.7715	0.7716	0.3466*** [0.0331]	0.3381*** [0.0320]	0.3306*** [0.0309]	0.3239*** [0.0289]
Info. Technology	-0.3100*** [0.0048]	0.9065	0.9048	0.9074	0.9114	0.2335*** [0.0148]	0.2282*** [0.0143]	0.2207*** [0.0139]	0.2084*** [0.0131]
Materials	-0.2377*** [0.0021]	0.8448	0.8452	0.8460	0.8453	0.2471*** [0.0161]	0.2389*** [0.0156]	0.2315*** [0.0151]	0.2203*** [0.0142]
Real Estate	-0.0852*** [0.0007]	0.8415	0.8434	0.8432	0.8452	0.2635*** [0.0197]	0.2555*** [0.0190]	0.2542*** [0.0183]	0.2501*** [0.0172]
Telecommunications	-0.2381*** [0.0011]	0.8587	0.8580	0.8672	0.8701	0.1254*** [0.0099]	0.1235*** [0.0096]	0.1189*** [0.0093]	0.1179*** [0.0087]
Utilities	-0.1155*** [0.0005]	0.9609	0.9608	0.9608	0.9599	0.0980*** [0.0037]	0.0946*** [0.0036]	0.0916*** [0.0036]	0.0868*** [0.0034]

Note: The results presented on the table are from the estimation of the WN-Type distributed lag predictive model for Bitcoin prices using realized volatilities of US sectoral stock returns singly as predictors, while simultaneously accounting for inherent persistence, endogeneity, conditional heteroscedasticity and structural breaks. The table comprises three panels: PANEL A presents the in-sample predictability of the realized volatility of US sectoral stock returns for log-transformed Bitcoin price; PANEL B presents the relative root mean square error that compares our WN-Type distributed lag model with the historical average model; while PANEL C presents the Clark and West (2007) test statistics that entails a pairwise comparison of our predictive model with the benchmark historical average model. Under Panels A and C, each cell contain the estimates and the corresponding standard errors in square brackets; while the *** indicates statistical significance at 1% level. Under the PANEL B, values less than unity indicate preference of our predictive model over the benchmark Historical average model; while under PANEL C, our predictive model is adjudged the preferred when the CW statistic is positive and statistically significant.

4.2. Economic Significance

In addition to the statistical-based forecast performance evaluation, we also conduct an economic-based forecast performance evaluation tool, drawing from Liu et al. (2019) study as well as Salisu et al. (2022). The economic-based measure is employed to ascertain whether, or not, the incorporation of Bitcoin prices as a predictor in our WN-type distributed lag model provides additional information that yields economic gains over the benchmark historical average model that ignores same. It is not unexpected for a typical mean-variance utility investors to optimize available portfolio among assets and/or investment options, in contrast to a risk free asset. The optimal weight, w_t, is defined as

$$w_t = \frac{1}{\gamma}\frac{\theta \hat{r}_{t+1} + (\theta - 1)\hat{r}^f_{t+1}}{\theta^2 \hat{\sigma}^2_{t+1}} \tag{3}$$

where γ represents the risk aversion coefficient; θ is a leverage ratio (Zhang et al. 2018) that is set between 1 and 10, given the assumption that investors usually maintain a margin account at 10% level $(\theta - 10)$; $\hat{r}_{t+1}$ is the realized volatility forecast at time $t+1$; $\hat{r}^f_{t+1}$ is a risk-free asset (we used the US Treasury bill rate); and $\hat{\sigma}^2_{t+1}$ is the estimate of the return volatility, which is estimated using a 30-day moving window of daily returns. The certainty equivalent return for investors' optimal weight (w_t) in equation (3) is defined as

$$CER = \overline{R}_p - 0.5(1/\gamma)\sigma^2_p \tag{4}$$

where $\overline{R}_p$ and σ^2_p are the out-of-sample period mean and variance, respectively, of the portfolio returns, $R_p = w\theta(r - r^f) + (1-w)r^f$. The associated portfolio return variance is defined as $Var(R_p) = w^2\theta^2\sigma^2$, where σ^2 denotes the excess return volatility. The economic significance is consequently determined by maximizing the objective function of a utility as equation in (5) below

$$\begin{aligned} U(R_p) &= E(R_p) - 0.5(1/\gamma)Var(R_p) \\ &= w\theta(r - r^f) + (1-w)r^f - 0.5(1/\gamma)w^2\theta^2\sigma^2 \end{aligned} \tag{5}$$

We report the portfolio returns, the associated volatility, as well as the certainty equivalent returns and the Sharpe ratio, which is computed as $SR = (R_p - r^f)\big/\sqrt{Var(R_p)}$. We judge the economic gains based on the model construct with the maximum returns, CER and SP; and minimum volatility (see Liu et al., 2019). Table 3 presents the economic significance results of incorporating Bitcoin price as a predictor in our WN-type distributed lag model framework for predicting realized volatility of US sectors' stocks (Composite, Consumer Discretionary, Consumer Staples, Energy, Financials, Health Care, Industrials, Information Technology, Materials, Real Estate, Telecommunication Services and Utilities) returns, when the leverage parameter is set to 6 and 8.

Table 3: Economic Significance

Sector Stock	Model	Returns	Volatility	CER	SR	Returns	Volatility	CER	SR
		$Gamma = 3$ and $Theta = 6$				$Gamma = 3$ and $Theta = 8$			
Composite	HA	0.1582	6.3683	0.1569	0.0518	0.2180	11.0590	0.2166	0.0573
	WN	0.3786	9.9721	0.3772	**0.1112**	0.4942	18.0405	0.4928	**0.1099**
Consumer Discretionary	HA	0.4717	78.2085	0.4686	0.0502	0.6115	139.5238	0.6085	0.0494
	WN	-2.4630	121.1427	-2.4670	-0.2263	-3.3025	215.4363	-3.3065	-0.2269
Consumer Staples	HA	-0.3913	10.6121	-0.3919	-0.1286	-0.5171	18.6920	-0.5177	-0.1260
	WN	0.2351	3.2580	0.2344	**0.1150**	0.3054	5.7784	0.3047	**0.1156**
Energy	HA	0.4169	50.9515	0.4145	0.0545	0.5448	90.7675	0.5424	0.0543
	WN	-3.8934	69.5225	-3.8960	-0.4702	-5.1918	123.5344	-5.1944	-0.4696
Financials	HA	-0.3992	12.8728	-0.4004	-0.1189	-0.5129	22.0292	-0.5140	-0.1151
	WN	-0.5381	37.2313	-0.5394	**-0.0927**	-0.7365	66.3159	-0.7378	**-0.0938**
Health Care	HA	-1.1088	29.4945	-1.1097	-0.2092	-1.4770	52.4270	-1.4778	-0.2078
	WN	-0.6532	29.7704	-0.6546	**-0.1248**	-0.8836	52.8847	-0.8851	**-0.1253**
Industrials	HA	-0.7841	20.0793	-0.7856	-0.1811	-1.0216	34.3436	-1.0230	-0.1790
	WN	-0.1195	33.2669	-0.1214	**-0.0255**	-0.1797	59.6844	-0.1816	**-0.0268**
Information Technology	HA	0.5007	12.2098	0.4972	0.1354	0.6604	21.5676	0.6569	0.1363
	WN	-0.1611	43.8404	-0.1648	-0.0285	-0.2399	79.7341	-0.2436	-0.0299
Materials	HA	-0.2289	19.4588	-0.2300	-0.0581	-0.3026	34.2687	-0.3038	-0.0564
	WN	-0.0533	18.5352	-0.0543	**-0.0188**	-0.0884	33.1728	-0.0895	**-0.0201**
Real Estate	HA	-1.3669	29.4363	-1.3680	-0.2570	-1.8207	52.2294	-1.8218	-0.2557
	WN	-0.9325	25.9896	-0.9334	**-0.1883**	-1.2618	46.2730	-1.2627	**-0.1895**
Telecommunication Services	HA	0.3155	73.2586	0.3129	0.0336	0.4156	130.2285	0.4130	0.0340
	WN	-0.8051	74.4048	-0.8076	-0.0965	-1.0965	133.5648	-1.0990	-0.0973
Utilities	HA	0.2550	6.5103	0.2536	0.0891	0.3363	11.6497	0.3349	0.0905
	WN	0.4724	10.3612	0.4710	**0.1382**	0.6230	18.6182	0.6216	**0.1380**

Note: HA is the historical average model while WN is the Westerlund and Narayan (2012, 2015) type distributed lag model that accommodates salient data features such as endogeneity, persistence, conditional heteroscedasticity and structural breaks. A given predictive model that incorporates Bitcoin (logged) as a predictor is said to yield economic gains over the compared benchmark whenever such model construct yields maximum returns, CER and SR; and minimum volatility. The figures in bold letterings are cases where our WN-type predictive model provides some economic gains over the benchmark historical average model. Also, the cases of negative SR indicate that the returns of the corresponding stocks are lower than the risk free asset used in the computation of economic significance; however, the decision remains based on the maximum SR.

Table 3 shows that our WN-type distributed lag model that incorporates Bitcoin price as a predictor variable provide higher economic gains but with higher risks (except in the cases of Consumer Staples and Real Estate) than the benchmark historical average model in all the cases except for Consumer discretionary, Energy, Information Technology and Telecommunications services, when the leverage parameter is set to 3. We also observe negative Sharpe Ratios in some cases (Consumer Discretionary, Consumer Staples, Energy, Financials, Health Care, Industrials, Information Technology, Materials, Real Estate and Telecommunication Services) indicating that the US sector stock returns are mostly less than the risk free asset they are being compared with. Another prominent observation is the achievement of high returns being associated with high risks. The stance of economic gains is not different when the leverage parameter is set to 8, as we find same feats across the US sector stocks. From the foregoing, the incorporation of Bitcoin price provides some economic gains irrespective of the set leverage parameter, with higher gains being mostly associated with higher risks. Conclusively, our predictive model performs better than the benchmark historical average model both statistically and economic-wise, in the in-sample and across out-of-sample forecast horizons.

4. Conclusion

In this study, we assess the nexus between realized volatility of US stock returns across different sectors and Bitcoin prices. This is in a bid to ascertain the predictive potential of Bitcoin prices for the realized volatility of US stock returns while controlling for possible biases arising from model mis-specification and/or variable omission by using the WN-type predictive model. The analyses are conducted for both the in-sample and out-of-sample periods as well as multiple forecast horizons. We employ both the relative RMSE and the pairwise Clark and West (2007) test statistics, to evaluate the forecast performance. Thereafter, we examine possible utility gains of observing Bitcoin prices when taking investment decisions about the US stock market.

We find evidence of at least three significant breaks using Bai-Perron (2003) multiple break point test with a maximum of five lags on the regression of realized volatility on Bitcoin, with one of them aligning with the period following the WHO announcement of the COVID-19 pandemic. Consequently, incorporating these observed breaks in the model framework that already accounts for other salient features such as endogeneity, persistence and conditional heteroscedasticity is hypothesized to improve the predictability outcomes. Our results are summarized as follows. First, we find an inverse relationship between Bitcoin prices and the realized volatility of US stocks. This is indicative of the fact that declining prices in the Bitcoin market could heighten the uncertainty in the US sector stock market due to improved trading in the latter. Second, on the out-of-sample forecast performance, we find consistent outperformance of our predictive model that accounts for Bitcoin prices as well as other observed salient data features over the benchmark historical average model that does not take cognisance of these information. Our conclusion remains the same across the different sectors of US stock market and multiple forecast horizons. Third, on the economic significance, incorporating Bitcoin prices as a predictor yields higher economic gains in a larger proportion of the US sector stocks under alternative assumptions about the leverage ratio. From the foregoing, we can conclude that observing Bitcoin prices when forecasting the realized volatility of US stocks not only will improve the forecasts but will also yield higher economic gains. Thus, investors seeking to maximize returns in US stock market are encouraged to pay attention to the price dynamics in the Bitcoin market as they have the ability to influence the volatility formation of US stocks significantly. Equally, practitioners and academics who are constantly involved in the analyses of financial markets may find our proposed model and the various conclusions insightful, particularly in terms of producing more accurate forecasts when analysing the riskiness of US stock market. Future research should consider whether the above findings can be generalized to other stock markets in Europe and Asia. In addition, it could be interesting to assess the response of emerging markets to the dynamics of the Bitcoin market, while also investigating whether markets' responses are symmetrical or asymmetrical, given the bullish and bearish nature of financial markets.

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