University of Pretoria
Department of Economics Working Paper Series

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Working Paper: 2020-67
July 2020

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Optimal Social Distancing in SIR based Macroeconomic Models

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July 23, 2020

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Abstract

The paper introduces voluntary social distancing to the canonical epidemiology model, integrated into a conventional macroeconomic model. The model is extended to include treatment, vaccination, and government-enforced lockdown. Infection-averse individuals face a trade-off between a costly social distancing and the risk of getting infected and losing next-period labor income. We find an individual's social distancing is proportional to the welfare loss she incurs when moving to the infected compartment. It increases in the individual's psychological discount factor but decreases in the probability of receiving a vaccination. Quantitatively, a laissez-faire social distancing flattens the infection curve that minimizes the economic damage of the epidemic. A government-enforced social distancing is more effective in flattening the infection curve but has a detrimental effect on the economy.

Key words:
COVID-19, lockdown, social distancing, macroeconomics, epidemics
JEL Classification:

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1. Introduction

At the time this paper is written, more than 15.4 million individuals are infected by COVID-19 worldwide and more than 631 thousand died while the spread of the epidemic shows no sign of slowing down.[1] However, many countries have already relaxed the strict lockdown measures that they implemented at the early stage of the epidemic to ease the pressure on the economy.[2] Controlling the spread of the infection has thus been mainly left for choices made at individual levels. Unfortunately, the canonical epidemiology models that are often adopted to track the spread of the COVID-19 epidemic do not have the necessary tool to account for individual behaviors despite some of the variables in these models largely depend on how individuals behave in the presence of the epidemic.

The present paper aims to contribute to fill this gap. It complements the recent macroeconomic literature that gives microfoundations to the canonical epidemiology models used to track the spreads of COVID-19 and assesses the spread of the outbreak and its macroeconomic impact. The paper in particular develops a SIR (Susceptible-Infected-Recovered) macroeconomic model where individuals face a trade-off between practicing a costly social distancing and increasing the risk of getting infected and losing next-period labor income. Their optimal decisions eventually determine the dynamics of the epidemic, aggregate income, and welfare. The model is further ex-

[1] COVID-19 was first reported in Wuhan, China, in December 2019.

[2] When the South Africa government had relaxed a five-week-long strict lockdown measure (including a ban on jogging, cycling, and dog-walking) in the 1st of May 2020, the number of confirmed COVID-19 cases in the country was less than six thousand, after ten weeks the country has breached the 300 thousand mark of COVID-19 cases.

tended to include treatment, vaccination, and government-enforced social distancing.

The basic model considers an economy that faces an epidemic where individuals are categorized into three compartments – Susceptible, Infected and Recovered; hence the name SIR. Initially, there are two types of individuals – susceptible and infected – while in the following periods some of the infected persons get recovered. Similar to conventional economic dynamic models, agents derive utility from consumption and leisure. In contrast, they derive utility from social closeness such as hugging, kissing, and shaking hands of their loved ones although this could expose susceptible individuals to the virus. Infected individuals work less time, due to sickness, and hence lose labor income. They could also die from the infection. However, if they get recovered they resume a normal life. Infected and recovered individuals practice the minimum social distancing, which is zero. While the latter develop immunity, the former have nothing to lose.

We thus model social distancing as costly to individuals but it does not involve consumption goods or time, which is in sharp contrast to the recent literature on macroeconomics and epidemics (e.g., Eichenbaum et al., 2020, Krueger et al., 2020). In particular, we incorporate social distancing into an otherwise standard utility function where individuals derive utility from consumption and leisure. In this context, individuals could optimally decide on social distancing along with the labor-leisure trade-off. Our rationale for providing microfoundations to SIR models does not rest upon individuals' consumption, work, or leisure activities. Instead, we assume susceptible individuals are infection-averse when it comes to social distancing. Accordingly, two different individuals may choose a similar bundle of consumption goods or leisure time but may experience different social distancing.

We modify the basic SIR model with laissez-faire social distancing to include treatment and vaccination (could be any other similar controlling mechanisms). In the SIVTR model (V and T stand for Vaccination and Treatment), individuals are categorized into five compartments every period: Susceptible, Infected, Vaccinated, Treated, and Recovered. Treatment is believed to decrease the infectivity of the epidemic as it often involves the identification and quarantining of infected individuals and increases the recovery rate of infected individuals. Vaccination or other controlling practices such as wearing masks, education, or washing hands, could significantly reduce susceptibility to infection as it reduces the number of susceptible individuals. We also extend the original SIR model to accommodate a government-enforced social distancing.

In the SLIR (Susceptible-Lockdown-Infected-Recovered) model, a fraction of susceptible individuals will leave the susceptible compartment starting from the initial period of the epidemic, which leads to a staggering job loss in the economy. We think of the latter as those individuals who work in industries such as hotels and tourism whose employment status is severely affected by the lockdown measure. The government may subsidize the resulting unemployment through lump-sum taxes, levied on the general population. But the government's revenue could quickly dwindle, as the infection soars and consequently people die, call in sick and their ability to pay taxes decreases. To cope up with the pressure of a declining economy, the government is allowed to relax the lockdown measure through time, which has been an empirical regularity. In such a scenario, with lump-sum taxes, we establish a second-best condition by equating the lifetime utility of individuals in lockdown to that of the recovered persons.

Among the findings, we show that a susceptible individual's current optimal social distancing is the difference between her value function (welfare) of having remained susceptible and being infected in the next period. It increases in her psychological discount factor but decreases at the probability of receiving a vaccination or her likelihood of developing immunity. Quantitatively, aggregate income, consumption, and welfare increase in laissez-faire social distancing but they decrease in government-enforced social distancing. The latter is more effective in flattening the curve but leads to a higher unemployment rate. If available, treatment and vaccination could affect aggregate welfare positively but in different ways. The availability of treatment has a positive influence on all individuals' welfare, including that of the susceptible individuals as it increases their likelihood of getting treatment (if they get infected) and hence getting recovered quickly. Whereas, the availability of vaccination pulls individuals out of the susceptible compartment from the outset and enables them to avoid costly social distancing.

Calibrating the model to the U.S. economy, a laissez-faire social distancing is found to have a strong impact on delaying and flattening the infection curve. It delays the peak period by about 20 more periods, flattens the curve at the peak by about 10 percentage pts, and decreases the death rate by more than 3 percentage pts from the baseline case of no social distancing. The decrease in the infection and fatality rates translates to a positive impact on the economy due to the boost in the labor supply. Aggregate income and consumption increase, which in turn leads to an increase in the aggregate welfare. During the early periods, when the infection and fatality rates are small, there is no much difference in the macroeconomic variables between practicing and not practicing laissez-faire social distancing. This would change quickly once the epidemic gains momentum, more people get infected and hence lose their labor income due to sickness and death. At the peak of the epidemic, there is a 20 percentage pts difference in aggregate income between practicing and not practicing social distancing and there is a permanent 5 percentage pts difference after herd immunity is achieved.

If treatment is available, it will have a relatively modest impact on the evolution of the outbreak. The infection curve flattens by about 1.84 percentage pts and the death rate decreases by 1.12 percentage pts, from the baseline case of no treatment is available. Although the impacts on aggregate income and consumption are relatively small, the impact on aggregate welfare could be quite important due to its positive impact on the lifetime welfare of all individuals, including susceptible individuals. However, if vaccination is available, it would have a much stronger influence on the dynamics of the epidemic and consequently on the economy. A 0.03% vaccination rate per period (similar to vaccinating 0.1 million susceptible individuals per week) cuts down the death rate by a 3 percentage pts and flattens the infection curve by about 4.5 percentage pts from the baseline case of no vaccination is available. Increasing the vaccination rate to 0.06 per period (similar to vaccinating 0.2 million susceptible individuals per week) could lead to herd immunity after only 2% of the population gets infected. Without vaccination, 73% of the population has to be infected to achieve herd immunity. With the 0.03% vaccination rate per period, aggregate income increases by 16 percentage pts, at the peak of the infection, from the baseline case of no vaccination is available.

The numerical simulation shows that, compared to a laissez-faire policy of do nothing, a government-enforced social distancing has a much stronger impact on the spread of the epidemic. With the latter, at the peak of the epidemic, only 0.74% of the susceptible individuals get infected and 2.43% of them die; however, with the laissez-faire social distancing, 11.43% of the susceptible individuals get infected and 17% of them die. The lockdown has a strong negative impact on the macroeconomy, due to losses in aggregate labor income, however. With our parametrization of the lockdown for the U.S. economy, aggregate income and welfare reduce by 46 and 75 percentage pts, respectively. When comparing between a more and a less strict lockdown, a more strict lockdown leads to a relatively higher welfare loss at the early stage of the epidemic; however, during and after the peak of the infection, it leads to a relatively lower welfare loss as some of the welfare loss are offset by the live-savings effects of the lockdown.

The work contributes to the very recent debate in the macroeconomic impact of the epidemic. In the last couple of months, many works have appeared that combine macroeconomic models with SIR models in response to the COVID-19 crisis. An incomplete list of these works includes Acemoglu et al. (2020), Alvarez et al. (2020), Atkeson (2020), Bodenstein et al. (2020), Chang and Velasco (2020), Eichenbaum et al. (2020), Farboodi et al. (2020), Garibaldi et al. (2020), Glover et al. (2020), Greenstone and Nigam (2020), Jones et al. (2020), Krueger et al. (2020), Toxvaerd (2020) and Fernandez-Vallaverde and Jones (2020).

Atkeson (2020) provides an early summary of SIR models from the perspective of macroeconomics. Jones et al. (2020) compare a social planner's mitigating incentives with that of private agents. They argue the planner's mitigation policy (that encourages working from home) could be much more effective in reducing the death rate despite that results in a significant drop in consumption. Acemoglu et al. (2020) focus on optimal targeted lockdown policy in a multi-group SIR model. While infection reduces in a strict and long lockdown of the most vulnerable group (the oldest group), this also enables to impose a lesser lockdown in the lower-risk group (the young). Bodenstein et al. (2020) look into the impact of public health measures such as social distancing or lockdown on the death rate in a model that combines a multi-sectoral model with the SIR model. Glover et al. (2020) examine the distributional impact of optimal mitigation policy across different groups, categorized in terms of age, sector, and health status.[3]

The current work is more closely related to the work of Eichenbaum et al. (2020), Farboodi et al. (2020), Krueger et al. (2020), and Toxvaerd (2020). Eichenbaum et al. (2020) and Krueger et al. (2020) attach individuals' consumption and labor activities to the contact rate that increases their likelihood of getting infected. Thus, a consumption tax could be considered as a containment policy. We share with them in our modeling approach to the extent that we introduce the SIR model to conventional macroeconomic models through the contact rate. We share with

[3] There are also other many recent macroeconomic works in the COVID-19 that abstract from the SIR model (e.g., Baker et al., 2020 and Barrot et al., 2020). Baker et al. (2020), for instance, examine how household spending responds to the COVID-19. Barrot et al. (2020) look at the impact of the weeks' long lockdown in France's and other European countries' output while focusing on the sectoral effects.

Farboodi et al. (2020), Basu et al. (2020), and Toxvaerd (2020) that agents in these models derive utility from social activity. Similar to them we focus on individual behaviors towards optimal social distancing, in contrast, we approach the problem from a macroeconomic point of view.

We organize the next sections as follows. Section 2 introduces the basic SIR model to the household problem. Section 3 models treatment and vaccination. In section 4, we introduce and examine a government-enforced social distancing. We calibrate the models in Section 5. Section 6 provides the numerical results and Section 7 concludes.

2. The SIR Model

We suppose an economy that faces an epidemic. There are in general three types of individuals, namely susceptible, infected, and recovered individuals, as in the standard epidemiological SIR models. For the susceptible individuals, the probability to remain susceptible in the next period is $1-p_t$, where p_t is the probability of getting infected. For the infected individuals, the probability to remain infected in the next period is $1 - \gamma - \iota$, where γ and ι are the probability of recovering and dying, respectively.

As in the standard representative household models, agents derive utility from consumption and leisure. In contrast to these models, susceptible individuals dislike social distancing. They derive utility from social closeness (such as kissing, hugging friends & relatives and shaking hands) although it increases the likelihood of getting infected, and thence, being off work and losing some of their earnings, and risk of dying in the next period. Infected and recovered individuals practice the minimum amount of social distancing, which is zero. While the latter develop immunity, the former has nothing to lose.

2.1. Household

At time $t = 0$ there are two types of individuals – susceptible and infected individuals. In the following periods, some of the infected individuals get recovered. Recovered individuals develop immunity to the virus and resume normal life. Denote infected individuals as 0, susceptible individuals as 1 and recovered individuals as 2. The problem for the infected and susceptible individuals can be represented as two state process $(\pi_{i,t})$, where i takes 0 or 1. At time t, an infected person is represented, by $\pi_{1,t} = 1$, and a non-infected person, by $\pi_{0,t} = 0$. The utility of the ith person is then given by:

$$U\left(C_{i,t}, L_{i,t}, \chi_{i,t}\right) = \frac{C_{i,t}^{1-\sigma} - 1}{1-\sigma} - \frac{L_{i,t}^{1+\theta}}{1+\theta} - \pi_{i,t}\frac{\chi_{i,t}^{2}}{2} \tag{1}$$

where $U_c' > 0$, $U_c'' < 0$, $U_l' < 0$, $U_l'' < 0$, $U_\chi' < 0$ and $U_\chi'' < 0$.

The budget constraint is given by:

$$C_{i,t} = w_t\left(\left(1 - \pi_{i,t}\right)\left(L_{0,t} - L_s\right) + \pi_{i,t}L_{1,t}\right) - M_t \tag{2}$$

where $C_{i,t}$ and $L_{i,t}$ are the ith individual consumption and leisure; w_t and M_t are the wage rate and lump-sum tax respectively. $\chi_{i,t}$ represents social distancing by a susceptible individual and L_s is work time lost due to sickness absence by an infected person. From (1), infected individuals $(\pi_{0,t} = 0)$ do not practice social distancing, and from (2), they do not work full time, $L_s \neq 0$.

The utility function for recovered individuals is given by,

$$U\left(C_{2,t}, L_{2,t}\right) = \frac{C_{2,t}^{1-\sigma} - 1}{1-\sigma} - \frac{L_{2,t}^{1+\theta}}{1+\theta} \tag{3}$$

subject to the budget constraint:

$$C_{2,t} = w_t L_{2,t} - M_t \tag{4}$$

The individual's consumption is simply her wage income minus lump-sum tax.

2.2. SIR

The *transmission risk* p_t is the probability of a susceptible individual encountering an infected individual and thence getting infected. We suppose it decreases in the individual's level of social distancing $\chi_{1,t}$ and has the following simple form:

$$p_t = 1 - a\chi_{1,t} \tag{5}$$

where $p_t \in [0, 1]$ and $a \in (0, 1]$ is a parameter.

In the typical SIR model, the total number of individuals infected at time t is the number of encounters between infected ($N_{0,t}$) and susceptible ($N_{1,t}$) individuals times the contact rate (β).

$$\beta N_{0,t} N_{1,t}$$

Our strategy of providing microfoundations to the SIR model is modifying the contact rate to account for a voluntary social distancing as follows:

$$\beta_t N_{0,t} N_{1,t} \tag{6}$$

where $\beta_t \equiv \beta p_t$ is the *effective* contact rate. The probability of recovering of an infected individual is γ and the total number of recovered individuals at time t is a fraction of infected people, $\gamma N_{0,t}$.[4]

In the SIR model, we have the following relations:

$$N_{1,t+1} = N_{1,t} - \beta_t N_{0,t} N_{1,t} \tag{7}$$

$$N_{0,t+1} = N_{0,t} + \beta_t N_{0,t} N_{1,t} - (\gamma + \iota) N_{0,t} \tag{8}$$

$$N_{2,t+1} = N_{2,t} + \gamma N_{0,t} \tag{9}$$

where $N_{1,t}$, $N_{0,t}$ and $N_{2,t}$ represent the number of susceptible, infected and recovered individuals in the economy, respectively. Eqs. (7)-(9) show the dynamics of susceptible, infected and recovered individuals. From (7)-(8), we see every period $\beta_t N_{0,t} N_{1,t}$ number of individuals leaves the susceptible compartment and joins the infected compartment in the next period. Similarly, from (8)-(9), $(\gamma + \iota) N_{0,t}$ number of individuals leaves the infected compartment, out of which $\gamma N_{0,t}$ number of individuals joins the recovered compartment and $\iota N_{0,t}$ number of individuals dies every period.

$\gamma + \iota$ is the removal rate and $1/(\gamma + \iota)$ is the mean periods that an infected

[4] Note that if we do not account for voluntary social distancing, $\chi_{1,t} = 0$, then $\beta_t = \beta$. We make the assumption, β and γ are constant, for simplicity.

individual remains in the infected compartment, leading to the basic reproduction number in the SIR model:

$$\mathbf{R}_0 = \frac{\beta_0}{\gamma + \iota}$$

An infected individual should at least transmit to more than one individual ($\mathbf{R}_0 > 1$) for the infection to have a first phase of an upward dynamics.

The size of the population at time t (N_t) is the total number of susceptible, infected and recovered individuals. At time $t+1$, this is equal to the population size at t net of infected individuals who died from the infection.

$$N_t = N_{1,t} + N_{0,t} + N_{2,t} \tag{10}$$

$$N_{t+1} = N_t - \iota N_{0,t} \tag{11}$$

$$D_{t+1} = D_t + \iota D_{0,t} \tag{12}$$

The last equation captures the dynamics for the death rate where D_t is the number of dead people at time t. We set initial population to be one ($N_0 = 1$) and assume zero population growth rate and zero *natural* death rate, with no loss of generality.

2.3. The Households' Problem

The lifetime problem of the agent who is susceptible at time t, recursively, is

$$V_1 = \max_{\{C_{1,t}, L_{1,t}, \chi_{1,t}\}} U\left(C_{1,t}, L_{1,t}, \chi_{1,t}\right) + \left(1 - p_t\right) \rho V_1' + p_t \rho V_0' \tag{13}$$

and that of the person who is infected at time t, is

$$V_0 = \max_{C_{0,t}, L_{0,t}} U\left(C_{0,t}, L_{0,t}\right) + \left(1 - \gamma - \iota\right) \rho V_0' + \gamma \rho V_2' \tag{14}$$

subject to (1) and (2). ρ is the discount rate; and, "$'$" indicates the next period value function.

Similarly, the problem of a recovered individual is to maximize

$$V_2 = \max_{C_{2,t}, L_{2,t}} U\left(C_{2,t}, L_{2,t}\right) + \rho V_2' \tag{15}$$

subject to her utility function and budget constraint. Implicit in (15), recovered individuals develop immunity to the infection and resume normal life.

2.4. Solution to the Household Problem

From the first order conditions of the susceptible individual and the budget constraint,

$$w_t C_{1,t}^{-\sigma} = L_{1,t}^{\theta} \tag{16}$$

$$C_{1,t} = w_t L_{1,t} - M_t \tag{17}$$

$$\chi_{1,t} = \rho a \left(V_1' - V_0'\right) \tag{18}$$

The first is the trade-off between the individual's consumption and leisure and the second is her budget constraint. The last equation captures the individual's optimal social distancing, which depends on the individual's discount rate ■, and the welfare loss she incurs when moving from the susceptible to the infected compartment.

Proposition 1. *(i) A susceptible individual's optimal social distancing is proportional to the welfare loss she incurs if she moves from the susceptible to the infected compartment. (ii) It increases in the individual's discount factor.*

The solution for the infected individual is,

$$w_t C_{0,t}^{-\sigma} = L_{0,t}^{\theta} \tag{19}$$

$$C_{0,t} = w_t \left(L_{0,t} - L_s\right) - M_t \tag{20}$$

and the solution for the recovered individual is

$$w_t C_{2,t}^{-\sigma} = L_{2,t}^{\theta} \tag{21}$$

$$C_{2,t} = w_t L_{2,t} - M_t \tag{22}$$

(19) and (21) show the labor-leisure trade-off for the infected and recovered individuals while (20) and (22) show their respective budget constraints. Infected individuals have the lowest individual consumption (20) due to time lost from sickness absentees.

2.5. Aggregate output and labour

Aggregate output is produced using aggregate labor:

$$Y_t = AL_t \tag{23}$$

where A is total factor productivity (TFP). Aggregate labor at time t is given by,

$$L_t = N_{0,t}\left(L_{0,t} - L_s\right) + N_{1,t}L_{1,t} + N_{2,t}L_{2,t}$$

which is the sum of labor supply by the infected, susceptible and recovered individuals in the economy at time t.

3. Treatment and Vaccination

We modify the SIR model to include treatment and vaccination (or any other controlling mechanisms that help to reduce the spread of the epidemic by removing some individuals from the susceptible compartment). When treatment is available, the infectivity of the epidemic is believed to decrease and the recovery rate to rise. The reduction in infectivity could happen as treatment often requires certain identification and quarantining of infected individuals. Vaccination or any other controlling practices such as wearing masks, education, or washing hands, could significantly reduce susceptibility to infection as it reduces the number of susceptible individuals.

3.1. SIVTR

In the SIVTR model, individuals can be categorized into five compartments: Susceptible, Infected, Vaccinated, Treated, and Recovered. We model treatment by letting ω number of infected individuals to receive treatment every period that decreases the infectivity by ε rate. We suppose ψ number of treated individuals leave the treatment room (or recover) and $1/\psi > 1/\gamma$, that is, the recovery period of individuals receiving treatment is shorter than that of individuals who do not receive treatment. We model vaccination letting υ fraction of susceptible individuals to be vaccinated every period. For simplicity, we assume that the vaccination or the control measure implemented will completely eliminate susceptibility to infection.

Then, following the approach of Feng et al. (2011), the SIVTR model could have the following form:

$$N_{1,t+1} = N_{1,t} - \beta_t \left(N_{0,t} + \varepsilon N_{T,t}\right) N_{1,t} - \upsilon N_{V,t} \tag{24}$$

$$N_{0,t+1} = N_{0,t} + (1-\omega)\, \beta_t (N_{0,t} + \varepsilon N_{T,t}) N_{1,t} - (\gamma + \iota)\, N_{0,t} \tag{25}$$

$$N_{V,t+1} = N_{V,t} + \upsilon N_{V,t} \tag{26}$$

$$N_{T,t+1} = N_{T,t} + \omega \beta_t (N_{0,t} + \varepsilon N_{T,t}) N_{1,t} - \psi N_{T,t} \tag{27}$$

$$N_{2,t+1} = N_{2,t} + \gamma N_{0,t} + \psi N_{T,t} \tag{28}$$

where $N_{T,t}$ and $N_{V,t}$ denote the number of individuals treated and vaccinated at period t respectively.

Eqs. (24)-(28) show the dynamics for susceptible, infected, vaccinated, treated and recovered individuals. From (24), every period, $\beta_t \left(N_{0,t} + \varepsilon N_{T,t}\right) N_{1,t}$ individuals leave the susceptible compartment and $1-\omega$ of these individuals join the infected compartment (25) while the rest join the treatment compartment (27). The term $\varepsilon \beta p_t N_{T,t} N_{1,t}$ captures the encounter of susceptible and treated individuals, which decreases infectivity by $\varepsilon \in (0,1)$ rate. From the treatment compartment, $\psi N_{T,t}$ individuals leave the treatment room every period and join the recovered compartment (28).

As shown in (24), $\upsilon N_{V,t}$ number of susceptible individuals leave the susceptible compartment every period and enter the vaccinated compartment (26). Eq. (28) presents the dynamics for the recovered individuals, those who leave the infection and treatment compartments.

The basic reproduction number for the SIVTR model is

$$\mathbf{R}_0 = (1 - v)\, \beta_0 \left(\frac{1 - \omega}{\gamma + \iota} + \frac{\omega \varepsilon}{\psi} \right) \tag{29}$$

The first term in the big bracket is the average number of periods that an infected individual spends in the infected compartment; the second is the fraction of infected individuals who receive treatment. $1/\psi$ is the average time an infected individual stays in the treatment compartment and it decreases by ε rate.

With the availability of vaccination, the lifetime utility of susceptible individuals would change. A susceptible individual receives vaccination with probability v and with the assumption that vaccination will eliminate susceptibility to infection, the lifetime utility of the person changes as follows:

$$V_1 = \max_{\{C_{1,t}, L_{1,t}, \chi_{1,t}\}} U\left(C_{1,t}, L_{1,t}, \chi_{1,t}\right) + (1 - v)\left[(1 - p_t)\, \rho V_1' + p_t \rho V_0'\right] + v \rho V_2' \tag{30}$$

Her optimal social distancing considers her likelihood of receiving vaccination and is summarized in the following Proposition:

Proposition 2. *A susceptible individual optimal social distancing,*

$$\chi_{1,t} = (1 - v)\, \rho a \left(V_1' - V_0'\right) \tag{31}$$

will reduce at the rate of the availability of a vaccine, v.

The value function for a vaccinated individual is similar to that of a recovered individual as both develop immunity and hence practice the minimum social distancing, which is zero. There is no change to the lifetime utility of infected and recovered individuals. With the availability of treatment, the lifetime utility of infected individuals would change though:

$$V_0 = \max_{C_{0,t}, L_{0,t}} U\left(C_{0,t}, L_{0,t}\right) + \rho\left[\left(1 - \gamma - \omega - \iota\right) V_0' + \gamma V_2' + \omega V_T'\right] \tag{32}$$

Infected individuals get a treatment with a probability of ω, get recovered with a probability of γ, remain sick and do not receive treatment, or die from the infection with a probability of ι, in the next period.

The lifetime utility of treated individual is

$$V_T = \max_{C_{T,t}, L_{T,t}} U\left(C_{T,t}, L_{T,t}\right) + \rho\left[\left(1 - \psi - \iota\right) V_T' + \psi V_2'\right] \tag{33}$$

For simplicity, we suppose there is no difference between an infected and treated individual in terms of labor supply and death rate. The only difference between the two is that individuals in the treatment compartment have a relatively higher recovery rate.

Aggregate labor supply at time t changes from the SIR model as it includes now treated and vaccinated individuals:

$$L_t = N_{T,t}\left(L_{T,t} - L_s\right) + N_{T,t}\left(L_{0,t} - L_s\right) + N_{1,t}L_{1,t} + N_{2,t}L_{2,t} + N_{V,t}L_{V,t} \tag{34}$$

where $L_{T,t}$ and $L_{V,t}$ are labor supply by treated and vaccinated individuals. One may note that the number of working time is similar for an infected and treated individual. Also, there is no difference in terms of labor supply between a vaccinated and a recovered person.

4. Lockdown

In this section, we suppose a government lockdown during the epidemic period that makes α_t number of susceptible individuals ($\alpha_t N_{1,t}$) unemployed. One may think of these individuals as those who work in industries (such as hotels and tourism) that are severely affected by the lockdown. Given that the main purpose of a lockdown is to cut down the number of susceptible individuals, we consider those individuals who are out of work also to be out of the susceptible compartment.

The government subsidizes the resulting unemployment through lump-sum taxes,

$$N_t M = \alpha_t N_{1,t} C_L \tag{35}$$

where M and C_L denote the lump-sump tax and the consumption of the individual who is affected by the lockdown respectively. $N_t M$ is the aggregate tax revenue, which will be used to subsidize the consumption of $\alpha_t N_{1,t}$ unemployed individuals. Note that initially, at $t = 0$, almost everyone is susceptible thus $N_{1,t} \approx N_t$. But, later on, because more and more people die from the infection, the size of the total population N_t will decline, resulting in declining government revenue. To hold a balanced budget, the government needs to relax the lockdown at the rate that keeps individual consumption constant. Considering that, we have from (35):

$$\alpha_t = \alpha \frac{N_t}{N_{1,t}} \tag{36}$$

α is the initial lockdown rate when $N_0 \approx N_{1,0} = 1$. Substituting (36) into the above, we get the consumption of an individual who loses her labor income due to government-enforced social distancing: $C_L = \frac{M}{\alpha}$. Because the individual is neither susceptible nor employed, $L_u = \chi_u = 0$ and her utility function is given by

$$U(C_L) = \frac{C_L^{1-\sigma} - 1}{1 - \sigma} \tag{37}$$

Second best condition can be obtained by equating the lifetime utility of this individual to that of a recovered person:

$$V_2 = V_L = U(C_L) + \rho V_L' \tag{38}$$

where V_L is the lifetime utility of the individual who loses her job due to the lockdown. Combining (15) and (38),

$$U(C_L) + \rho V_L' = U(C_2, L_2) + \rho V_2' \tag{39}$$

which equates the lifetime utility of a recovered person to that of an unemployed person. The sufficient condition for (39) to be satisfied is

$$U(C_L) = U(C_2, L_2) \tag{40}$$

One easily solves the level of lump-sum tax M associated to a given α, after substituting (3), (4) and (37) into (40):

$$M^* = \alpha \left((1-\sigma) \left(\frac{C_2^{1-\sigma} - 1}{1-\sigma} - \frac{L_2^{1+\theta}}{1+\theta} \right) + 1 \right)^{\frac{1}{1-\sigma}} \tag{41}$$

where M^* is the optimal lump-sum tax that each working individual pays, and C_2 and L_2 are given by (21) and (22) respectively.

5. Calibration

We calibrate the baseline model for the COVID-19 and the U.S. economy. A period is a week as in Eichenbaum et al. (2020). We let $\theta = 1$; estimates for the Frisch elasticity of labor supply θ often range between 0.5 and 2. We set $\sigma = 2$ for the curvature of the utility function and $\rho = 0.96\hat{}(1/52)$ for the weekly discount rate. We compute $A = 24$, using a $\$50,000$ per year income target and 40 weekly work hours. $L_s = 0.1$ that implies a 10% less consumption for individuals who do not work full time, a regularity in the incomplete market literature. For the SIR model, we assume $M = 0$.

Values for the COVID-19 parameters are largely varied between estimates and quickly change. We mainly rely on data from the Center for Disease Control and Prevention (CDC).[5] The national U.S. infection fatality rate among people infected with the COVID-19 is about 1.3%. We suppose a 18 days recovery time for infected individuals that implies $\frac{1}{\gamma+\iota} = 18/7$ removal weeks in our model. This gives $\gamma =$

[5] https://www.cdc.gov/coronavirus/2019-ncov/hcp/COVIDSurge.html

0.32. An average initial reproduction number of $\mathbf{R}_0 = 2.2$ implies a contact rate of $\beta = 0.86$. Initial population size is $P = 330 \times 10^6$ which is standardized to one ($N_0 = 1$). We start with 50 infected individuals, $N_{0,0} = 50/P$ and zero recovered and death rate, $D_0 = N_{2,0} = 0$.

For the SIVTR model, we set ω and ε at 5.5%, which is the total percentage of all COVID-19 cases that are hospitalized.[6] The average length of hospital stay ranges from 8 days (with no ICU), 10 days (with ICU and without ventilators) to 16 days (with ICU and ventilators), which is about 11 days or 1.57 weeks on average. This implies $\psi = 1 \div 1.57 = 63\%$. Apparently, there is no value for vaccination ν thus we start with some small number such as $\nu = 1/3300$, which is equivalent to vaccinating 100 thousand people weekly, and then experiment on the level of a vaccination rate that is required to achieve herd immunity at a very small infection rate.

We calibrate α, the fraction of initial susceptible individuals that leave the susceptible compartment due to government-enforced social distancing, based on the resulting unemployment. We then calibrate the lump-sum taxes M corresponding to these values from (41). The U.S. went on lockdown in March 2020 to prevent the further spread of the epidemic. According to the U.S. Bureau of Labor Statistics, the number of Americans drawing unemployment benefit at the end of May 2020 was 20.9 million people. This is equivalent to 0.5% of susceptible individuals leaving the susceptible compartment per period, in our model.[7] We experiment between $\alpha = 0.001$ and 0.005. The table below lists the full calibrated values.

[6] https://www.cdc.gov/coronavirus/2019-ncov/hcp/COVIDSurge.html

[7] Dividing 21 million by the total number of susceptible individuals, 330 million, gives 0.06 and dividing that by 12 gives about 0.005.

Table 1: Baseline values

Preference	$\sigma = 2$; $\theta = 1$; $\rho = 0.96\hat{}(1/52)$; $a = 1$
Technology and policy	$A = 24$; $L_s = 0.1$; $M = 0$
SIR	$\beta = .86$; $\gamma = 0.32$; $\iota = 0.013$
SIVTR	$\psi = 0.63$; $\varepsilon = \omega = 5.5\%$; $\nu = 1/3300$
Lockdown	$\alpha = 0.001$, $\alpha = 0.005$
Baseline population	$P = 330 \times 10^6$; $N_0 = 1$; $N_{V,0} = N_{T,0} = N_{2,0} = D_0 = 0$
	$N_{0,0} = 50/P$; $N_{1,0} = 1 - N_{0,0}$

6. Results and Discussion

6.1. Baseline SIR

We start by examining the epidemiological SIR model (Figure 1). Figure 1a depicts the dynamics of susceptible, infected, and recovered individuals. Figure 1b is similar to Figure 1a except that it includes the population and death dynamics. Initially, almost all individuals are susceptible.[8] It takes a while for the epidemic to build momentum as shown in the curve for the susceptible individuals, which is

[8] Only 50 individuals out of 330 million are infected at $t = 0$.

almost flat for the first thirty periods. During these periods, the number of infected and recovered individuals is close to zero. But once the number of infected individuals starts to rise, the number of susceptible individuals will decline sharply. And, the number of recovered individuals will rise quickly because as more and more people get infected, more and more people get recovered. At the peak of the epidemic, more than 21% of susceptible individuals get infected. Herd immunity could be achieved after 88% of susceptible individuals are infected. And, the death toll from the infection could pass more than 20% of the population.[9]

Figures 1c and 1d show the dynamics for aggregate consumption and labor supply that are largely determined by the dynamics of the outbreak. During the early stages of the epidemic, labor is mainly supplied by susceptible individuals, as there are only a few infected and recovered persons. As the number of susceptible individuals decreases, following the increase in the infection rate, labor supply also decreases, which in turn leads to a decline in consumption. The macroeconomic variables start to stabilize once herd immunity is achieved or the epidemic dies out.

6.2. Laissez Faire Social Distancing

Figure 2 compares the economic and epidemiological impacts of a laissez-faire social distancing to the baseline case of no social distancing. The former has a significant impact on the epidemics, through delaying and flattening the infection curve (Figures 2a and 2b). While it takes about 20 more periods to reach the peak

[9] The dynamics of the population and the fatality of the infection behave similarly but conversely. During the early stages of the outbreak, the latter is almost zero. The associated curve starts to incline later on, following the increase in the infection rate, eventually, it stabilizes as the epidemic dies out.

with voluntary social distancing, the infection rate decreases by about 10 percentage pts at its peak. Herd immunity could be achieved with a 15% lesser infection rate. And, the death rate declines by more than 3 percentage pts from the baseline.

The effect on the economy is positive. Aggregate income, consumption, and welfare increase compared to the baseline (Figures 2c-2f). The decrease in the infection and death rates increase aggregate labor supply, which in turn increases aggregate consumption and welfare. As shown in Figure 2e, the difference between the macroeconomic variables with and without social distancing follows the path of the infection curve. At the early stage of the epidemics, when the infection rate is too low, it is close to zero. However, at the peak of the epidemic, aggregate income is higher by more than 24 percentage pts compared to the baseline. The gap then decreases as the infection rate slows down while it remains constant once herd immunity has achieved. The latter represents the long term macroeconomic effect of voluntary social distancing, which is the result of the decline in the fatality rate.

6.3. Treatment and Vaccination

As shown in Figure 3, treatment has a relatively smaller effect on the spreads of the outbreak, particularly when compared to other controlling measures.[10] It flattens the infection curve by only 1.84 percentage pts while it decreases the death rate by 1.12 percentage pts (Figures 3a and 3b). The high recovery rate also implies that herd immunity could be achieved relatively quickly.

How does that translate to the economy? The impact on aggregate income and

[10] In this and the next sections, laissez-faire social distancing is assumed in all of the numerical simulations.

consumption is modest as shown in Figures 3c-3e, which entirely depends on the impacts of treatment on the infection and death rates. At the early stage of the epidemic, there is no difference between aggregate income and consumption, with or without treatment. But at the peak of the infection, aggregate income is higher by 6 percentage pts from the baseline case of no treatment (Figure 3e).

The impact of treatment on welfare is more important for two reasons (Figure 3f). First, treated individuals have relatively higher welfare because of their high recovery rate. Second, the lifetime welfare of susceptible individuals is higher with the availability of treatment. Because, if they get infected, they could receive treatment and quickly recovered. The same works for infected individuals, they are better off with the prospect of receiving treatment in the future.

Even a small vaccination rate greatly influences the dynamics of the outbreak. Figure 4 demonstrates the effects of a 0.03% vaccination rate per period on the epidemics, vis-à-vis the baseline case of no vaccination. It decreases the death rate by 3 percentage pts and flattens the infection curve by about 4.5 percentage pts (Figures 4a and 4b). Herd immunity could be achieved at a much lesser infection rate (by 13 percentage pts) than the baseline. Increasing the vaccination rate to 0.06 per period, which is equivalent to vaccinating 200 thousand individuals per period, will have a tremendous impact on the outbreak. Herd immunity will be achieved after only 2% of the population gets infected.

Figures 4c-4f show the macroeconomic effects of vaccination. As shown in Figure 4e, the impact on aggregate income and consumption mainly follows that of the impact on the infection curve. Aggregate income increases by 16 percentage pts at the peak of the infection, and by about 5 percentage pts permanently after herd immunity is achieved, compared to the case of no vaccination. However, its influence in aggregate welfare rather starts from the outset (Figure 4f). The intuition is that vaccinated individuals develop herd immunity, leave the susceptible compartment, and hence do not incur any more disutility from social distancing.

6.4. Government–Enforced Social Distancing

The numerical simulation shows that government-enforced social distancing could be among the most effective controlling mechanisms of the spread of the outbreak. Figure 5 compares a laissez-faire (do nothing) policy with two different lockdown levels – when $\alpha = 0.001$ and $\alpha = 0.005$. Figure 5a captures a quite interesting dynamics of susceptible individuals under lockdown. During the early stages, more individuals leave the susceptible compartment being in lockdown than being infected. During the latter stages, however, more people leave the susceptible compartment being infected than being in lockdown. The latter roughly matches the U.S. unemployment data. The rates of infection at the peak of the epidemic are 0.74%, 8.95%, and 11.43% for the lockdown levels of $\alpha = 0.005$, $\alpha = 0.001$, and do nothing (Figure 5b). The respective death rates are 2.43%,17%, and 14.75% (Figure 5c).

The lockdown has a strong negative impact on the macroeconomy, however (Figures 5d-5f). Despite savings life, it leads to job loss and thence a loss in aggregate labor income. From Figures 5d and 5e, aggregate income and consumption decrease. A 0.005 lockdown rate reduces aggregate income by 46 percentage pts while a 0.001 lockdown rate reduces it by about 10 percentage pts, at the end of the simulation periods (Figure 5f).

The dynamics of aggregate welfare is different from that of aggregate income and consumption as it includes the consumption of individuals under lockdown whose income comes from government transfer (Figure 5g). Figure 5h shows the percentage loss in aggregate welfare due to lockdown could go up to more than 75% pts, depending on the level of the lockdown. However, a more strict lockdown doesn't always mean a bigger welfare loss. Particularly, at later stage, a more strict lockdown could mean a lower welfare loss, as some of the negative job-loss effects are offset by the positive life-saving effects.

7. Final Remark

The paper provided an alternative framework of the SIR (Susceptible-Infected-Recovered) epidemiology model integrated into the standard economic dynamic model through a voluntary social distancing. The rationale for providing microfoundations to the SIR models does not rest upon individuals' consumption, work, or leisure activities but on infection-averse individuals who have a taste for non-pecuniary social closeness. In addition to leisure and consumption, individuals care for social closeness (e.g., hugging, kissing, socializing) although these could cost their life or income. Accordingly, two different individuals may choose a similar bundle of consumption goods or leisure time but may experience different social distancing. In their leisure choice, one person may go out to a beach for three hours and the other may stay at home watching The Wolf of Wall Street for the same amount of time, for instance.

Susceptible individuals face a trade-off between practicing social distancing and increasing their likelihood of being infected and thence losing labor income in the following periods. Infected individuals work less time, due to sickness, and hence lose some labor income while recovered individuals resume a normal life. Infected and recovered individuals do not practice social distancing. While the latter develop immunity, the former have nothing to lose. Optimal individual-level social distancing determines the dynamics of the epidemic, which in turn determine the dynamics of the macroeconomic variables.

An individual's optimal social distancing is the difference between her value function of remaining in the susceptible compartment and moving to the infected compartment in the next period. It increases in her psychological discount factor but decreases at the probability of receiving a vaccination or her likelihood of developing immunity. From the numerical simulation, a laissez-faire social distancing is important in terms of delaying and flattening the infection curve that minimizes the economic damage from the outbreak. A government-enforced social distancing or a lockdown is highly effective in flattening the infection curve. But it would have a detrimental effect on the economy, through a negative job-loss effect. Treatment and vaccination positively influence aggregate welfare but through different mechanisms. The former increases aggregate welfare by increasing individuals' likelihood of getting recovered quickly. The latter pulls out individuals of the susceptible compartment from the outset and enables them to avoid a costly social distancing.

The paper is part of the primary efforts to provide microfoundations to the canonical epidemiology model, used to track the spread of the recent outbreak, and to integrate it into conventional macroeconomic models. A simple approach was adopted to deal with the problem in a tractable manner, without loss of generality. The qualitative results, however, should be read as illustrative and caution should be taken while interpreting the results from the numerical simulations. A strong quantitative prediction of the course of the epidemic could be obtained through adopting a more elaborated version of the model, which considers the different stages of the outbreak such as asymptomatic and symptomatic cases, different severity of illness (non-life-threatening cases and cases that require ICU admission). The work can also be extended to include more detailed non-pharmaceutical interventions (e.g., a ban on gathering, stay at home orders and the closures of industries and school), and the financing of the health sector.

Acknowledgements

The author is grateful to the comments from the participants of the Mini Online Economics Conference (26 May 2020), University of Hull, particularly to Parantap Basu and Keshab Bhattarai. The usual disclaimer applies.

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Figures

Figure 1a: Baseline SIR for susceptible, infected and recovered persons

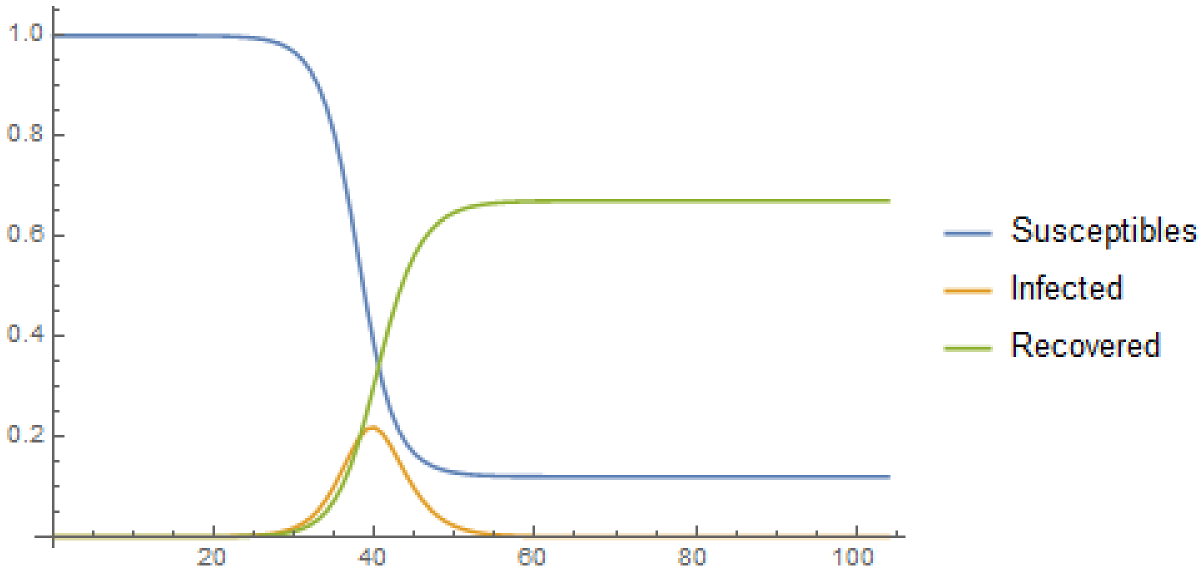

Figure 1b: Baseline SIR including death and population dynamic

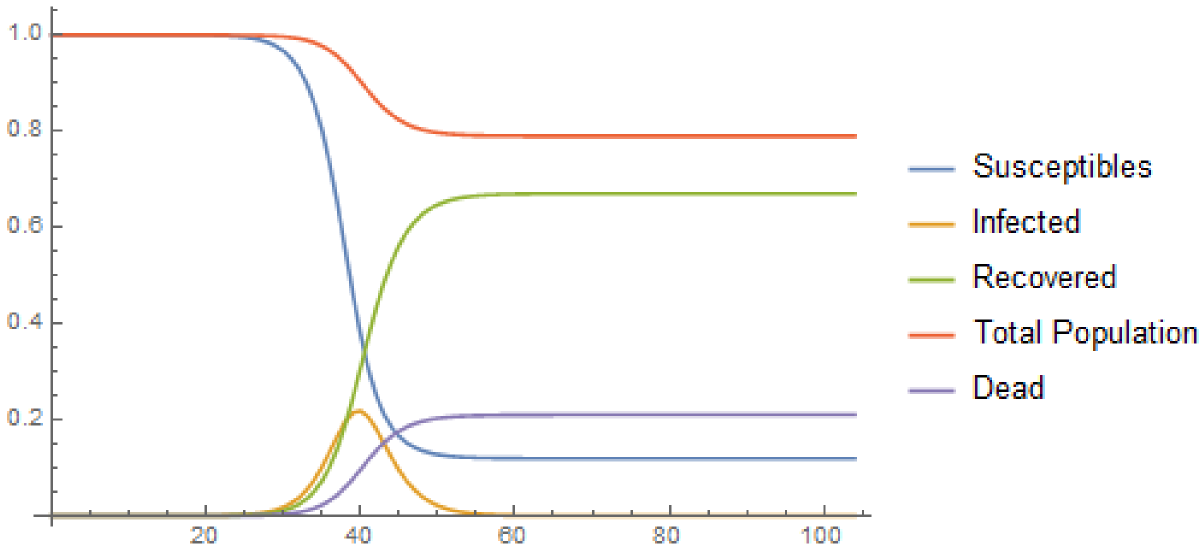

Figure 1c: Baseline SIR for the dynamics of aggregate consumption

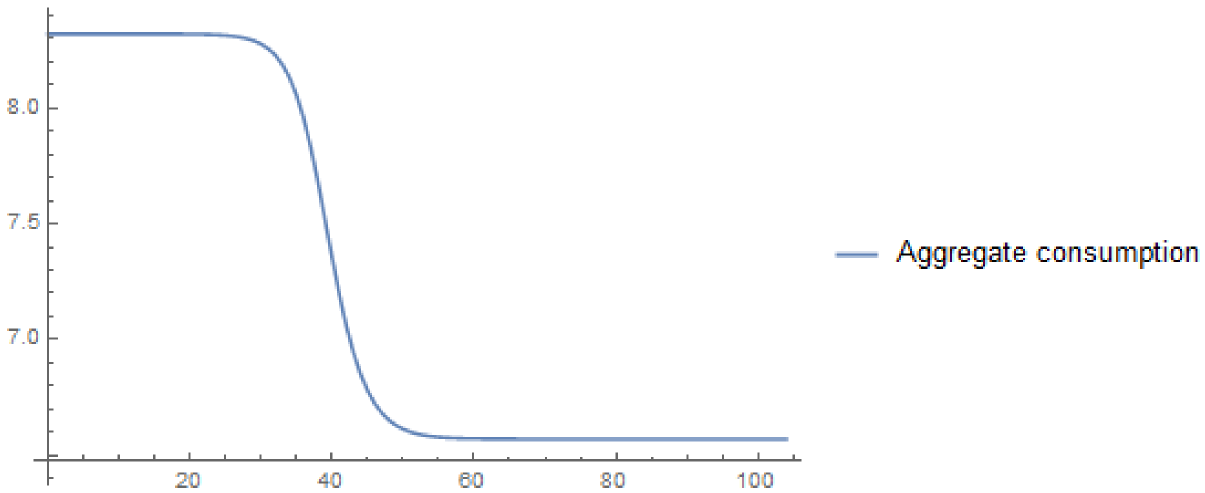

Figure 1d: Baseline SIR for the dynamics of aggregate labour supply

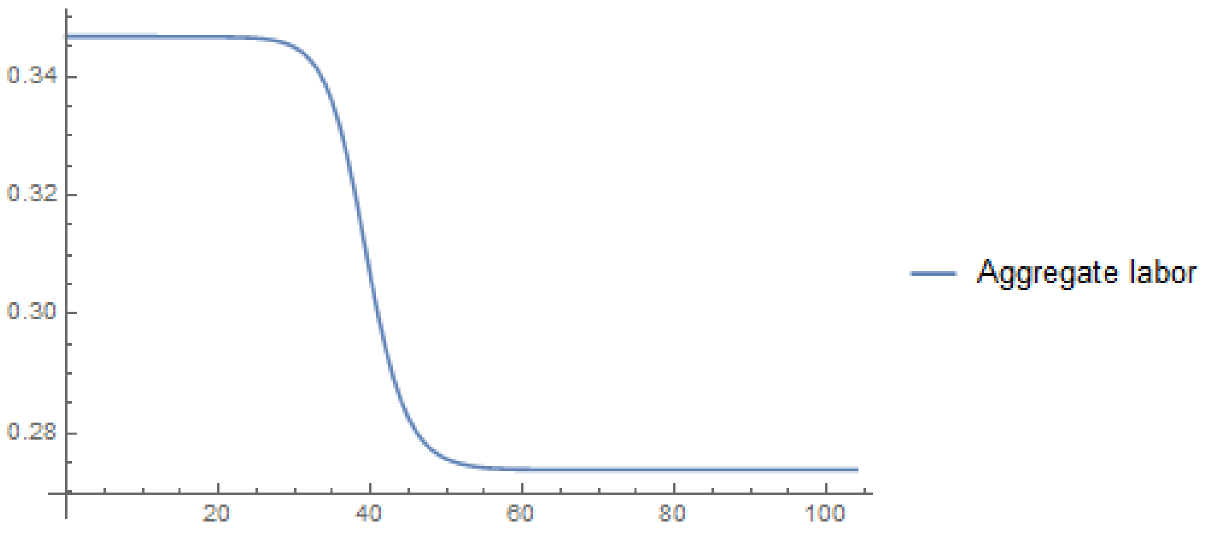

Figure 2a: Infection dynamics with laissez-faire social distancing

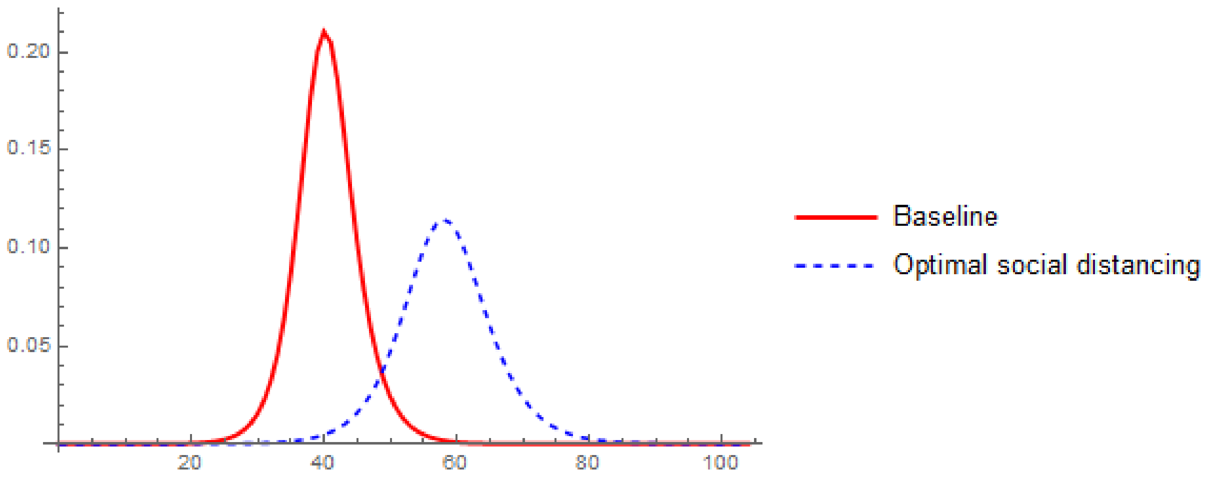

Figure 2b: Death dynamics with laissez-faire social distancing

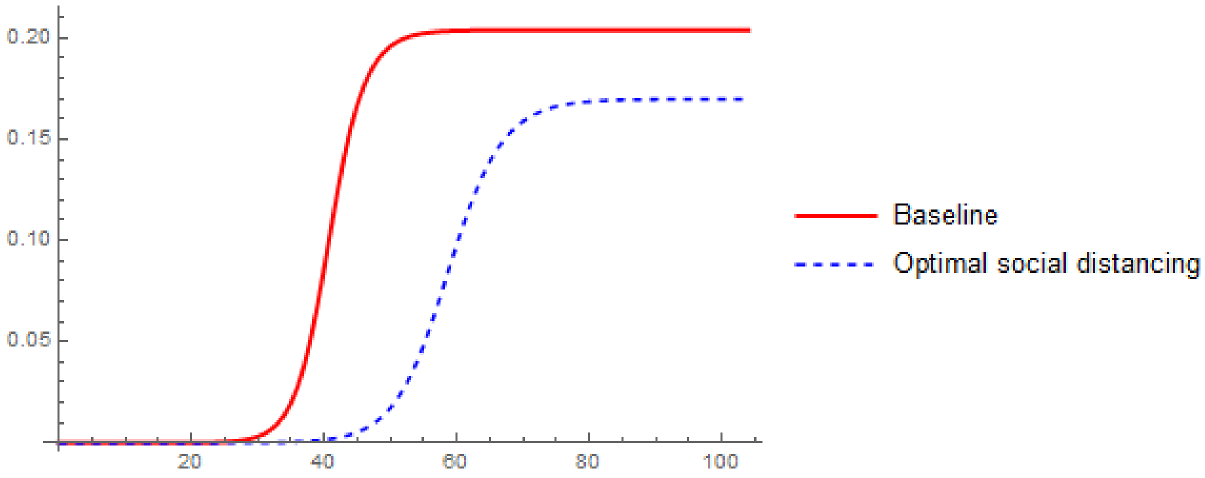

Figure 2c: Aggregate consumption dynamics and laissez-faire social distancing

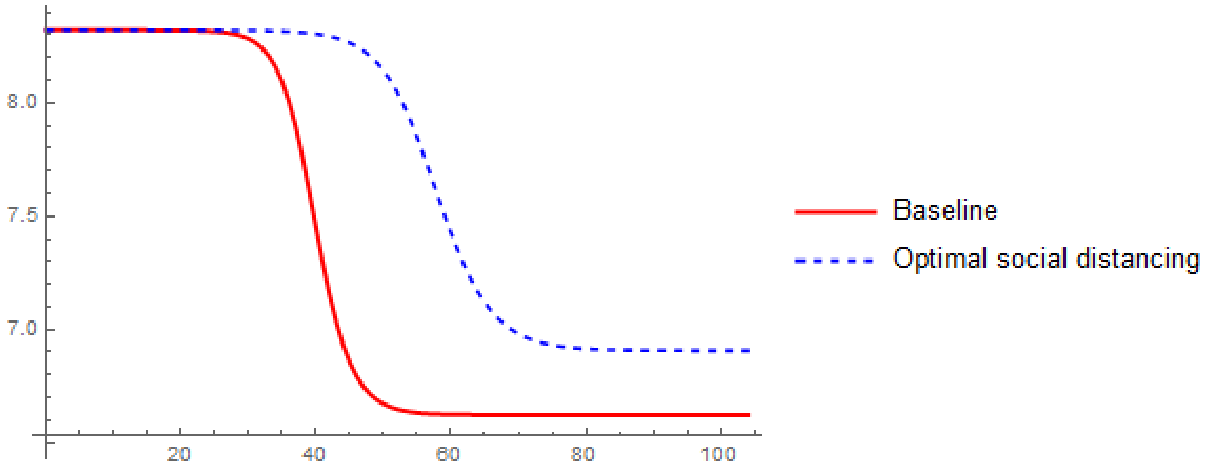

Figure 2d: Aggregate Labour dynamics and laissez-faire social distancing

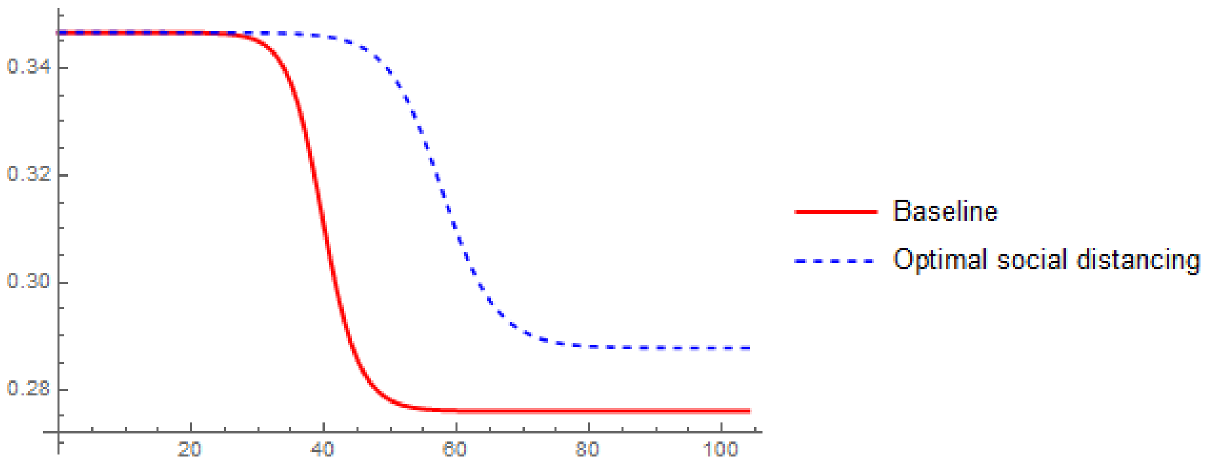

Figure 2e: Percentage differences in aggregate consumption and income, with and without optimal social distancing

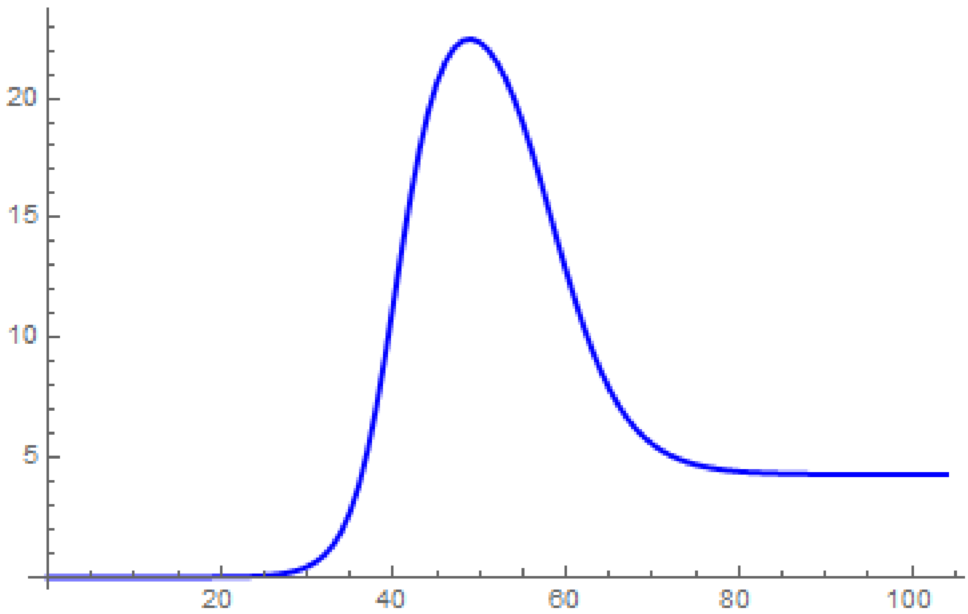

Figure 2f: Aggregate welfare dynamics and laissez-faire social distancing

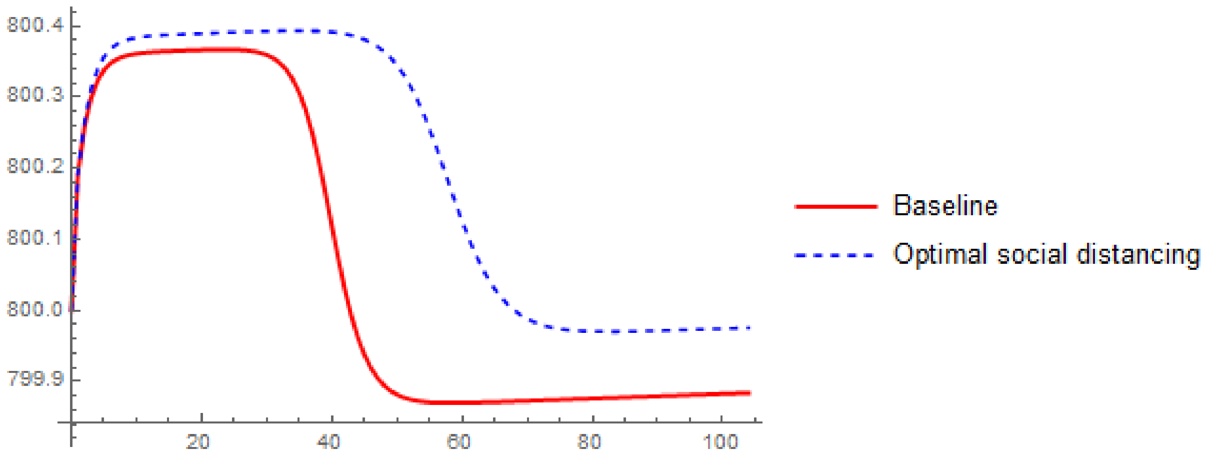

Figure 3a: Infection dynamics with and without treatment

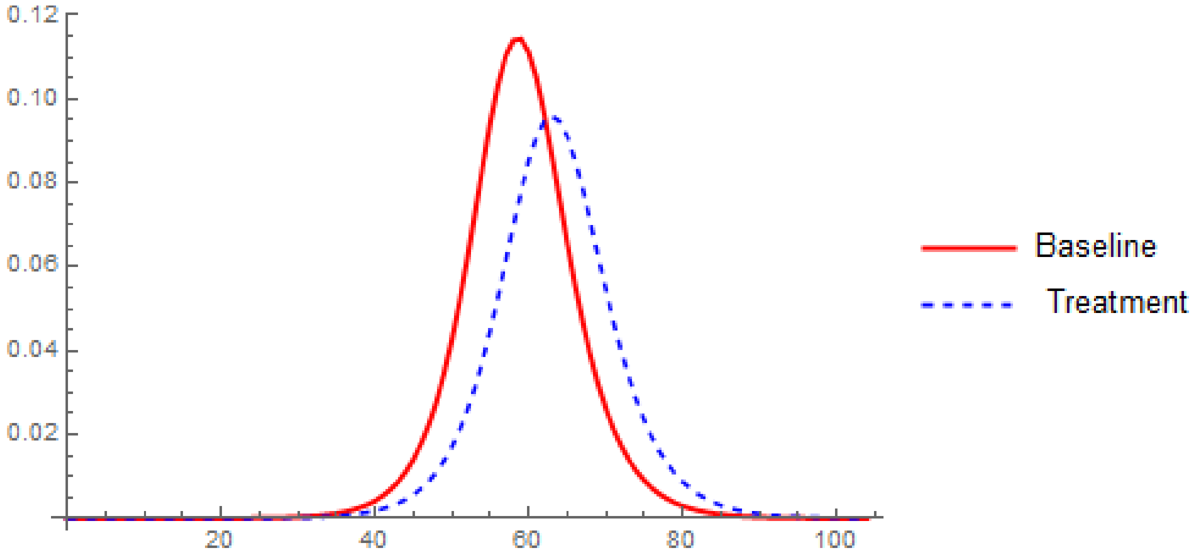

Figure 3b: Death dynamics with and without treatment

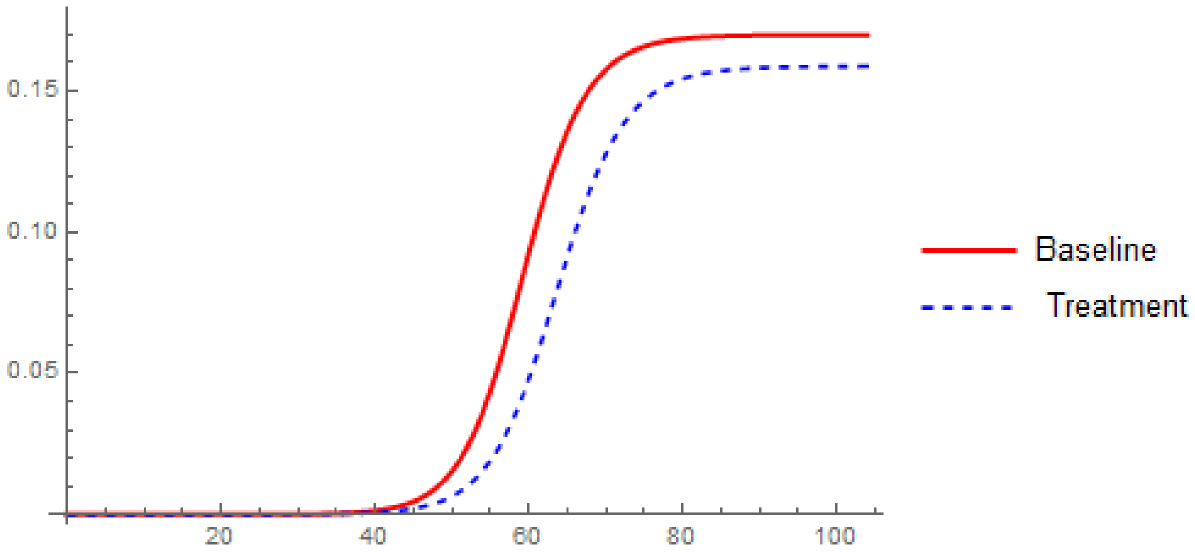

Figure 3c: Aggregate consumption with and without treatment

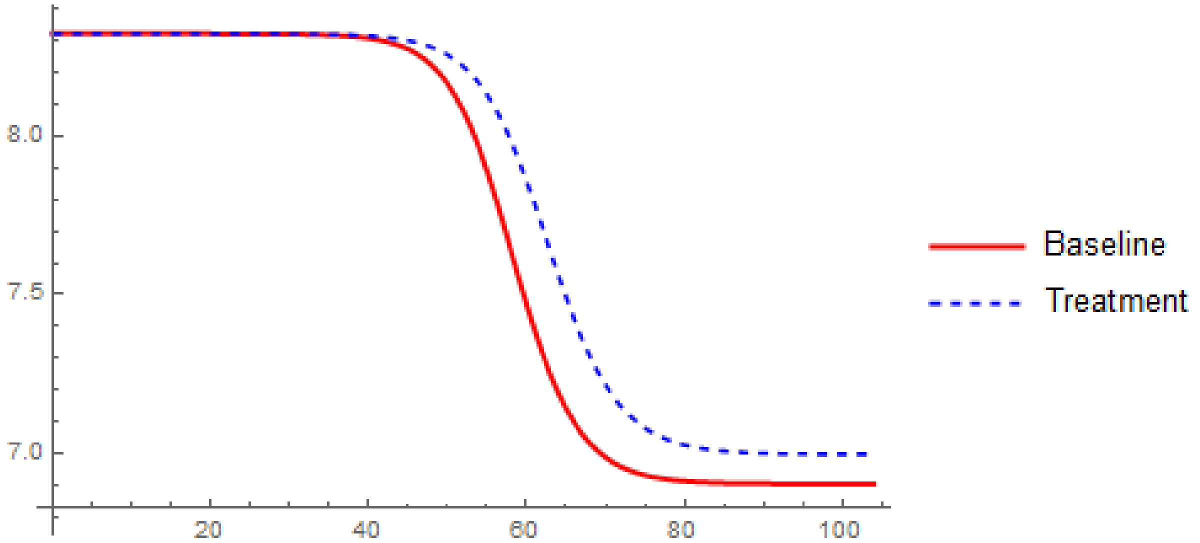

Figure 3d: Aggregate labour with and without treatment

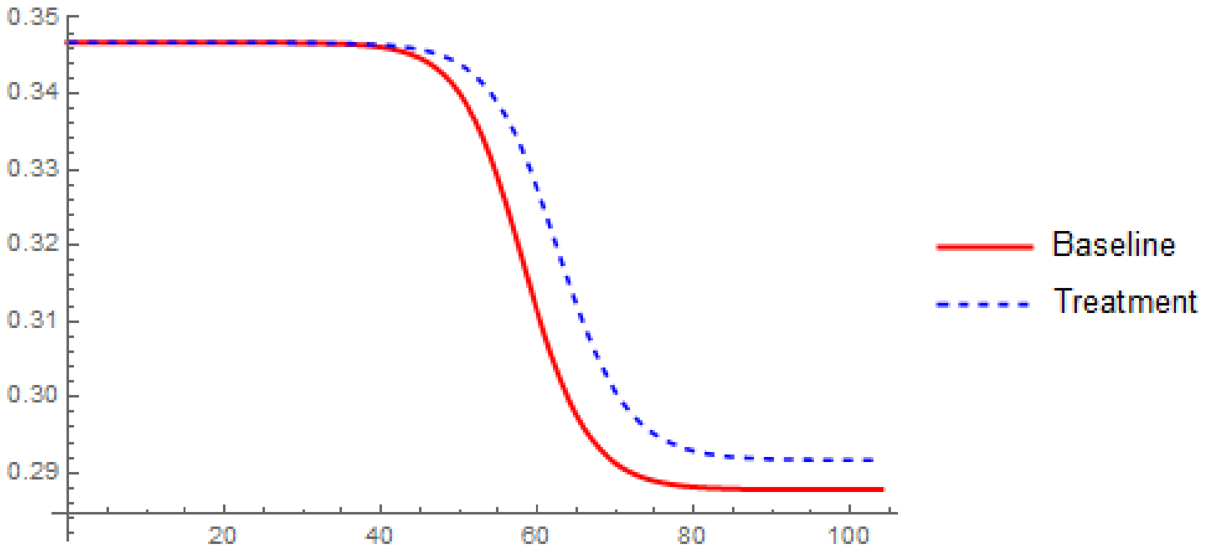

Figure 3e: Percentage differences in consumption and income with and without treatment

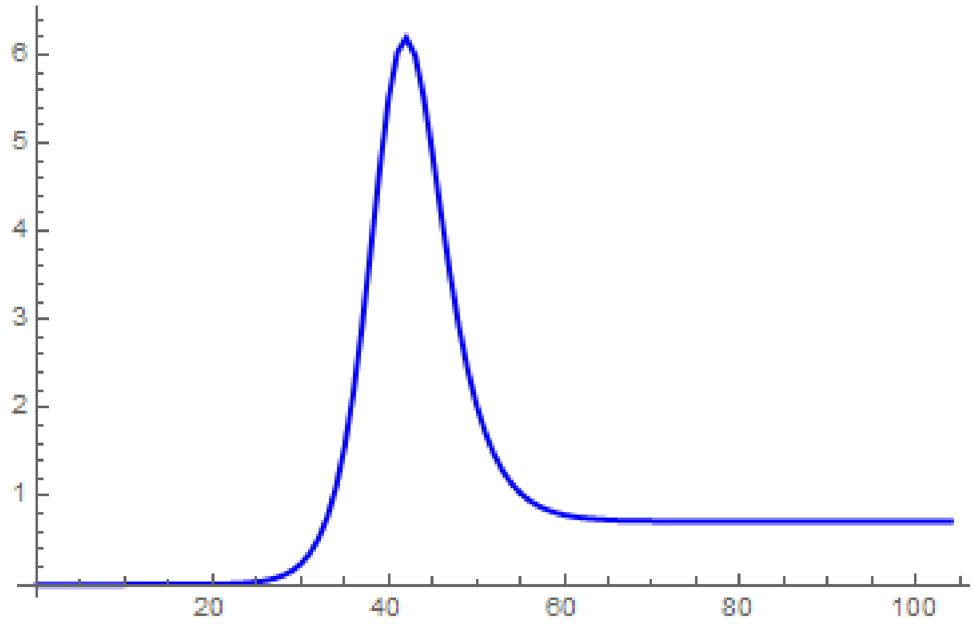

Figure 3f: Aggregate welfare dynamics with and without treatment

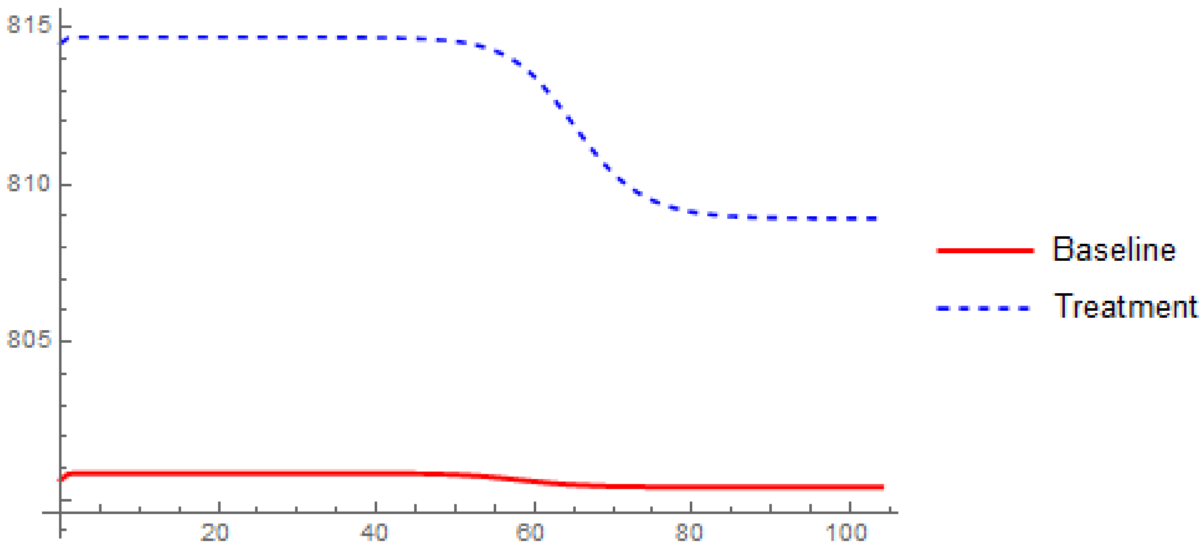

Figure 4a: Infection dynamics with and without vaccination

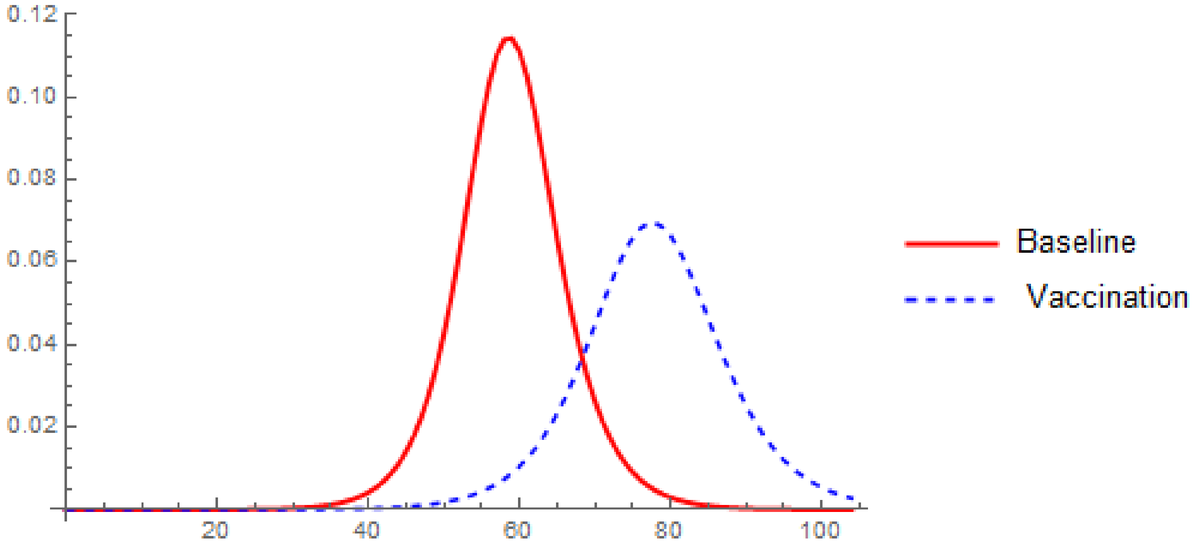

Figure 4b: Death dynamics with and without treatment

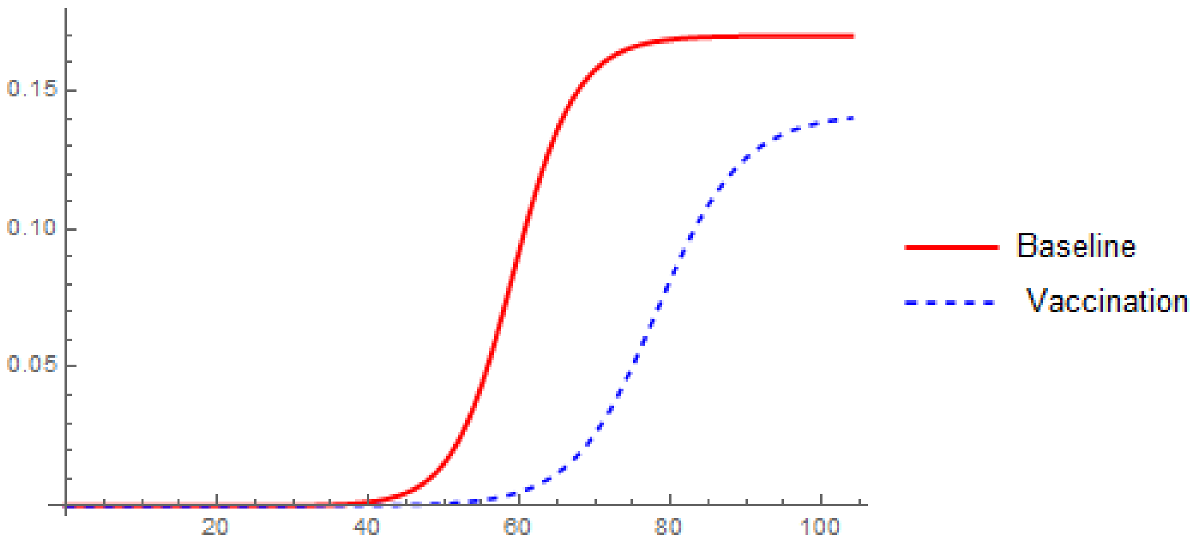

Figure 4c: Aggregate consumption with and without vaccination

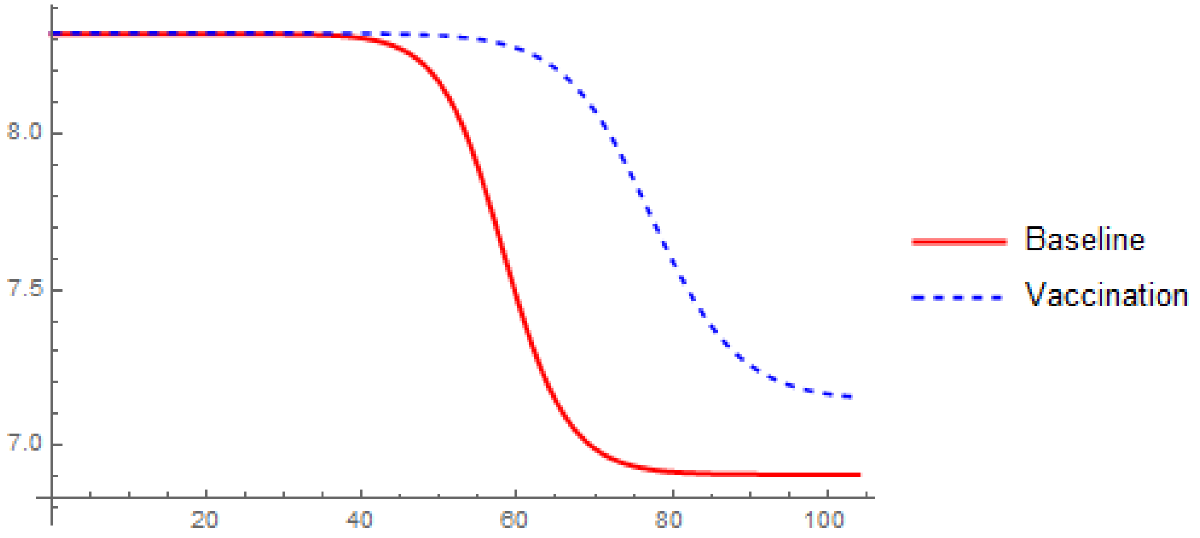

Figure 4d: Aggregate labour with and without vaccination

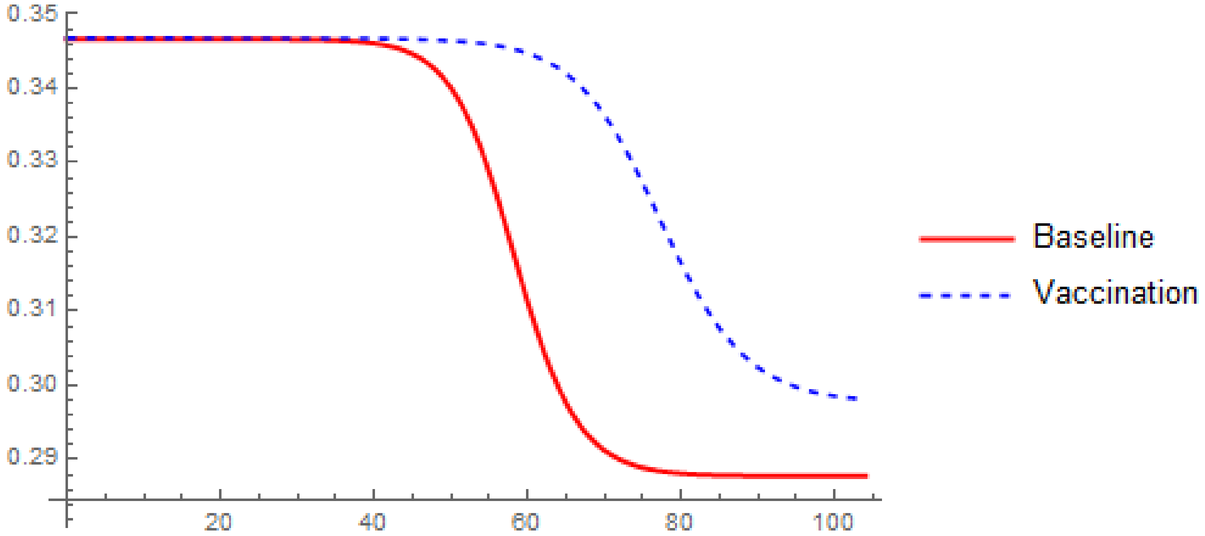

Figure 4e: Percentage differences in consumption and income, with and without vaccination

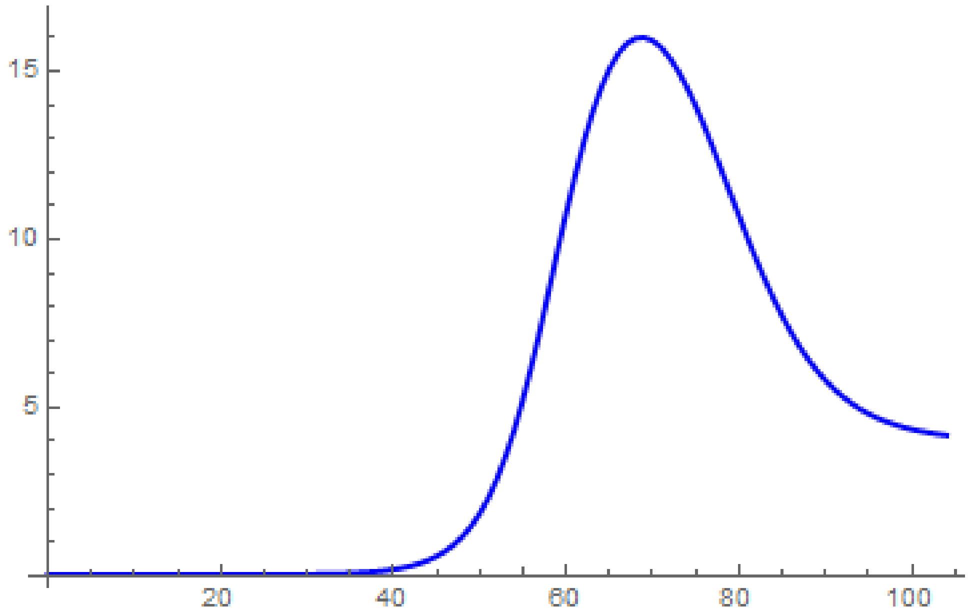

Figure 4f: Aggregate welfare dynamics with and without vaccination

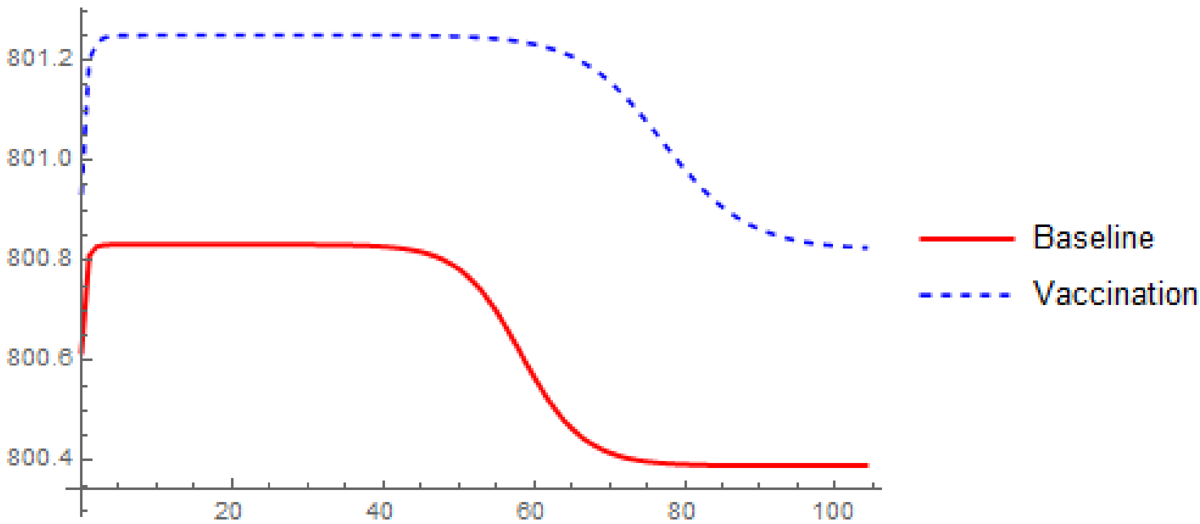

Figure 5a: Dynamic of susceptible individuals with government-enforced social distancing

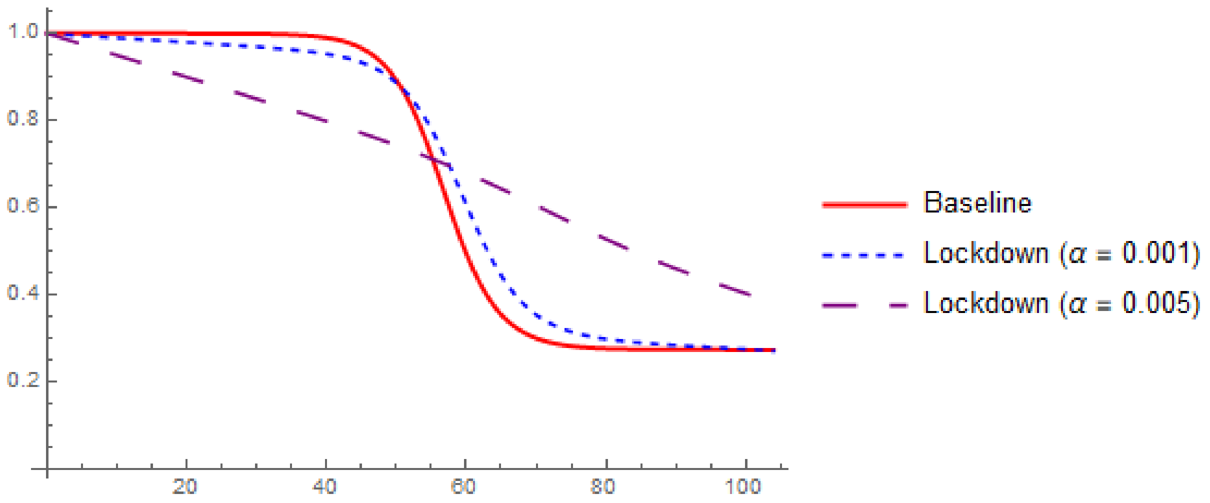

Figure 5b: Infection dynamics with government-enforced social distancing

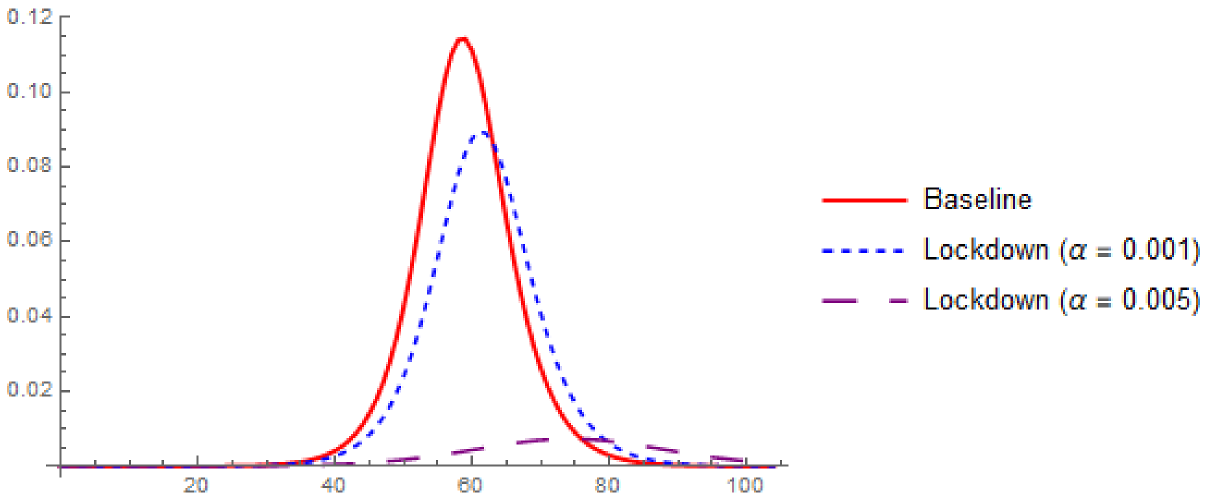

Figure 5c: Death dynamics with government-enforced social distancing

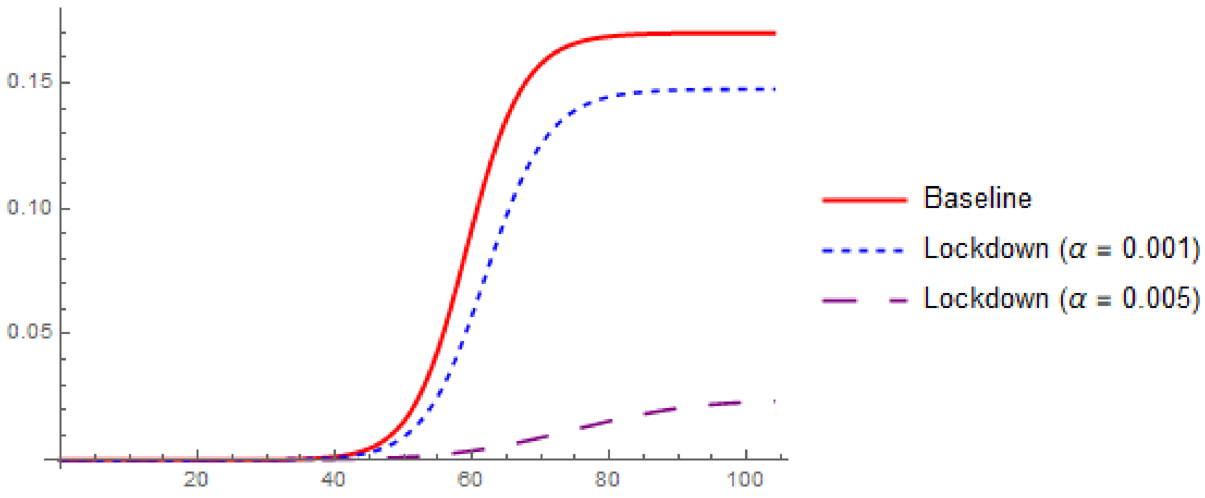

Figure 5d: Aggregate consumption dynamics with government-enforced social distancing

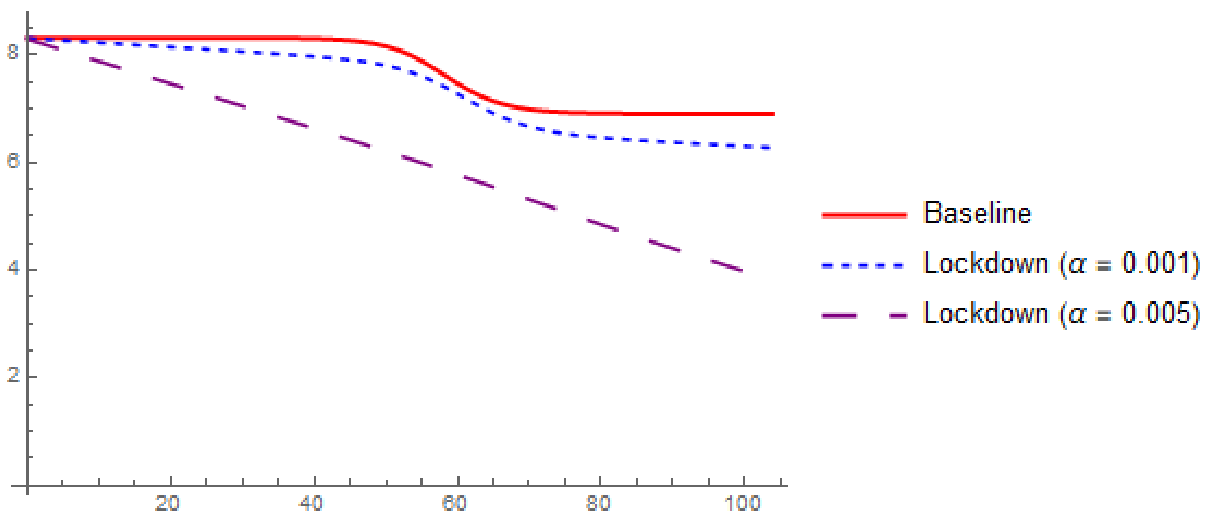

Figure 5e: Aggregate labour dynamic with government-enforced social distancing

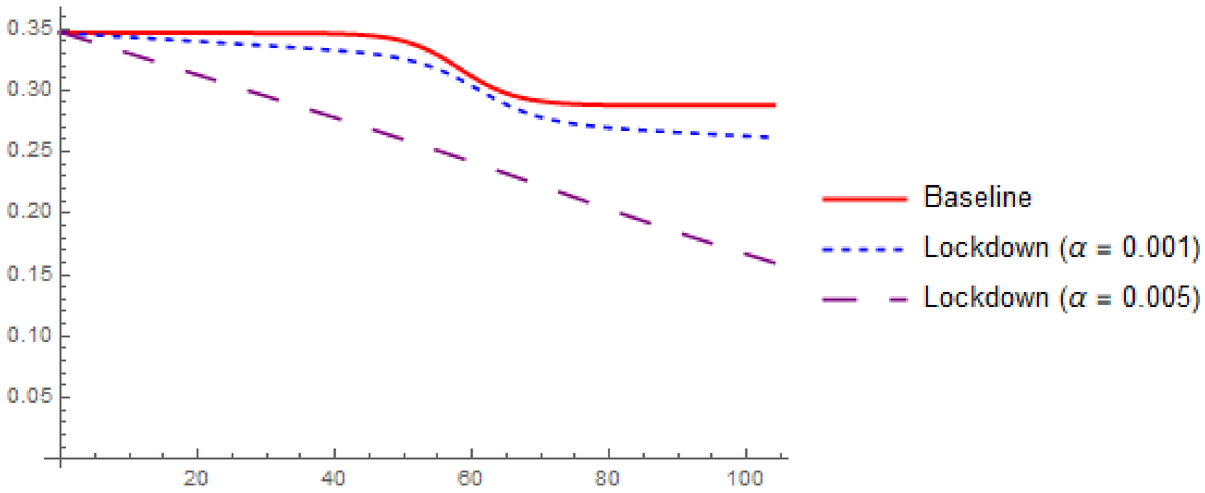

Figure 5f: Percentage differences in consumption and income, with and without government-enforced social distancing

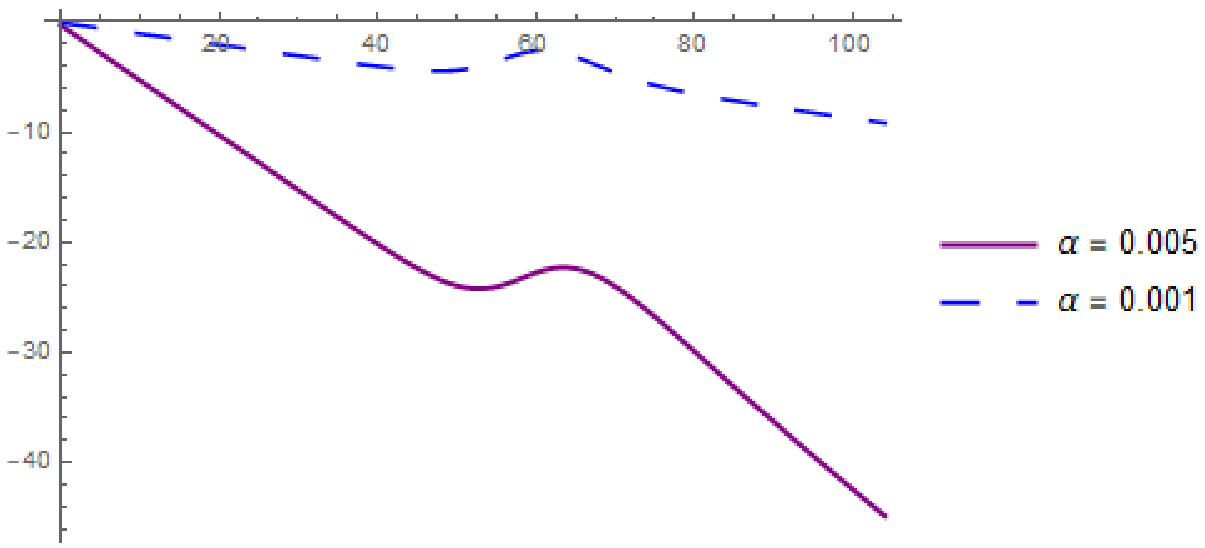

Figure 5g: Aggregate welfare with government-enforced social distancing

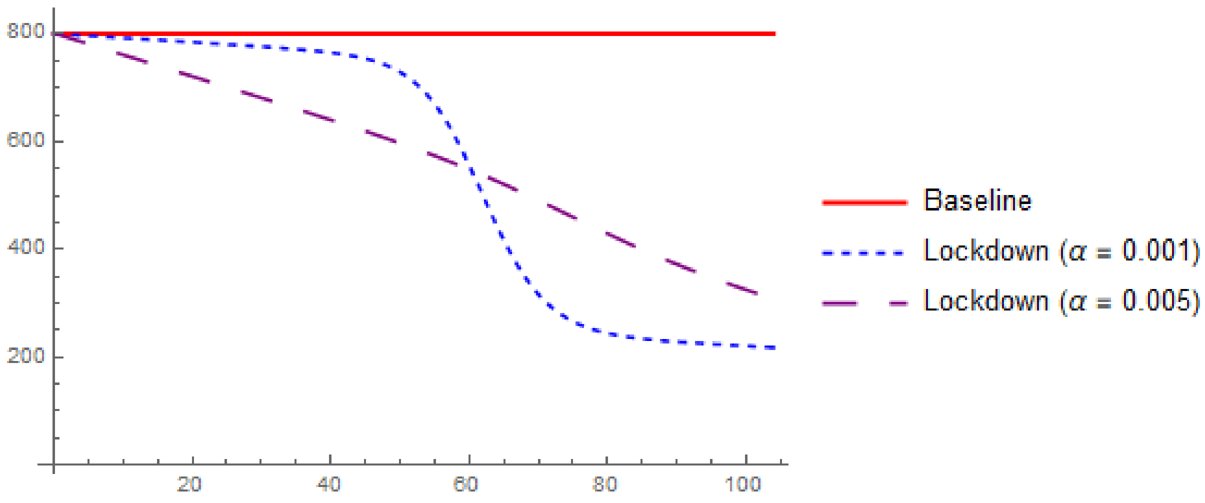

Figure 5h: Percentage differences in aggregate welfare with and without government-enforced social distancing