Evolving United States Stock Market Volatility: The Role of Conventional and Unconventional Monetary Policies
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Abstract
Despite the econometric advances of the last 30 years, the effects of monetary policy stance during the boom and busts of the stock market are not clearly defined. In this paper, we use a structural heterogenous vector autoregressive (SHVAR) model with identified structural breaks to analyze the impact of both conventional and unconventional monetary policies on the U.S. stock market volatility. We find that contractionary monetary policy enhances stock market volatility, but the importance of monetary policy shocks in explaining volatility evolves across different regimes and is relative to supply shocks (and shocks to volatility itself). In comparison to business cycle fluctuations, monetary policy shocks explain a greater fraction of the variance of stock market volatility at shorter horizons, as in medium to longer horizons. Our basic findings of a positive impact of monetary policy on equity market volatility (being relatively stronger during calmer stock markets periods) is also corroborated by analyses conducted at the daily frequency based on an augmented heterogenous autoregressive model of realized volatility (HAR-RV) and a multivariate k-th order nonparametric causality-in-quantiles framework, respectively. Our results have important implications both for investors and policymakers.

Keywords: Stock Market Volatility; Conventional and Unconventional Monetary Policies; Structural Breaks; Structural Heterogenous Vector Autoregressive Model; Multivariate Nonparametric higher-Order Causality-in-Quantiles Test; Intraday Data

JEL Codes: C22, C32, E32, E52, G10

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1. Introduction

Stock market volatility has always been a closely monitored market parameter for both investors and policymakers. As far as market participants is concerned, stock market volatility has ample implications for portfolio diversification, pricing of derivative securities and risk management (Granger and Poon, 2003; Rapach et al., 2008). At the same time, policymakers are interested in the adverse effects of equity returns volatility, capturing uncertainty on real activity, as widely discussed during the Global Financial Crisis (GFC) of 2007-2009 (Castelnuovo et al., 2017; Gupta et al., 2018, 2019, 2020a), and during the ongoing coronavirus pandemic (Caggiano et al., 2020; Gupta et al., 2020b). Naturally, determining the underlying factors that drive stock market volatility is a challenging task. In this regard, besides business cycles (see Choudhry et al., (2016), Atanasov (2018), Demirer et al., (2019), Bouri et al., (2020) for a detailed discussion), one of the main determinants of stock market volatility is considered to be monetary policy decisions, as proposed by Schwert (1989) over three decades back, and more recently by Lobo (2002), Andersen et al., (2003, 2007), Bomfim (2003), Chen and Clements (2007), Farka (2009), Konrad (2009), Vahamaa and Aijo (2011), Zare et al., (2013), Nyakabawo et al., (2018), and Gupta et al., (2020c), among others.¹

Theoretically, the effect of monetary policy decisions on stock prices is twofold; monetary policy stance alters investors’ expectations about future cash flows and in parallel modifies the cost of capital, i.e., the real interest rate which is used to discount the future cash flows and/or the risk premium associated with holding stocks (Bernanke and Blinder, 1992; Bernanke and Kuttner, 2005; Maio, 2014). The effect may be even more significant when it is unexpected; an (unexpected) increase in the monetary policy-related interest rate (traditionally considered as “bad news”) impacts negatively stock prices and/or returns, which in turn leads to higher stock market volatility, as suggested by the “leverage effect” (Gospodinov and Jamali, 2012, 2018).

¹ From an out-of-sample perspective, the role of interest rate has been discussed as early as Glosten et al., (1993), and more recently by, Engle and Rangel (2008), Asgharian et al., (2013), Conrad and Loch (2015) besides others.
Our study extends the aforementioned literature evaluating the impact of not only conventional but also unconventional monetary policy decisions at the United States (U.S.). In doing so, we study the effect of monetary policy shocks on the realized volatility (RV) of stock returns using a Structural Vector Autoregressive (SVAR) model, controlling for the role of output growth on stock volatility. Our sample covers the period November, 1985 to April, 2020, over which the U.S. stock market has undergone multiple regime changes, and economic recessions, besides the usage of both conventional and unconventional monetary policy instruments. To cater for the existence of different regimes, we conduct structural break tests on the mean and volatility of the variables under consideration to explicitly identify certain sub-samples that characterize regime changes.

Unlike other economies, the U.S. sample is suitable for studying the effect of both conventional and unconventional monetary policy initiatives of stock market volatility, given the wide use of monetary tools of both categories and the magnitude of the stock market. In the wake of the “Great Recession” of 2007-2009 associated with the GFC, the Federal Reserve (Fed) responded by reducing the Federal funds rate, as a standard conventional monetary policy response, to the extent that the policy rate reached the physical zero lower bound (ZLB) of nominal interest rates, and hence left little room for further cuts to provide stimulus to the wider economy. Facing this limitation, the Fed introduced Quantitative Easing (QE) measures, i.e., unconventional monetary policies, to implement a further monetary stimulus. QE covered actions that expanded the Fed’s balance sheet such as large-scale asset purchases (LSAP), and those that change the maturity composition of the Fed’s bond portfolio, i.e. the Maturity Extension program also known as “Operation Twist”. Another powerful instrument of the central bank’s unconventional toolkit is a measure known as Forward Guidance; a verbal assurance to the public regarding the intended monetary policy stance. Understandably, unlike conventional monetary policy decisions which typically involves changes in the policy rate, unconventional monetary policy involves a gamut of options, also recently observed following the outbreak of the COVID-19 pandemic. Naturally, unconventional monetary policy measures such as the QE poses a challenge for econometric analysis, since there is no single policy instrument whose variation reflects unconventional policy steps.
Thus, the first step to analyzing the impact of unconventional monetary policy measures on stock market volatility involves the creation of a corresponding metric. QE measures have been primarily modeled as binary indicators used, among others, for event study regressions not easily implementable in a conventional VAR framework. Likewise, QE steps are likely to be endogenous depending on the state of the economy, and cannot simply be modeled as dummy variables only. Previous studies use various approaches to capture unconventional monetary policy decisions in the context of a VAR model by using long-term interest rates, spreads between long and short-term interest rate, measures on the quantity of money, and also endogenizing the dummy within the VAR model, by what is referred to as the Qualitative (Qual) VAR model, and the shadow short rate (SSR; see Meinusch and Tillmann (2016) for a detailed discussion). In this regard, the SSR is the nominal interest rate that would prevail in the absence of its (physical) effective zero lower bound, derived from the term structure of the yield curve. In our model we cater for monetary policy measures through the use of the SSR, which adheres closely to the historical behaviour of the Federal funds rate and summarizes the macroeconomic effects of both conventional and unconventional monetary policies without the physical limit of the zero bound (Wu and Xia, 2016).

To the best of our knowledge, this is the first paper to provide a structural analysis on the evolving effect of both conventional and unconventional monetary policy on U.S. stock market volatility, studying explicitly each measure’s effect on a sub-period analysis framework, identified through structural break tests. In contrast, the use of a full-fledged time-varying model could result in a loss of estimation accuracy when employed in a period of constant coefficients under a continuous change framework, implied by the random walk assumption in time-varying parameters (Bataa et al., 2016; Bataa and Park, 2017). In addition, besides the monthly data analysis which is our primary focus, we follow the suggestion of Nakamura and Steinsson (2018a, b) that monetary policy shocks are better identified using high-frequency data and conduct two additional daily data exercises. In the first case, we analyze the impact of monetary policy shocks on both stock returns and volatility (an issue with a significant research attention as discussed in Kishor and Marfatia, 2013; Simo-Kengne et al., 2016; Caraiani and Călin, 2018; 2020; Paul, 2020). In doing so, we control for other macroeconomic factors, developing a multivariate version of the $k$-th
order nonparametric causality-in-quantiles test of Balcilar et al., (2018). The specific test
controls for underlying nonlinearity and regime changes on the entire conditional
distribution of returns and volatility. Secondly, we use the Heterogenous Autoregressive
(HAR) model of realized volatility (HAR-RV) of Corsi (2009), given its ability to capture
important “stylized facts” of financial-market volatility such as long memory and multi-
scaling behavior, with the daily RV derived from S&P500 futures intraday (5-minute
interval) data, to check for the daily impact of monetary policy shocks, also controlling for
macroeconomic surprises. These two analyses are the first of their kind in the evaluation of
the impact of monetary policy shocks on U.S. stock market volatility.

The remainder of the paper is organized as follows: Section 2 lays out the SVAR model
and the associated identification of the structural breaks following the novel unit root testing
approach of Kejriwal et al. (2020). Section 3 discusses the data and conducts univariate data
analysis involving unit roots and structural breaks, with Section 4 presenting the results
(from the monthly- and daily data-based models). Finally, Section 5 concludes the paper.

2. Methodologies

While the relevant literature in studying the relationship between monetary policy and
the stock market is vast with a plethora of methodological approaches, the workhorse of
macroeconomic estimation still remains the SVAR model. In this paper, we compare the
typical SVAR model with the one proposed by Bataa et al., (2013); an iterative trained
model that takes into account structural breaks during the estimation of model’s coefficients.
The first part describes the model and the structural break test of Kejriwal et al. (2020) used
to identify the exact structural break dates, while the second part describes the methodology
employed for computing the confidence intervals of impulse response functions and the
forecast error decomposition. While outlining our model, we also discuss our data.

The model used for the analysis builds on a typical SVAR model, with no intercepts or
seasonal dummy variables, without loss of generality:

\[ A_0 x_t = \sum_{i=1}^{24} A_i x_{t-i} + \varepsilon_t \]  

(1)

where \( x_t = (\Delta_1 IP_t, \Delta_1 SSR_t, S&P500vol_t) \); \( \Delta_1 IP_t \) is the first difference of the logarithm of
the industrial production index, \( \Delta_1 SSR_t \) the first difference of the Shadow Short Rate (SSR)
and \( \text{S&P500 vol}_t \) the monthly realized volatility (RV) of the S&P500 index. The \( \mathbf{\varepsilon}_t = (\varepsilon_{IP,t}, \varepsilon_{SSR,t}, \varepsilon_{\text{S&P500 vol},t}) \) denotes the diagonal variance matrix \( E(\mathbf{\varepsilon}_t, \mathbf{\varepsilon}_t') = \mathbf{\Sigma} \) of structural shocks with variances of production, monetary policy and volatility \( \sigma^2_{IP}, \sigma^2_{SSR}, \sigma^2_{\text{S&P500 vol}} \), respectively. The ordering of the variables is standard in the context of a monetary SVAR (Walsh, 2017), with the slow-moving real activity variable, i.e., the growth of industrial production ordered before the monetary policy instrument, and the fast-moving financial market variable, i.e., the stock returns RV ordered after the SSR. In other words, monetary policy shocks impact output growth with a lag, while volatility is affected contemporaneously.

We assume a lag length of 24 months to allow for relatively long time responses to shocks. To counter the curse of dimensionality, while retaining a reasonable number of lags to observe long-run effects, we follow the HAR approach of Corsi (2009) that is very popular in finance in measuring long-lagged effects (or the so-called long-memory effect), building a structural heterogeneous VAR (SHVAR) model of the form:

\[
A_0 x_t = \Psi_1 \Delta_1 x_{t-1} + \Psi_2 \Delta_3 x_{t-1} + \Psi_3 \Delta_6 x_{t-1} + \Psi_4 \Delta_{12} x_{t-1} + \Psi_5 \Delta_{24} x_{t-1} + \mathbf{\varepsilon}_t \tag{2}
\]

where \( \Delta_k = (1 - L^k) \) is the conventional lag operator. Thus, the specification of equation (2) allows for short, medium and long term dynamics, represented in changes over the previous month, quarter, six months, one year and two years, respectively. Although the choice of these study periods is relatively arbitrary, it allows for reasonable flexibility within a specification with relatively smooth effects over lags. A conventional lag order choice based on Akaike Information Criterion (AIC) or Bayes Information Criterion (BIC) would result in a relative smaller lag order; specifically we find 1 lag for the BIC and 2 lags on the AIC as optimal. The proposed SHVAR is basically a restricted VAR, where responses over 24 lags are counted with only 15 coefficients. In other words, the restricted SVAR of (1) can be recovered from (2) as in:

\[
A_1 = \sum_{i=1}^{5} \Psi_j, A_2 = A_3 = \sum_{j=2}^{5} \Psi_j, A_4 = A_5 = \sum_{j=3}^{5} \Psi_j, A_7 = A_8 = \cdots = A_{12} = \sum_{j=4}^{5} \Psi_j, A_{13} = A_{14} = \cdots = A_{24} = \Psi_5 \tag{3}
\]
These relationships imply 57 coefficient restrictions in each SHVAR equation, tested against a general form SVAR. After imposing restrictions, estimates from the SHVAR can be used to conduct conventional SVAR analyses, such as the examination of impulse responses and variance decompositions. Based on the recursive ordering of the structural model (1), the individual equations can be written as:

\[
\Delta_1 IP_t = \sum_{i=1}^{24} a_{1i} x_{t-1} + \varepsilon_{IP,t} \quad (4)
\]

\[
\Delta_1 SSR_t = a_{21} \Delta_1 IP_t + \sum_{i=1}^{24} a_{2i} x_{t-1} + \varepsilon_{SSR,t} \quad (5)
\]

\[
S&P500vol_t = a_{31} \Delta_1 IP_t + a_{32} \Delta_1 SSR_t + \sum_{i=1}^{24} a_{3i} x_{t-1} + \varepsilon_{S&P500vol,t} \quad (6)
\]

To ascertain the stability of the SHVAR model, we perform unit root tests following the novel procedure of Kejriwal et al. (2020). The proposed methodology is a new sequential bootstrap procedure for detecting multiple shifts in a time series driven by non-stationary volatility. The assumed volatility process can accommodate discrete breaks, smooth transition variation as well as trending volatility. In other words, the aforementioned approach disentangles persistence shifts from a trend function and is able to detect changes in persistence that are not narrowly stated as means shifts. Moreover, it overcomes the homoskedasticity assumption of Kejriwal (2020) or the stability and regime-wise I(0) hypothesis of Bai and Perron (1998) (and Bataa et al. (2014) in the presence of breaks) that may overestimate/underestimate the aggressiveness of the monetary policy stance towards stock volatility. Following Kejriwal et al. (2020), the basic steps of the test are as follows:

- Determine the number of breaks using a sequential bootstrap procedure to test the null hypothesis of $l$ versus $l+1$ breaks, based on generalized least squares (GLS) heteroskedasticity consistent estimators;
- Based on the detected number of breaks, estimate autoregressive regressions for each regime (period between breaks);
- Perform a Wald test on the null hypothesis that the break is subject to mean shifts against the alternative of mean and persistence shifts, based on a robust standard error estimator;
- Based on the selected model, the largest (across regimes) estimated sum of the autoregressive parameters is computed along with equal-tailed 90% confidence
intervals based on the procedure of Andrews and Guggenberger (2014), which is uniformly valid over stationary and non-stationary regions and robust to conditional (though not unconditional) heteroskedasticity. We use the BIC to select the number of lags within each regime with a maximum of 12 lags;

- Perform unit root tests allowing for non-stationary volatility (Cavaliere and Taylor, 2009) based on a wild bootstrap Augmented Dickey Fuller test, as a robustness check on model’s stationarity.

The empirical literature postulates that shift changes in the mean could be a result of simple changes in volatility (Blanchard and Simon, 2001; Bataa et al., 2014) and not of the mean itself. To account for this effect, we also test for structural changes in volatility and include the detected breaks in the volatility process of the SHVAR. Given the iterative nature of the model, volatility is based on an autoregressive model of order one on the error term for each equation (4)-(6).

While impulse response functions (IRFs) and forecast error variance decomposition (FEVD) are commonly used in the relevant literature, their application in the presence of shifting regimes can be problematic, due to the changing coefficients of the model. To overcome this drawback we estimate IRFs and FEVD for each regime independently. One and two standard deviation confidence bands (approximate 68% and 95% confidence intervals) are estimated using a recursive wild bootstrap procedure, where artificial bootstrap data are generated on 2,000 replications, and the SHVAR coefficients are re-estimated over the sub-periods given by the (variable-specific) coefficient break dates, treated as known. The use of the wild bootstrap process takes into account volatility changes, which may affect confidence intervals. The same approach is repeated for the FEVDs.

3. Data and Preliminary Analysis

The industrial production index is derived from the database of the Federal Reserve Bank of St. Louis. The RV is based on the sum of squared daily log-returns of the S&P500 index following Andersen and Bollerslev (1998), with the raw closing price data compiled from Thomson Reuters Datastream. Note that measuring volatility using RV provides an observable and unconditional metric of volatility, which is otherwise a latent process. Conventionally, the time-varying volatility is modeled and the fit assessed using various
generalized autoregressive conditional heteroscedastic (GARCH) models, under which the conditional variance is a deterministic function of model parameters and past data. Alternatively, some recent papers consider stochastic volatility models, where the volatility is a latent variable that follows a stochastic process. Irrespective of whether we use GARCH or SV models, the underlying estimate of volatility is not model-free as in the case of RV. Since our sample period includes the ZLB period, we use SSR obtained from the two-factor shadow rate term structure model (SRTSM) of Krippner (2013, 2015),\(^2\) which in turn allows us to study the impact of conventional and unconventional monetary policy shocks on the RV of S&P500 returns without worrying about explicitly modelling the ZLB. Note that, Wu and Xia (2016) also develop estimates of US SSR based on a three-factor SRTSM model, but we decided to use the one proposed by Krippner (2013, 2015) due to instability issues during estimation, highlighted by Krippner (2020). In addition, the U.S. SSR data of Wu and Xia (2016) are available after 1990, while we can start from 1985 using Krippner’s (2013, 2015) approach. Our analysis covers the period November, 1985 to April, 2020, with the start and the end date determined by the availability of SSR data at the time of writing this paper.

Our sample extends over three decades covering a variety of changes in the stance of monetary policy in the U.S., including the Great Moderation period of the Volker administration, the GFC of 2007-2009, the QE program of the Federal Reserve and the onset of the COVID-19 pandemic with its effect on the global economy and the Forward Guidance initiative. We are particularly interested in whether changes in monetary policy stance are reflected at the stock market, the duration and the transmission time of this relationship. Since the focus of this paper is on structural breaks as well, the data is allowed to determine whether the relationship over recent months do, indeed, differ from earlier periods. The changing features over our sample appear particularly evident at the right-hand panel of Figure 1 which depicts month to month changes (after logarithmic transformations of industrial production). As a preliminary step to ensure the stability of the SHVAR model,

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\(^2\) The SSR data for the US is available for download from the website of Dr. Leo Krippner at: https://www.ljkmfa.com/test-test/united-states-shadow-short-rate-estimates/.
the remainder of this section examines the unit root properties of our series, using tests robust to structural breaks.

Figure 1. Time series of S&P500 realized volatility, (Log of) Industrial Production and Shadow Short Rate in levels and first differences

Given the obvious existence of structural breaks from the visual inspection of the series, we perform the unit root test of Kejriwal et al., (2020). The structural breaks
specification requires for a trade-off between the detection of the maximum possible number of breaks, while preserving an adequate number of observations between two given breaks (regime) to allow for the consistent estimation of model’s parameters. To achieve this two-fold goal, we allow for a maximum of 5 breaks over our sample, while we assume that each regime should include at least 15% of the total number of observations. Thus, smaller regime periods are dropped. This leaves a period of December 1990 to April 2015 for detecting structural breaks, rejecting the structural break of the COVID pandemic in early 2020; a period that could be characterized as an exception from normal periods with a big drop in industrial production induced from an exogenous source to the entire economy, but without significant changes accounted to monetary policy decisions. Although we do not mark the onset of the pandemic as a structural break, we choose to keep the observations in the respective last regime and not to remove the pandemic period altogether, in order to obtain coefficient values that adhere as close as possible to the actual production level. The results for the existence of the unit root test are reported in Table 1.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Breaks</th>
<th>Break dates</th>
<th>Pure mean shifts</th>
<th>Largest AR sum</th>
<th>90% band</th>
<th>Unit root decision</th>
<th>ADF p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
</tr>
<tr>
<td>Panel A: Levels</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial Production</td>
<td>0</td>
<td>-</td>
<td>No</td>
<td>0.99</td>
<td>[0.99,1.00]</td>
<td>I(1)</td>
<td>1.00</td>
</tr>
<tr>
<td>Shadow Interest Rate</td>
<td>0</td>
<td>-</td>
<td>No</td>
<td>0.99</td>
<td>[0.99,1.01]</td>
<td>I(1)</td>
<td>0.91</td>
</tr>
<tr>
<td>S&amp;P500 volatility</td>
<td>0</td>
<td>-</td>
<td>No</td>
<td>0.35</td>
<td>[0.13,0.56]</td>
<td>I(0)</td>
<td>0.39</td>
</tr>
<tr>
<td>Panel B: First differences</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ(Industrial Production)</td>
<td>2</td>
<td>2000M12</td>
<td>Yes</td>
<td>0.73</td>
<td>[0.57,0.89]</td>
<td>I(0)</td>
<td>0.01*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2008M08</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ(Shadow Interest Rate)</td>
<td>0</td>
<td>-</td>
<td>No</td>
<td>0.44</td>
<td>[0.37,0.51]</td>
<td>I(0)</td>
<td>0.00*</td>
</tr>
<tr>
<td>Panel C: Volatility</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_{ IPs}$</td>
<td>2</td>
<td>1998M11</td>
<td>Yes</td>
<td>-0.11</td>
<td>[-0.54,0.31]</td>
<td>I(0)</td>
<td>0.00*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2008M09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_{ SSR}$</td>
<td>0</td>
<td>-</td>
<td>No</td>
<td>0.02</td>
<td>[-0.07,0.11]</td>
<td>I(0)</td>
<td>0.00*</td>
</tr>
<tr>
<td>$\sigma^2_{ S&amp;P500vol}$</td>
<td>0</td>
<td>-</td>
<td>No</td>
<td>-0.14</td>
<td>[-0.36,0.07]</td>
<td>I(0)</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: This table reports the empirical results based on monthly observations over the period 1985:11-2020:04. Panel A reports the results on levels and Panel B on first differences of the variables. Column (1): the variable name; column (2): the estimated number of breaks; column (3): the estimated break dates; column (4): the outcome of the test for the null hypothesis of pure mean shifts; column (5): the OLS estimate of the largest sum of the AR coefficients across the estimated regimes; column (6): Andrews and Guggenberger (2014) 90% confidence band for the largest sum of the AR coefficients, where the existence of values equal to 1 or above
is an indication of non-stationarity; column (8): the \( p \)-value of the wild bootstrap ADF test of Cavaliere and Taylor (2009), provided as a robustness test. Volatility estimates of Panel C are obtained recursively based on the errors of an autoregressive model of order one [AR(1)] on the variables of Panel B, where all variables are found stationary in the mean equation. I(1) denotes non-stationarity, while I(0) denotes a stationary variable. * denotes rejection of the null hypothesis of the existence of a unit root process at the 5% level of significance.

As we observe from Panel A of Table 1, both industrial production and the SSR are non-stationary in levels, while the S&P500 RV is stationary. An ADF-test (based on the wild bootstrap approach of Cavaliere and Taylor, 2009) fails to reject the null hypothesis of non-stationarity for all variables. No structural breaks are detected as well. In contrast, when we examine the first differences of the log of industrial production and the SSR variables, they are both found to be stationary and we detect 2 breaks for the former. In fact, the detected breaks are actually mean shifts and not simply a change in the persistence of the series. The ADF-test also corroborates to the stationary nature of all variables, despite the existence of the structural breaks. In Panel C, we report the respective unit root tests on the volatility of each variable, based on the extraction of volatility with an autoregressive model with one lag. The results coincide with that of Panel B, with a slight divergence on break dates. We choose to keep the structural break dates of growth rates in output for volatility series, since the estimators of the regime 1998M11-2000M12 would be biased due to the small number of degrees of freedom.

The first detected structural break coincides with the bursting of the “dot.com” bubble in the stock market and the consequent recession of the early 2000 of the U.S. economy. The second break is associated with the GFC, and the QE program of the Fed. The method detects both breaks with great accuracy, almost at the beginning of their formation. Interestingly, we would expect for the detection of breaks in S&P 500 volatility, but we do not reach to such a finding, presumably due to the highly aggregated monthly frequency observation that smooths out volatility spikes in the stock market of limited temporal period. Again the COVID-19 pandemic period seems to provide a great spike in volatility, but it lies at the end of our sample and we cannot mark it as a separate regime. Further examination with more data observations could provide additional insight, but we leave it for future research when the data will be available.
4. Empirical findings

4.1. Main analysis results

In the previous section, we determine the structural breaks to impose in the SHVAR model, and ensure that all variables are stationary. In our case, we identify 3 regimes: December 1985 to November 1998, December 1998 to August 2008 and September 2008 to April 2020. We are interested in measuring the response of stock volatility to shocks in monetary policy stance over these regime periods. Coefficients are estimated separately according to the existence of structural breaks. More specifically, coefficients of equation (4) referring to the industrial production growth rate are estimated independently between regimes, while the coefficients for changes in interest rates (5) and stock volatility (6) are constant throughout. Moreover, we allow for volatility breaks in the equation of industrial production. Though our focus is the impact of monetary policy shocks on stock market volatility, we also analyze the impact of a supply shock, i.e., the shock to industrial production growth rate, and also the effect of stock market volatility shocks.

We depict cumulative IRFs for a shock at industrial production (supply shock) in Figure 2. The continuous (red) line depicts the IRF of each coefficient regime, along with the one (red dashed) and two (red closed-dashed) standard deviation bands. The shaded areas in each graph provide IRFs over the full sample with constant coefficients for one and two standard deviations, respectively. To aid comparisons, all shocks across regimes are one standard deviation with respect to the entire sample.

As we observe, a positive supply shock decreases interest rates as expected. This negative relationship is especially apparent at the last regime, given the over-sensitive interest rates during this period. The decrease of interest rates for the first and the second period is smaller in size for the model with changing coefficients, than the model with constant parameters. Moreover, a supply shock has a positive effect on stock volatility – a result in line with theory, i.e., higher economic activity leads to more trading and higher volatility (Marfatia et al., 2020). The bands of the shock are narrower for the first two regimes and increase at the last period, while all IRFs are different than zero only for the varying coefficients model. The model implies that the transmission mechanism for the effects of supply shocks on stock volatility has also changed; especially after the 2007-2009
GFC. In particular, supply shocks have more immediate and longer lasting effects than previous periods, given that traders monitor macroeconomic conditions more closely after 2008.

**Figure 2.** Response to a shock in the industrial production growth

*Note:* Each graph shows the cumulated impulse response to a one standard deviation shock, estimated over the whole sample with no breaks. Each of the three columns represents a sub-sample as defined by the break dates and includes one (red dashed line) and two (red close-dashed line) standard deviation confidence bands. The background shaded areas provide corresponding confidence intervals around the responses (dotted line) for a constant parameter model estimated over the whole sample period for one and two standard deviations, respectively.

In Figure 3, we depict the cumulative IRFs for a one standard deviation positive shock to SSR growth rates. As we observe, a shock in interest rates consistently reduces output growth, with dominant effects observed during the last sub-sample, which is not surprising given the efforts of the Fed to boost the economy through expansionary monetary policy in the wake of the GFC and the COVID-19 pandemic, resulting in a strong negative association between output and monetary policy. Moving to the response of stock volatility to an interest rate shock, we observe a significant positive response over the first regime that is followed by a short-term smaller response over the second period. In contrast, the effect over the third period is significant and positive that reaches up to 40 basis points after 20 periods. Thus,
our approach traces the effect of contractionary policy to be more persistent before 1998 and after 2008, suggesting a smaller effect of monetary policy over the stock market uncertainty during the second period of our study. The positive effect on volatility due to a contractionary monetary policy, considered a bad news, is in line with a leverage effect, which results in lower stock returns and higher volatility following an increase in the monetary policy rate. We return to this issue later when examining the FEVDs. As we observe over all periods, the constant coefficients model tends to underestimate the response of stock volatility, and hence highlights the need to account for structural breaks in the series.

![Figure 3](image)

**Figure 3.** Response to a shock in the changes in SSR  
**Note:** See Notes to Figure 2.

Finally, in Figure 4 we depict the response of all variables on a one standard deviation volatility shock. The uncertainty increase in the stock market has a small and insignificant

---

3 Following Canova and Gambetti (2010), we also estimated a full-fledged TVP-VAR model to detect relatively stronger influence of monetary policy shocks on stock market volatility towards the end of the sample. Complete details of these results are available upon request from the authors. In addition, we used the SHVAR model to estimate the impact of monetary policy (SSR) shocks on RV of the Euro Area (Eurostoxx50), Japan (Nikkei225) and the United Kingdom (UK, FTSE100) stock markets, with data derived from the same sources as that of the US, barring the industrial production, which was obtained from the main economic indicators of the OECD database. The effect was found to be significant only for the UK, but instead of a positive impact a negative sign was observed on RV following an expansionary monetary policy, possibly due to a contraction of trading volume. Complete details of these results are available upon request from the authors.
negative effect over the first two periods for output and monetary policy, suggesting an independence of stock volatility to the aforementioned variables. In contrast, the constant parameter model detects a negative effect on production that should be attributed to overlooking structural breaks of the sample. The effect of a stock volatility shock on output growth and monetary policy decisions over the third period is negative and significant, in contrast with the response reported by the constant parameter model. Overall, the results of the SHVAR model adhere closer to the real option theory of uncertainty (Bernanke, 1983; Pindyck, 1991; Dixit and Pindyck, 1994; Bloom, 2009)\(^4\) than the typical SVAR model, particularly in the sub-period characterizing the Great Recession and the Coronavirus outbreak, and is in line with the large existing literature on the effect of uncertainty shocks on the U.S. economy (see Bleich et al., 2013; Gupta et al., 2018; Christou et al., 2020).

\[\text{Figure 4. Responses to a shock in stock market volatility} \]
\[\text{Note: See Notes to Figure 2.}\]

\[^4\text{The theory suggests that decision-making is affected by uncertainty because it raises the option value of waiting. In other words, given that the cost associated with wrong investment decisions are very high, uncertainty makes firms and, in the case of durable goods, also consumers more cautious. As a result, economic agents postpone investment, hiring and consumption decisions to periods of lower uncertainty (which results in cyclical fluctuations in macroeconomic aggregates). In other words, uncertainty is expected to negatively impact consumption, investment and overall output, to which the central bank responds by reducing interest rate.}\]
To clarify the evolution of stock volatility over the three decades of the sample due to various shocks, in Table 2 we provide the FEVDs of each variable. Since both coefficient and volatility shifts are relevant for FEVDs, we report the estimated FEVDs for each regime and across the entire sample. We employ four alternative forecast horizons: 1 month, 6 month, two year and five year to illustrate short, medium and long-run results.

Concentrating on stock market volatility, which is the focus of our paper, we find that the role of monetary policy shocks in explaining the variance of stock volatility ranges between 9.7% to 12.4% in the first sub-sample, 9.6% to 30.5% in the second sub-sample, and from 3.5% to 9.6% in the last sub-period, with the explanatory power moving downwards at longer horizons under this third regime, unlike the first two. Interestingly, the medium to longer-run effect of monetary policy is quite strong when the stock market is relatively calm, as during the second sub-sample. During the post-GFC period, the role of output growth becomes highly important at longer horizons (jumping from 0.78% to 36.1%) in explaining the variability of volatility, which is understandable due to the deep recession of the U.S. economy faced namely the Great Recession, and more recently the one associated with the Coronavirus pandemic, which is found to have a long-term explanatory power for the RV. In general, for the first two sub-sample and the very short-run in the third period, monetary policy shocks dominate supply shocks in explaining the variance of stock market uncertainty. Further, the differences between the SHVAR and the constant SVAR model are more intense upon the examination of FEVDs, as the latter fails to pick up the nuances of the evolving role of monetary policy in explaining stock market volatility.

As a final issue, it must be pointed out that stock volatility, capturing uncertainty, has made an important contribution to production and interest rates during the last regime associated with GFC and the Coronavirus pandemic, with the effect increasing with the forecasting horizon.
<table>
<thead>
<tr>
<th>Regimes</th>
<th>Supply shock ($\varepsilon_{i\rho}$)</th>
<th>Monetary policy shock ($\varepsilon_{SSR}$)</th>
<th>Volatility shock ($\varepsilon_{RV}$)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$h=1$</td>
<td>$h=6$</td>
<td>$h=24$</td>
</tr>
<tr>
<td><strong>Industrial Production</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>85M12-98M11</td>
<td>100 (0)</td>
<td>91.5 (5.3)</td>
<td>86.5 (7.2)</td>
</tr>
<tr>
<td>98M12-08M08</td>
<td>100 (0)</td>
<td>89.2 (10.4)</td>
<td>62.2 (11.5)</td>
</tr>
<tr>
<td>08M09-20M04</td>
<td>100 (0)</td>
<td>48 (15.8)</td>
<td>39.1 (16.8)</td>
</tr>
<tr>
<td>Ignoring breaks</td>
<td>100 (0)</td>
<td>69 (12.2)</td>
<td>66.5 (11.5)</td>
</tr>
<tr>
<td><strong>Shadow Short Rate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>85M12-98M11</td>
<td>0.4 (0.2)</td>
<td>1.3 (0.6)</td>
<td>1.9 (0.8)</td>
</tr>
<tr>
<td>98M12-08M08</td>
<td>0.79 (0.4)</td>
<td>1.6 (0.9)</td>
<td>1.8 (1)</td>
</tr>
<tr>
<td>08M09-20M04</td>
<td>0.83 (0.4)</td>
<td>5 (1.6)</td>
<td>21.6 (7.4)</td>
</tr>
<tr>
<td>Ignoring breaks</td>
<td>0.87 (3.7)</td>
<td>4.7 (6.3)</td>
<td>6.2 (7.6)</td>
</tr>
<tr>
<td><strong>Realized Volatility</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>85M12-98M11</td>
<td>0.37 (0.2)</td>
<td>5.8 (4.8)</td>
<td>6.2 (4.3)</td>
</tr>
<tr>
<td>98M12-08M08</td>
<td>0.75 (0.4)</td>
<td>6.1 (4.5)</td>
<td>4.7 (2.4)</td>
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<tr>
<td>08M09-20M04</td>
<td>0.78 (0.5)</td>
<td>14.7 (6.3)</td>
<td>29.8 (13.9)</td>
</tr>
<tr>
<td>Ignoring breaks</td>
<td>0.83 (10.3)</td>
<td>15.9 (10.7)</td>
<td>19.1 (10.9)</td>
</tr>
</tbody>
</table>

**Note:** All entries are in percentages; Bootstrap standard deviation provided in parenthesis.
4.2 High-Frequency Results

As a second step into the evaluation of the causal effect of monetary policy shocks \((\text{FFRshock})\) on the stock market, we extend our analysis at the daily frequency by using a multivariate \(k\)-th order nonparametric causality-in-quantiles test and a HAR-RV model, with the RV derived from S&P500 futures intraday 5-minute interval data. In doing so, we control for macroeconomic surprise \((\text{MS})\) on the financial market, using the daily macroeconomic index of Scotti (2016). The details of the multivariate higher-order causality-in-quantiles test are described in the Appendix of the paper, while the HAR-RV model of Corsi (2009) that is used here is standard, and is augmented by the metrics of monetary policy and macroeconomic shocks. Specifically, we use the model of the form:

\[
RV_t = c_0 + c_1 RV_{t-1} + c_2 RV_{5,t-1} + c_3 RV_{22,t-1} + c_4 \text{FFRshock}_t + c_5 \text{MS}_t + \epsilon_t \tag{7}
\]

where \(RV_{5}\) and \(RV_{22}\) are moving 5- and 22-day averages of RV and \(\epsilon\) the error term.

The S&P500 index is derived from Datastream, while for the monetary policy surprise \((\text{FFRshock})\) we use the monetary policy shock measure by Nakamura and Steinsson (2018a). The authors construct a monetary policy shock dataset on changes in the prices of federal funds futures rates over a 30-minute window around FOMC announcements.\(^5\) The daily macroeconomic index of Scotti (2016) is constructed based on a dynamic factor model and is a weighted composite index of business conditions estimates of quarterly Gross Domestic Product (GDP), monthly industrial production (IP), employees on non-agriculture payroll, monthly retail sales and the monthly Institute of Supply Management (ISM) manufacturing index. The weights are then used to average individual surprises to construct the macroeconomic surprise index.\(^6\) The period of analysis in this case is 1\(^{st}\) February, 1995 to 29\(^{th}\) November, 2019, based on data availability. As far as the data for the daily RV of stock returns are concerned, they were obtained from Risk Lab\(^7\). Risk Lab collects trades at their highest frequencies available and then cleans them using the prevalent national best

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\(^5\) The original and updated data (by Miguel Acosta and Joe Saia) are available from the website of Professor Emi Nakamura at: [https://eml.berkeley.edu/~enakamura/papers.html](https://eml.berkeley.edu/~enakamura/papers.html), and at: [https://www.acostamiguel.com/research](https://www.acostamiguel.com/research), i.e., webpage of Miguel Acosta.

\(^6\) The data are available from the website of Dr. Chiara Scotti at: [https://sites.google.com/site/chiarascottifr/frb/research?authuser=0](https://sites.google.com/site/chiarascottifr/frb/research?authuser=0).

\(^7\) The data are available from the website of Professor Dacheng Xiu at Booth School of Business, University of Chicago: [https://dachxiu.chicagobooth.edu/#risklab](https://dachxiu.chicagobooth.edu/#risklab).
bid and offer (NBBO) that are available, up to every second. The estimation procedure for realized volatility follows Xiu (2010), and is based on the quasi-maximum likelihood estimates (QMLE) of volatility built on moving-average models (MA(q)), using non-zero returns of transaction prices sampled up to their highest frequency available, for days with at least 12 observations. In this paper, we used the realized volatility estimates based on 5-minute subsampled returns of the futures data of the S&P500. The analysis based on the spot data covers 3rd January, 1996 to 29th November, 2019, based on the availability of the FFRshock and MS data.

The plots in Figure 5 show the $k$-th order (in our case returns and its squared value as volatility) nonparametric causality-in-quantiles test statistic over the conditional quantiles, along with the 5% and 10% critical values, with rejection of the null of non-causality occurring when the test statistic exceeds the critical values of 1.96 and 1.645 respectively. As it is obvious from Figure 5, irrespective of the use of a bivariate model (involving stock returns and FFRshock) or a multivariate model with MS as the control, the FFRshock causes stock returns at the extreme quantiles, while the effect on volatility is relatively stronger as it spans virtually the entire conditional distribution (characterizing various conditional states of volatility) of the squared returns, i.e., volatility. The large values of the test statistic at the lower quantiles of volatility highlights stronger explanatory power of monetary policy shocks when stock market variance is relatively low, and hence is in line with FEVD result obtained from the SHVAR for the second sub-sample, which is basically characterized by a relatively calm stock market as seen from Figure 1. Moreover, when we analyzed the extended HAR-RV model, $c_4$ produced a value of 0.3944 with a $p$-value of 0.0398, i.e., the FFRshock produced a positive significant effect on RV. Further, conducting the $L+1$ versus $L$ sequentially determined structural break test of Bai and Perron (2003), we obtained a break point at 21st August, 2002 for the augmented HAR-RV model. The corresponding coefficients for the first and second sub-samples were 0.4859 ($p$-value: 0.1000) and 0.1110 ($p$-value: 0.0439) respectively, again suggesting positive and significant effect, in particular in the second sample which involved the second and third sub-periods in the SHVAR model. Overall, our high frequency analyses confirms the positive and time-evolving (as captured by the conditional quantiles) impact of monetary policy shocks on volatility.
5(a). Bivariate causality from FFRShock to stock returns

5(b). Trivariate causality from FFRShock to stock returns, controlling for macroeconomic shocks.

5(c). Bivariate causality from FFRShock to squared stock returns (volatility)
5(d). Trivariate causality from FFRShock to squared stock returns (volatility), controlling for macroeconomic shocks

![Image of causality-in-quantiles test results]

**Figure 4.** $k$-th Order Multivariate Nonparametric Causality-in-Quantiles Test Results

5. Conclusions

In this paper, we evaluate the effect of both conventional and unconventional monetary policy shocks on U.S. stock market volatility, based on a structural heterogeneous VAR (SHVAR) model that takes into account structural breaks in the evolution of the series namely, growth of industrial production, changes in the shadow short rate (a metric for both conventional and unconventional monetary policies), and the realized volatility (RV) of the S&P00 index. We identify three regimes (December 1985 to November 1998, December 1998 to August 2008, and September 2008 to April 2020) based on structural break tests over the monthly period of November, 1985 to April, 2020. Our empirical findings suggest that contractionary monetary policy increases stock market volatility, with unconventional monetary policy shocks having a stronger impact on RV relative to conventional policy measures. Further, compared to supply shocks, monetary policy plays a bigger part in explaining the variance in stock market volatility, especially during calmer periods of the equity market. During episodes of heightened uncertainty, business cycles shocks take over particularly in the longer-run, though shocks to stock market volatility itself is the most important structural shock in explaining the volatility of the S&P500 RV. The fact that stock market volatility is impacted more strongly during calm periods, and is affected positively by monetary policy increases, was also confirmed based on a multivariate version of $k$-th order causality-in-quantiles test and an extended heterogeneous autoregressive RV (HAR-
RV) model applied to daily data, following the empirical literature that high frequency data should could identify the effect of monetary policy shocks more precisely. In essence, we find that monetary policy shocks are quite important relative to supply shocks in explaining stock market volatility, but the role is evolving and non-constant, and hence would require a structural model with regime changes to obtain accurate inferences.

Our results have important implications for both investors and policymakers. In particular, market agents need to pay relatively close attention to monetary policy shocks when the stock market is less volatile rather than business cycle fluctuations when it comes to portfolio diversification decisions. At the same time, our analysis provides an insight better insight for policymakers on the interplay between monetary policy stance and stock market volatility.

As part of future analysis, it would be interesting to extend our study to other stock markets as well as to emerging markets, such as the BRICS (Brazil, Russia, India, China and South Africa).

References


APPENDIX: Multivariate k-th order Nonparametric Causality-in-Quantiles Test

In this section, a multivariate extension of the bivariate k-th order nonparametric causality-in-quantile test of Balcilar et al., (2018) is discussed.

To start with, we denote the dependent variable (S&P500 returns) as $Y_t$, the predictor variable in question (say $FFR_{shock}$) as $X_t$, and $k$ possible additional predictors contained in $W_t = (w_{1,t}, w_{2,t}, …, w_{2,k})'$, used as possible control variables, which in our case is only MS. Therefore, the multivariate quantile causality is defined using: $Y_{t-1} = (y_{t-1}, …, y_{t-p})'$, $X_{t-1} = (x_{t-1}, …, x_{t-p})$ and $W_{t-1} = (w_{1,t-1}, …, w_{1,t-p}, …, w_{k,t-1}, …, w_{k,t-p})'$. Following the notation $Z_t = (Y_t, X_t, W_t)'$, the conditional distribution of $y_{t-1}$ given $Z_{t-1}$ and $y_{t-1}$ given $Z_{t-1} \setminus X_t \equiv V_t \equiv (Y_t', W_t')'$ can be denoted by $F_{y_t|Z_{t-1}}(y_t|Z_{t-1})$ and $F_{y_t|Z_{t-1}\setminus X_t}(y_t|Z_{t-1}\setminus X_{t-1})$, respectively, where $Z_{t-1} \setminus X_{t-1}$ implies the information set which does not include $X_t$. Let also the $\theta$-th conditional quantile of $y_t$, given the information set $\cdot$, be denoted by $Q_\theta(y_t|\cdot)$. Following the framework in Nishiyama et al., (2011) and Jeong et al., (2012), Granger non-causality in quantile (GNCQ) is defined as: $x_t$ does not cause $y_t$ in the $\theta$-th quantile, if:

$$Q_\theta(y_t|Z_{t-1}) = Q_\theta(y_t|Z_{t-1} \setminus X_{t-1}) \tag{A1}$$

while Granger causality in quantile (GCQ) is defined as: $x_t$ is a prima facie cause of $y_t$ in the $\theta$-th quantile, if:

$$Q_\theta(y_t|Z_{t-1}) \neq Q_\theta(y_t|Z_{t-1} \setminus X_{t-1}). \tag{A2}$$

Therefore, Eq. (A1) and Eq. (A2) can be equivalently expressed as:

$$H_0: \quad P\{F_{y_t|Z_{t-1}}(Q_\theta(Z_{t-1} \setminus X_{t-1})|Z_{t-1}) = \theta\} = 1 \tag{A3}$$
$$H_1: \quad P\{F_{y_t|Z_{t-1}}(Q_\theta(Z_{t-1} \setminus X_{t-1})|Z_{t-1}) = \theta\} < 1 \tag{A4}$$

where the $\theta$-th quantiles are denoted as $Q_\theta(Z_{t-1}) \equiv Q_\theta(y_t|Z_{t-1})$ and $Q_\theta(Z_{t-1} \setminus X_{t-1}) \equiv Q_\theta(y_t|Z_{t-1} \setminus X_{t-1})$, which satisfy $F_{y_t|Z_{t-1}}(Q_\theta(Z_{t-1})|Z_{t-1}) = \theta$ with probability one.
In order to construct the test, we consider metric: \( J = \{\epsilon_t E(\epsilon_t | Z_{t-1}) f_Z(Z_{t-1})\} \), where \( f_Z(Z_{t-1}) \) is the marginal density. The regression error \( \epsilon_t \) emerges based on the null in Eq. (A3), which can only be true if and only if \( E[1\{y_t \leq Q_\theta(Z_{t-1} \setminus X_{t-1}) | Z_{t-1}\}] = \theta \) or equivalently \( 1\{y_t \leq Q_\theta(Z_{t-1} \setminus X_{t-1})\} = \theta + \epsilon_t \), where \( 1\{\cdot\} \) is the indicator function. Thus, the metric \( J \) can be specified as:

\[
J = E[\{F_{y_t|Z_{t-1}} \{Q_\theta(Z_{t-1} \setminus X_{t-1}) | Z_{t-1}\} - \theta\}^2 f_Z(Z_{t-1})]
\]  

(A5)

The empirical counterpart of Eq. (A5) based on Jeong et al., (2012) is constructed as follows:

\[
\hat{J}_T = \frac{1}{T(T-1)h^{(k+2)p}} \sum_{t=p+1}^{T} \sum_{s=p+1}^{T} K\left(\frac{Z_{t-1} - Z_{s-1}}{h}\right) \hat{\epsilon}_t \hat{\epsilon}_s
\]  

(A6)

where \( K(\cdot) \) is the kernel function with bandwidth \( h \), \( T \) represents the sample size, \( p \) simply denotes the lag order and \( \hat{\epsilon}_t \) is simply the unknown regression estimate, which is constructed as:

\[
\hat{\epsilon}_t = 1\{y_t \leq \hat{Q}_\theta(Z_{t-1} \setminus X_{t-1})\} - \theta
\]  

(A7)

where \( \hat{Q}_\theta(Z_{t-1}) \) is an estimate of the \( \theta \)-th conditional quantile. Following similar arguments in Jeong et al., (2012), \( Th^p \hat{J}_T \overset{d}{\rightarrow} N(0, \sigma^2) \). In general, causality in conditional means (1st moment) implies causality in higher order moments, but not vice versa. Thus, a sequential testing approach for causality in \( k \)-th moment is adopted as follows:

\[
\begin{align*}
H_0: \quad & P\left\{F_{y_t^k|Z_{t-1}} \{Q_\theta(Z_{t-1} \setminus X_{t-1}) | Z_{t-1}\} = \theta\right\} = 1 \quad k = 1, 2, \ldots, K \\
H_1: \quad & P\left\{F_{y_t^k|Z_{t-1}} \{Q_\theta(Z_{t-1} \setminus X_{t-1}) | Z_{t-1}\} = \theta\right\} < 1 \quad k = 1, 2, \ldots, K
\end{align*}
\]  

(A8)  

(A9)

The test statistic is formulated as in Eq. (A6) by replacing \( y_t \) with \( y_t^k \). It is important to note that \( J \geq 0 \), i.e. the equality holds if and only if \( H_0 \) in Eq. (A3) or Eq. (A8) is true, while \( J > 0 \) holds under the alternative \( H_1 \) in Eq. (A4) or Eq. (A9). We, therefore, consider a re-scaled version using:
\[ \hat{\sigma}_0 = \sqrt{2\theta(1-\theta)} \sqrt{\frac{1}{T(T-1)h^{(k+2)p}}} \sqrt{\frac{1}{\sum_{t=p+1,t\neq s}^T} K^2 \left( \frac{Z_{t-1} - Z_{t-1}}{h} \right)} \]

and establish that

\[ t = \frac{\hat{f}_T}{T^{-1}h^{-(k+2)p/2}\sigma_0} \xrightarrow{d} N(0,1) \]

The \( \theta \)-th quantile of \( y_t \), \( \hat{Q}_0(Z_{t-1} \mid X_{t-1}) \) is estimated as \( \hat{Q}_0(Z_{t-1} \mid X_{t-1}) = \text{inf}\{y_t: \hat{F}_{y_t|Z_{t-1} \mid X_{t-1}}(y_t|Z_{t-1} \mid X_{t-1}) \geq \theta\} \), where

\[ \hat{F}_{y_t|Z_{t-1} \mid X_{t-1}}(y_t|Z_{t-1} \mid X_{t-1}) = \frac{\sum_{s=p+1,s \neq t}^T K \left( \frac{V_{t-1} - V_{s-1}}{h} \right) 1\{y_s \leq y_t\}}{\sum_{s=p+1,s \neq t}^T K \left( \frac{V_{t-1} - V_{s-1}}{h} \right)} \]

In implementing this test, on the basis of our model specifications, we have: \( (r_t)^{(m)} = m(Z_{t-1}) + \epsilon_t \), where \( r_t \) represents the S&P500 log-returns. Causality in mean is defined as \( m = 1 \) while causality in variance is defined as \( m = 2 \).

The empirical implementation of the tests involves the specification of three main parameters: the bandwidth \( (h) \), the lag order \( (p) \), and the kernel types for \( K(\cdot) \) and \( L(\cdot) \). The lag order \( (p) \) is selected based on the BIC, with \( (h) \) determined by the leave-one-out least-squares cross-validation, and we use Gaussian kernels for \( K(\cdot) \) and \( L(\cdot) \).