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The Behavior of Real Interest Rates: New Evidence from a "Suprasecular" Perspective*

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Abstract:

We examine the temporal dynamics of the historical series of real interest rates for a sample of six European countries (Italy, France, Germany, Holland (the Netherlands), Spain pre-1730 and post-1800, and the United Kingdom), the United States and Japan stretching back to the 14th century using fractional integration techniques. We estimate the fractional integration parameter *d* using the Whittle function in the frequency domain as proposed in Dahlhaus (1989) and implemented by Robinson (1994) for the linear case and Cuestas and Gil-Alaña (2016) for the non-linear case in terms of Chebyshev time polynomials. We find evidence of short memory, persistence, and anti-persistence. In the linear case, we find evidence of persistence for France and the United Kingdom and evidence of anti-persistence for Spain pre-1730, Germany, and Italy, while for Holland (the Netherlands), Japan, Spain post-1800, and the United States the evidence favors the short memory hypothesis. Non-linear trend stationarity, however, is found for Spain pre-1739, Germany, Holland (the Netherlands), Japan, Spain, the United Kingdom, and the United States. Among these countries, evidence of anti-persistence is detected for Spain pre-1730, Germany, Holland (the Netherlands), Japan, and the United Kingdom, while Spain post-1800 and the United States exhibit short-memory behavior. Thus, the vast majority of the findings, in sharp contrast with most of the extant literature, support the hypothesis that the behavior of real interest rates is non-linear trend stationary driven by a prolonged damped oscillatory dynamics and not by a high degree of persistence.

JEL Classification: C22; C58

Keywords: Anti-persistence; long memory; short memory; Chebyshev polynomials; fractional integration; non-linearity

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1. Introduction

The real interest rate has occupied a central place in modern macroeconomics and in policy discussions for more than a century, since the seminal work of Fisher (1930). Fisher's theory of interest rates argues that a positive one-to-one relationship exists between nominal interest rates and the expected inflation rate and that the causality runs from the expected inflation rate to nominal interest rates. In the Fisherian framework, the expected real interest rate—the real cost of consuming now rather than later—is the difference between the nominal interest rate and the expected rate of inflation. As such, the expected real rate of interest is a key determinant of saving, investment, and all other intertemporal decisions.

An important macroeconomic question concerns the stationarity of the real interest rate. Whether the real interest rate follows a mean-reverting stationary process has important theoretical implications, since the validity of a variety of economic hypotheses and models depends on a stationary real interest rate (Rose, 1988; Neely and Rapach, 2008). These theories and models include the intertemporal Euler equation implied by consumption-based intertemporal models of asset prices (Lucas, 1978), the Fisher equation (Fisher, 1930), neoclassical growth models (Koopmans, 1965), investment models (Tobin, 1965), term-structure models (Modigliani and Shiller, 1973) and so on.

A stationary real interest rate converges to a long-run equilibrium value, determined by the growth of potential output, population growth, the rate of time preference, and risk aversion of economic agents. All macroeconomic models developed for monetary policy analysis and forecasting incorporate relationships between real GDP, inflation, and the real interest rate (Taylor, 2000). From a monetary policy viewpoint, Taylor (1993) argues that a stationary real interest rate direct bears on the ability of the central bank to effectively implement monetary policy by controlling the real interest rate. That is, if the real interest rate is stationary, then any change in the real interest rate resulting from monetary policy actions will only exert a

transitory effect and permanent shocks to the nominal interest rate and expected inflation must cancel out. This inference corresponds to long-run neutrality of money as the inflation rate will generate no permanent effect on interest rates. In contrast, if the real interest rate is nonstationary, then monetary policy changes in the real interest rate will cause permanent, rather than transitory, effects. That is, permanent shocks to the nominal interest rate and transitory shocks to expected inflation will result in permanent shocks to the real interest rate.

Rose (1988) is the first to examine the question of stationarity of real interest rates. Using post-war data from 18 OECD countries and applying the conventional Dickey-Fuller (1979) test, Rose (1988) finds that the nominal interest rate contains a unit root whereas the inflation rate does not, which means that any linear combination of a stationary inflation rate and a non-stationary nominal interest rate is non-stationary. Thus, the nominal interest rate minus the inflation rate (= real interest rate) is nonstationary by definition.

Following this controversial result, much research effort examines the stochastic properties of the real interest rate with varying degree of success. Using unit-root and cointegration methods, for example, Mishkin (1992), Wallace and Warner (1993), Evans and Lewis (1995), Crowder and Hoffman (1996), Atkins and Coe (2002), Granville and Mallick (2004) provide evidence in support of stationarity real interest rates, while Shapiro and Watson (1988), MacDonald and Murphy (1989), King, et al. (1991), Galí (1992), Goodwin and Grennes (1994), Mishkin and Simon (1995), Koustas and Serletis (1999), Atkins and Serletis (2003) and Rapach and Weber (2004) generally corroborate Rose (1988) finding of a unit root or nonstationary real interest rates.

These findings obtained from conventional unit-root tests are problematic from a theoretical perspective and can potentially complicate policy prescriptions (Smallwood and Norrbin, 2008). Recent advances in time-series econometrics allow researchers to revisit the question using tools that explore departures from the linear and dichotomous unit-root methods,

including long memory and non-linearity. These developments provide a better understanding of the dynamic behavior of the real interest rate. Lai (1997), Tsay (2000), Mignon and Lardic (2003), Gil-Alana (2003), Sun and Phillips (2004), Kasman et al. (2006), Smallwood and Norrbin (2008), Neely and Rapach (2008), McMillan and Wohar (2010), Norrbin and Smallwood (2011) and Gil-Alana et al. (2017) consider fractional models and typically find that the real interest rate is mean-reverting, although highly persistent.

A clear advantage of the fractional integration approach over conventional unit-root tests is that the former permits a wider range of mean-reverting behavior (Granger and Joyeux, 1980).[1] Moreover, as emphasized by Diebold and Inoue (2001) and Gadea and Mayoral (2006), long-memory models provide useful approximations to non-linear and structural-break models, which can also generate the observed dynamics of the real interest rate. Kapetanios, et al. (2003), Million (2004), Lanne (2006), Christopoulos and Leon Ledesma (2007), Koustas and Lamarche (2010), and Norrbin and Smallwood (2011) advocate threshold autoregressive (TAR) and smooth transition (STAR) models to characterize the asymmetric dynamics of the data and generally, while rejecting the unit-root hypothesis, find that real interest rates are probably persistent processes.

The failure to reject a false null hypothesis of a unit root may reflect the low power of conventional tests in samples with a short data span (Perron, 1991). The connection between the power of the unit-root tests and the time span covered by the sample size is often recognized in the literature, especially the literature testing the validity of, for example, PPP (e.g., Lothian and Taylor, 1996) . As explained by Campbell and Perron (1991), the power of unit-root tests

[1] In this regard, an alternative approach could be to calculate the degree of persistence in the data, based on the sum of the autoregressive coefficients, ρ, in an autoregressive representation of the real interest rate data, rather than simply trying to determine if the series is I(0) or I(1), following Rapach and Wohar (2004). These authors computed 95 percent confidence intervals for the persistence parameter ρ using grid-bootstrap and subsampling procedures applied to quarterly nominal long-term government bond yield and CPI inflation rate data for 13 industrialized countries for 1960-1998, and reported that the lower bounds of the 95 percent confidence interval of ρ for the tax-adjusted ex-post real interest rate are often greater than 0.90, while the upper bounds are almost all greater than unity. In other words, Rapach and Wohar (2004) detected strong evidence of persistence in real interest rates.

"depends very little on the number of observations per se but is rather influenced […] by the span of the data" (Campbell and Perron, 1991, 13). At an intuitive level, finding significant mean reversion requires a realization of a process that crosses its mean quite regularly within the sample; increasing the sampling frequency does not change this in-sample mean reversion, whereas a longer history of the time series presents more instances of crossing the mean.

Thus, an answer to the low power problem involves increasing the power of the unit-root tests by increasing the length of the sample period. Gil-Alana et al. (2017) and Sekiouaa and Zakane (2007) are among the few that recognize this problem in the context of real interest rates. Gil-Alana et al. (2017) address this issue by examining the time-series behavior of U.S. short- and long-run real ex-post interest rates within a long-memory approach using a long span of monthly (1871:01-2015:04) and annual (1800-2013) data. Their results suggest that U.S. real interest rates are not as persistent as suggested in the extant literature.

Specifically, Gil-Alana et al. (2017), using the linear methodology, find that (a) monthly real interest rates display long-memory, I(d), behavior with significantly positive value of d, but significantly below 0.5, and (b) annual real interest rates, conversely, display I(0) behavior, that is, they do not follow long-memory dynamics. Importantly, however, Gil-Alana et al. (2017) do not find evidence of non-linearity in the deterministic trends of the real interest rates. Using a recursive approach, however, they confirm the findings for the monthly rates (i.e., the estimated values of *d* again imply long memory), but for the annual rates, they find evidence of two structural breaks (1916 and 1946). They find again, however, evidence that annual real interest rates do not display long memory for the post-war period. Thus, even after accounting for structural breaks and time-varying persistence, their findings identify less persistent than suggested by prior literature.

Sekiouaa and Zakane (2007) test for a unit root in monthly real interest rates using the GLS version of the Dickey-Fuller (1979) test (Elliott et al., 1996) for the United States, United

Kingdom, France, and Japan spanning the period from 1876 to 2003. The exact sample period for each country is 1876:01 to 2001:06 for the United States, 1934:01 to 2003:07 for the United Kingdom, 1916:01 to 2003:07 for France, and 1923:01 to 2001:08 for Japan. Sekiouaa and Zakane (2007) find that the unit-root hypothesis is rejected at the 1-percent level of significance. This suggests tests based on long data series possess more power to reject a unit root than those using short samples.

Given the relevance of the time span, where long data series possess more power to reject a unit root than those using short samples, an even longer-term look might prove warranted. We revisit the issue of mean reversion of the real interest rate by using the historical data from the Bank of England (Staff Working Paper No. 845) compiled by Schmelzing (2020). The data, which use archival, printed primary and secondary sources, reconstruct global real interest rates on an annual basis going back to the 14th century. This dataset represents the most comprehensive history of the *ex-post* real (inflation adjusted) interest rate.

Schmelzing (2020) finds that real interest rates, despite temporary stabilizations during the periods 1550-1640, 1820-1850, and 1950-1980, display a pattern of continuous decline stretching back to the deep monetary crises of the "Great Bullion Famine" of the late Middle Ages. This downward trend has persisted across monetary regimes, through armed conflicts, pandemics, famines, and political, religious, and financial revolutions. Schmelzing (2020) speculates that this declining trend may indicate that the world as a safer whole emerged through the development of state institutions over the centuries. Such institutions, by protecting property rights, developing an impartial judicial system, and regulating markets, define the "rules of the game" and shape incentive of individuals and businesses. From the lender's viewpoint, this means that less risk exists that the loaned money will not be repaid, implying a reduction in the risk premium.

Jordà et al. (2020) recently used the Schmelzing (2020) dataset to examine the consequences of pandemics in the long-term. In contrast, we focus mainly on the long-run behavior of real interest rates. Gil-Alana et al. (2017), using data over the last two centuries, find that the U.S. real interest rates are not as persistent as suggested in the extant literature. Does the low persistence found by Gil-Alana et al. (2017) for the U.S. real interest rates also characterize other countries? More importantly, does this dynamic behavior hold over a "suprasecular" span of eight centuries? We employ a history of real interest rates stretching back to the 14th century to shed light on this problem. Our analysis not only represents an exercise in cliometrics, the famed chapter of "modern" economy history that aims to increase our knowledge of the past by combining economic theory with newly acquired data, but also contributes, by questioning earlier results, to the dismissal of irrelevant hypotheses (Fogel, 1966).

A rather different picture emerges from the commonly accepted one. In particular, we do not find that the real interest rates of the United States, the United Kingdom, Spain, Germany, France, Italy, Holland (the Netherlands), and Japan are characterized by a high degree of persistence (Neely and Rapach, 2008). On the contrary, our results provide historical evidence of mean reversion of real interest rates in three diverse modes: short memory, long-memory, and the anti-persistence phenomenon.[2] The latter outcome appears as the prevalent mode. Persistent and anti-persistent time-series processes are two well-known examples of time-series models with hyperbolic decay (i.e., the autocovariances decay hyperbolically, Beran, 1994). In the time domain, persistent processes show a positive long-range dependence

[2] Anti-persistence does not appear as a common phenomenon in economics. Long-memory properties are mostly focused on persistence. Anti-persistence, however, does appear in few cases. Otway (1995) finds evidence of anti-persistence in the gold-silver exchange rates of the medieval Florentine Republic. Shiryaev (1999) finds that implied and realized volatility of the Standard & Poor's 500 index exhibit anti-persistence. Caporale et al. (2018) find that the VIX index in normal periods exhibits anti-persistence. Caporale, et al. (2020) find evidence of anti-persistence in European stock markets. Dimitrova et al. (2019) find several episodes of significant anti-persistent memory in the BTC-USD and S&P500 from the mid-2010 to the early 2019.

between the observations, while anti-persistent often reverse direction and exhibit strong negative autocorrelations. In terms of the spectral density, persistence shows a singularity of the spectrum at the origin, while anti-persistent time series exhibit zero spectral density at the origin (Dittmann and Granger, 2002). Persistent process trend locally while anti-persistent process avoid trends, switching signs more frequently than a random process (i.e., it is more volatile than the random walk, Beran, 1994).

The method uses the fractional integration approach and estimates *d* using the Whittle function in the frequency domain as proposed in Dahlhaus (1989) and implemented by Robinson (1994) for the linear case and Cuestas and Gil-Alana (2016) for the non-linear case.[3] For each model, we also consider two error structures, white-noise and autocorrelated errors. Limiting the analysis to linear deterministic trends or white-noise error structures could produce spurious results, if the trend exhibits a nonlinear structure and the error-disturbance term exhibits autoregressive effects.

Extending the time span of the data is not without a cost, as the probability that the data generating process (DGP) remains time invariant may significantly decrease. Thus, the need to account for this problem in the context of the historical real interest series is most relevant since "epochal" changes in institutional settings (the transition from feudal to modern economies, from metallic monetary standards to fiat currency monetary regimes, from absolutist monarchies to democratic republics) may induce changes in those trends, which, in turn, may lead to changes in the equilibrium values of the real interest rates. In fact, Schmelzing (2020) proposes three historical "epochal" periods in which the trend for real interest rates could have changed. These are: 1) the "post-Bullion famine" period, beginning in 1494 after the second

[3] The Robinson (1994) methodology has several distinguishing features compared with other procedures. First, it permits the determination of the degree of integration of a univariate series independently of its stationary or nonstationary nature. Second, the test statistic has a standard null limit distribution and this standard behavior holds independently of the inclusion or non-inclusion of deterministic trends, which is crucial in the case of testing for unit roots in real interest rates over a "suprasecular" span. Concerning the statistical properties, the Robinson (1994) approach delivers tests that are the most efficient ones in the Pitman sense against local departures.

monetary contraction identified by Day (1978), and the resumption of Balkan mining output; 2) the "North-Weingast" period (in reference to the well-known transaction cost theory of institutions articulated by North and Weingast, 1989), which posited a key institutional revolution in late seventeenth century Britain, which enabled the emergence of credible debt mechanisms (1694); and 3) the "post-Napoleonic" period, beginning in 1820, after the Congress of Vienna and the establishment of the modern state system.

These are not abrupt, sharp structural breaks, but smooth evolutions from old to new paradigms, permeated by Schumpeterian 'creative destruction' episodes. They occur over centuries, not years or even decades. We account for this aspect of our analysis by incorporating into the estimation process non-linear deterministic trends based on the Chebyshev time polynomials.[4] Chebyshev polynomials are cosine functions of time and provide a flexible approximation of deterministic trends (Bierens, 1997). In particular, they can approximate highly nonlinear trends with rather low-degree polynomials (Bierens, 1997). In fact, any function of time can be approximated arbitrarily close by a linear function of Chebyshev polynomials. Bierens (1997) uses Chebyshev polynomials in the context of unit-root testing against nonlinear trend stationarity.

The outline of the paper is as follows. Section 2 briefly recalls the I(d) model and outlines the testing procedures. Section 3 examines the real interest rates of six European countries (Italy, France, Germany, Holland (the Netherlands), Spain pre-1730 and post-1800, the United Kingdom, Japan, and the United States) by means of linear and non-linear fractionally integrated techniques in the "suprasecular" framework. Section 4 presents the main empirical results, while Section 5 contains some concluding comments and policy implications.

[4] This is important, since the appearance of (spurious) long memory can be generated by the presence of structural breaks (see Kellard et al. (2015) for a detailed discussion in this regard).

2. Methodology

We consider two fractional integration models. We estimate the fractional integration parameter d using the Whittle function in the frequency domain as proposed in Dahlhaus (1989) and implemented by Robinson (1994) for the linear case and Cuestas and Gil-Alaña (2016) for the non-linear case. In the latter case, we allow for non-linear deterministic trends in the form of Chebyshev polynomials in time to account for the significant disruptions observed in the temporal evolution of real interest rates.

This method permits the testing of any real value for the fractional integration parameter, including short memory ($d = 0$), persistent memory ($0 < d < 0.5$), nonstationary values ($d \geq 0.5$), and anti-persistent memory ($-0.5 < d < 0$). Persistent and anti-persistent processes are two types of processes that exhibit hyperbolic decay. If $0 < d < 0.5$, the autocorrelations are all positive and decay monotonically and hyperbolically. The spectral density is concentrated at low frequencies and tends to infinity when the frequency tends to zero.

The data exhibit persistence or long memory in the sense that positive (negative) shocks are on average followed by positive (negative) shocks. If $-0.5 < d < 0$, the autocorrelations are negative and decay hyperbolically, but not monotonically; rather, in a prolonged damped oscillatory way. The spectral density at zero frequency equals zero. The data exhibit anti-persistence in the sense that positive (negative) shocks are on average followed by negative (positive) ones and the series returns to its mean level more rapidly than does a white noise series. When $0.5 \leq d < 1$, the process is non-stationary, but mean reverting. When $d \geq 1$ the process is explosive, non-stationary, and non-mean reverting. For $d = 0$, the process exhibits "short memory" and has a short-term dependency structure, but no structure of long-term dependency, corresponding to a stationary and invertible ARMA. The autocorrelation structure

of a short-memory process is geometrically bounded, while its spectral density is bounded for all frequencies.

The first model is the standard linear model of the form advocated, for example, in Gil-Alana and Robinson (1997). The model incorporates two equations. The first accommodates the deterministic terms, while the second expresses the conventional fractional integration model, i.e.,

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1,2,... \tag{1}$$

with

$$(1-L)^d x_t = u_t, \qquad t = 1,2,..., \tag{2}$$

where y_t is the observed time series, β_0 and β_1 are the coefficients corresponding, respectively, to the intercept and linear time trend, L is the lag operator ($Lx_t = x_{t-1}$), and x_t is I(d), where d refers to the order of integration. The operator $(1-L)^d$ is the fractional filter defined by means of the gamma function $\Gamma(\cdot)$

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)}, \tag{3}$$

where the parameter d can assume any real value.

The second model accounts for non-linearity and is designed to capture smooth, evolutionary changes rather than sharp, abrupt changes in real interest rates. This model employs an extension of the linear method to the non-linear case, replacing the linear regression model in equation (1) by a non-linear model based on Chebyshev polynomials in time defined by:

$$y_t = \sum_{i=0}^{m} \theta_i P_{i,N}(t) + x_t, \qquad t = 1,2,..., \tag{4}$$

where m indicates the order of the Chebyshev polynomial $P_{i,N}(t)$ defined as

$$P_{i,N}(t) = \sqrt{2}\cos\left[i\pi(t-0.5)/N\right]; \quad t = 1,2,...,N; \quad i = 1,2,.... \tag{5}$$

with $P_{0,N}(t)=1$.[5] From equation (4), if $m = 0$, the model contains only an intercept; if $m = 1$, it contains an intercept and a linear trend; and if $m > 1$, the model becomes non-linear, where the higher the value of m is, the higher is the non-linear structure. The parameters $\theta_i, i=1,2...m$ are the non-linear parameters where the significance of $m > 1$ parameters implies non-linearity of the time series. An issue that immediately arises is the optimal value of m. Cuestas and Gil-Alaña (2016) argue that if one combines equations (2) and (4) in a single equation, standard t-tests will remain valid with an I(0) error term by definition. Then, the choice of m will depend on the significance of the Chebyshev coefficients.

3. Empirical results

We examine the time series of real interest rates for the following countries: Spain pre-1730 (1400-1729), Spain post-1800 (1800-2018), France (1387-2018), Germany (1326-2018), Holland (the Netherlands) (1400-2018), Italy (1314-2018), Japan (1742-2018), the United Kingdom (1314-2018), and the United States (1776-2018). The real interest rate is the ex-post real interest rate, computed as the difference between the nominal bond yield and the rate of inflation.[6] We use the real interest rates in levels and not logarithms. Table 1 reports the summary statistics for the real interest rate of the individual countries. Most of the real interest rate series exhibit negative skewness and a high degree of kurtosis. The Jarque-Bera test rejects the null hypothesis of normality for all series at any reasonable level of significance. According

[5] The test is appealing in the sense that it is robust to testing between linear persistence and non-linear persistence in a fractional integration framework. Other non-linearity tests assume either stationarity I(0) or non-stationarity I(1) of the series, but the Cuestas-Gil-Alana (2017) test is more general, allowing for any real value of d for the degree of integration of the series.

[6] Most empirical studies employ the *ex-post* rate to investigate the stationarity properties of the real interest rate. The *ex-ante* interest rate, on the other hand, is the relevant rate, since economic decisions inherently depend on this rate, which equals the difference between the nominal interest and the expected inflation rate. The *ex-ante* real rate, however, is unobservable in contrast to the *ex-post* rate. The difference between the two represents a measurement error, which is typically a stationary process (white noise under rational expectations) and is unlikely to affect the results profoundly (Kapetanios et al., 2003).

to Fang et al. (1994), this significant deviation from normality may indicate non-linear dynamics.

3.1 Testing the order of integration of real interest rates in a linear framework

As is standard practice in the literature, we conducted an array of unit-root tests, which included the Augmented Dickey-Fuller (Dickey and Fuller, 1979), the GLS-Dickey-Fuller (Elliott et al., 1996), the Phillips-Perron (Phillips and Perron, 1988), the KPSS (Kwiatkowski et al, 1992), and the Ng and Perron (2001) tests. These results are available from the authors. All these tests indicate that the real interest rates are stationary. These tests, however, only reveal whether the data are I(0) or I(1), that is, stationary or contain a unit root. They do not speak to the nature of the stationarity and mean reversion (long memory, short memory, anti-persistence). The linear model given by (1) and (2), instead, can detect whether mean-reversion is fast or slow, whether the decay is hyperbolic or exponential, and whether the hyperbolic decay occurs monotonically or in the form of prolonged damped oscillations.

For the linear model, we estimate the fractional integration model under three different assumptions for the deterministic terms, namely a model with no deterministic terms, a model with only a constant (intercept), and a model with both constant and a linear trend. Table 2 displays the results under the assumption that the error term, u_t, is uncorrelated. We note that the time trend is not required for Spain pre-1730 and post-1800, Italy, Japan, and the United States. We also note that the estimated value of d is less than 1 in all nine cases, which rejects the null hypothesis of a unit root. The mean reversion property is confirmed in all cases. Thus, exogenous shocks to real interest rates will die out in the long run. For Spain post-1800, the upper value in the 95% interval is strictly greater than 0.5, implying that we cannot strictly reject the hypothesis of stationarity. Furthermore, for Spain pre-1730 and Japan, we cannot reject the null hypothesis of short memory. In all the remaining cases, the evidence favors long-memory.

Table 3 displays the results under the assumption of an autocorrelated error term, specifically following an AR(1) process. We prefer the latter results as they enrich the structure of the model. In Table 3, the time trend is significant in all nine cases. Table 4 reports the estimated coefficients. The coefficient estimate on the time trend is significant and, except for Japan, negative. This generally supports the findings of Schmelzing (2020), who argues that global interest rates exhibit a persistent downward trend over the past five centuries. Focusing on the fractional integration estimates, we find evidence of anti-persistence ($-0.5 < d < 0$) for Spain pre-1730, Germany and Italy, short memory ($d = 0$) for Spain post-1800, Holland (the Netherlands), Japan, and the United States, and long memory ($0 < d < 0.5$) for France and the United Kingdom.

3.2 Testing the order of integration of real interest rates in a non-linear framework

Next, we apply the approach of Cuestas and Gil-Alana (2016) to account for non-linearities along the time trend of the real interest rates of our sample of countries. We restrict the Chebyshev polynomial in time to θ_i, i = 0, 1, 2, 3. In particular, as the non-linear order increases, the intensity of non-linearity increases. Cuestas and Gil-Alana (2016) indicate that a second-order ($m = 2$) and a third-order non-linearity structure ($m = 3$) sufficiently infer non-linearity following Chebyshev polynomial in time. In general, the statistical significance of the Chebyshev coefficients informs the choice of the value of m.

Tables 5 and 6 present the results. Table 5 assumes that the error term is uncorrelated, while Table 6 relaxes the assumption of no correlation. Assuming an uncorrelated error term, we find evidence of non-linear components in the deterministic trend only for Holland (the Netherlands), and the United Kingdom. Permitting autocorrelation, however, we find non-linearity in seven of the nine series. The non-linearity is quadratic for Spain post-1800 and Japan, and of third order for Spain pre-1730, Germany, Holland (the Netherlands), the United

Kingdom, and the United States. In contrast, Gil-Alana et al. (2017) provide no evidence of non-linearities in the U.S. data. France and Italy fail to display a non-linear pattern. Thus, for these two countries, we adopt the results in Table 4 that allows only a linear deterministic trend. In this context, we conclude that Italy exhibits anti-persistence while France exhibits persistence.

In the non-linear context and in the countries where the Chebyshev coefficients for $m > 1$ are significant, we find anti-persistence for Spain pre-1730, Germany, Holland (the Netherlands), Japan, and the United Kingdom, while short-memory is evident for Spain post-1800, France, Italy, and the United States. The finding of short-memory for the U.S. series confirms the Gil-Alana et al. (2017) finding with annual data. We find no evidence of long memory using the non-linear model with Chebyshev polynomials. We note, however, that we can reduce the degree of fractional integration by artificially increasing the order of the Chebyshev polynomials. Cuesta and Gil Alana (2016) also find this phenomenon and is consistent with the literature on fractional integration and non-linearity (Diebold and Inoue, 2001; Granger and Hyung, 2004), which argues that fractional integration and non-linearity are intimately related issues.

4. Conclusion

We examine the historical "suprasecular" series of real interest rates (Schmelzing, 2020) for evidence of unit roots, persistence, short memory and anti-persistence using fractional integration techniques. We estimate the fractional integration parameter d using the Whittle function in the frequency domain as proposed in Dahlhaus (1989) and implemented by Robinson (1994) for the linear case and Cuestas and Gil-Alana (2016) for the non-linear case. In the latter case, we allow for non-linear deterministic trends in the form of Chebyshev polynomials in time to account for the significant "epochal" evolution of real interest rates. For

each case of linear and non-linear models, we generate estimates under white-noise and autocorrelated processes of the error term.

We establish, however, that the fractional integration estimates for the autocorrelated process dominate the estimates of the white-noise process of the error term, following the statistical significance of the autoregressive components. Evidence from linear and non-linear fractional integration models indicate that the real interest rate is mean-reverting and stationary for all countries. Thus, the historical evidence rules out the unit-root case, meaning that shocks to real interest rates do not exhibit permanent effects. We cannot, however, rule out the case for persistence and anti-persistence.

Persistence and anti-persistence are two versions of hyperbolic decay processes. A persistent process implies that another positive (negative) movement is statistically more likely to follow a positive (negative) movement. Conversely, an anti-persistent process implies that a negative (positive) movement is statistically more likely to follow positive (negative) movement. Thus, equilibrium emerges through prolonged damped oscillations.

Contrary to much of the literature of the past several decades using relatively recent data, we find relatively scant evidence of persistence of real interest rates over centuries of data. In the linear case with autocorrelated errors, we find evidence of mild persistence for France and the United Kingdom and evidence of anti-persistence for Spain pre-1730, Germany and Italy, while for Holland (the Netherlands), Japan, Spain post-1800, and the United States, the evidence favors the short-memory hypothesis. In the non-linear model with autocorrelated errors, which combines fractional integration and non-linearity in a single framework, we find non-linearity for Germany, Holland (the Netherlands), Japan, Spain pre-1730 and post-1800, the United Kingdom, and the United States. France and Italy do not support non-linearity. For these two countries, the linear model still applies, which finds persistence in the case of France and short memory in the case of Italy. Among the countries for which we cannot reject non-

linearity, we find no evidence of persistence. We do detect evidence of anti-persistence for Spain pre-1730, Germany, Holland (the Netherlands), Japan, and the United Kingdom, while Spain post-1800 and the United States exhibit short-memory behavior. Thus, the findings do not demonstrate that persistence of real interest rates represents a "stylized" fact (Neely and Rapach, 2008) after extending the analysis to eight centuries of data. The nonlinear effect as well as the autocorrelation effect significantly affect the degree of persistence of the nine countries by inducing less persistence.

What are the economic implications of our findings of short memory, long memory, and anti-persistence? Short memory expresses quantitatively market rationality, as the current value reflects the entire history of the series. Thus, the efficient market hypothesis remains a valid paradigm for Spain post-1800 and the United States even in the "suprasecular" framework. Persistence and anti-persistence, on the other hand, completely alienate the hypothesis of market efficiency, since they signal that markets process information very slowly. An anti-persistent market, in particular, manifests itself with temporal patterns of overreacting reversals. Interestingly, any sequence of uniformly distributed random variables exhibits anti-persistence properties (Otway, 1995). This suggests that in an anti-persistent market, real interest rates may have been set potentially in a random manner, which provides evidence supporting the noisy market hypothesis in capital markets.

As part of future research, an interesting question to ask would be, besides political regime changes, if movements in real interest rates is a monetary phenomenon in this historical data set, given that evidence of this was detected by Rapach and Wohar (2005) in post-World War II data for multiple industrialized economies,[7] based on tests of multiple structural breaks

[7] Based on the Bai and Perron (2003) multiple structural breaks tests, Rapach and Wohar (2005) found significant evidence of structural breaks in the mean of the ex-post real interest rates in each of the 13 countries, and also observed that the breaks in the mean inflation rate often coincided with breaks in the mean of the ex-post real interest rate for each country's data. Furthermore, increases (decreases) in the mean inflation rate were almost always associated with decreases (increases) in the mean ex-post real interest rate. Note that the application of these tests for international interest rate data can also be found in the work of Olson et al. (2012).

applied to the ex-post real interest rates and inflation rates of these economies. This is an important issue, since we would be able to accept or reject the hypothesis of whether central banks change monetary policy and inflation through persistent effects on the real rate of interest.

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Table 1: Descriptive statistics

Country	Mean	St. Dev.	Skewness	Kurtosis	Jarque-Bera
Italy	3.47	24.09	-12.27	248.67	1798309***
United Kingdom	10.77	13.68	0.73	3.45	69.33***
Holland (the Netherlands)	5.09	10.00	-0.62	8.01	686.67***
Germany	5.25	7.81	-1.12	11.71	2332.86***
France	7.57	12.73	-0.83	8.79	956.88***
United States	2.91	5.98	-0.24	4.99	40.98***
Spain pre-1730	5.87	7.59	-0.21	3.81	11.02***
Spain post-1800	7.41	16.75	0.84	7.51	212.20***
Japan	0.60	13.99	-0.80	6.71	168.21***

Note: The sample periods are Spain pre-1730: 1400-1729; Spain post-1800: 1800-2018; France: 1387-2018; Germany: 1326-2018; Holland (the Netherlands): 1400-2018; Italy: 1314-2018; Japan: 1742-2018; the United Kingdom: 1314-2018; the United States: 1776-2018.

Table 2: Estimates of *d* and 95 Confidence Bands with No Autocorrelated Errors

Country	No deterministic terms	With an intercept	With an intercept and a linear time trend
Spain pre-1730	-0.13 (-0.16, -0.07)	**-0.16 (-0.22, -0.07)**	-0.18 (-0.24, -0.09)
Spain post-1800	**0.42 (0.33, 0.55)**	0.40 (0.31, 0.54)	0.38 (0.26, 0.53)
France	0.29 (0.24, 0.34)	0.26 (0.21, 0.31)	**0.19 (0.13, 0.26)**
Germany	0.24 (0.18, 0.32)	0.21 (0.15, 0.28)	**0.17 (0.09, 0.26)**
Holland (the Netherlands)	0.15 (0.11, 0.20)	0.13 (0.09, 0.17)	**0.07 (0.03, 0.13)**
Italy	**0.18 (0.12, 0.25)**	0.18 (0.12, 0.25)	0.17 (0.10, 0.24)
Japan	**0.10 (-0.01, 0.26)**	0.10 (-0.01, 0.26)	0.10 (-0.02, 0.26)
United Kingdom	0.35 (0.32, 0.39)	0.30 (0.27, 0.33)	**0.18 (0.15, 0.23)**
United States	0.31 (0.20, 0.45)	**0.29 (0.18, 0.43)**	0.29 (0.17, 0.44)

Note: In bold, the most adequate specification in relation to the deterministic terms. The estimated model is $y_t = \beta_0 + \beta_1 t + x_t, (1-L)^d x_t = u_t, t = 1,2,...$, where u_t is white noise.

Table 3: Estimates of *d*: Autocorrelated Errors

Country	No deterministic terms	With an intercept	With an intercept and a linear time trend
Spain pre-1730	-0.14 (-0.18, -0.09)	-0.21 (-0.29, -0.10)	**-0.25 (-0.34, -0.13)**
Spain post-1800	0.30 (0.20, 0.43)	0.27 (0.17, 0.40)	**0.12 (-0.01, 0.34)**
France	0.26 (0.20, 0.32)	0.22 (0.18, 0.27)	**0.07 (0.00, 0.15)**
Germany	0.04 (-0.02, 0.11)	0.03 (-0.01, 0.08)	**-0.10 (-0.17, -0.04)**
Holland (the Netherlands)	0.12 (0.07, 0.19)	0.08 (0.04, 0.14)	**0.02 (-0.05, 0.08)**
Italy	0.03 (-0.03, 0.12)	0.03 (-0.02, 0.11)	**-0.02 (-0.09, -0.08)**
Japan	-0.16 (-0.27, 0.01)	-0.15 (-0.28, 0.01)	**-0.20 (-0.32, 0.03)**
United Kingdom	0.35 (0.31, 0.38)	0.29 (0.26, 0.32)	**0.13 (0.10, 0.18)**
United States	0.13 (-0.03, 0.34)	0.11 (-0.02, 0.28)	**0.04 (-0.11, 0.29)**

Note: In bold, the most adequate specification in relation to the deterministic terms. The estimated model is $y_t = \beta_0 + \beta_1 t + x_t, (1-L)^d x_t = u_t, t = 1,2,...$, where u_t is AR(1).

Table 4: Estimates of the coefficients: Autocorrelated Errors

Country	d (95% CI)	Intercept (t-value)	Time trend (t-value)
Spain pre-1730	-0.25 (-0.34, -0.13)	6.4328 (23.42)	-0.0034 (-2.08)
Spain post-1800	0.12 (-0.01, 0.34)	18.8316 (6.49)	-0.1039 (-4.69)
France	0.07 (0.00, 0.15)	17.4943 (14.10)	-0.0312 (-9.45)
Germany	-0.10 (-0.17, -0.04)	8.7376 (27.06)	-0.0101 (-12.12)
Holland (the Netherlands)	0.02 (-0.05, 0.08)	9.9143 (11.69)	-0.0154 (-6.57)
Italy	-0.02 (-0.09, -0.08)	8.6493 (5.50)	-0.0145 (-3.75)
Japan	-0.20 (-0.32, 0.03)	-0.8889 (-1.10)	0.0143 (2.19)
United Kingdom	0.13 (0.10, 0.18)	26.7181 (19.35)	-0.0442 (-13.58)
United States	0.04 (-0.11, 0.29)	5.1101 (6.04)	-0.0185 (-3.00)

Notes: The table presents the estimates of *d* together with the 95-percent confidence interval, and the estimates of the linear trend.

Table 5: Estimates of the Non-linear Chebyshev I(d) Model: White-Noise Errors

Country	d	θ_0	θ_1	θ_2	θ_3
Spain pre-1730	-0.19 (-0.27, -0.10)	**5.879** **(38.67)**	0.307 (1.61)	-0.108 (-0.52)	-0.288 (-1.31)
Spain post-1800	0.35 (0.22, 0.52)	5.231 (1.02)	**6.058** **(1.76)**	2.366 (0.79)	-1.596 (-0.60)
France	0.19 (0.13, 0.26)	**7.757** **(5.65)**	**5.691** **(5.23)**	-0.412 (-0.41)	0.743 (0.78)
Germany	0.15 (0.07, 0.25)	**5.227** **(7.44)**	**1.988** **(3.40)**	0.694 (1.26)	0.678 (1.30)
Holland (the Netherlands)	0.01 (-0.04, 0.08)	**5.097** **(12.64)**	**2.557** **(6.42)**	**1.301** **(3.28)**	**1.567** **(3.96)**
Italy	0.16 (0.09, 0.24)	3.354 (1.43)	2.673 (1.38)	-0.360 (-0.75)	0.631 (0.36)
Japan	0.09 (-0.03, 0.25)	0.698 (0.47)	-0.866 (-0.66)	-0.906 (-0.71)	-0.500 (-0.40)
United Kingdom	0.03 (-0.03, 0.09)	**10.750** **(25.55)**	**9.205** **(22.71)**	**3.680** **(9.19)**	**1.149** **(2.90)**
United States	0.28 (0.14, 0.44)	**3.240** **(2.20)**	1.625 (1.54)	0.838 (0.89)	-0.257 (-0.30)

Note: In bold, the most adequate specification in relation to the Chebyshev coefficients. The estimated model is $y_t = \sum_{i=0}^{m} \theta_i P_{i,N}(t) + x_t$, $(1-L)^d x_t = u_t, t = 1, 2, \ldots$, where u_t is white noise and $m \leq 3$.

Table 6: Estimates of the non-linear Chebyshev I(d) model: Autocorrelated errors

Country	d	θ_0	θ_1	θ_2	θ_3
Spain pre-1730	-0.29 (-0.39, -0.17)	**5.884** **(64.30)**	**0.301** **(2.35)**	-0.101 (-0.70)	**-0.294** **(-1.85)**
Spain post-1800	-0.01 (-0.17, 0.26)	**7.431** **(7.88)**	**7.385** **(7.73)**	**3.391** **(3.53)**	-0.894 (-0.93)
France	0.06 (-0.01, 0.15)	**7.619** **(11.96)**	**5.689** **(9.62)**	-0.503 (-0.87)	0.745 (1.31)
Germany	-0.23 (-0.30, -0.14)	**5.261** **(68.76)**	**1.990** **(19.82)**	**0.736** **(6.65)**	**0.675** **(5.66)**
Holland (the Netherlands)	-0.11 (-0.18, -0.03)	**5.066** **(25.47)**	**2.522** **(11.09)**	**1260** **(5.28)**	**1.535** **(6.21)**
Italy	-0.03 (-0.12, 0.06)	**3.479** **(4.63)**	**2.998** **(3.84)**	-1.278 (-1.61)	0.865 (1.08)
Japan	-0.26 (-0.40, -0.06)	**0.680** **(2.43)**	**-0.937** **(-2.48)**	**-0.921** **(-2.18)**	-0.560 (-1.22)
United Kingdom	-0.14 (-0.20, -0.08)	**10.810** **(71.04)**	**9.290** **(51.51)**	**3.765** **(19.66)**	**1.225** **(6.11)**
United States	-0.09 (-0.32, 0.26)	**2.908** **(11.68)**	**1.216** **(4.37)**	**0.650** **(2.25)**	**-0.525** **(-1.76)**

Note: In bold, the most adequate specification in relation to the Chebyshev coefficients. The estimated model is $y_t = \sum_{i=0}^{m} \theta_i P_{i,N}(t) + x_t$, $(1-L)^d x_t = u_t, t = 1, 2, \ldots$, where u_t is AR(1) and $m \leq 3$.