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Abstract

I construct and partially characterize the solution of a life-cycle model of fertility choice and human capital accumulation. Because children take time to raise, women face a trade-off between between lifetime earnings and childbearing. The model implies that (i) earnings must drop discontinuously at the time of a birth; (ii) age at first birth and human capital will be positively correlated; and (iii) a permanently higher demand for skill causes women to delay first births. I show that the second of these predictions holds in a sample of South African women drawn from the first wave of the National Income Dynamics Study.

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1. Introduction

1.1 Motivation

Over the long run, economic growth is a powerful force for poverty reduction. This fact motivates macroeconomists and some policymakers to seek policies that may increase the rate of economic growth. In order to describe such policies, one needs to think in terms of a model in which the growth rate is determined in equilibrium - i.e. an "endogenous growth" model. An important class of these models, which I briefly describe below, has a natural link to fertility rates.

Economists have developed two broad types of such "endogenous growth" models. One type, most closely associated with the work of Romer (1986), Romer (1990) and Aghion and Howitt (1992), emphasizes the role of firms in productivity growth. The other type, originally developed in Uzawa (1965) and popularized by Lucas (1988), highlights the accumulation of skills in the workforce - i.e., of human capital - as a key force driving long-run growth.

Because human capital is "embodied" in people, its accumulation raises not only the productivity of the workforce, but also the opportunity cost of time. And since bearing and raising children requires much parental time, there is a natural connection between fertility and economic growth, via labor supply decisions. Further, because investing in the human capital of one's children is a potential substitute for investing in physical capital, the time allocation decisions of the workforce affect not just current output (through labor supply), but also future output and productivity (through savings and consumption choices).

Aggregate data on economic growth, labour inputs, and various summary measures of fertility are widely available, for most countries. But a serious problem with the use of aggregate data is that it cannot deliver evidence on causality, even if the aggregation procedures used to measure factor inputs were uncontroversial.[1]

Thus, it may be worthwhile to start from the opposite end of the spectrum - i.e., from a microeconomic perspective. That is, one could start by modelling the labor supply, fertility, and consumption choices of individual workers, and aggregate them later. However, in order to use such data to estimate e.g. the costs of childbearing, one first needs to spell out the implications of an economic model of those choices. That is what I do in this paper.

The key force that drives the model here is the opportunity cost of a potential mother's time. In any model of fertility choice, it is necessary to assume that parental time is a major component of the cost of childbearing.[2] Measuring those opportunity costs is a necessary step in

[1] An easy way to see this point is to think of performing a growth accounting exercise on data generated by a Solow model with exogenous technological progress. One would attribute a nonzero fraction of growth to capital accumulation, even though the only cause of growth in that model is technology. The accumulation of capital is itself a consequence of the increases in technological productivity, so "adjusting" for it makes no sense.

[2] Without this assumption, it would be impossible to rationalize the fact that in the cross-section, richer parents

understanding the relationship between fertility and growth. These costs differ from person to person, because people vary in how their time is valued - in the labor market, certainly, but potentially also by their co-parents or extended families. These opportunity costs cannot be directly observed, for the same reason.

There are at least two important components to these time costs. One is the immediate loss of earnings from reduced labor supply, at given wages. However, workers' wages tend to rise over the course of their careers, although the extent to which they do depends on other factors, such as a worker's own education (Mincer (1958), Becker (1993); Heckman et al. (2006)). This lost wage growth is another, potentially large, component of the cost.

1.2 Main Results

In this paper, I construct and partially characterize the solution of a life-cycle model of fertility choice and human capital accumulation. Because children take time to raise, women face a trade-off between between lifetime earnings and childbearing. The model implies that:

(i) earnings must drop discontinuously at the time of a birth (this is Proposition 1);

(ii) age at first birth and human capital will be positively correlated (this is Proposition 2); and

(iii) a permanently higher demand for skill causes women to delay first births (this is Proposition 3).

Focusing on the timing rather than the level of fertility has the additional advantage that one can use data on incomplete fertility histories, which may be more readily available than completed fertility histories - even in contexts such as many African countries, where vital registration systems are weak. Completed fertility histories are also - by construction - several decades out of date, so models that can use more current data would be useful.

In section 4., I demonstrate that prediction (ii) holds in a sample of South African women drawn from the first wave of the National Income Dynamics Study. The relationship between education and age at first birth is hardly unique to South Africa, though: for example, Bongaarts et al. (2017) shows that across more than 40 countries in West Africa, East Africa, and Latin America, higher level of education are associated with later first births.

The model's prediction (i) - that women's earnings should drop sharply at the time of a first birth - has recently been shown (by Kleven et al. (2019b)) to hold in Danish data spanning more than three decades. Because the measurements they perform require highly detailed administrative data, it has been difficult to demonstrate the existence of these "child penalties", although Kleven et al. (2019a) show that they can be observed in several other European countries.

tend to have fewer children (at least, not without resorting to the assumption that children are "inferior goods").

1.3 Contribution to Literature

That completed fertility rates - i.e. the total number of children born to a mother over the course of her lifetime - are negatively correlated with income seems to be a very robust finding (see e.g. Jones et al. (2011)), at least after the Industrial Revolution. There is also evidence going back at least to the 19th century (e.g. Aaronson et al. (2014)) that more highly educated mothers have fewer children, on average.

It is natural to view both of these empirical patterns as arising from a model like the one presented here; that is, one where the opportunity cost of women's time is a major determinant of fertility choices. While there have been some attempts to measure the time costs of childbearing - Adda et al. (2017), for example, claim to find that in Germany, "skill atrophy" accounts for about a quarter of the total costs - but because there is not a common theoretical framework for measurement, it is unclear how to reconcile or compare estimates across different contexts.

Empirical studies in economics have typically focused on estimating the causal effects of observable variables - such as pronatalist subsidies (Cohen et al. (2013); Malkova (2018)), or the prevalence of childhood diseases (Bleakley and Lange (2009)) - on other observable outcomes, like fertility choices. The model I propose here is necessary in part because the opportunity costs of childbearing are not observable.

The main contribution of this paper is methodological: it suggests a method for estimating those costs. The technical aspects of the model (the use of continuous time, and the specification of how childcare requirements decline with age) are also potentially useful for future work, because they reduce the computational burden necessary to solve the model. Indeed, Hotz et al. (1997) highlights the computational intractability of discrete-time models of fertility choice, so the use of continuous time in this paper may represent a real advance.

2. Model

2.1 Preferences and Constraints

I construct a variation on a canonical model (Ben-Porath (1967)) of human capital accumulation over the life cycle, modified to include time costs of childcare and the option to choose the timing of a birth.

Preferences

Consider the forward-looking decisions of a woman who lives forever and discounts the future at rate ρ. She cares about the paths of her consumption (c_t) and parity (k_t) over time, ordered by the utility functional

$$\mathscr{U} = \int_0^\infty e^{-\rho t} u(c_t, k_t) dt \tag{1}$$

where $u(c,k)$ is increasing and weakly concave in both arguments. She can have at most one child; let T be the time (mother's age) at which this occurs. She can allocate her time to one of three mutually exclusive activities: working, investing in human capital, or childcare.

Time Costs of Childcare

Childcare takes less time for older children. If a newborn requires a fraction $\psi_0 < 1$ of her time, a child of age a requires only $\psi_0 e^{-\gamma a}$. Of course, before the first birth, no childcare is required ($\psi_t = 0$).

Investing in Human Capital

This woman can choose to spend a fraction s_t of her time investing in human capital, h_t. The gross gain in human capital is $\phi(h_t s_t)$, where ϕ is strictly increasing and strictly concave. I also assume ϕ satisfies the Inada condition $\lim_{x \to 0^+} \phi'(x) = \infty$, so that it's always optimal to invest for a nonzero amount of time. Human capital also depreciates at rate δ, so its law of motion is

$$\dot{h}_t = \phi(h_t s_t) - \delta h_t. \tag{2}$$

At each instant t, the feasibility constraint $0 \leq s_t \leq 1 - \psi_t$ must hold. Time not spent on childcare or human capital investment is spent working.

Borrowing and Saving

She finances consumption by borrowing and saving in a capital market where the (net) interest rate is r. Her net holding of physical capital is y_t, with the intertemporal budget constraint

$$\dot{y}_t = r y_t + w h_t [1 - s_t - \psi_t] - c_t. \tag{3}$$

2.2 Life-Cycle Problem

The primitives of the model are

- the subjective rate of time preference, ρ;
- the (flow) utility function for consumption and children, $u(c,k)$;
- the market interest rate, r;
- the decay rate of the childcare time burden, γ;

- the initial level of childcare burden, $\psi_0 \in (0,1)$;
- the depreciation rate of human capital, δ; and
- the production function for human capital, $\phi(\cdot)$.

Given these elements, we can break up the life-cycle problem of a woman with initial levels of human capital h_0 and physical capital y_0 into three related sub-problems. Suppose she has a first birth at age T.

Before T, she must choose (at each instant) how much time to spend investing in human capital and how much to save in the form of physical capital. Similarly, after her first birth, she must again choose an optimal path for consumption and time allocation.

Let $V_0(y_0,h_0|T,y',h')$ be the woman's maximal discounted utility before the first birth (when parity is $k=0$), subject to the terminal conditions $h_T = h'$ and $y_T = y'$, for given (y',h'). And let $V_1(y,h)$ be the woman's maximal discounted utility after the first birth (when parity is $k=1$), beginning from a given initial state (y,h). Then, given the levels of physical and human capital to hold at the moment of the first birth - call these (y_T,h_T) - her life-cycle utility is

$$\widehat{V}(T,h_T,y_T|y_0,h_0) = V_0(y_0,h_0|T,y_T,h_T) + e^{-\rho T}V_1(y_T,h_T).$$

Of course, T, h_T and y_T should themselves be chosen optimally. So her maximal lifetime utility is

$$V(y_0,h_0) = \max_{T,y_T,h_T} \widehat{V}(T,h_T,y_T|y_0,h_0).$$

I give the first-order necessary conditions characterizing V_0 and V_1 below, and I spell out some properties of the optimal choice of (T,h_T,y_T).

2.3 Human Capital Investment and Time Allocation

2.3.1 Before a Birth

$$V_0(y,h|T,y',h') \quad = \quad \max_{(s_t,c_t)_t} \int_0^T e^{-\rho t}u(c_t,0)dt \tag{4}$$

subject to the laws of motion (2) and (3), the initial condition $(y_0,h_0) = (y,h)$ and the terminal condition $(y_T,h_T) = (y',h')$. If we let λ_t be the costate for physical capital, and μ_t the costate for human capital, the current-value Hamiltonian for this problem is

$$\mathscr{H} = u(c_t,0) + \lambda_t\{ry_t + wh_t[1-s_t] - c_t\} + \mu_t\{\phi(h_t s_t) - \delta h_t\}$$

with the corresponding first-order conditions holding at each time t:

$$[c_t] \quad u_c(c_t, 0) = \lambda_t \tag{5}$$

$$[s_t] \quad -\lambda_t w h_t + \mu_t h_t \phi'(h_t s_t) \geq 0 \text{ (with equality if and only if } s_t < 1) \tag{6}$$

$$[\lambda_t] \quad \dot{\lambda}_t = \rho \lambda_t - r\lambda_t \tag{7}$$

$$[\mu_t] \quad \dot{\mu}_t = (\rho + \delta)\mu_t - \max\{\lambda_t w, \mu_t \phi'(h_t)\} \tag{8}$$

Notice also that by the envelope theorem, the marginal disutility of h_T is

$$\frac{\partial V_0}{\partial h_T} = -e^{-\rho T} \mu_T$$

i.e. the current marginal value of a unit of human capital at time T, discounted back to $t = 0$. Accumulating more human capital by time T is a cost, since it implies lower earnings and thus lower consumption. Similarly the marginal disutility of retaining more physical capital at T is

$$\frac{\partial V_0}{\partial y_T} = -e^{-\rho T} \lambda_T.$$

2.3.2 After a Birth

The woman's problem after the first birth is different because the need to provide childcare decreases the amount of time available for work and human capital investment, although this burden decreases as the child ages. The consumption-smoothing aspects of the problem are unchanged, though.

The discounted utility of starting a post-birth period with (y, h) is

$$V_1(y, h) = \max_{(c_t, s_t)} \int_0^\infty e^{-\rho t} u(c_t, 1) dt \tag{9}$$

subject to the laws of motion (2) and

$$\dot{y}_t = r y_t + w h_t [1 - s_t - \psi_t] - c_t, \tag{10}$$

the initial condition $(y_0, h_0) = (y, h)$, and the feasibility constraint $0 \leq s_t \leq 1 - \psi_t$. Notice that although the law of motion for human capital is unchanged, the law of motion for physical capital (i.e. the intertemporal budget constraint) *is* different after a birth, because only a fraction $1 - s_t - \psi_t$ is now available for work.

The current-value Hamiltonian is

$$\mathscr{H} = u(c_t, 1) + \lambda_t\{ry_t + wh_t[1 - s_t - \psi_t] - c_t\} + \mu_t\{\phi(h_t s_t) - \delta h_t\}$$

and the first-order conditions are

$$[c_t] \quad u_c(c_t, 1) = \lambda_t \tag{11}$$

$$[s_t] \quad -\lambda_t w h_t + \mu_t h_t \phi'(h_t s_t) \geq 0 \text{ (with equality if and only if } s_t < 1 - \psi_t) \tag{12}$$

$$[\lambda_t] \quad \dot{\lambda}_t = \rho\lambda_t - r\lambda_t \tag{13}$$

$$[\mu_t] \quad \dot{\mu}_t = (\rho + \delta)\mu_t - [1 - \psi_t]\max\{\lambda_t w, \mu_t \phi'(h_t[1 - \psi_t])\}. \tag{14}$$

The transversality conditions are

$$\lim_{T\to\infty} e^{-\rho T}\mu_T h_T = \lim_{T\to\infty} e^{-\rho T}\lambda_T y_T = 0.$$

2.4 Birth Timing

We can now state the woman's birth timing problem - noting that birth timing has to be jointly chosen with (y_T, h_T) - as follows:

$$(T^*, y_T^*, h_T^*) \quad = \quad \arg\max_{T, y_T, h_T} V_0(y_0, h_0 | T, y_T, h_T) + e^{-\rho T} V_1(y_T, h_T) \tag{15}$$

The first-order conditions for h_T and y_T are

$$[h_T] \quad \frac{\partial V_0}{\partial h_T}(y_0, h_0 | T, y_T, h_T) + e^{-\rho T}\frac{\partial V_1}{\partial h}(y_T, h_T) = 0 \tag{16}$$

$$[y_T] \quad \frac{\partial V_0}{\partial y_T}(y_0, h_0 | T, y_T, h_T) + e^{-\rho T}\frac{\partial V_1}{\partial y}(y_T, h_T) = 0. \tag{17}$$

Since $\frac{\partial V_0}{\partial h_T} = -e^{-\rho T}\mu_T$ and $\frac{\partial V_0}{\partial y_T} = -e^{-\rho T}\lambda_T$, we see that the path for both costates will be continuous at the time of the first birth (even if T is not chosen optimally). This fact alone leads to a first empirical prediction of the model: for those women who are not investing "full time" - i.e. they are choosing to work for some of their time both before and after a birth - earnings must drop discontinuously at the time of a birth, as the arrival of a child reduces her available time.

PROPOSITION 1. *If the solution for human capital investment, s_t, is interior both before and after the first birth, earnings must fall discontinously at T:*

$$\lim_{t\uparrow T} wh_t[1-s_t] = w[h_T - (\phi')^{-1}(\mu_T)] > w[h_T - (\phi')^{-1}(\mu_T) - h_T\psi_0] = \lim_{t\downarrow T} wh_t[1-s_t-\psi_T]$$

3. Special Case: Linear Utility

To say more, I'll specialize to the case where

$$u(c,k) = c + \beta k$$

for some $\beta > 0$. I also assume $\rho = r$, so that there is no systematic trend in consumption over the life-cycle. An immediate consequence of this is that $\lambda_t = 1$, regardless of wealth levels.

Notice also that the intertemporal budget constraint can be integrated into present-value form, to yield

$$e^{-rT}y_T - y_0 = \int_0^T e^{-rt} wh_t[1-s_t]dt - \int_0^T e^{-rt}c_t dt \text{ before } T\text{, and} \tag{18}$$

$$\lim_{S\to\infty} e^{-rS}y_S - y_T = \int_T^\infty e^{-r(t-T)} wh_t[1-s_t-\psi_t]dt - \int_T^\infty e^{-r(t-T)}c_t dt \text{ after } T. \tag{19}$$

That is, with no constraints on borrowing, the present value of consumption has to be equal to lifetime wealth:

$$\int_0^\infty e^{-rt}c_t dt = y_0 + W_0(h_0|T,h_T) + e^{-rT}W_1(h_T).$$

As above, we can treat the sub-problems of choosing a path of time use and consumption over the intervals $[0,T)$ and $[T,\infty)$ as inputs into the "upper-level" problem of choosing T, h_T and y_T optimally.

Let $W_0(h,|T,h')$ be the maximal present value of earnings before the first birth, given the date of the first birth T and the terminal condition that human capital must be equal to h' at T. That is, let

$$W_0(h|T,h') = \max_{(s_t)_t} \int_0^T e^{-rt} wh_t[1-s_t]dt$$

subject to the law of motion (2), the initial condition $h_0 = h$ and the terminal condition $h_T = h'$. Similarly let $W_1(h)$ be the maximal present value of earnings after a first birth. Formally, we have

$$W_1(h) = \max_{(s_t)_t} \int_0^\infty e^{-rt)} w h_t [1 - s_t - \psi_t] dt$$

subject to the law of motion (2), and the initial condition $h_0 = h$.

Then, with this utility function, we have

$$V_0(y, h|T, y', h') = W_0(h|T, h') + y - e^{-rT} y'$$

and

$$V_1(y, h) = W_1(h) + y + \rho^{-1}\beta.$$

Then lifetime utility is

$$\begin{aligned} \widehat{V}(T, h_T, y_T|y_0, h_0) &= \{W_0(h|T, h_T) + y_0 - e^{-rT} y_T\} + e^{-rT}\{W_1(h_T) + y_T + \rho^{-1}\beta\} \\ &= y_0 + \widehat{W}(T, h_T|h_0) + \rho^{-1} e^{-\rho T}\beta \end{aligned} \tag{20}$$

where $\widehat{W}(T, h_T|h_0) = W_0(h|T, h_T) + e^{-rT} W_1(h_T)$ is lifetime earnings. (Recall that $r = \rho$.) This formulation makes the tradeoff that determines birth timing clear: marginal delays of the first birth results in foregone utility of $e^{-\rho T}\beta$. On the other hand, delaying the first birth may increase lifetime earnings as it allows for more human capital to be built up before the onset of childbearing.

3.1 Covariance Between Birth Timing and Human Capital

We can use the second-order conditions for the maximization of $\widehat{V}(T, h_T, y_T)$ to sign the covariance between a woman's age at first birth and the level of human capital she accumulates by that age.

PROPOSITION 2. *Women who have children later will accumulate more human capital by the time they have their first birth. That is,* $\frac{\partial h_T}{\partial T} > 0$.

Proof. We showed above that with the utility function $u(c, k) = c + \beta k$, and assuming optimal behavior before and after the first birth, lifetime utility simplifies to

$$\widehat{V}(T, h_T, y_T|y_0, h_0) = y_0 + \widehat{W}(T, h_T|h_0) + \rho^{-1} e^{-\rho T}\beta$$

The first-order condition for h_T is

$$\frac{\partial \widehat{W}}{\partial h_T} = \frac{\partial W_0}{\partial h_T}(T, h_T|h_0) + e^{-rT}\frac{\partial W_1}{\partial h_T}(h_T)$$

which we may take to implicitly define a function $h_T^*(T)$, the relationship between age and human capital at first birth. Of course, this relationship depends on preferences (β), the skill price w, and the initial level of human capital h_0.

Differentiating with respect to T, we get

$$\frac{\partial^2 W_0}{\partial h_T \partial T} + \frac{\partial^2 W_0}{\partial h_T^2}\frac{\partial h_T^*}{\partial T} - re^{-rT}\frac{\partial W_1}{\partial h_T} = 0. \tag{21}$$

Since $\frac{\partial W_0}{\partial h_T} = -e^{-rT}\mu_T$ (by the envelope theorem applied to W_0), we have

$$\frac{\partial^2 W_0}{\partial h_T \partial T} = re^{-rT}\mu_T - e^{-rT}\dot{\mu}_T.$$

Then in (21) we have

$$\begin{aligned} 0 &= re^{-rT}\mu_T - e^{-rT}\dot{\mu}_T + \frac{\partial^2 W_0}{\partial h_T^2}\frac{\partial h_T^*}{\partial T} - re^{-rT}\frac{\partial W_1}{\partial h_T} \\ &= \frac{\partial^2 W_0}{\partial h_T^2}\frac{\partial h_T^*}{\partial T} - r\left\{-e^{-rT}\mu_T + e^{-rT}\frac{\partial W_1}{\partial h}\right\} - e^{-rT}\dot{\mu}_T \\ &= \frac{\partial^2 W_0}{\partial h_T^2}\frac{\partial h_T^*}{\partial T} - e^{-rT}\dot{\mu}_T \end{aligned} \tag{22}$$

where we used the first-order condition for h_T to eliminate the middle term. Thus,

$$\frac{\partial h_T^*}{\partial T} = \frac{e^{-rT}\dot{\mu}_T}{\partial^2 W_0/\partial h_T^2}. \tag{23}$$

In section A2 of the Appendix, I show that $W_1''(h_T) = 0$ if child costs decay exponentially. Then, the second-order conditions for the maximization of $\widehat{V}$ imply

$$\frac{\partial^2 \widehat{V}}{\partial h_T^2} = \frac{\partial^2 W_0}{\partial h_T^2} + e^{-rT}\frac{\partial^2 W_1}{\partial h^2} = \frac{\partial^2 W_0}{\partial h_T^2} < 0.$$

Finally, notice that $\dot{\mu}_T < 0$ because (as shown in Appendix A2), the marginal value of human capital at the time of first birth must be

$$\mu_T = w\left[\frac{1}{\rho+\delta} - \frac{\psi_0}{\rho+\delta+\gamma}\right] < \frac{w}{\rho+\delta}$$

and $\dot{\mu}_T = (\rho + \delta)\mu_T - w$. Thus, in (23) we have $\frac{\partial h_T^*}{\partial T} > 0$. □

The intuition here is that the arrival of a birth does two things: it immediately reduces the "demand" for human capital, but it also starts a long-run recovery process (since child time costs decrease with the child's age). A forward-looking woman would want to build up human capital before a birth, in order to meet this future need.

3.2 Effects of Permanent Wage Differences on Birth Timing

Let $W(T|h_0) = \max_{h_T} \widehat{W}(T, h_T|h_0)$ be the maximal value of lifetime earnings, for a given age at first birth. Notice that the optimal timing of a birth is

$$T^* = \arg\max_T W(T|h_0) + y + \rho^{-1} e^{-\rho T} \beta$$

for which the first-order condition is $W'(T|h_0) - \beta e^{-\rho T} = 0$. Now, differentiate with respect to β and rearrange to obtain:

$$\frac{\partial T^*}{\partial \beta} = \frac{e^{-\rho T^*}}{W''(T^*) + \beta \rho e^{-\rho T^*}} < 0$$

because, by the second-order condition, $W''(T^*) + \beta \rho e^{-\rho T^*} < 0$ at an interior solution for T^*. Thus, women with a stronger preference for childbearing - i.e. a higher β - will have earlier first births.

Now, this result can be pushed further by noting that $W(T|h_0)$ is proportional to the skill price w. So the above proof also shows that T^* is decreasing in β/w.

PROPOSITION 3. *Women with either a stronger preference for children (β), or those who face a lower skill price w, will have earlier first births. That is, $\frac{\partial T}{\partial(\beta/w)} > 0$.*

So, a prediction of this model is that women who face higher skill prices will tend to have later first births. Moreover, Proposition 2 implies they will accumulate more human capital early in life.

3.3 Willingness to Pay for Births

This specification of preferences allows for an easy definition of the marginal willingness to pay for accelerating a birth: the foregone utility of waiting slightly longer is $\beta e^{-\rho T}$. At an interior optimum, this is equal to the marginal increase in earnings from delaying, $W'(T)$. To estimate $W'(T)$, we would need not just information on the timing of births, but also on lifetime earnings.

This observation also suggests a different interpretation of the "child penalty" as measured by Kleven et al. (2019b); the *instantaneous* earnings loss due to a birth is a lower bound on the full cost of childbearing, since children impose time costs over many years. On the other hand, the logic of revealed preference implies that for those women who choose to bear children, the total cost (i.e. cumulative earnings losses) must be outweighed by the benefits of parenthood.

4. Empirical Results

Here I show that age at first birth and education - a common proxy for human capital - are indeed positively correlated in a sample of South African women.

4.1 The National Income Dynamics Study (NIDS)

I use data from the National Income Dynamics Study (NIDS), a panel survey of the non-institutional population of South Africa. At present, there are five waves of NIDS publicly available. NIDS is conducted by the Southern Africa Labour and Development Research Unit (SALDRU) at the University of Cape Town. Respondents were questioned about their labor market histories, social and demographic characteristics, health, educational attainment, and assets; household-level questionnaires were also administered covering dwelling conditions, amenities, subsistence agriculture, and expenditures.

Wave 1 of NIDS was a stratified and clustered by design, with the strata corresponding to district councils. Racial minorities - white, “coloured” and Indian respondents - were oversampled, with weights post-stratified after the completion of fieldwork to match census estimates of for race - age - sex - province cells. African South Africans constitute the overwhelming majority of the country’s population (roughly 80%) and are, on the whole, poorer and have quite different fertility patterns than South Africans of other races. Consequently I focus my analysis on this population.

Fieldwork for the first wave was completed during 2008. 28,247 persons were interviewed for Wave 1, residing in 7,301 households.

4.2 Sample Restrictions and Data Cleaning

For the analysis here, I drop all racial minorities, focusing only on African women. I map the an individual’s education (pre-cleaned by the NIDS team) into several categories:

- no schooling,
- primary school or less,
- some high school,

- matric,
- some postsecondary (including several types of diplomas or certificates), and
- 3-year university degree or higher.

The last category accounts for a tiny minority of cases.

4.3 Relationship Between Age at First Birth and Education

Table 1 displays some summary statistics for the distribution of age at first birth within several education groups. Figure 1 plots those same conditional densities. I restrict myself to women born in 1973 or later (the median birth year for this sample), because I am concerned about recall error for older persons. I also discard the few observations where a woman is recorded to have had a birth at age 10 or younger.

We can see that in this sample, women tend to have first births in their early 20s. Compared to some European countries, where the average age at first birth is typically in the late 20s or even early 30s, this is young.

More interesting is that the mean age at first birth is indeed highest for those in the highest education category ("some postsecondary"), at about 23 years old. In fact, the mean age at first birth is monotonically increasing in education when we order the education categories as in Table 1. This is certainly consistent with the model described above and in particular with Proposition 2. Perhaps surprisingly, the dispersion in age at first birth is quite similar across education categories; as we see in Figure 1, the shape of the conditional densities doesn't seem to differ much (only the location shifts).

Education	**Mean**	**Std. Dev.**	**Observations**
primary school or less	20.3	4.04	277
some high school	20.6	3.64	1089
matric	21.6	3.69	422
some postsecondary	22.7	3.77	168

Table 1: Summary statistics for age at first birth, by education category. Sample consists of all African women in NIDS Wave 1 born in 1973 or later.

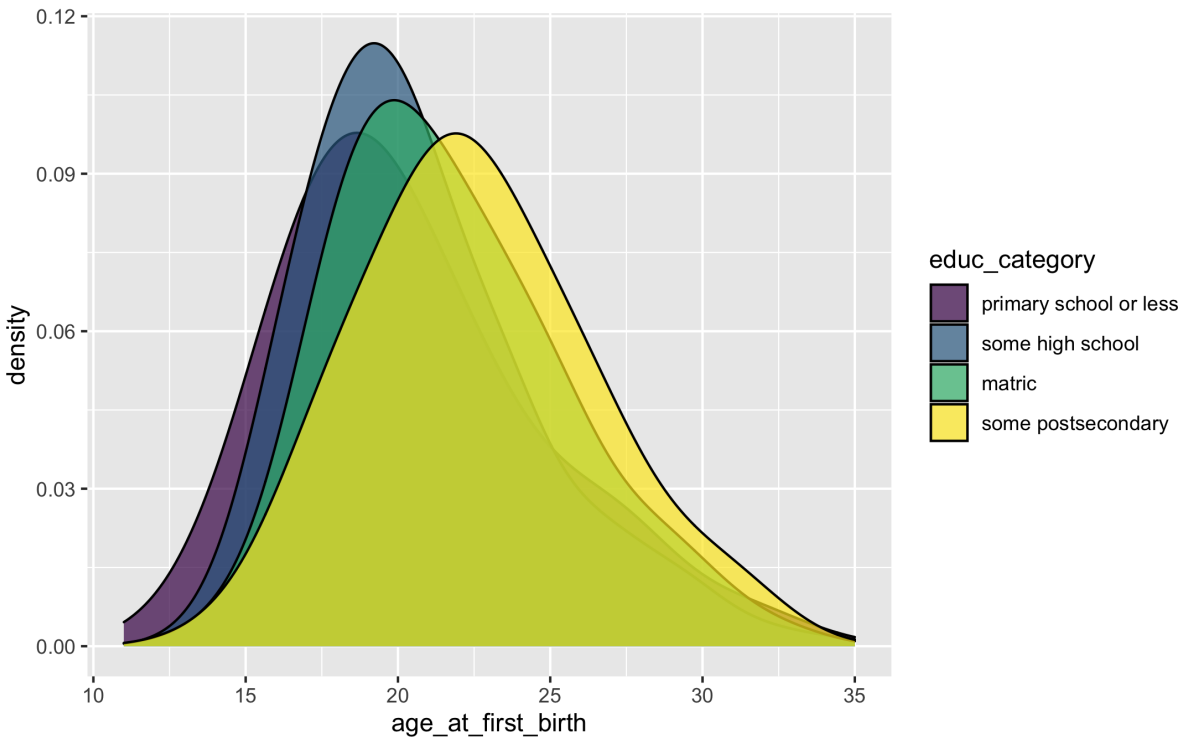

Figure 1: The distribution of age at first birth, by education category. Sample consists of all African women in NIDS Wave 1 born in 1973 or later.

5. Conclusion

Taking a life-cycle perspective on fertility and career choices leads naturally to a model in which the main cost of childbearing is maternal time. In this paper I show how this perspective can rationalise several widely observed patterns: that earnings drop sharply at the time of a birth, and that women with delay births to accumulate more education.

The implications of this model are not exhausted by this paper, though. Proposition 3 suggests that certain labour market trends (e.g. "skill-biased technical change") may drive changes in the cross-sectional distribution of fertility timing. For example, women of different education levels may be differentially affected by changes in the relative demand for skilled labour. However, a complete exploration of that topic raises complex econometric issues and is beyond the scope of this present work.

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Appendices

A Technical Appendix: Solution for $W_1(h)$

Here, we give some properties of the solution for the present value of earnings *after* a birth has occurred. Note that the properties stated here *do* depend on the parametric assumptions of exponential decay (for child time costs) and of Cobb-Douglas production.

We want to characterize $W_1(h)$, defined by

$$W_1(h) = \max_{(s_t)} \int_0^\infty e^{-\rho t} w h_t [1-\psi_t] dt$$

subject to the law of motion (2), the initial condition $h_0 = h$, and the feasibility constraint $0 \leq s_t \leq 1-\psi_t$. The current-value Hamiltonian is

$$\mathscr{H} = w h_t [1 - s_t - \psi_t] + \mu_t \{\phi(h_t s_t) - \delta h_t\}$$

and the first-order conditions are

$$[s_t] \quad -w h_t + \mu_t h_t \phi'(h_t s_t) \geq 0 \text{ (with equality if and only if } s_t < 1-\psi_t) \tag{24}$$

$$[\mu_t] \quad \dot{\mu}_t = (\rho+\delta)\mu_t - [1-\psi_t] \max\{w, \mu_t \phi'(h_t[1-\psi_t])\}. \tag{25}$$

The transversality condition is that $\lim_{T\to\infty} e^{-\rho T} \mu_T h_T = 0$.

A1 Steady State

We can show there is a unique steady state for this problem directly. Suppose there is a level of human capital $\overline{h}$, a marginal value of human capital $\overline{\mu}$, and a fraction of time $\overline{s}$ such that

$$0 = \phi\left(\overline{h}\overline{s}\right) - \delta\overline{h} \tag{26}$$

$$\begin{aligned} 0 &= (\rho+\delta)\overline{\mu} - \lim_{t\to\infty}[1-\psi_t]w \\ &= (\rho+\delta)\overline{\mu} - w \end{aligned} \tag{27}$$

$$w = \overline{\mu}\phi'\left(\overline{h}\overline{s}\right) \tag{28}$$

where we used the assumption that $\psi_t \longrightarrow 0$ as $t \longrightarrow \infty$. With $\phi(x) = x^\alpha$ for some $\alpha \in (0,1)$, we have

$$
\begin{aligned}
\overline{\mu} &= \frac{w}{\rho+\delta} \\
\overline{h} &= \delta^{-1}\left(\frac{\alpha}{\rho+\delta}\right)^{\alpha/(1-\alpha)} \\
\overline{s} &= \delta\left(\frac{\alpha}{\rho+\delta}\right)
\end{aligned}
$$

A2 Marginal Value of Human Capital at First Birth

We can say more about the initial (marginal) value of human capital $\mu_0 = W_1'(h_0)$. We can multiply the costate equation for human capital by the integrating factor $e^{-(\rho+\delta)t}$ to obtain

$$\frac{d}{dt}e^{-(\rho+\delta)t}\mu_t = -w[1-\psi_t]e^{-(\rho+\delta)t}$$

and integrating forward from an initial time t up to a later one, say T, we obtain

$$e^{-(\rho+\delta)T}\mu_T - e^{-(\rho+\delta)t}\mu_t = -\int_t^T we^{-(\rho+\delta)s}[1-\psi_s]ds.$$

If we let $T \longrightarrow \infty$ and $t \longrightarrow 0$, we obtain (using the transversality condition):

$$\mu_t = \int_0^\infty we^{-(\rho+\delta)s}[1-\psi_s]ds.$$

When $\psi_t = \psi_0 e^{-\gamma t}$, carrying out the integration gives us that

$$\mu_0 = w\left[\frac{1}{\rho+\delta} - \frac{\psi_0}{\rho+\delta+\gamma}\right]. \tag{29}$$

Two important implications of this result are:

- The initial marginal value of human capital does not depend on the level of human capital. That is, $W_1''(h_0) = 0$.
- At the moment of first birth (here, when "$t = 0$"), the marginal value of human capital will be below its steady-state value of $\overline{\mu} = w/(\rho+\delta)$.