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Conventional and Unconventional Monetary Policy Rate Uncertainty and Stock Market Volatility: A Forecasting Perspective

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Abstract

Theory suggests the existence of a bi-directional relationship between stock market volatility and monetary policy rate uncertainty. In light of this, we forecast volatilities of equity markets and shadow short rates (SSR) – a common metric of both conventional and unconventional monetary policy decisions, by applying a bivariate Markov-switching multifractal (MSM) model. Using daily data of eight advanced economies (Australia, Canada, Euro area, Japan, New Zealand, Switzerland, the UK, and the US) over the period of January, 1995 to March, 2021, we find that the bivariate MSM model outperforms, in a statistically significant manner, not only the benchmark historical volatility and the univariate MSM models, but also the Dynamic Conditional Correlation-Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) framework, particularly at longer forecast horizons. This finding confirms the bi-directional relationship between stock market volatility and uncertainty surrounding conventional and unconventional monetary policies, which in turn has important implications for academics, investors and policymakers.

Keywords: Shadow short rate uncertainty, Stock market volatility, Markov-switching multifractal model (MSM), Forecasting.
JEL Codes: C22, C32, C53, D80, E52, G15.

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1 Introduction

As discussed in detail by Poon and Granger (2003), and Rapach et al., (2008), modeling and forecasting of volatility is a pertinent issue due to several reasons : Firstly, when volatility is interpreted as uncertainty, it becomes a key input to investment decisions and portfolio choices. Secondly, volatility is the most important variable in the pricing of derivative securities. To price an option, one needs reliable estimates of the volatility of the underlying assets. Thirdly, financial risk management according to the Basle Accord established in 1996 and later Basel III in 2009 also requires modeling and forecasting of volatility as a compulsory input to risk-management for financial institutions around the world. Finally, financial market volatility, as witnessed during the Global Financial Crisis (GFC), and the recent outbreak of the COVID-19 pandemic (Salisu et al., 2021a), can have wide repercussions on the economy as a whole, via its effect on real economic activity (Caggiano et al., 2020; Gupta et al., 2021; Salisu et al., 2021b) and public sentiment (Baker et al., 2020, Cox et al., 2020). Hence, estimates of market volatility can serve as a measure for the vulnerability of financial markets and the economy, and can help policymakers design appropriate policies. Given the multi-dimensional importance of appropriate modeling and accurate forecasting of the process of volatility, not surprisingly, the associated literature is huge based on wide-array of univariate and multivariate models, as well as macroeconomic, financial and behavioral predictors.[1]

In this regard, a series of recent studies (see for instance, Antonakakis et al., (2017), Kaminska and Roberts-Sklar (2018), Gupta and Wohar (2019), Paule-Vianez (2020), Baker et al., (forthcoming)) have depicted both in- and out-of-sample predictive ability of monetary policy rate uncertainty (volatility) for stock market volatility of the Euro Area, the United Kingdom (UK) and the United States (US). Theoretically, this finding obtained by these studies should not come as a surprise, given that the monetary policy rate (i.e. short-term risk-free) is a key factor for pricing many securities and derivatives, and hence, there should be a strong link between monetary policy rate uncertainty and equity return volatility, at both short- and long-run. This is understandable, since, according to basic present value models, the variance of equity prices is directly linked to the conditional variances of future discount rates, which are in turn, the explicit functions of expected risk-free interest rates and risk premia.

At the same time, stock market volatility may also impact monetary policy rate uncertainty, resulting in a compromise of the effectiveness of monetary policy. The argument is that in the wake of especially the Zero Lower Bound (ZLB) situations as observed during GFC and now under the ongoing coronavirus outbreak, low interest rate policies may be destabilizing given that they destabalize the behaviour of investors towards riskier strategies (Rajan, 2006). If investor behaviour follows a 'toddler's tantrum', then investors would expect

[1] See, among others, Engle and Rangel (2008), Rangel et al., (2011), Asgharian et al., (2013), Engle et al., (2013), Ben Nasr et al., (2014, 2016), Conrad et al., (2014), Conrad and Loch (2015), Fang et al., (2020), Liu and Gupta (2020), Liu et al., (2020), Salisu et al., (2020, forthcoming), Demirer et al., (2021), Salisu and Gupta (2021)).

that the central bank will provide additional monetary stimulus during increased financial market turmoil, thereby leading to an increased monetary policy rate uncertainty. Empirically, this line of reasoning has been validated for the United States (US) and internationally (for Asian economies, Canada, the Euro area, Japan and the United Kingdom (UK)) by Valera et al., (2017), Donzwa et al., (2019), Hassani et al., (2020) and Hkiri et al., (2021).[2] In other words, just like monetary policy rate uncertainty can lead to increased equity market volatility, the opposite is supposed to hold as well, implying that stock market volatility and monetary policy rate uncertainty are endogenous to each other.

Given the underlying endogeneity between stock market volatility and monetary policy rate uncertainty, from an econometric perspective, we primarily rely on a bivariate version of the Markov-switching multifractal (MSM) model, to analyze the predictive relationship between these two variables. We motivate the suitability of applying bivariate MSM model as follows. Research on long memory and structural changes in volatility has discussed the connection between these phenomena, and has suggested that, in fact, volatility persistence may be due to switching of regimes in the volatility process (Diebold, 1986; Lamoureux and Lastrapes, 1990). Hence, it could be very difficult to distinguish between true and spurious long memory processes. This ambiguity motivates us to consider the MSM framework, which, despite allowing for a large number of regimes, is more parsimonious in parameterization than other alternative models. Moreover, it is well-known to give rise to apparent long memory over a bounded interval of lags (Calvet and Fisher, 2004) and it has limiting cases in which it converges to a 'true' long memory process. In addition to the bivariate MSM model, we also consider its univariate version, and the Dynamic Conditional Correlation-Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) method (Engle, 2002), as our benchmark model. With possible bi-directional causality, our expectations are that the bivariate MSM should outperform its univariate counterpart, and also the DCC-GARCH, given the superiority of the econometric structure of the MSM framework in terms of its ability to correctly distinguish between persietence and regime-changes.

While we present in-sample analysis too, our main focus is out-of-sample forecasting of stock returns and monetary policy rate volatility, since in-sample predictability does not necessarily guarantee forecasting gains. Furthermore, Campbell (2008) stresses that the ultimate test of any predictive model (in terms of the econometric methodologies and the predictors used) is in its out-of-sample performance. While high-frequency predictions of stock volatility is highly important from the perspective of Value-at-Risk calculations required for the design of investment portfolios (Ghysels and Valkanov, 2012), the same for monetary policy rate volatility is a pertinent issue for policy authorities. This is because international evidence has shown that uncertainty with monetary policy decisions negatively impact economic activity (Istrefi and Mouabbi, 2018; Husted et al., 2020). In light of this, daily forecasts of monetary policy uncertainty can be fed into mixed data sampling (MIDAS) models to nowcast the

[2] The reader is also referred to an unpublished earlier work by Xu (2007).

low-frequency measures of macroeconomic variables capturing the state of the economy (Bańbura et al., 2011), and in turn can allow policymakers to design expansionary policies ahead-of-time in case a recession is expected. Naturally, besides the importance of out-of-sample predictability from a statistical perspective, real-time forecasts of the volatility of the two variables of concern, rather than a full-sample-based predictive analysis, are more valuable for investors and policymakers in making their respective decisions in an optimal manner.

In this paper, for the first time in the literature, we concentrate on the forecasting of stock market and monetary policy rate volatilities, with the latter capturing associated uncertainties of central bank decisions simultaneously using a bivariate MSM framework, besides the univariate MSM and DCC-GARCH, relative to the benchmark of historical volatility. For our forecasting exercise, we consider the eight countries/regions namely, Australia, Canada, Euro area, Japan, New Zealand, Switzerland, the UK, and the US over the daily period of January, 1995 to March, 2021. Note that, the choice of these mature equity markets is primarily motivated by their importance in the global economy, with these countries representing nearly two-third of global net wealth, and nearly half of world output (Das et al., 2019). In addition to this, since our period of analysis involves both conventional and unconventional monetary policy regimes, we need to consider an uniform metric of monetary policy rate which can appropriately capture both traditional and non-traditional monetary policy decisions. In this regard, we use the Shadow Short Rate (SSR), which in turn is only available for these countries or regions under investigation. The SSR is based on models of the term-structure, which essentially removes the effect that the option to invest in physical currency (at an interest rate of zero) has on yield curves, resulting in a hypothetical "shadow yield curve" that would exist if the physical currency were not available. The process allows one to answer the question: "what policy rate would generate the observed yield curve if the policy rate could be taken negative?" The "shadow policy rate" generated in this manner, therefore, provides a measure of the monetary policy stance after the actual policy rate reaches zero. The main advantage of the SSR is that it is not constrained by the Zero Lower Bound (ZLB), and thus allows us to combine the data from the ZLB period with that of the non-ZLB era, and in turn to use it as the common metric of monetary policy stance across the conventional and unconventional monetary policy episodes.

The remainder of the paper is organized as follows: Section 2 provides information on the structure of the MSM model and its estimation. Section 3 presents the data and the empirical results. Section 4 concludes the paper.

2 Multifractal models

Most financial markets models are based on additive structure of asset returns dynamics, and models with multiplicative operations have been introduced recently under the heading of multifractal models. The development of the multifractal approach goes back to Benoit Mandelbrot's work on the turbulent dissipation in the 1970s. Financial markets display some similarities to fluid turbulence, for example, both turbulence and financial fluctuations are characterized by intermittency at all scales, and it is known to occur from the large scale of injection to the small scale of dissipation, which can be modeled by multifractal processes.

Mandelbrot et al., (1997) first introduced the multifractal apparatus into finance, adapting the approach of Mandelbrot (1974) to an asset-pricing framework. This multifractal model of asset returns (MMAR) assumes that returns r_t follow a compound process, in which an incremental fractional Brownian motion is subordinate to the cumulative distribution function of a multifractal measure. However, the practical applicability of MMAR suffers from the non-causal nature of the time transformation and non-stationarity due to the inherent restriction to a bounded interval. These limitations have been overcome by the development of an iterative version of the multifractal models, including the Markov-switching multifractal model (MSM), cf. Calvet and Fisher (2004) and Lux (2008). In this approach, asset returns volatilities are conceived as hierarchical multiplicative processes with heterogeneous components at different lifetimes. Specifically, MSM models asset returns as:

$$r_t = \sigma \left(\prod_{i=1}^{k} M_t^{(i)} \right)^{1/2} \cdot \epsilon_t, \tag{1}$$

with ϵ_t drawn from a standard normal distribution, and the instantaneous volatility is determined by the product of k volatility components or multipliers $M_t^{(1)}$, $M_t^{(2)}$ ..., $M_t^{(k)}$, with a constant scale parameter σ. In addition, M_t can be drawn from either a discrete distribution, e.g., a binomial distribution in Calvet and Fisher (2004), or a continuous distribution, e.g., lognormal distribution in Lux (2008). Each volatility component is renewed at time t with probability γ_i depending on its rank within the hierarchy of multipliers or remains unchanged with probability $1 - \gamma_i$. Calvet and Fisher (2004) propose to specify the transition probabilities as:

$$\gamma_i = 1 - (1 - \gamma_1)^{(b^{i-1})}, \tag{2}$$

with parameters $\gamma_1 \in (0, 1)$ and $b \in (1, \infty)$; In contrast, without introducing additional parameters, Lux (2008) proposes $\gamma_i = 2^{(k-i)}$. Both specifications guarantee convergence of the discrete-time multifractal process to a limiting continuous-time version with random renewals of the multipliers.

This novel approach preserves the hierarchical structure of MMAR, but dispenses with its restriction to a bounded interval. While this model is asymptotically "well-behaved" (i.e., it shares all the convenient properties of Markov-switching processes), it is still capable of capturing some important properties of financial markets time series data, namely, volatility clustering and the power-law behaviour of the autocovariance function of absolute moments:

$$Cov(|r_t|^q, |r_{t+\tau}|^q) \propto \tau^{2d(q)-1}. \tag{3}$$

Eq. (3) implies multifractal models are rather characterized by only 'apparent' long-memory with an approximately hyperbolic decline of the autocorrelation of absolute powers over a finite horizon and exponential decline thereafter. In particular, approximately hyperbolic decline as expressed in eq. (3) holds only over an interval $1 \ll \tau \ll b^k$, with b the parameter of the transition probabilities of eq. (2), and k being the number of hierarchical cascade levels.

2.1 Bivariate multifractal models

In order to study the interactions and comovements among financial assets, multifractal models can be easily extended to multivatiate setting without imposing much restrictions such as bivariate models. Calvet et al., (2006) assume that instantaneous volatility is composed of heterogenous frequencies, and model the bivariate asset returns r_t as:

$$r_t = \sigma \otimes [g(M_t)]^{1/2} \otimes \epsilon_t. \tag{4}$$

Here, r_t, σ, and u_t are all bivariate vectors: $r_t = \begin{bmatrix} r_{1,t} \\ r_{2,t} \end{bmatrix}$, $\sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}$, $\epsilon_t = \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix}$, and $\otimes$ denotes element by element multiplication. σ is the vector of constant scale parameters (the unconditional standard deviation); ϵ_t is a 2×1 vector whose elements follow a bivariate standard normal distribution, with an unknown correlation parameter ρ. $g(M_t)$ is the vector of the products of multifractal volatility components, i.e. $g(M_t) = \begin{bmatrix} g(M_{1,t}) \\ g(M_{2,t}) \end{bmatrix}$, with each element defined as in the univariate case:

$$g(M_{n,t}) = \prod_{i=1}^{k} M_{n,t}^{(i)}, \tag{5}$$

as the product of volatility components for asset n, and the bivariate volatility components at frequency i of series $n = 1, 2$:

$$M_t^{(i)} = \begin{bmatrix} M_{1,t}^{(i)} \\ M_{2,t}^{(i)} \end{bmatrix}. \tag{6}$$

$M_t^{(i)}$ are drawn from the bivariate binomial distribution $M = (M_1,\ M_2)'$, with M_1 taking values $m_1 \in (1, 2)$ and $2 - m_1$, and M_2 taking values $m_2 \in (1, 2)$ and $2 - m_2$. While the framework by Calvet et al., (2006) allow for variation of the correlation (say, ρ_m) between components M_1 and M_2, they report that a correlation ρ_m equal to one is never rejected in their empirical applications. We, therefore, restrict this parameter to unity to economize on the number of parameters to be estimated.

In addition, whether or not a volatility component (new arrival) being updated for the individual multifractal processes is governed by the transition probabilities, we use $\gamma_i = 2^{(k-i)}$ as in Lux (2008). The correlation of arrivals between the two series is characterized by a parameter $\lambda \in [0, 1]$, i.e., the probability of a new arrival at hierarchy level i for one time series given a new arrival in the other time series is $(1 - \lambda)\gamma_i + \lambda$. New arrivals are independent if $\lambda = 0$ and simultaneous if $\lambda = 1$.

2.2 Filtering via simulation

The extension to bivariate multifractal model poses challenges for the estimation of the model parameters. For a binomial distribution of the multipliers $M^{(i)}_{n,t}$, with both assets being characterized by the same number k of multipliers, the bivariate model has the Markov-switching structure with a total of $(2^2)^k = 4^k$ different states, i.e., m^i, with $i = 1, 2, \ldots, 4^k$. The likelihood function of such a Markov-switching model is defined in the conventional way as follows:

$$\begin{aligned} f(r_1, \cdots, r_T; \theta) &= \prod_{t=1}^{T} f(r_t | r_1, \cdots, r_{t-1}) \qquad (7) \\ &= \prod_{t=1}^{T} \left[f(r_t | M_t = m^i) \cdot \sum_{i=1}^{4^k} P(M_t = m^i | r_1, \cdots, r_{t-1}) \right]. \end{aligned}$$

with θ being the vector of parameters. The transition matrix A is composed of the conditional probabilities $a_{ij} = P(M_{t+1} = m^j | M_t = m^i)$ with $i, j = \{ 1, 2 \ldots 4^k \}$, with the conditional probability defined as: $\pi^i_t = P(M_t = m^i | r_1, \cdots, r_t)$.

In general, it is always possible to implement the maximum likelihood estimation via Eq. (7) when the dimension size of the transition matrix A is reasonable. Since the large degree of heterogeneity of volatility trajectories that can be modelled with a relatively large number of k is one of most attractive features of the multifractal approach, therefore, with a larger number of k, the numerous multiplications with the transition matrix A within an optimization step pose computational constraints on this straight forward approach of Eq. (7). One can easily see the computational complexity of evaluating the $4^k \times 4^k$ elements of the transition matrix at each time-step for the maximum likelihood estimation. In practice, it hardly works for the number of multipliers larger than 5, i.e., $k > 5$, due to the capacity of the current personal computer. [3]

In order to reduce the computational burden, Calvet et al., (2006) propose a simulation-based maximum likelihood (SML) approach using a particle filter with sampling/importance resampling (SIR), cf. Rubin (1987), Pitt and Shephard (1999). Instead of explicitly evaluating the exact $4^k \times 4^k$ elements within the transition matrix, the particle filter uses an approximation to the prediction probability density $P(M_t = m^i_t | r_{t-1})$, by using the discrete support of a finite

[3] For arbitrary n-variate processes it would amount to $(2^n)^k$, moving on to trivariate models, the dimension of the transition matrix would become 8^k, which could only be handled with much smaller k. Also, the variants of MSM with a continuous distribution of multipliers could not be estimated via maximum likelihood approach at all due to their infinite state spaces.

number B of particles. Denoting by $m^{(b)}$ the volatility state of any particle $b = 1, \ldots, B$, the one-step-ahead conditional probability is approximated by:

$$\pi_t^i \propto f(r_t | M_t = m^i) \frac{1}{B} \sum_{b=1}^{B} P(M_t = m^i | M_{t-1} = m^{(b)}). \tag{8}$$

As can be seen, Eq (8) provides a discrete approximation of the conditional densities by filtering the particles. This approximation not only simplifies the maximum likelihood estimation by avoiding evaluation of the infeasible dimensions of the transition matrix, but also provides a practical solution for multi-step forecasting. For instance, to conduct one-step ahead forecast, we complete the approximation by simulating each $m^{(b)}$ one-step forward and re-weighting using an importance sampler as follows:

1. Simulate the Markov chain one-step-ahead to obtain $\hat{M}_{t+1}^{(1)}$ given $M_t^{(1)}$. Repeat B times to generate draws $\hat{M}_{t+1}^{(1)}, \hat{M}_{t+1}^{(2)}, \ldots, \hat{M}_{t+1}^{(B)}$.

2. This preliminary step only uses information available at date t, and must therefore be adjusted to account for the new return. Drawing a random number q from 1 to B with probabilities of:

$$P(q = b) = \frac{f(r_{t+1} \mid M_{t+1} = m^{(b)})}{\sum_{i=1}^{B} f(r_{t+1} \mid M_{t+1} = m^{(i)})}. \tag{9}$$

3. We then select $M_{t+1}^{(1)} = \hat{M}_{t+1}^{(q)}$, and repeat B times to obtain B draws to get the new $M_{t+1}^{(1)}, \ldots M_{t+1}^{(B)}$, which have been adjusted to account for the new realizations.

This recursive procedure provides a discrete approximation to Bayesian updating, which is computationally convenient in large state spaces. We will follow the procedure for the volatility forecasting in the next section.

3 Data and Empirical Results

3.1 Data Description

We have collected the SSRs, which as indicated in the introduction is a common monetary policy instrument that captures both conventional and unconventional monetary policy decisions for the eight countries/regions, namely Australia, Canada, Euro area, Japan, New Zealand, Switzerland, the UK, the US, and their corresponding Morgan Stanley Capital International (MSCI) stock markets indices in US dollars. The SSR estimates used in this paper are derived from the works of Krippner (2013, 2015) and are considered to be an improvement over those obtained by Wu and Xia (2016), as discussed in detail by Krippner (2020). Besides the robust methodology used by Krippner (2013, 2015) in the estimation of the SSRs, the SSR estimates are available at daily rather than monthly frequency, and also for eight countries/regions instead of three (i.e., the Euro area, the UK and the US), as in the case of Wu and Xia (2016). Both SSR and MSCI time series data cover the period from 02/01/1995 to 26/03/2021, with the start and end dates being driven by the availability of the SSR estimates at the time of writing this paper. We calculate daily MSCI stock index returns as the log difference $r_t = \ln(MSCI_t) - \ln(MSCI_{t-1})$, where $MSCI_t$ is the MSCI stock index. While, SSR change is calculated as the difference of SSR_t, i.e., $r_t = SSR_t - SSR_{t-1}$. While the MSCI indexes are obtained from Datastream, the SSRs are derived from the website of Dr. Leo Krippner.[4]

Table 1 reports the descriptive statistics of SSR and MSCI returns for the 8 countries/regions. In general, we observe the SSR mean returns are all negative, and their standard deviations are relatively larger than ones of MSCI index returns. We also observe that most of MSCI index returns exhibit negative skewness except for Japan, and all countries have considerable positive excess kurtosis. The autocorrelations for the squared returns at different time lag are also reported, and one can see not only the significant autocorrelation at short time lag of 5 days but also for longer horizon of 100 days, which are evident of apparent long term dependency of squared returns as proxy of volatility. Table 1 also reports the pertinent ARCH tests and the Augmented Dickey-Fuller (ADF, Dickey and Fuller (1981)) statistics in the bottom two rows, which depicts evidence of significant volatility clustering, and the stationarity of the empirical data.

3.2 Empirical Findings

We separate each time series data into two subsets (i.e., in-sample data used for estimation, and out-of-sample data for forecasting assessment). We estimate the in sample data from 02/01/1995 to 15/08/2008 when Lehman Brothers filed bankruptcy, and then perform out-of-sample evaluation of volatility using data from 16/08/2008 to 26/03/2021, based on the DCC-GARCH, and the univariate and bivariate multifractal models. The in-sample DCC -GARCH estimates are shown in Table 2. Specifically, we estimate the bivariate DCC-GARCH model of Engle (2002), which is an extension of the conventional GARCH model of Engle (1982). With the univariate GARCH (1, 1) formulated as: $r = \mu + \epsilon_t$, and $\epsilon_t | I_{t-1} \sim N(0, \sigma_t)$, the volatility process follows: $\sigma_t = \omega + \alpha \cdot \epsilon^2_{t-1} + \beta \cdot \sigma^2_{t-1}$. The dynamic conditional correlation (DCC) has a non-linear GARCH-type specification: $Q_t = (1 - a - b)\bar{Q} + a\epsilon_t \cdot \epsilon'_{t-1} + bQ_{t-1}$, where a and b are the so-called news and decay coefficients, respectively. $\bar{Q} = E[\epsilon_t \cdot \epsilon'_{t-1}]$ is the unconditional variance and covariance matrix of the standardized residuals (the unconditional covariance) and ρ_{12} represents the unconditional correlation coefficient in the matrix $\bar{Q}$.[5] The GARCH parameters for individual time series

[4] https://www.ljkmfa.com/.

[5] We also studied the DCC-GARCH of Tse and Tsui (2002), where: $R_t = (1 - a - b)R + a\epsilon_t \cdot \epsilon'_{t-1} + bR_{t-1}$, with $R_t = diag(Q_t)^{-1} Q_t diag(Q_t)^{-1}$. However, we found qualitatively similar results, which are available upon request from the authors.

have their usual interpretation. Though most of GARCH persistent parameter β estimates for the MSCI indexes returns are within a reasonable range, i.e., around 0.9, the GARCH reaction parameter estimates α for the SSR returns are much higher, and is indicative of relatively more spikes in volatility.

Table 3 reports univariate and bivariate models estimates for SSR and MSCI indexes returns for the eight countries/regions. Note that the vector of multifractal model parameters consists of $\{m_{1,i}, \sigma_i, \rho, \lambda\}$ (i stands for SSR and MSCI). We adopt a similar two-stage procedure proposed by Calvet et al., (2006), which combines an ML estimator for the first group of parameters $\{m_{1,i}, \sigma_i\}$ with an SML estimator for the second group $\{\rho$ and $\lambda\}$. The latter are obtained through the particle filter approach keeping the first set of parameters at their ML-estimated values, and we then maximize the simulated likelihood using the Nelder-Mead algorithm. The two-stage approach provides a reduction in computation time against a complete SML approach, and it also makes the choice of larger number of cascade level k feasible. Note that, use $k = 8$ which is consistent with the existing literature. The first four of these parameters could be identified by an estimator for a univariate multifractal model, while the remaining ones require the complete bivariate data set. In terms of fractality of volatility as measured by the parameter m_1, we find that SSR returns exhibit stronger persistence than the MSCI returns for each pair of countries/regions. The unconditional volatility estimate of σ reveals that SSR returns are much volatile than those of the MSCI indexes. In terms of correlation of innovations ρ, most of countries exhibit strong positive correlations, except the cases of Canada and New Zealand. Since the correlation across markets often pertains to arrival of new volatility components, empirical estimates of λ show significant degree of co-movement for all pairs of SSR and MSCI returns, with the result for the US appearing to be more pronounced. These findings are in line with the possible bi-directional relationship outlined in the introduction between stock market volatility and monetary policy rate uncertainty, and corroborates the need to use a bivariate multifractal approach.

Table 4 to Table 6 present the out-of-sample forecasting performances at different horizons with a range 1 day to 100 days. We report relative mean square error (RMSE) and relative mean absolute error (RMAE), i.e., MSE and MAE divided by the respective statistics of the naive volatility predictor (derived using historical volatility). Therefore, any value smaller than 1 indicate an improvement relative to historical volatility. A glance of Table 4 reveals that the volatility forecasting performance of the DCC-GARCH models are quite accurate in most of the short-term horizons, specifically for one- and five-steps-ahead forecasts. However, its performance deteriorates at longer horizons, i.e., for 50- and 100-days-ahead forecasts in particular.

We now turn to Table 5 and Table 6, which reports the forecasting performances of the univariate and bivariate multifractal models. Though our results are not entirely homogeneous, in general, one can observe that accurate forecasts are obtained when we incorporate the information contents of both SSR and MSCI returns within the bivariate multifractal models. The fact that these gains are statistically significant relative to the univariate version of the multifractal model is also strongly confirmed by the dominant number of significant Diebold and Mariano (1995) test statistics reported in Table6. This observation confirms the underlying theoretical relationship that is likely to exist in both directions between the volatility process of the stock and SSR returns, as discussed in detail in the introduction.

We then assessed the forecasting performances across the bivariate multifractal and the DCC-GARCH model, which are also included in Table 6. In general, the bivariate multifractal model outperforms the DCC-GARCH in a statistically significant manner at long horizons under under both RMSE and RMAE criteria, and at all horizons under the latter. The results are expected, especially if we recall the autocorrelations of squared returns reported in Table 1, which show that there are still sizeable autocorrelations at the 100-day lag as can be seen from the ACF(100). Alternatively, while the traditional GARCH-type models are not able to capture the existence of significant long-term dependence (i.e., long-memory) present in the data, the multifractal models produce better forecasting performances by their ability to capture the same in a genuine, i.e., non-spurious-manner, by allowing for regime-switching cascades.

4 Concluding remarks

Theoretically it is expected that stock market volatility and the monetary policy rate uncertainty share a bi-directional relationship. Given this, we forecast equity market volatility and do the same for the shadow short rate (SSR), capturing both conventional and unconventional monetary policy decisions, of eight advanced economies (Australia, Canada, Euro area, Japan, New Zealand, Switzerland, the UK, and the US) using bivariate Markov-switching multifractal (MSM) model over the daily period of January, 1995 to March, 2021. We find that, in line with the theoretical expectations, besides in-sample evidence, the bivariate MSM method generate smaller forecasts errors in a statistically significant manner, relative to not only the benchmark historical volatility and the univariate MSM models, but also the Dynamic Conditional Correlation-Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) framework, particularly at longer forecast horizons.

Our results have important implications for academics, investors and policymakers. First, from the perspective of a financial economist, our analyses provide support to the theoretical claim of two-way Granger causality between stock market volatility and the uncertainty surrounding conventional and unconventional monetary policy. Further, the superior performance of the bivariate MSM model, highlights the need to capture long-memory and structural breaks simultaneously, when forecasting stock market and monetary policy rate volatilities based on information contained in these two endogenous variables for each other. Second, this latter finding also suggests that investors can improve their portfolio allocation, pricing of derivative securities and risk management by accommodating the role of monetary policy uncertainty into their models of volatility that are primarily multivariate in nature and capture long-memory and regime changes, i.e., via the usage of a bivariate MSM model. Finally, since stock market volatility provides high-frequency forecasts of monetary policy rate uncertainty, which in turn is known to contain leading information for economic activity, policymakers can nowcast low-frequency macroeconomic variables, and design appropriate policy responses in advance. In the current context of the COVID-19 pandemic which has resulted in tremendous stock market volatility, our findings become even more crucial for investors and policymakers.

As part of future research, it would be interesting to extend our analyses to emerging economies, conditional on the availability of SSR estimates for these countries.

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Table 1: Descriptive statistics

	Australia		Canada		EU		Japan	
	SSR	MSCI	SSR	MSCI	SSR	MSCI	SSR	MSCI
Mean	-0.18	0.02	-0.14	0.03	-0.11	0.02	-0.07	0.00
S.D	2.67	1.42	3.07	1.34	1.72	1.30	1.78	1.37
Min	-24.28	-15.98	-24.99	-14.25	-14.37	-14.82	-11.39	-9.51
Max	15.64	8.81	33.67	12.21	10.89	11.06	13.98	12.27
Skewness	-0.56	-0.80	0.16	-0.92	0.08	-0.43	0.42	0.02
Kurtosis	9.80	12.98	13.51	16.59	6.41	12.78	7.81	7.66
ACF(5)	0.18	0.25	0.25	0.3	0.26	0.25	0.18	0.12
ACF(100)	0.07	0.04	0.15	0.03	0.05	0.05	0.03	0.04
ARCH	542.80	542.80	1772.16	1772.16	568.56	568.56	1657.71	1657.71
ADF	-31.93[2]	-57.30[1]	-37.18[3]	-31.71[1]	-23.42[2]	-39.20[3]	-29.28[2]	-62.14[1]

	New Zealand		Switzerland		UK		USA	
	SSR	MSCI	SSR	MSCI	SSR	MSCI	SSR	MSCI
Mean	-0.18	0.01	-0.09	0.03	-0.12	0.01	-0.12	0.03
S.D	2.33	1.35	1.61	1.15	2.38	1.29	2.42	1.19
Min	-36.68	-15.76	-14.67	-11.33	-16.90	-14.21	-17.31	-12.92
Max	19.24	11.03	13.24	9.74	15.73	12.16	11.82	11.04
Skewness	-1.07	-0.44	0.62	-0.20	0.04	-0.39	-0.31	-0.45
Kurtosis	17.94	10.40	9.21	9.42	7.81	14.44	6.12	14.24
ACF(5)	0.08	0.09	0.33	0.23	0.28	0.28	0.19	0.32
ACF(100)	0.04	0.02	0.01	0.05	0.07	0.06	0.02	0.04
ARCH	492.58	492.58	961.86	961.86	724.97	724.97	659.21	659.21
ADF	-25.56[1]	-58.32[6]	-20.96[4]	-39.94[1]	-25.61[1]	-32.86[4]	-30.39[6]	-61.42[1]

Note: This table reports a range of descriptive statistics for the shadow short rate (SSR) returns of 8 countries/regions: Australia, Canada, Eurozone, Japan, New Zealand, Switzerland, UK, US, and their corresponding MSCI stock index returns. The specific statistics reported are the mean returns, its standard deviation, minimum (min.) and maximum (max.) returns, skewness and kurtosis of returns, followed by ACF(5)and ACF(100) which are the autocorrelations of the squared returns at 5 and 100 lags respectively. ARCH is the heteroskedasticity test: we filter the returns series using an autoregressive model of an order of 12 and then implement the Lagrange-Multiplier test examining the null-hypothesis of no-ARCH. The LM test statistic and the p-value (in parenthesis) are reported. The last row contains an Augmented Dickey and Fuller (ADF, 1981) unit root test. The optimal lag length in the ADF regression is chosen using the Schwarz Information criterion (SIC); we begin with a maximum of eight lags and then use the SIC to select the optimal lag length. The optimal lag lengths are reported in square brackets beside the ADF test statistic.

Table 2: In-sample estimates for the DCC GARCH(1,1) model

	Australia		Canada		EU		Japan	
	SSR	MSCI	SSR	MSCI	SSR	MSCI	SSR	MSCI
μ	-0.083	0.056	-0.005	0.088	-0.073	0.064	-0.204	0.005
	(0.049)	(0.018)	(0.044)	(0.016)	(0.029)	(0.014)	(0.031)	(0.020)
ω	2.544	0.038	0.218	0.014	0.810	0.011	0.528	0.030
	(0.349)	(0.010)	(0.047)	(0.004)	(0.075)	(0.003)	(0.101)	(0.008)
α	0.357	0.073	0.214	0.076	0.509	0.092	0535	0.075
	(0.072)	(0.017)	(0.056)	(0.020)	(0.067)	(0.012)	(0.057)	(0.009)
β	0.358	0.901	0.779	0.917	0.182	0.901	0.397	0.912
	(0.174)	(0.025)	(0.044)	(0.023)	(0.109)	(0.013)	(0.065)	(0.010)
ρ_{12}	-0.258		-0.016		-0.017		-0.043	
	(0.021)		(0.037)		(0.091)		(0.058)	
a	0.011		0.012		0.023		0.010	
	(0.004)		(0.034)		(0.006)		(0.003)	
b	0.988		0.984		0.971		0.986	
	(0.005)		(0.052)		(0.010)		(0.004)	

	New Zealand		Switzerland		UK		USA	
	SSR	MSCI	SSR	MSCI	SSR	MSCI	SSR	MSCI
μ	-0.194	0.047	-0.012	0.057	-0.088	0.052	-0.022	0.057
	(0.037)	()0.019	(0.025)	(0.015)	(0.038)	(0.015)	(0.044)	(0.014)
ω	1.337	0.041	0.520	0.033	1.388	0.018	1.627	0.008
	(0.107)	(0.010)	(0.037)	(0.007)	(0.190)	(0.005)	(0.214)	(0.002)
α	0.179	0.088	0.810	0.098	0.321	0.086	0.211	0.064
	(0.036)	(0.038)	(0.038)	(0.011)	(0.076)	(0.014)	(0.032)	(0.011)
β	0.784	0.891	0.022	0.875	0.626	0.899	0.784	0.930
	(0.120)	(0.046)	(0.027)	(0.014)	(0.110)	(0.018)	(0.030)	(0.012)
ρ_{12}	-0.012		0.046		-0.047		-0.053	
	(0.051)		(0.029)		(0.262)		(0.281)	
a	0.007		0.012		0.015		0.020	
	(0.003)		(0.005)		(0.009)		(0.007)	
b	0.988		0.982		0.982		0.976	
	(0.005)		(0.008)		(0.015)		(0.011)	

Note: This table reports the in-sample estimates for DCC-GARCH model for the shadow short rate (SSR) returns of the 8 countries/regions: Australia, Canada, Eurozone, Japan, New Zealand, Switzerland, UK, US, and their corresponding MSCI stock indexes. GARCH parameters estimates $(\mu, \omega, \alpha, \beta)$ are obtained via estimating the univariate GARCH model for each SSR and MSCI returns individually: $r = \mu + \epsilon_t$, and the volatility process $\sigma_t = \omega + \alpha \cdot \epsilon_{t-1}^2 + \beta \cdot \sigma_{t-1}^2$. The DCC parameter estimates (a and b) in $Q_t = (1-a-b)\bar{Q} + a\epsilon_t \cdot \epsilon_{t-1}' + bQ_{t-1}$ are obtained by joint estimation for each pair of SSR and MSCI returns.

Table 3: In-sample estimates of the multifractal models

	Australia		Canada		EU		Japan	
	SSR	MSCI	SSR	MSCI	SSR	MSCI	SSR	MSCI
$m_{1,i}$	1.324	1.233	1.386	1.270	1.309	1.258	1.357	1.228
	(0.019)	(0.011)	(0.028)	(0.005)	(0.012)	(0.007)	(0.012)	(0.005)
σ_i	2.893	1.277	3.615	1.143	1.619	1.060	1.904	1.392
	(0.022)	(0.010)	(0.028)	(0.013)	(0.013)	(0.012)	(0.022)	(0.011)
ρ	0.036		0.012		0.100		0.136	
	(0.017)		(0.013)		(0.021)		(0.022)	
λ	0.165		0.157		0.150		0.135	
	(0.002)		(0.011)		(0.031)		(0.045)	

	New Zealand		Switzerland		UK		USA	
	SSR	MSCI	SSR	MSCI	SSR	MSCI	SSR	MSCI
m_1	1.433	1.270	1.405	1.248	1.333	1.245	1.317	1.267
	(0.027)	(0.007)	(0.029)	(0.005)	(0.017)	(0.010)	(0.021)	(0.004)
σ	2.718	1.360	1.564	1.143	2.276	1.096	2.617	1.021
	(0.022)	(0.010)	(0.029)	(0.005)	(0.017)	(0.005)	(0.017)	(0.007)
ρ	-0.016		0.058		0.092		0.043	
	(0.021)		(0.024)		(0.041)		(0.007)	
λ	0.150		0.149		0.150		0.270	
	(0.0011)		(0.022)		(0.012)		(0.047)	

Note: This table reports the in-sample estimates of the parameters of the univariate and bivariate multifractal model for the shadow short rate (SSR) returns of the 8 countries/regions: Australia, Canada, Eurozone, Japan, New Zealand, Switzerland, the UK, and the US, and their corresponding MSCI stock index returns. The estimation is based on the two-step estimation, that is, m_1 and σ for SSR and stock index returns are obtained by maximizing the univariate likelihood, and in the second-step, we obtain ρ and λ by maximizing the simulated bivariate likelihood given the estimates of the first-step. Standard errors for the second step estimates are computed as in Calvet et al., (2006, Appendix A.4).

Table 4: Volatility forecasts: DCC-GARCH model

		Australia		Canada		EU		Japan	
Horizon		SSR	MSCI	SSR	MSCI	SSR	MSCI	SSR	MSCI
	RMSE								
1		0.542	0.745	0.425	0.769	0.656	0.843	0.463	0.833
5		0.967	0.807	0.654	0.823	0.930	0.859	0.909	0.895
10		0.998	0.849	0.718	0.908	0.992	0.898	1.204	0.923
20		0.999	0.883	0.759	0.973	0.998	0.962	1.477	0.998
50		0.999	0.976	1.179	1.114	0.998	1.039	1.866	1.038
100		0.999	0.992	1.768	1.083	0.999	1.013	1.933	1.017
	RMAE								
1		0.735	0.943	0.293	0.953	0.709	1.060	0.492	0.839
5		0.964	0.947	0.394	0.979	0.986	1.072	0.904	0.866
10		0.993	0.964	0.465	1.029	1.007	1.096	1.205	0.889
20		0.997	0.986	0.584	1.082	1.010	1.142	1.526	0.936
50		0.997	1.000	0.994	1.188	1.011	1.183	1.807	1.007
100		0.997	1.002	1.730	1.233	1.012	1.183	1.849	1.057

		New Zealand		Switzerland		UK		USA	
		SSR	MSCI	SSR	MSCI	SSR	MSCI	SSR	MSCI
	RMSE								
1		0.966	0.805	0.585	0.834	0.545	0.830	0.582	0.774
5		1.391	0.860	0.858	0.858	0.913	0.848	0.924	0.834
10		1.561	0.906	0.946	0.892	0.983	0.894	0.986	0.913
20		1.724	0.935	0.995	0.941	0.999	0.953	1.034	1.021
50		1.808	0.971	1.031	0.981	1.052	1.013	1.075	1.139
100		2.042	0.998	1.044	0.993	1.099	1.020	1.096	1.110
	RMAE								
1		0.490	0.943	0.520	0.916	0.631	1.031	0.692	0.937
5		0.861	0.958	0.985	0.932	0.956	1.042	0.948	0.97
10		1.318	0.973	1.095	0.949	0.998	1.059	1.005	1.018
20		1.546	0.991	1.131	0.969	1.007	1.082	1.021	1.076
50		1.603	1.035	1.146	0.996	1.008	1.075	1.033	1.191
100		1.788	1.070	1.153	1.007	1.008	1.044	1.040	1.228

Note: This table reports the multi-horizon volatility forecast performances based on the DCC-GARCH(1,1) model for the shadow short rate (SSR) returns of 8 countries/regions: Australia, Canada, Eurozone, Japan, New Zealand, Switzerland, the UK, and the US and their corresponding MSCI index returns. We report the relative MSE (RMSE) and relative MAE (RMAE) measurements, computed by dividing the MSE and MAE estimates by the pertinent MSE and MAE of the naive volatility predictor (using historical volatility), therefore any values smaller than 1 indicate an improvement against historical volatility.

Table 5: Volatility forecasts: Univariate multifractal model

		Australia		Canada		EU		Japan	
Horizon		SSR	MSCI	SSR	MSCI	SSR	MSCI	SSR	MSCI
	RMSE								
1		0.678	0.892	0.417	0.835	0.734	0.926	0.558	0.896
5		0.914	0.919	0.554	0.871	0.913	0.938	0.823	0.933
10		0.950	0.945	0.578	0.912	0.961	0.944	0.937	0.939
20		0.902	0.941	0.533	0.946	0.988	0.954	0.962	0.965
50		1.027	0.973	0.549	0.995	1.066	0.971	0.993	0.982
100		1.041	1.001	0.568	1.021	1.089	0.984	1.044	0.993
	RMAE								
1		0.713	0.949	0.285	0.908	0.821	0.963	0.511	0.819
5		0.788	0.956	0.356	0.903	0.987	0.957	0.662	0.839
10		0.805	0.964	0.366	0.919	1.026	0.965	0.719	0.843
20		0.795	0.979	0.359	0.936	1.039	0.979	0.759	0.854
50		0.827	0.998	0.362	0.975	1.096	0.991	0.759	0.870
100		0.844	1.056	0.376	1.019	1.129	1.026	0.788	0.872

		New Zealand		Switzerland		UK		USA	
		SSR	MSCI	SSR	MSCI	SSR	MSCI	SSR	MSCI
	RMSE								
1		0.739	0.877	0.676	0.909	0.680	0.937	0.692	0.844
5		0.899	0.908	0.867	0.926	0.863	0.949	0.907	0.898
10		0.931	0.917	0.927	0.936	0.899	0.955	0.962	0.912
20		0.979	0.933	0.999	0.948	0.952	0.966	1.005	0.937
50		1.051	0.970	1.034	0.979	0.986	0.998	1.059	1.011
100		1.097	1.029	1.043	0.996	1.034	1.087	1.087	1.038
	RMAE								
1		0.447	0.945	0.691	0.891	0.74	0.949	0.709	0.878
5		0.584	0.954	0.892	0.899	0.902	0.948	0.843	0.888
10		0.610	0.949	0.983	0.904	0.938	0.953	0.865	0.908
20		0.636	0.961	1.057	0.912	0.965	0.966	0.890	0.931
50		0.698	0.981	1.073	0.931	1.014	0.981	0.925	1.003
100		0.637	1.015	1.129	0.954	1.064	0.997	0.946	1.046

Note: This table reports the multi-horizon volatility forecast performances based on the univariate multifractal model for the shadow short rate (SSR) returns of 8 countries/regions: Australia, Canada, Eurozone, Japan, New Zealand, Switzerland, the UK, and the US and their corresponding MSCI index returns. We report the relative MSE (RMSE) and relative MAE (RMAE) measurements, computed by dividing the MSE and MAE estimates by the pertinent MSE and MAE of the naive volatility predictor (using historical volatility), therefore any values smaller than 1 indicate an improvement against historical volatility.

Table 6: Volatility forecasts: Bivariate multifractal model

		Australia		Canada		EU		Japan	
Horizon		SSR	MSCI	SSR	MSCI	SSR	MSCI	SSR	MSCI
	RMSE								
1		0.659*	0.852*	0.404*$^\sharp$	0.815*	0.739	0.915*	0.570	0.862*
5		0.898*$^\sharp$	0.871*	0.539*$^\sharp$	0.833*	0.904*$^\sharp$	0.925*	0.819$^\sharp$	0.894*
10		0.955$^\sharp$	0.891*	0.567*$^\sharp$	0.872*$^\sharp$	0.953*$^\sharp$	0.933*	0.938$^\sharp$	0.901*$^\sharp$
20		0.899$^\sharp$	0.920*	0.460*$^\sharp$	0.942*$^\sharp$	0.978*	0.947*$^\sharp$	0.965$^\sharp$	0.952*$^\sharp$
50		1.029	0.971$^\sharp$	0.445*$^\sharp$	0.983*$^\sharp$	1.056*	0.970$^\sharp$	0.998$^\sharp$	0.990$^\sharp$
100		1.037	1.012*	0.463 *$^\sharp$	1.005*$^\sharp$	1.080*	0.985$^\sharp$	1.046$^\sharp$	1.001$^\sharp$
	RMAE								
1		0.721$^\sharp$	0.930*$^\sharp$	0.275*$^\sharp$	0.886*$^\sharp$	0.820	0.966$^\sharp$	0.495*	0.805*$^\sharp$
5		0.797$^\sharp$	0.936*$^\sharp$	0.338*$^\sharp$	0.886*$^\sharp$	0.967*$^\sharp$	0.960$^\sharp$	0.638*$^\sharp$	0.825*$^\sharp$
10		0.816$^\sharp$	0.944*$^\sharp$	0.349*$^\sharp$	0.899*$^\sharp$	1.003*	0.970$^\sharp$	0.698*	0.829*$^\sharp$
20		0.805$^\sharp$	0.959*$^\sharp$	0.342*$^\sharp$	0.913*$^\sharp$	1.012*	0.988$^\sharp$	0.740*$^\sharp$	0.843*$^\sharp$
50		0.848$^\sharp$	1.010	0.344*$^\sharp$	0.940*$^\sharp$	1.070*	1.005$^\sharp$	0.743	0.860*$^\sharp$
100		0.870$^\sharp$	1.058	0.357*$^\sharp$	0.974*$^\sharp$	1.105*	1.035$^\sharp$	0.769	0.863*$^\sharp$

		New Zealand		Switzerland		UK		USA	
		SSR	MSCI	SSR	MSCI	SSR	MSCI	SSR	MSCI
	RMSE								
1		0.720*$^\sharp$	0.861*	0.608*	0.884*	0.645*	0.917*	0.699	0.815*
5		0.880*$^\sharp$	0.892*	0.836*$^\sharp$	0.902*	0.838*$^\sharp$	0.930*	0.907$^\sharp$	0.874*
10		0.913*$^\sharp$	0.887*$^\sharp$	0.932$^\sharp$	0.917*	0.887*$^\sharp$	0.937$^\sharp$	0.973$^\sharp$	0.901*$^\sharp$
20		0.967*$^\sharp$	0.918*$^\sharp$	1.037	0.936*	0.952$^\sharp$	0.955	1.025$^\sharp$	0.948$^\sharp$
50		1.033*$^\sharp$	0.972	1.040	0.984	1.013$^\sharp$	0.979$^\sharp$	1.065$^\sharp$	0.996*$^\sharp$
100		0.981*$^\sharp$	1.034	1.051	1.008	1.072$^\sharp$	0.989$^\sharp$	1.092	1.017*$^\sharp$
	RMAE								
1		0.436*$^\sharp$	0.933*$^\sharp$	0.678*$^\sharp$	0.895$^\sharp$	0.738	0.954$^\sharp$	0.706*	0.859*$^\sharp$
5		0.568*$^\sharp$	0.943*$^\sharp$	0.913$^\sharp$	0.904$^\sharp$	0.903$^\sharp$	0.952*$^\sharp$	0.822$^\sharp$	0.869*$^\sharp$
10		0.594*$^\sharp$	0.937*$^\sharp$	0.995$^\sharp$	0.912$^\sharp$	0.944$^\sharp$	0.959$^\sharp$	0.853*$^\sharp$	0.883*$^\sharp$
20		0.623*$^\sharp$	0.949*$^\sharp$	1.068$^\sharp$	0.924$^\sharp$	0.977$^\sharp$	0.978$^\sharp$	0.877*$^\sharp$	0.902*$^\sharp$
50		0.680*$^\sharp$	0.972*$^\sharp$	1.090$^\sharp$	0.954$^\sharp$	1.014	1.001$^\sharp$	0.903*$^\sharp$	0.951*$^\sharp$
100		0.624*$^\sharp$	1.005*$^\sharp$	1.131$^\sharp$	0.974$^\sharp$	1.073	1.016$^\sharp$	0.936*$^\sharp$	0.985*$^\sharp$

Note: This table reports the multi-horizon volatility forecast performances based on the bivariate multifractal model for the shadow short rate (SSR) returns of 8 countries/regions: Australia, Canada, Eurozone, Japan, New Zealand, Switzerland, UK, US and their corresponding MSCI index returns. We report the relative MSE (RMSE) and relative MAE (RMAE) measurements, computed by dividing the MSE and MAE estimates by the pertinent MSE and MAE of the naive volatility predictor (using historical volatility), therefore any values smaller than 1 indicate an improvement against historical volatility. $^\sharp$ indicates an improvement of the bivariate multifractal model against the DCC-GARCH model at 5% level; * indicates an improvement of bivariate multifractal model against the univariate model at 5% level; All comparisons are based on the test statistics of Diebold and Mariano (1995).