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Time-Varying Spillovers between Housing Sentiment and Housing Market in the United States∗

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Abstract

This paper investigates spillovers between the housing sentiment index of Bork et al. (2020), common factors in US real housing returns and their volatility (derived from a time-varying dynamic factor model with stochastic volatility), GDP growth and real interest rates, using the time-varying parameter vector autoregressive version of the Diebold and Yılmaz (2012, 2014) methodology. We find that in contrast to spillovers from the common factor in housing returns, reverse spillovers are relatively weak. Net spillovers from the common factor of housing returns to housing sentiment and GDP increase durably after the Global Financial Crisis. This suggests that, while a shock to housing prices is likely to have a significant impact on housing sentiment and the economy, a purely exogenous shock to housing sentiment, may in itself have little impact on housing returns and volatility.

Keywords: Common Housing Market Movements, Sentiment, Time-Varying Spillovers.

JEL codes: C32, R31.

∗The views expressed in this paper are those of the authors and do not necessarily reflect those of the Organisation for Economic Co-operation and Development (OECD) or the governments of its member countries.
1 Introduction

Economic analysts and forecasters have for decades monitored consumer and business confidence to assess current economic conditions and prospects. Recently, Bork et al. (2020) have constructed a housing sentiment index for the United States, based on the University of Michigan consumer survey, and shown that it outperforms a range of traditional fundamental determinants in predicting house price developments. Nevertheless, the nexus between sentiment, housing prices, and fundamentals is complex. Sentiment influences buying and selling decisions and thereby house prices. However, sentiment also reflects fundamentals, such as income and interest rates. Moreover, it is also likely to be affected by contemporary and past house price developments, as house price expectations usually contain an important extrapolative component.

In this paper, we study the spillovers between housing sentiment, common factors in US real housing returns and volatility, GDP growth and real interest rates, in order to assess the impact of exogenous changes in sentiment on house prices and better understand the interactions between sentiment and the fundamental determinants of house prices. Bork et al. (2020) noted that housing market sentiment could also predict state-level housing returns, while suggesting the same holds for housing returns volatility. Hence, we consider simultaneously in our analysis both the first and second-moments of housing returns. Following Del Negro and Otrok (2007), who pointed out that the US housing market across the states cannot be considered as a single housing market, we analyse the common movements of housing returns, rather than those of an aggregate house price index. But unlike their constant parameter framework, we follow Del Negro and Otrok (2008) and Gupta et al. (2020b) and use a time-varying dynamic factor model with stochastic volatility (TVP-DFM-SV). We then use a Bayesian time-varying parameter vector autoregressive model (TVP-VAR), in an extension of the Diebold and Yılmaz (2012, 2014) methodology, which has become popular for studying economic and financial spillovers.

The model we use has five key advantages in the context of this study. First, the VAR structure allows capturing the complex dynamics of the system. Second, the identification of shocks is data-driven, rather than requiring parameter restrictions, which would be theoretically challenging to specify, due to contemporaneous interactions between variables and the lack of strong theoretical grounds for structural restrictions. In the model we are using, the forecast error variance decomposition is invariant to the ordering of the variables. The method uses the
historical co-variance structure of the variables to avoid the need to impose specific identifying restrictions on shocks. Third, the Bayesian estimation adjusts the parameters of the model in a timely manner as behaviours evolve over time. TVP-VAR estimates have been shown to be superior to rolling-windows, which have been widely used in the literature, as they avoid the arbitrary choice of the size of the window and discarding part of the sample. They are also less prone to excessive persistence and less sensitive to outliers (Gabauer and Gupta, 2018; Antonakakis et al., 2018, 2020). Fourth, the model allows for heteroskedastic volatility to capture changes in the variance of innovations, which yields more robust parameter estimates when volatility varies over time (Nakajima et al., 2011). Finally, the use of a FAVAR model allows the incorporation of common return and volatility components in US state-level housing prices. This is preferable to using national aggregates, which can be strongly influenced by idiosyncratic developments in large states (Del Negro and Otrok, 2007).

The role of psychology in housing markets has been widely documented (Mayer and Sinai, 2009; Shiller, 2008). In particular, past appreciation of house prices has been found to often act as a bubble builder, due to extrapolative house price expectations, while mean-reversion acts as a bubble burster (Abraham and Hendershott, 1996). Households tend to form excessive expectations of long-term house prices in upswings, as they fail to anticipate the mean-reversion generally observed in the data (Case et al., 2014; Armona et al., 2019). The event causing a change in sentiment and triggering the reversal in expectations may be trivial (Kindelberger and Aliber, 2005). Hence, one may expect sentiment to contain substantial information for future house price developments, beyond economic and housing market fundamentals. Indeed, Bork et al. (2020) show that their housing sentiment index explains a large share of house price variation and improves forecasts, compared to models based on fundamental house price drivers. A number of other papers point to an important role of various housing sentiment indicators in housing market developments. Lambertini et al. (2013), using data from the University of Michigan consumer survey, provide evidence on the importance of news and consumers’ beliefs for US housing market dynamics. Ling et al. (2015) show that changes in homebuyers, homebuilders and lenders’ sentiments predict house price appreciation in subsequent quarters, beyond the impact of changes in lagged price changes, fundamentals and market liquidity. Gupta et al. (2019) show that taking into account housing sentiment allows forecasting US home sales growth more accurately than using only information contained in economic variables. Hence, monitoring housing sentiment is important for market analysts and policymakers.
However, relations between housing sentiment, housing prices, and fundamentals are complex. This paper aims at shedding light on the dynamic connections between housing sentiment, real housing returns and volatility, and two of their main fundamental determinants, economic activity (measured by real GDP growth) and real interest rates. We find strong spillovers from housing returns to sentiment, but weak spillovers in the opposite direction. Net spillovers from the common factor of housing returns to housing sentiment and GDP increase durably after the Global Financial Crisis (GFC). The paper is organised as follows. Section 2 briefly describes the data and the methodology. Section 3 describes the empirical results. Section 4 concludes.

2 Data & Methodology

The analysis is performed on quarterly data covering the period 1986Q1-2017Q4, with the start date made to correspond with Del Negro and Otrok (2007) to cover a homogenous period in the US housing market post financial liberalization. Pre-sample data going back to 1975 are used in deriving Bayesian priors (Del Negro and Primiceri, 2015). Our measure of housing sentiment is the index built by Bork et al. (2020), using time series data from the consumer surveys of the University of Michigan. Housing sentiment is defined as the general attitude of households about house buying conditions. Bork et al. (2020) focus on the survey responses relative to the underlying reasons why households believe that it is a bad or good time to buy a house. Partial least squares are used to aggregate the information contained in ten time series into an easy-to-interpret index of housing sentiment.

Housing returns and volatility are measured by common factors extracted from state-level Federal Housing Finance Agency (FHFA) house price indices, using a Bayesian TVP-DFM-SV, as in Gupta et al. (2020a). Seasonally-adjusted real GDP (in billions of chained 2012 dollars) is obtained from the FRED database of the Federal Reserve Bank of St. Louis, and converted to quarter-on-quarter growth rate. As far as the real interest rates are concerned, we subtract the Consumer Price Index (CPI)-based annualized inflation rate from the policy rate, with CPI (1982-1984=100) data derived from the FRED database. Note that, the policy rate is the estimated shadow rates derived from a three-factor shadow rate term structure model (SRTSM) of Wu and Xia (2016) over the period of 1975Q3 to 2015Q4. The remainder of the period (2016Q1 to 2017Q4), we use the Federal funds rate obtained from the FRED database, given that the Federal Reserve increased the Federal funds rate by 50 basis points in December.

1The details of the TVP-DFM-SV model is presented in the Appendix of our paper.
2015, thus coming out of the ZLB-situation.

Housing returns and volatility are measured by common factors extracted from state-level Federal Housing Finance Agency (FHFA) house price indices, using a Bayesian dynamic factor model, as in Del Negro and Otrok (2007) and Gupta et al. (2020a).

Table 1 shows summary statistics for all variables. We find that all variables except the Real Interest Rate and the Housing Sentiment Index are significantly skewed and/or exhibit excess kurtosis and are thus non-normally distributed at least at the 1% significance level (Real Housing Returns Common Factor does not show kurtosis). Furthermore, the ERS unit-root test statistics indicate that each series is stationary at least at the 10% significance level. Finally, the weighted portmanteau test detect that all employed time series are significantly autocorrelated and exhibit ARCH errors at the 1% significance level.

[Insert Table 1 around here]

To compute spillover indices, we use a refined version of the methodology introduced by Diebold and Yılmaz (2012, 2014), based on a Bayesian factor augmented time-varying parameter VAR model with heteroskedastic volatility (Antonakakis et al., 2020)\(^2\). We estimate a TVP-VAR with a lag length of one as suggested by the Bayesian Information Criterion (BIC) which can be written as follows:

\[
y_t = \Phi_t y_{t-1} + \epsilon_t \quad \epsilon_t \sim N(0, \Sigma_t) \quad (1)
\]

\[
vec(\Phi_t) = vec(\Phi_{t-1}) + \nu_t \quad \nu_t \sim N(0, R_t) \quad (2)
\]

where \(y_t\) represents an \(N \times 1\) conditional volatilities vector, \(y_{t-1}\) is an \(N \times 1\) lagged conditional vector, \(\Phi_t\) is an \(N \times N\) dimensional time-varying coefficient matrix and \(\epsilon_t\) is an \(N \times 1\) dimensional error disturbance vector with an \(N \times N\) time varying variance-covariance matrix, \(\Sigma_t\). The parameters \(vec(\Phi_t)\) depend on their own past values \(vec(\Phi_{t-1})\) and on an \(N^2 \times 1\) dimensional error vector \(\nu_t\) with an \(N^2 \times N^2\) variance-covariance matrix, \(R_t\).

Assuming covariance stationarity, the VAR process has the following MA (\(\infty\)) representation, \(y_t = \sum_{j=0}^{\infty} \Psi_{jt} \epsilon_{t-j}\), where \(\Psi_{jt}\) is a lag polynomial matrix of coefficients that can be calculated recursively. Koop et al. (1996) and Pesaran and Shin (1998) show that the generalization forecast error variance decomposition (GFEVD) of a variable can be computed from

\(^2\)To save space, we provide a simplified presentation of the methodology. Details can be found in Antonakakis et al. (2020).
components attributable to shocks to the different variables in the system for a given forecast horizon \( H \) as:

\[
\phi_{ijt}(H) = \frac{\sum_{i=1}^{N} \sum_{h=0}^{H} (t_i \Psi_{ht} \sum_t \Sigma_l \Psi_l')^2}{\sum_{j=1}^{N} \sum_{h=1}^{H} (t_i \Psi_{ht} \sum_t \Sigma_l \Psi_l'} t_i^2),
\]

(3)

where \( t_i \) corresponds to a zero vector with unity on the \( i \)th position, \( \Psi_{ht} \) is a \( N \times N \) matrix, and \( \phi_{ijt}(H) \) denoting the contribution of the variable \( j \) of the system to the variance of the forecast error of variable \( i \) at the forecast horizon \( H \). Given that own- and cross-variable variance contributions do not necessarily add up to one, the effect attributable to each variable is standardized as:

\[
\tilde{\phi}_{ij,t}(H) = \frac{\phi_{ij,t}(H)}{\sum_{j=1}^{N} \phi_{ij,t}(H)},
\]

(4)

where \( \sum_{j=1}^{N} \phi_{ij,t} = 1 \) and \( \sum_{i,j=1}^{N} \phi_{ij,t} = N \).

Based upon the GFEVD multiple connectedness measures can be derived:

\[
TO_{jt} = \sum_{j=1,i\neq j}^{N} \tilde{\phi}_{ij,t}(H)
\]

(5)

\[
FROM_{jt} = \sum_{i=1,i\neq j}^{N} \tilde{\phi}_{ij,t}(H)
\]

(6)

\[
NET_{jt} = TO_{jt} - FROM_{jt}
\]

(7)

\[
TCI_t = N^{-1} \sum_{j=1}^{N} TO_{jt} \equiv N^{-1} \sum_{j=1}^{N} FROM_{jt}.
\]

(8)

\[
NPDC_{ij,t} = \tilde{\phi}_{ij,t}(H) - \tilde{\phi}_{ji,t}(H)
\]

(9)

As mentioned previously \( \tilde{\phi}_{ij,t}(H) \) illustrates the impact a shock in variable \( j \) has on variable \( i \). Hence, Equation (5) represents the aggregated impact a shock in variable \( j \) has on all other variables which is defined as the total directional connectedness to others whereas Equation (6) illustrates the aggregated influence all other variables have on variable \( j \) that is defined as the total directional connectedness from others. Thus, Equation (7) represents the net total directional connectedness which provides us with information whether a variable is a net transmitter or a net receiver of shocks. If variable \( j \) is a net transmitter (receiver) of shocks – and hence driving (driven by) the network – it means that the impact of variable \( j \) on others is larger (smaller) than the influence all others have on variable \( j \), \( NET_{jt} > 0 \) (\( NET_{jt} < 0 \).
Equation (8) demonstrates the total connectedness index which equals the average impact one variable has on all others or all others have on one. If this measure is relatively high (low) it indicates that the market risk is high (low). Finally, Equation (9) provides information on the bilateral level - between variable \( j \) and \( i \). The so-called net pairwise directional connectedness \((NPDC_{ij,t})\) exhibits whether variable \( j \) is driving variable \( i \) or vice versa. If \( NPDC_{ij,t} > 0 \) \((NPDC_{ij,t} < 0)\), variable \( j \) is dominating (dominated by) variable \( i \).

### 3 Empirical results

Spillovers estimated over the whole sample are displayed in Table 2. The \( ij \)th entry is the estimated contribution to the forecast error variance of variable \( i \) originating in shocks to variable \( j \). The diagonal elements \((i = j)\) measure spillovers from past shocks to the same variable, while the off-diagonal elements \((i \neq j)\) capture spillovers from shocks to other variables. The row sums excluding the main diagonal elements (labelled ”From others”) report the total spillovers to the variable in the row, whereas the column sums (labelled ”To others”) report the total spillovers from the variable in the column. The difference between each variable’s (off-diagonal) column sum and the same variable’s row sum is the net spillovers of the respective variable to all other variables. The TCI measures the contribution of spillovers between variables to the total forecast error variance of the system.

![Insert Table 2 around here]

Spillovers account for a substantial 33.6% of the forecast error variance of the system, with the main source being the common factor of housing returns and the main receivers being sentiment and GDP. The common factor of housing returns strongly affect sentiment, while reverse spillovers are relatively muted.

Shocks to sentiment explains only 6.2% of the forecast error variance of the common factor of housing returns, while shocks to the latter explain 15.0% of the forecast error variance of sentiment. Similarly, shocks to housing returns affect GDP much more than the reverse. Spillovers between housing sentiment and the common factor of housing volatility are relatively important in both directions.

The TCI index is relatively stable over the sample period, although it increases somewhat in the wake of the 2008 GFC. The directions of the net spillovers remain mostly unchanged over the whole sample period (Figure 1). The most notable evolution is the lasting increase in
net spillovers from the common factor of housing returns to housing sentiment after the GFC. Net spillovers to housing sentiment, and to a lesser extent GDP, increase markedly after the GFC.

[Insert Figure 1 around here]

This development is mainly driven by the larger pairwise spillovers from the common factor of housing returns to both housing sentiment and GDP after the GFC (Figure 2).

[Insert Figure 2 around here]

4 Conclusion

In this paper, we have used a TVP-VAR version of the the Diebold and Yılmaz (2012, 2014) methodology to measure spillovers between the housing sentiment index of Bork et al. (2020), the common factors of US real housing returns and their volatility (derived from a TVP-DFM-SV), GDP growth and real interest rates. Two main conclusions emerge from the analysis. First, spillovers predominantly run from from the common factor of housing returns to housing sentiment and GDP. Second, spillovers from the common factor of housing returns increase durably after the GFC. This suggests that, while housing prices now have a greater economic influence on the economy than prior to the GFC, an exogenous shock to housing sentiment, may in itself have little impact on housing returns and volatility.
References


Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Real GDP Growth</th>
<th>Real Housing Returns SV Common Factor</th>
<th>Real Housing Returns Common Factor</th>
<th>Real Interest Rate</th>
<th>Housing Sentiment Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.696</td>
<td>1.104</td>
<td>0.029</td>
<td>1.247</td>
<td>0.000</td>
</tr>
<tr>
<td>Variance</td>
<td>0.564</td>
<td>8.389</td>
<td>11.229</td>
<td>0.006</td>
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</tr>
<tr>
<td>Skewness</td>
<td>-0.399***</td>
<td>-0.746***</td>
<td>-0.176</td>
<td>0.271</td>
<td></td>
</tr>
<tr>
<td>(0.033)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.102)</td>
<td>(0.511)</td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.283***</td>
<td>0.897***</td>
<td>-0.176</td>
<td>0.271</td>
<td></td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.038)</td>
<td>(0.783)</td>
<td>(0.349)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JB</td>
<td>80.368***</td>
<td>15.676***</td>
<td>2.896</td>
<td>0.919</td>
<td></td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.038)</td>
<td>(0.235)</td>
<td>(0.632)</td>
<td></td>
<td></td>
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<tr>
<td>ERS</td>
<td>-4.057***</td>
<td>-2.677***</td>
<td>-1.70***</td>
<td>-2.776***</td>
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</tr>
<tr>
<td>(0.000)</td>
<td>(0.091)</td>
<td>(0.091)</td>
<td>(0.006)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q(12)</td>
<td>37.282***</td>
<td>490.166***</td>
<td>668.045***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q²(12)</td>
<td>10.429*</td>
<td>228.903***</td>
<td>466.201***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.098)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Table 2: Averaged Connectedness Table

<table>
<thead>
<tr>
<th></th>
<th>Real GDP Growth</th>
<th>Real Housing Returns SV Common Factor</th>
<th>Real Housing Returns Common Factor</th>
<th>Real Interest Rate</th>
<th>Housing Sentiment Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real GDP Growth</td>
<td>76.3</td>
<td>6.8</td>
<td>10.4</td>
<td>0.5</td>
<td>6.0</td>
</tr>
<tr>
<td>Real Housing Returns SV</td>
<td>3.9</td>
<td>53.4</td>
<td>12.5</td>
<td>9.7</td>
<td>20.6</td>
</tr>
<tr>
<td>Real Housing Returns</td>
<td>4.8</td>
<td>12.0</td>
<td>71.6</td>
<td>5.4</td>
<td>6.2</td>
</tr>
<tr>
<td>Real Interest Rate</td>
<td>0.2</td>
<td>11.2</td>
<td>6.9</td>
<td>78.7</td>
<td>3.0</td>
</tr>
<tr>
<td>Housing Sentiment Index</td>
<td>6.7</td>
<td>23.5</td>
<td>15.0</td>
<td>2.8</td>
<td>52.0</td>
</tr>
<tr>
<td>Contribution TO others</td>
<td>15.6</td>
<td>53.5</td>
<td>44.8</td>
<td>18.4</td>
<td>35.9</td>
</tr>
<tr>
<td>NET directional connectedness</td>
<td>-8.1</td>
<td>6.8</td>
<td>16.4</td>
<td>-3.0</td>
<td>-12.1</td>
</tr>
</tbody>
</table>

Notes: Results are based on a TVP-VAR model with lag length of order one (BIC) and a 12-step-ahead generalized forecast error variance decomposition.
Figure 1: Dynamic Total Connectedness & Net Total Directional Connectedness

Figure 2: Net Pairwise Directional Connectedness
A The Generalized Dynamic Factor Model

In this section we present a generalized dynamic factor model (DFM) that is employed to decompose the real housing returns in all states into a common (or national) factor and an idiosyncratic (or state-specific) factor. The DFM is often used to tease out the common movements among multiple time series, and has become a standard tool since the work by Stock and Watson (1989). We generalize the standard DFM with constant parameters to one that allows for time-varying loading parameters and the stochastic volatility (TVP-DFM-SV). As such, the generalized TVP-DFM-SV captures important time-varying comovements among multiple time series. Formally, our model specification closely follows Del Negro and Otrok (2008), and is specified as follows:

\[ r_{i,t} = \beta_{i,t} \cdot f_t + e_{i,t} \]  \hspace{1cm} (10)

Here, \( r_{i,t} \) is the first-difference of the natural log of the real house price for state \( i \) at time \( t \). \( f_t \) is the national factor that affects all house prices at time \( t \), and \( \beta_{i,t} \) is the time-varying loading parameter of this national factor. \( e_{i,t} \) is the idiosyncratic factor.

The common factor and the idiosyncratic factors are assumed to be independent from each other. Therefore, the variance decomposition of our model is given by:

\[ Var(r_{i,t}) = \beta_{i,t}^2 \cdot Var(f_t) + Var(e_{i,t}) \]  \hspace{1cm} (11)

Note that either the time-varying loading parameters or the stochastic volatility of the factors enables the factors contributions to the total variations of each variable to vary over time.

Following the standard practice in this literature, we model the common factor \( f_t \) using a stationary AR(\( p \)) process:

\[ f_t = \phi_1 f_{t-1} + \phi_2 f_{t-2} + \ldots + \phi_p f_{t-p} + \exp(h_f^t) \cdot \varepsilon_f^t \] \hspace{1cm} (12)

where \( \varepsilon_f^t \sim i.i.d.N(0,\sigma_f^2) \). Therefore, the shock to the factor has a stochastic volatility, and its time-varying volatility is governed by \( \exp(h_f^t) \).

To keep the model parsimonious, we employ a driftless random walk process to capture the time variation of the volatility:

\[ h_f^t = h_f^{t-1} + \sigma_h^f \cdot \xi_f^t, \quad \xi_f^t \sim i.i.d.N(0,1) \] \hspace{1cm} (13)

The factor loading \( \beta_{i,t} \) varies over time, and is also assumed to follow a random walk process:

\[ \beta_{i,t} = \beta_{i,t-1} + \sigma_\beta \cdot \eta_{i,t}; \quad \eta_{i,t} \sim i.i.d.N(0,1) \] \hspace{1cm} (14)
Here shocks to the loading parameters in different series are assumed to be orthogonal to each other.  

The idiosyncratic factor follows a stationary AR($q$) process:

\[ e_{i,t} = \phi_{i,1}e_{i,t-1} + \phi_{i,2}e_{i,t-2} + \cdots + \phi_{i,q}e_{i,t-q} + \exp(h_{i,t}) \cdot \varepsilon_{i,t} \]  

where \( \varepsilon_{i,t} \sim i.i.d.N(0,\sigma_i^2) \). The stochastic volatility of the idiosyncratic factor follows a random walk process:

\[ h_{i,t} = h_{i,t-1} + \sigma_h^i \cdot \xi_{i,t}, \quad \xi_{i,t} \sim i.i.d.N(0,1) \]  

Here we assume that the shocks to the stochastic volatility in different factors are independent from each other. This assumption simplifies the estimation algorithm.

As usual, some normalizations of the factor rotations are needed before the model can be identified and estimated. The loading parameters and the variance of the shock to the common factor are not separately identifiable. We choose to set \( \sigma_f^2 = 1 \) to achieve the identification. Following Del Negro and Otrok (2008) we also restrict that the time-varying volatility all start from zero for the same identification purpose. We demean each series before the estimation since the means of factors are not separately identifiable. Finally, following works such as Neely and Rapach (2011) and Bhatt et al. (2017), we set \( p = q = 2 \) to keep the model parsimonious.

We estimate this TVP-DFM-SV model using the Monte Carlo Markov Chain (MCMC) Bayesian estimation method. Specifically, we employ the well-established Gibbs-Sampling algorithm by breaking the model into several blocks and sampling sequentially from posterior conditional densities. The idea of the Gibbs-Sampling algorithm is that when the algorithm converges after the initial burn-in draws, these random draws from the conditional densities altogether constitute a good approximation of the underlying joint densities. Applying the law of large numbers, the numerical integration can be easily taken to obtain the marginal densities of the parameters and the state variables of our interest. Most blocks in the model are linear and Gaussian, and as a result the standard algorithms in Kim and Nelson (1999) are readily applicable. The stochastic volatility introduces a non-Gaussian feature into the model. We apply the procedure proposed in Kim et al. (1998) that utilizes a mixture of normal densities to approximate the underlying non-Gaussian distribution in order to simulate the stochastic volatility. This procedure has been widely used in the literature, see e.g., Stock and Watson (2007) and Primiceri (2005). For further details on the Gibbs-Sampling estimation algorithm, the reader is referred to Gupta et al. (2020b).