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Redistribution, Inequality, and Efficiency with Credit Constraints*

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Abstract

We develop a model that characterizes the joint determination of income distribution and macroeconomic aggregate dynamics. We identify multiple channels through which alternative public policies such as transfers, consumption and income taxes, and public investment will affect the inequality–efficiency trade off. Some policy changes can affect net income inequality both directly, and indirectly by inducing structural changes in the private-public capital ratio. This in turn influences market inequality and determines the distribution of the next period's investment and net income. Income tax and transfers have both a direct income effect and an indirect substitution effect, whereas the consumption tax has only the latter. After developing some theoretical propositions summarizing these policy tradeoffs, we present extensive numerical simulations motivated by the South African National Development Plan 2030, the objective of which is to tame soaring inequality and increase per capita GDP. Our numerical simulations illustrate how the judicious combination of these policies may help achieve these targets. The simulations also suggest that the sharp decline in private-public capital ratio coupled with high degree of complementarity between the public and private capitals could be behind the persistence of market inequality in South Africa during the last two decades.

Key words: Redistribution policies; Incomplete Capital Market; Idiosyncratic shocks; Efficiency; Inequality

JEL Classification: D31, O41

1 Introduction

The relationship between redistribution, equity and efficiency is a complex issue. Rising inequality may hurt growth, but subsequent efforts to tame it could be

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worse.[1] Taxes and social grants intended to promote equity may have a "leaky bucket" effect – welfare loss when transferring resources from the rich to the poor – the classical trade-off between equity and efficiency (Okun, 1975). It is often suggested that redistributive policies in the form of productive public investment could be "win-win" strategies as they lead to both equity and efficiency. But how effective are these policies, particularly when compared to other redistributive policies such as grants and transfers? Although pure redistributive policies may provide incentives to reduce individual efforts and savings, they could also have a positive efficiency effect, insofar as they provide resource-poor households with self-insurance, enabling them to relax the severe resource constraints by which they are restricted (see, for instance, Benabou, 2005). The fact that such policies (taxes, transfers, and public investment) are not mutually exclusive, but may be constrained by requiring a revenue-neutral budget, can pose a dilemma as policy makers evaluate the tradeoffs between them.

The present paper identifies various mechanisms whereby taxes and government expenditures relate to the inequality-efficiency nexus. In particular, it analyzes the role of income taxes and consumption taxes, social grants (transfers) and public investment on *gross* (pre tax and transfers) and *net* (post tax and transfers) income inequality, economic growth, and welfare.[2] We are also interested in identifying more effective combinations of optimal policies in terms of promoting equity, growth, and welfare. Our approach accords with recent empirical work that emphasizes the role of redistribution, gross and net inequality on economic growth (e.g., Berg et al., 2018, Grundler and Scheuermeyer, 2018) and provides a framework for future similar work.

We develop a general equilibrium model in which both aggregate and distributional dynamics are endogenously determined, while the government engages in redistribution of resources through taxes, transfers, and productive public investment. Individuals live for three periods; first as a youth, then as an adult, and finally as a retiree. In the spirit of Benabou (2005) and Cagetti and De Nardi (2006), households operate privately-owned firms. Individuals are heterogeneous in terms of their initial capital, and credit markets are incomplete. With respect to the credit market we consider two cases: (i) where it is missing, thereby precluding the opportunity to borrow, and (ii) where credit is *partially* available. In the latter case, youth consumption is financed by borrowing against future income, though only to a suboptimal degree, due to limited credit avail-

[1] There is a vast literature that started in the 1990s, identifyung different channels through which inequality could harm growth. Galor and Zeira (1993) and Benabou (1996, 2000, 2002), for instance, argue that inequality has a negative impact on growth if the credit market is missing or incomplete. Because, in non-egalitarian societies, relatively more high-return investment opportunities could be forgone by resource-poor households than would otherwise be so. Alesina and Rodrik (1994), Persson and Tabellini (1994) and Alesina et al., (2018) associate high inequality to rising populism and hence demands for inefficient redistribution. While De la Croix and Doepke (2002) link inequality and growth through differential fertility between poor and rich parents, Alesina and Perotti (1996) and Benhabib and Rustichini (1996) relate high inequality to sociopolitical instability.

[2] "Gross" income inequality is often referred to as "market" inequality; we shall use the two terms interchangeably.

ability. During adulthood, individuals work and earn income that is used to (i) pay off their debt incurred as youth, (ii) for current consumption, and (iii) for savings. The latter inclusive of returns will then be used to finance old-age consumption.

Firm level production is specified as a two-step process and is subject to idiosyncratic productivity shocks. In the first stage, public and private capital are combined using a constant elasticity of substitution production function. This is then combined via a Cobb-Douglas technology with inelastically supplied labor, the productivity of which is augmented by aggregate private capital to render the long-run marginal product of capital constant, thereby enabling the economy to sustain a long-run equilibrium of endogenous growth (e.g., Romer, 1986, Barro, 1990, Futagami et al., 1993, Turnovsky 1997). The government employs consumption and income taxes to finance productive government expenditure, as well as a direct transfer program. Endogenous inequality dynamics is generated due to the presence of marginal diminishing returns to investment at the individual level and the missing or imperfect credit market (e.g., Loury, 1981, Benabou, 2000, 2002).

The credit constraint causes initial individual productivity differences to persist. With diminishing returns to investment, the resource-poor have a higher marginal product than do the rich, which manifests itself as differences in growth rates between them. In the *absence* of a credit market the inequality dynamics generated in this way drives the dynamics of the macroeconomic aggregates, including the aggregate growth rateOne of the novel features of the current paper is that, in contrast, with an *imperfect* credit market and the associated limited borrowing opportunities, the dynamics of the macroeconomic aggregates also influence the dynamics of inequality. That is, there is direct *bidirectional* causality between the dynamics of inequality and the macroeconomic aggregates, in contrast to the extant literature that is characterized by a *unidirectional* dynamics.[3] The underlying intuition is straightforward. With partial availability of credit, individuals are allowed to borrow against their next period's income, for young age consumption. Consequently, their current investment decision (that forms the next period's capital) depends not only on their current income, but also on their next period's income, leading to a contemporaneous relation between capital and income. Therefore, the next period's aggregate capital depends on both the current, and the next period's, distribution and aggregate income.

[3] Two general approaches to analyzing the relationship between growth and inequality can be identified in the literature. The first is the so-called "representative consumer theory of distribution" as it was named by Caselli and Ventura (2000), which is based on complete markets, so that all agents have identical access to all markets. In this case one can exploit the aggregation procedures due to Gorman (1959), in which case the causality is uni-directional in that macroeconomic aggregates influence income distribution but not vice versa; see also Turnovsky (2020) for an extensive discussion of this approach and many diverse applications. In contrast the approach adopted by Benabou (2000, 2002), Getachew (2010, 2012, 2016), Bandyopadhyay and Tang, (2011), Getachew and Turnovsky (2015) and Basu and Getachew (2019) is based on incomplete markets and the causality is reversed; inequality dynamics now drive the dynamics of the macroeconomic aggregates. See Turnovsky (2015) for further discussion.

These imply that the next period's distribution of income also depends on the next period's aggregate capital, which in turn depends on the current aggregate capital and its distribution.

Another important contribution of the current work is to identify the channels through which fiscal policy impacts the inequality-growth nexus. The first is a *substitution effect* with respect to gross income distribution. This effect depends upon the change in the composition of private vs. public capital in the production process. An increase in either the income tax rate, the consumption tax rate, or a decline in the transfer, reduces the private-public capital ratio, and conversely. The second is a direct *income effect* with respect to net income distribution. This arises through the differential effect the income tax and transfer have on net income, and is unaffected by the consumption tax. There is a third indirect channel, manifested through the interaction between gross and net inequality. A change in the fiscal structure could change the private-public capital ratio and influence individual gross income that in turn determines the distribution of the next period investment and net income.

The extent to which a change in the private-public capital ratio impacts gross income inequality is determined by the elasticity of substitution between the two capital goods in production. Specifically, an increase in public investment will decrease or increase gross income inequality, according to whether the elasticity of substitution between public and private capital is greater than, or less than, unity. This is because public capital that is highly substitutable for private capital provides poor households with the opportunity to circumvent their restricted capacity to invest, due to their limited ability to borrow, by engaging in factor substitution. This, in turn leads to a more equitable distribution of wealth, and a subsequent positive impact on productive efficiency. By contrast, public investment having a low degree of substitution with private capital (i.e. is complementary) will exacerbate inequality and have an adverse effect on productive efficiency. This is because public investment will now disproportionately favor the rich who, because they own much of the private capital in the economy, obtain lower marginal returns on their investments.

The polar case of the missing credit market can be solved analytically, enabling us to summarize some of the main results pertaining to the impact of policy on income inequality and growth by four formal propositions. These all stem directly from the effects of the income tax, consumption tax, and transfers on the substitution and income effects, noted above. Thus, for example, if the elasticity of substitution between the capital goods exceeds unity, raising the income tax will reduce both gross and net income inequality, while if it is less than unity gross inequality will still rise, but now the income effect on net income inequality will be offsetting. With respect to growth, if public investment is suboptimal, the direct effect of raising the income tax rate is to increase the growth rate through boosting public investment. Other policies have contrasting effects, as a result of which by choosing them judiciously a variety of competing policy objectives can be simultaneously attained.

We supplement these formal analytical results with numerical simulations, based on calibrating the model to approximate the South African economy, where public capital is found to be suboptimal, and complementary to the private capital good in production.[4] The simulations illustrate several of the tradeoffs that the alternative policy instruments entail. An increase in either the income tax or consumption tax will boost public investment and increase the growth rate, and welfare, while leading to higher gross income inequality. At the same time, increasing the income tax will reduce net inequality directly, and thereby moderate some of the increase in gross income inequality due to the substitution effects. However, this is not so for the consumption tax that has only a direct substitution effect. Thus, for example, our simulations suggest that a 2 percentage point increase in the consumption tax could increase net inequality by up to around 1.7%, whereas a similar increase in the income tax could increase it by only up to 1.2%.

Increasing the transfer (at the expense of public investment) will reduce both gross and net inequality, but by putting pressure on the (already) suboptimal public investment, it results in a negative growth rate and causes a significant welfare loss. A superior alternative could be to couple a smaller increase in transfers with a similar increase in the income tax, leaving public investment little affected. In this case, growth is barely affected and net inequality decreases by up to 2.8%. The net impact on growth is positive (reflecting the decline in inequality) but marginal. With little change in the ratio of private to public capital, such a policy change will also have a relatively very small effect on gross income inequality.

A one-to-one substitution of the income tax by a consumption tax is shown to have a more positive impact on growth, and hence welfare. For example, a $2.12\%pts$ reduction in the income tax, accompanied by an identical increase in the consumption tax, could boost both private and public savings rates and hence increase welfare up to 7.7%. However, such a policy reduces the private-public capital ratio, thereby increasing gross income inequality through the substitution effect. In addition, through the income effect, it increases net inequality.

A better outcome (in terms of inequality) may be obtained if the $2.12\%pts$ increase in the consumption tax is accompanied by a lesser reduction in the income tax (by $1.12\%pts$) and an increase in transfers (by $1\%pts$). In this case growth will be slightly improved due to the improvement in income distribution. As in the preceding case, gross income inequality rises due to a decrease in the capital ratio but at a much lesser rate. More importantly, net inequality significantly declines (by up to 1.4%), which also indirectly mitigates the rise in gross inequality.

When evaluating the model *vis-a-vis* the South African National Develop-

[4] Public infrastructure is in general expected to be highly complementary in South Africa, particularly when compared to other middle income countries, due to the country's unique experience of an apartheid system. Before the democracy in 1994, South Africa was divided into four provinces, where white South Africans (that constitute about 10% of the population) live, that represented most of the infrastructure development in the country. The ten homelands, where the majority of black South Africans live, are rather characterized with limited infrastructure development.

ment Plan's 2030 (NDP's 2030), expanding the transfer and increasing the consumption tax is found to be a relatively superior policy in moving the economy in the direction of the NDP's 2030 targets.[5] Increasing the consumption tax by 2.12%*pts* and transfer by 3.18%*pts* (from its current average value 4.12%) decreases net inequality by up to 6.79%, for instance.[6] It may lead to a long-run growth rate of up to 4.38%and an increase in GDP per capita of up to 235% (the target is to increase it by140%), over approximately 20 years. It leads to a lower consumption to GDP ratio (due to the increase in consumption tax) but the net welfare effect is positive (that comes from the boost in growth and reduction in inequality), which increases up to 5.3%.

Most of our simulations are based on the estimated low elasticity of substitution between private and public capital. This implies that the sharp decline in private-public capital ratio, after the democracy in 1994, could explain the persistence of market inequality in South Africa (Figure 1). [7] It also suggests that if the two public goods were substitutes rather than complements, the policy would have resulted in a decline in gross income inequality. This is confirmed by simulations based on assuming an elasticity of substitution of 1.539. It also suggests that a potentially fruitful strategy to reducing gross income inequality could be to redirect government investment to areas such as public transport, which are likely to be substitutes for private transport, rather than to complements as appears to have been the case.

The work is related to three strands of literature. First and most directly, it relates to the work on infrastructure, growth, and inequality (e.g., García-Peñalosa and Turnovsky, 2007; Chatterjee and Turnovsky, 2012; Getachew, 2010, 2012; Getachew and Turnovsky, 2015; Turnovsky, 2015). However, unlike the current paper, these studies consider contrasting and quite extreme credit market environments. García-Peñalosa and Turnovsky (2007) and Chatterjee and Turnovsky (2012) have examined the impact of growth-enhancing fiscal policies on inequality within a *complete* market framework following the line of Caselli and Ventura (2000). Getachew (2010, 2012) and Getachew and Turnovsky (2015) have studied the role of public investment on inequality and growth in models with *missing* credit markets.

The paper is also closely related to the work on growth and inequality with imperfect credit markets, although this literature does not incorporate the role of public capital (e.g., Loury, 1981; Galor and Zeira, 1993; Banerjee and Newman, 1993; Aghion and Bolton, 1997; Piketty, 1997; Aghion, et al., 1999; Benabou, 1996, 2000, 2002 ; Seshadri and Yuki, 2004; Bandyopadhyay, 2011; Halter et al., 2014).

[5] The South African government has launched NDP 2030 in 2012, with a goal of reducing the high levels of inequality and poverty in the country. NDP's 2030 targets are to reduce the Gini coefficient from 69 to 60 by 2030 and to increase per capita income from 50,000 Rand ($3,344) to 120,000 Rand ($8,026) (South African Government, 2012).

[6] This figure is much lower than the average government social spending in low-income and middle-income countries between 1990 and 2009, which was in the order of 9.3%, reported by Haile and Nino-Zarazua (2017).

[7] Private public capital ratio in 2011 is reduced by more than 240 percent than it was in 1994.

The third strand of literature to whoch this paper is related deals with the implications of incomplete insurance markets for savings behavior and the distribution of wealth (e.g., Aiyagari, 1994; Krusell and Smith, 1998, 2006; Castaneda et al., 2003; Heathcote, 2005; Alonso-Ortiz and Rogerson, 2010; Benhabib et al., 2011, 2015). The current paper, however, abstracts from the concentration of wealth, precautionary savings, or the effects of aggregate shocks that are at the center of most of this work. The paper rather combines credit constraints and diminishing marginal returns to investment to generate a rich distributional dynamics. The model's simple representation of the structure of heterogeneity (household operated firms) enables to get a closed form tractable solutions,[8] which in most cases uncharacterstic of this literature, that help to derive optimal growth policies in such environments. [9]

The remainder of the paper is organized as follows. Section 2 presents the model. Sections 3 and 4 discuss the dynamics and the steady state in the absence of borrowing, when many of the results can be obtained analytically. Section 5 sets out the dynamics and steady-state implication of the model where the credit market is imperfect, and limited borrowing is permitted. Section 6 describes the calibration of the model, while Section 7 presents numerical simulations for the general model. Section 8 concludes, with technical details being relegated to the Appendix.

2 The Model

2.1 Preferences and Technology

We consider an overlapping generations (OLG) model with a continuum of heterogeneous households, $i \in (0, 1)$. Every period, a household comprises three individuals: a young agent, an adult, and an old agent. The ith household of the initial generation, at time $t = 0$, is endowed with private capital $k_{i,0}$ and has access to g_0 units of a non-rival and a non-excludable public productive input. In the second period of their life, agents are also endowed with a unit of non-leisure time that they supply inelastically to earn income. In the spirit of Benabou (2005) and Angeletos and Calvet (2006), each household operates its

[8] Cagetti and De Nardi (2006) also consider similar entrepreneurial form; however, in their model entrepreneurship is a choice variable where households possess both entrepreneurial and worker ability. Their focus, on identifying the role of credit constraints in the determination of entrepreneurial decisions, is different from us as well.

[9] While there are many other ways of reducing heterogeneity, recent developments show that developing tractable models with such strategy can be useful in macroeconomic analysis. Krusell et al. (2011) consider an environment with no-trade equilibria where autarky is induced with "maximally tight" credit constraints to study asset pricing. Broer et al. (2020) apply that in a New Keynesian model to study the interaction between inequality and monetary policy in a tractable manner. Heathcote et al. (2017) develop a tractable model with zero net supply assets where production only necessitates labor supply to quantify risk-sharing and study how heterogeneity in productivity and preference influence the optimal degree of tax progressivity. See Ragot (2018) for discussion on this class of models and other types of models with reduced heterogeneity.

own firm, using its own labor and capital.[10] Individuals are allowed to borrow and finance their current consumptions while they are young, and to repay the loan (with interest) during adulthood. The remaining income will be used for consumption during adulthood and saving, which, in turn is used to finance old age consumption. Population remains constant over time.

The utility of an agent of the ith household who is born at time t is

$$U_{i,t} \equiv \ln c_{i,t}^{y} + \beta \ln c_{i,t+1}^{a} + \beta^{2} \ln c_{i,t+2}^{o} \tag{1}$$

where $c_{i,t}^{y}$, $c_{i,t+1}^{a}$ and $c_{i,t+2}^{o}$ represent young age, adulthood, and old age consumptions of the agent, respectively. The agent maximizes its utility subject to the budget constraints:[11]

$$(1+\tau_c)c_{i,t}^{y} = q_{i,t} \tag{2}$$

$$c_{i,t+1}^{a}\left(1+\tau_c\right) + s_{i,t+1}^{a} + q_{it}R_{i,t+1} = \left(1-\tau_y\right) w_{i,t+1} + T_{t+1} \tag{3}$$

$$c_{i,t+2}^{o} = s_{i,t+1}^{a} R_{i,t+2} \tag{4}$$

together with the borrowing constraint

$$q_{it}R_{i,t+1} \leq \phi\left[(1-\tau_y)w_{i,t+1} + T_{t+1}\right] \tag{5}$$

where $q_{i,t}$ is the loan used to finance young age consumption; $w_{i,t}$ denotes labor income paid to the adult; $R_{i,t+1}$ and $R_{i,t+2}$are the respective interest rates (returns to capital) at time t, and t+1; $s_{i,t+1}^{a}$ denotes the individual's household adulthood saving, which will be used for capital accumulation in the next period.[12] We assume that labor income and consumption are taxed at the rates $\tau_y$and τ_c respectively.[13] In addition, T_t denotes government transfers distributed uniformly to each household. To sustain an equilibrium constant balanced growth rate, we assume that it is set as a proportion, m, of aggregate labor income, $T_t = mw_t$.[14] Thus, equations (2) - (4) represent the respective budget constraints facing the three members of the household. Equation (5) describes the liquidity constraint facing the young. It asserts that an individual can borrow only a proportion, ϕ, of the present value of his labor income (after-tax income plus transfer), in the spirit of Aiyagari (1994), Jappelli and Pagano (1994) and De Gregorio (1993).

Each household owns a firm that produces output, $y_{i,t}$, using the production function:

$$y_{i,t} = a_1 \varepsilon_{i,t}\left(\alpha\left(g_t\right)^{\rho} + (1-\alpha)\left(k_{i,t}\right)^{\rho}\right)^{(1-\theta)/\rho}\left(\bar{l}_{i,t}k_t\right)^{\theta} \tag{6}$$

where $\varepsilon_{i,t}$ represents the idiosyncratic shocks, k_t is the economy-wide capital that captures spillover effects in the economy and interacts with labor, and g_t

[10] One may consider the capital stock of the household is owned by the old member of the household. As a result, all income due to capital accrues to the old agent.

[11] Note that, variables with(out) subscript i represent individual (aggregate) variables.

[12] Capital is installed a period earlier.

[13] For simplicity and without any loss of generality we assume that the income from capital is untaxed.

[14] We impose the restrictions $0 < \tau_y < 1,\ 0 < \tau_c < 1,\ 0 < m < 1$

denotes public capital that is available to all households. Similar to Castaneda et al. (2003) and Cageti and De nardi (2006), labor is exogenously supplied. $\bar{l}_{i,t} = 1$. This is a two-level production function, in which at the first level, private capital and public capital combine in a CES production technology with an elasticity of substitution $\delta = 1$. This aggregate is then combined in a Cobb-Douglas technology with labor measured in efficiency units to produce final output. We assume that the two capital goods are cooperative in production, (i.e. $\partial^2 y_{i,t} \partial g_t > 0$ which implies $\rho < (1-\theta)$, i.e. $\delta < \theta^{-1}$, thus imposing an upper bound on the elasticity of substitution. We assume that the wage rate received by the adult member is determined by the marginal product of labor, and with $\bar{l}_{i,t} = 1$, this implies $w_{i,t} = \theta y_{i,t}$. The rate of return to private capital is then determined residually and equals$R_{i,t} = (1-\theta) y_{i,t}$.[15] Individual capital endowment, $k_{i,0}$, is assumed to be lognormally distributed: $\ln k_{i,0} \sim N\left(\mu_0, \sigma^2_{k,0}\right)$, while the idiosyncratic shocks are also assumed to be lognormally distributed: $\varepsilon_{i,t} \ln \sim N\left(-\frac{\upsilon^2}{2}, \upsilon^2\right)$ and uncorrelated with endowments.

We assume that both public and private capital depreciate fully within the period. In view of the fact that we are considering a three-period overlapping generations model in which the time unit is a generation of approximately 25 years, this assumption is not only not as restrictive as may at first appear, but indeed is appropriate.[16] It implies the following capital market equilibrium condition:

$$k_{i,t+1} = s_{i,t} \tag{7}$$

where $s_{i,t}$ is the net savings of the household.

Note that by allowing individuals to invest in privately owned firms, and have certain access to a loan market (during young age), we are creating a form of credit market *imperfection*. Doing so enables us to generalize many earlier models with *missing* credit and capital markets (e.g., Loury, 1981, Benabou, 2000, 2002, 2005, Getachew and Turnovsky, 2015).

2.2 Solution to the Household Problem

When the borrowing constraint is not binding, household $i's$ optimization problem leads to the following consumption levels by young members of its household:

$$(1+\tau_c) c^y_{i,t} = \frac{\chi\theta}{R_{i,t+1}} \left[(1-\tau_y) y_{i,t+1} + m y_{t+1}\right] \tag{8}$$

where $\chi \equiv \left[1+\beta+\beta^2\right]^{-1}$. With the liquidity constraint, the consumption of the young individual in the ith household at time t is reduced to

$$(1+\tau_c) c^y_{i,t} = \frac{\phi\theta}{R_{i,t+1}} \left[(1-\tau_y) y_{i,t+1} + m y_{t+1}\right] \tag{9}$$

[15] That is,$R_{i,t} = (y_{i,t} - \theta \bar{l}_{i,t} w_t)/k_{i,t} = (1-\theta) y_{i,t}/k_{i,t}$

[16] For example, assuming an annual depreciation rate of 8% implies a depreciation of 88% after 25 years.

where $\phi < \chi$. In that case, the consumption and saving of the individual upon reaching adulthood at time $t{+}1$, and his resulting consumption upon reaching old age at time $t{+}2$, are respectively

$$(1+\tau_c)c^a_{i,t+1} = \frac{(1-\phi)\theta}{1+\beta}\left[(1-\tau_y)y_{i,t+1} + my_{t+1}\right] = \frac{1}{\beta}s^a_{i,t+1} \tag{10}$$

$$(1+\tau_c)c^o_{i,t+2} = R_{i,t+2}\frac{\beta\theta}{1+\beta}(1-\phi)\left[(1-\tau_y)y_{i,t+1} + my_{t+1}\right] = R_{i,t+2}s^a_{i,t+1} \tag{11}$$

Dividing (10) by (11) we see that the inter-temporal trade-off in marginal utility between adulthood and old age consumptions is independent of the borrowing cost, reflecting the fact that individuals take out loans only while they are young.

The ith household's total saving (wealth) at time t is the saving by the adult member of the household, net of the debt of the young. The saving function of the adult (born at time $t-1$) obtained from (10) is

$$s^a_{i,t} = \frac{\beta}{1+\beta}(1-\phi)\,\theta\left((1-\tau_y)\,y_{i,t} + my_t\right) \tag{12}$$

The debt of the young at time t is simply their consumption at date t, given by (9). Thus, subtracting this from (12) yields the net wealth of the household at date t:

$$s_{i,t} = \frac{\beta}{1+\beta}(1-\phi)\,\theta\left((1-\tau_y)\,y_{i,t} + my_t\right) - \frac{\phi}{R_{t+1}}\theta\left((1-\tau_y)\,y_{i,t+1} + my_{t+1}\right) \tag{13}$$

Thus, the household's investment decision is seen to depend upon both current and future household members' income.[17]

2.3 The government budget and equilibrium

The government uses consumption and income tax revenues, earned in period t to finance transfers in that period, $m\theta y_t$, and public investment, g_{t+1}, in period $t+1$. Aggregating over the individual households, this is described by

$$g_{t+1} + m\theta y_t = \tau_c\left(c^y_t + c^a_t + c^o_t\right) + \tau_y\theta y_t \tag{14}$$

where $c^y_t c^a_t c^o_t$ are aggregates at time t, obtained by summing over (9) - (11).

Intertemporal equilibrium in this economy is defined by: (i) the individuals' optimality conditions, (9) - (11), and savings and investment functions, (7) and (13); (ii) the economy-wide government budget, (14); and (iii) product market equilibrium conditions:

$$y_t = c^y_t + c^a_t + c^o_t + k_{t+1} + g_{t+1} \tag{15}$$

[17] Note that, although capital is installed a period earlier, (**??**) implies a contemporaneous relationship between investment in capital and income. As we see later in Section 5, this leads to a bidirectional causality between inequality and aggregate dynamics.

k_{t+1} is obtained by summing over (7) and (13). In equilibrium, aggregate output is allocated between aggregate consumptions across the generations, public and private investment. With full depreciation of capital in the economy, the latter also represent the next period's capital stocks of the economy.

3 Missing Credit Market ($\phi = 0$)

The case $\phi = 0$ represents missing credit and capital markets, as in Getachew and Turnovsky (2015) and others. It is convenient to begin with this case, since it can be solved analytically, facilitating the underlying intuition. In Section 5 below, we generalize our analysis by allowing for restricted borrowing, although this generalization can be performed only numerically.

3.1 Households optimal decision

Setting $\phi = 0$, the equilibrium decisions of individuals at time t are given by:

$$s_{i,t}^{a} = \nu\theta\tilde{y}_{i,t} \tag{16}$$

$$(1+\tau_c)\, c_{i,t}^{a} = (1-\nu)\,\theta\tilde{y}_{i,t} \tag{17}$$

$$(1+\tau_c)c_{i,t}^{o} = R_{i,t}s_{i,t-1} \tag{18}$$

$$\tilde{y}_{i,t} \equiv (1-\tau_y)\, y_{i,t} + my_t \tag{19}$$

where $\nu \equiv \beta/(1+\beta)$ and $[ERR : mgroupChr : nParams = 1, isTop = 1, iOp = 0x02DC]_{i,t} = \theta[ERR : mgroupChr : nParams = 1, isTop = 1, iOp = 0x02DC]_{i,t}$ represents the individual disposable labor income – after tax labor income plus transfer. In the absence of borrowing, consumption by the young agent is zero. Adult household members consume a fraction of their after tax labor income and save the rest for old age consumption. Old agents in the household consume their total saving plus returns. Optimal individual consumption and savings increase with the transfer, but they decrease with the income tax, whereas the consumption tax has no effect on individual savings.

3.2 Aggregates

Summing over (16) – (19), yields the capital and consumption aggregates at time t

$$k_{t+1} = (1-\tau_y+m)\,\nu\theta y_t \tag{20}$$

$$(1+\tau_c)\, c_t^{a} = (1-\nu)\,(1-\tau_y+m)\,\theta y_t \tag{21}$$

$$(1+\tau_c)c_t^{o} = \int R_{i,t}s_{i,t-1}di = \int R_{i,t}k_{i,t}di = (1-\theta)\, y_t \tag{22}$$

Condition (20) represents the aggregate capital investment at $t+1$, and incorporates the capital market clearing condition $k_{t+1} = s_t$. Eq. (21) is aggregate consumption of the adults as a fraction of disposable aggregate income. Aggregating total saving plus return in the previous period gives current consumption for the old agents (22). With consumption by the young agent being zero,

$c_t^y = 0$, and accordingly, the government budget constraint under the missing capital market is given by (14), but with the corresponding consumption tax revenue $\tau_c c_t^y = 0$.

Note also that given m, τ_y, and τ_c are constant, the fraction of GDP invested in infrastructure is also constant, ensuring a long-run balanced growth path. Substituting (20)-(22) into (15), we obtain

$$g_{t+1} = \psi y_t \tag{23}$$

where

$$\psi \equiv \frac{\tau_c}{1+\tau_c}\left[(1-\nu)\left(1-\tau_y+m\right)\theta + (1-\theta)\right] + (\tau_y - m)\theta \tag{24}$$

From the restrictions on τ_y, τ_c, m, noted in footnote 13, it is straightforward to see $\psi < 1$. However, m should be sufficiently small relative to τ_y for ψ to be positive. Therefore, the following additional restriction in the policy parameters is necessary and sufficient to guarantee a positive level of public investment, $\psi > 0$,

'$m - \tau_y < \frac{\tau_c(1-\theta v)}{\theta(1+\tau_c \nu)}$

This implies that if $\tau_c = 0$, the rate of government expenditure on transfers should be less than its rate of income tax. But with additional revenue from the consumption tax, the government transfer could exceed the income tax rate as long as the transfer rate is kept sufficiently small (vis-a-vis τ_y and τ_c).

3.3 Constancy of aggregate ratios

Combining (20) and (23), the aggregate private-public capital ratio, is given by,

$$\frac{k_{t+1}}{g_{t+1}} = \frac{(1-\tau_y+m)\nu\theta}{\psi} \equiv \varphi \tag{25}$$

which is constant. This condition no longer holds under partially available credit (Section 5) where the capital ratio displays transitional dynamics, before it converges to its long run equilibrium value. Combining (25) with (24), we may write:

$$\varphi = \varphi(\tau_y, \tau_c, m; \nu, \theta)\,, \; Where \;\; \varphi_{\tau_y} < 0,\; \varphi_{\tau_c} < 0,\; \varphi_m > 0\,,\; \varphi_{\tau_y} = -\varphi_m \tag{26}$$

Since taxes are used to finance public capital, the private-public capital ratio (φ) varies inversely with τ_c and τ_y. In contrast, by reducing the available funds for public investment, an increase in the fraction of income, m allocated to transfers, increases the ratio, φ.

The aggregate consumption-output ratio is derived from (21) – (22)

$$\frac{c_t}{y_t} = \frac{c_t^a + c_t^o}{y_t} = \frac{(1-\nu)\left(1-\tau_y+m\right)\theta + (1-\theta)}{(1+\tau_c)} \tag{27}$$

Also, from (20), one can readily compute the aggregate output-capital ratio:

$$\frac{y_t}{k_{t+1}} = \frac{1}{(1-\tau_y+m)\,v\theta} \tag{28}$$

Therefore, at the aggregate level, public and private capital (g_{t+1} and k_{t+1}, respectively) grow at the same rate at all times, while consumption (c_t) and income (y_t) also grow at the same rate but with a one period delay.

As we shall discuss more in Section 4, the macroeconomic equilibrium is characterized by transitional dynamics. This is despite the fact that production is characterized by a one-sector Ak technology, as in Romer (1986), where the economy is always on its balanced growth path. The difference is due to (i) the presence of idiosyncratic technological shocks, coupled with (ii) the absence of borrowing.

4 Equilibrium dynamics of inequality, capital, and output

In this section we characterize the equilibrium dynamics of inequality and key aggregates. An appealing property of the log-normal distribution is that it facilitates aggregation, details of which are provided in Appendix A. There we show that the macroeconomic equilibrium is summarized by the following relationships, where the suppression of the index i identifies aggregates, and inequality is measured by variances across the distribution of agents.

1. **Dynamics of inequality**

$$\sigma_{k,t+1}^2 = \sigma_{c,t}^2 = \sigma_{\tilde{y},t}^2 = \ln\left(\lambda^2\left(e^{\sigma_{y,t}^2} - 1\right) + 1\right) < \sigma_{y,t}^2 \tag{29}$$

where $\lambda \equiv (1 - \tau_y),\ \lambda_{\tau_y} < 0,\ \lambda_m < 0$ and

$$\sigma_{y,t}^2 = \left((1-\theta)/\rho\right)^2 \ln\ z_t + \upsilon^2 \tag{30}$$

$$z_t \equiv \frac{x_t^2}{(x_t+1)^2}\left(e^{\rho^2\sigma_{k,t}^2} - 1\right) + 1 \tag{31}$$

$$x_t \equiv (1-\alpha)\,\alpha^{-1}\varphi^{\rho} e^{\sigma_{k,t}^2\rho(\rho-1)2} \tag{32}$$

Substituting (32) into (31) and thence into (30) and (29), yields an autonomous dynamic system determining the evolution of wealth inequality, $\sigma_{k,t+1}^2$. Once $\sigma_{k,t}^2$ is known, these three equations immediately determine, $\sigma_{y,t}^2$, $\sigma_{\tilde{y},t}^2$, and $\sigma_{c,t}^2$. The distribution of the next-period capital ($\sigma_{k,t+1}^2$) is similar to that of the current consumption distribution ($\sigma_{c,t}^2$) and the distribution of net income ($\sigma_{\tilde{y},t}^2$), but it exceeds the current gross income distribution,($\sigma_{y,t}^2$), due to the transfer program. This is because the function $\lambda(\tau_y, m) < 1$ reflects the proportion of income subject to idiosyncratic risk, which is reduced, to the extent to which agents receive identical transfers ($m > 0$). It is intuitive that next-period's capital investment and current consumption have similar distributions. This is because current individual savings are simply the next-period capital investment, due to complete depreciation and zero adjustment cost of capital. Since both consumption and saving are linear functions of individual disposable income, the two have similar distributions, whereas the latter determines the dynamics of capital. In the absence of transfers, $\lambda = 1$ and (29) implies

$$\sigma^2_{k,t+1} = \sigma^2_{c,t} = \sigma^2_{\tilde{y},t} = \sigma^2_{y,t} \tag{33}$$

Otherwise, they are related as in (29).

2. **Dynamics of aggregate output**

$$y_t = a_2 \varphi^\theta g_t \left(x_t + 1\right)^{(1-\theta)/\rho} z_t^{(1-\theta)(1-\theta-\rho)/\left(2\rho^2\right)} \tag{34}$$

where $a_2 \equiv a_1 \alpha^{(1-\theta)\rho}$.

3. **Dynamics of aggregate private capital accumulation**

$$k_{t+1} = \left(1 - \tau_y + m\right) \nu \theta y_t \tag{35}$$

4. **Dynamics of public capital accumulation**

$$g_{t+1} = \psi a_2 \varphi^\theta g_t \left(x_t + 1\right)^{(1-\theta)/\rho} z_t^{(1-\theta)(1-\theta-\rho)/\left(2\rho^2\right)} \tag{36}$$

The source of the dynamics is the heterogeneity in the initial endowments of capital, $\sigma^2_{k,0}$. If $\sigma^2_{k,0} = 0$, then (29) and (30) reduce to $\sigma^2_{y,t} = \upsilon^2$, $\sigma^2_{k,t+1} = \sigma^2_{c,t} = \sigma^2_{\tilde{y},t} = \ln \lambda^2 (e^{\upsilon^2} - 1) + 1)$, which depend only upon the exogenous variance in the productivity shocks and the policy parameters inλ. Aggregate output reduces to $y_t = a_2 \varphi^\theta g_t \left(\left(1-\alpha\right) \alpha^{-1} \varphi_t^\rho + 1\right)^{(1-\theta)/\rho}$ from which k_{t+1}, g_{t+1} follow.

4.1 Transitional dynamics

Taking logarithms of equations (35) and (36), we see that during the period $(t, t{+}1)$, private and public capital grow at the common rate

$$\gamma^g_{t+1} \equiv \gamma^k_{t+1} \equiv \ln g_{t+1} - \ln g_t = \ln(\psi a_2 \varphi^\theta) + \left(\frac{1-\theta}{\rho}\right) \ln\left(x_t + 1\right) \quad + \frac{1}{2}\left(1 - \frac{\rho}{1-\theta}\right)\left(\sigma^2_{y,t} - \upsilon^2\right) \tag{37}$$

while during the same period output and consumption grow at the common rate

$$\gamma^y_{t+1} \equiv \ln y_{t+1} - \ln y_t = \ln(\psi a_2 \varphi^\theta) + \left(\frac{1-\theta}{\rho}\right) \ln\left(x_{t+1} + 1\right) \quad + \frac{1}{2}\left(1 - \frac{\rho}{1-\theta}\right)\left(\sigma^2_{y,t+1} - \upsilon^2\right) \tag{38}$$

so that the growth rate of output leads that of capital by one period.

From equations (17) to (36) it is evident that the transitional dynamics of the economy are driven entirely by the evolution of income inequality. This causality contrasts sharply with that obtained in the class of inequality-growth models developed by Caselli and Ventura (2000), and

Turnovsky and García-Peñalosa (2008), for example, where their underlying assumptions permit exact aggregation as pioneered by Gorman (1959).[18] Under those assumptions, the macroeconomic equilibrium is determined independently of the distribution across agents, while the distribution is then determined by returns to capital and labor generated by the aggregates.[19] The reversal of the causality here arises because of a combination of two factors: (i) the absence of a credit market, and (ii) diminishing returns to individual investment. The inability to borrow implies that productive investment opportunities may be foregone. With diminishing returns to investment, the poor have a higher marginal product than do the rich. Therefore, greater inequality is associated with a loss of productive efficiency, leading to lower growth.

4.2 Local stability

With the transitional dynamics being driven by the evolution of inequality, the condition for local stability can be established by considering equations (29)-(32), from which one can derive

$$\frac{\partial \sigma_{k,t+1}^2}{\partial \sigma_{k,t}^2} = \left(\frac{\lambda^2 e^{\sigma_{y,t}^2}}{e^{\sigma_{k,t+1}^2}}\right) \frac{(1-\theta)^2}{\rho} \left(\frac{z_t - 1}{z_t}\right) \left((\rho - 1)\frac{1}{1+x_t} + \rho \frac{e^{\rho^2 \sigma_{k,t}^2}}{e^{\rho^2 \sigma_{k,t}^2} - 1}\right) \equiv D_{t+1} \tag{39}$$

It is straightforward to show that $D_t > 0$, in which case $D_t < 1$ is necessary and sufficient for the evolution of inequality to be locally stable, from which the stability of the rest of the aggregate economy, including the growth rate follows.

4.3 Steady state

Assuming that the stability condition $0 < D < 1$ is met, the steady-state equilibrium net income inequality, $\sigma_{\tilde{y}}^2 = \sigma_k^2$, gross income inequality, σ_y^2, are determined jointly (together with x) by[20]

$$\sigma_k^2 = \ln\left(\lambda^2 \left(e^{\sigma_y^2} - 1\right) + 1\right) \tag{40}$$

$$\sigma_y^2 = \left((1-\theta)/\rho\right)^2 \ln\left[\frac{x^2}{(x+1)^2}\left(e^{\rho^2 \sigma_k^2} - 1\right) + 1\right] + \upsilon^2 \tag{41}$$

$$x \equiv (1-\alpha)\,\alpha^{-1} \varphi^{\rho} e^{\sigma_k^2 \rho(\rho-1)2} \tag{42}$$

[18] The key assumptions are homogeneity of the underlying utility functions and perfect factor markets, in which all agents earn the same rates of return.

[19] Caselli and Ventura (2000) characterized these models as embodying the "representative consumer theory of distribution". It is very versatile and many applications are discussed in detail by Turnovsky (2020).

[20] We drop the subscript t to denote steady state-variables.

from which the steady-state growth rate, γ, is then obtained

$$\gamma = \ln((1-\tau_y+m)\nu\theta a_2\varphi^{\theta-1}) + \left(\frac{1-\theta}{\rho}\right)\ln(x+1) + \frac{1}{2}\left(1-\frac{\rho}{1-\theta}\right)\left(\sigma_y^2-\upsilon^2\right) \tag{43}$$

These equations define the long-run tradeoffs between gross income inequality, (σ_y^2), net income inequality = capital investment inequality, (σ_k^2), and the equilibrium growth rate (γ). This raises the question of the channels through which tax and expenditure policies may influence these tradeoffs. To examine this issue, it is convenient to write the solutions for the equilibrium inequality in the form

$$\sigma_k^2 = f\left(\lambda(\tau_y,m),\varphi(\tau_y,\tau_c,m)\right) \tag{44}$$

$$\sigma_y^2 = h\left(\lambda(\tau_y,m),\varphi(\tau_y,\tau_c,m)\right) \tag{45}$$

These two equations reveal that in general there are two channels through which policy affects the dynamics of inequality. The first is a direct *income effect* described by the term $\lambda(\tau_y,m)$. From (29) it is seen that a change in λ(due to a change τ_y or m or both) will have an impact on the distribution of after-tax income $(\sigma_{\tilde{y},t}^2)$ and thence on the subsequent dynamics. This effect depends upon the allocation of some fraction of GDP to transfers, and in its absence $(m=0)$ this term reduces to $\lambda\equiv 1$ and this effect disappears. The second is an indirect *substitution effect* via changing the composition of the public-private inputs in the production process. A change in the fiscal structure changes the private-public capital ratio (φ) that in turn influences individual before-tax income, and determines the distribution of the next period investment and income. An increase in either tax rate or a decline in transfer decreases φ, and vice versa. From (44)-(45) we see that in general the income tax and the transfer give to both a direct income effect and a substitution effect, whereas the consumption tax has only the latter effect.

To examine these effects and the growth-inequality tradeoffs they generate, from the steady-state conditions (23), we derive the following:

$$\frac{\partial\sigma_k^2}{\partial\lambda} = \frac{2\lambda(e^{\sigma_y^2}-1)}{e^{\sigma_k^2}}\frac{1}{(1-D)} > 0; \quad \frac{\partial\sigma_y^2}{\partial\lambda} = \frac{2(e^{\sigma_y^2}-1)}{\lambda e^{\sigma_y^2}}\left(\frac{D}{1-D}\right) > 0 \tag{46}$$

$$\frac{\partial\sigma_k^2}{\partial\varphi} = \frac{\lambda^2 e^{\sigma_y^2}}{(1-D)}\left(\frac{\rho}{\varphi}\right)\left(\frac{1-\theta}{\rho}\right)^2\frac{2x}{z(1+x)^3}(e^{\rho^2\sigma_k^2}-1); \quad \frac{\partial\sigma_y^2}{\partial\varphi} = \frac{e^{\sigma_k^2}}{\lambda^2 e^{\sigma_y^2}}\frac{\partial\sigma_k^2}{\partial\varphi} \tag{47}$$

Thus, an increase in the fraction of income that is subject to idiosyncratic risk increases inequality. In contrast, the extent to which an increase in the ratio of private to public capital impacts inequality is determined by the elasticity of substitution between the two capital goods. This is because public capital serves both as a substitute and a complement to private capital, depending upon its specific nature. If the elasticity of substitution exceeds unity, and public capital is easily substituted for private capital, this will tend to benefit the poorer households with relatively low endowments of private capital as it relaxes credit constraints that impede their investment. Inequality with therefore tend to decline. However, if public capital is primarily complementary to private capital, which is owned disproportionately by the rich, who have a relatively lower marginal product due to diminishing returns to investment, the benefits of more public investment will accrue to them and inequality will increase.

Combining the responses summarized in (25) with the direct impact of taxes and expenditures we may derive the following propositions:

Proposition 1: Suppose transfers $m > 0$. An increase in the income tax rate, τ_y, will (i) reduce λ, the direct effect of which is to reduce income inequality; (ii) reduce φ, the effect of which is to reduce income inequality, if and only if the elasticity of substitution between the public and private capital goods,$\delta > 1$. In this case both effects are reinforcing and income inequality declines. If $\delta < 1$ the two effects are offsetting and the net effect on income inequality depends upon which effect is dominant.

Proposition 2: An increase in the transfer rate, m, will (i) reduce λ, the direct effect of which is to reduce income inequality; (ii) increase φ, the effect of which is to reduce income inequality if and only if the elasticity of substitution between the public and private capital goods,$\delta < 1$. In this case both effects are reinforcing and income inequality declines. If $\delta > 1$ the two effects are offsetting and the net effect on income inequality depends upon which effect is dominant.

Proposition 3: An increase in the consumption tax rate, τ_c, will reduce φ, the effect of which is to reduce income inequality if and only if the elasticity of substitution between the public and private capital goods,$\delta > 1$.

Differentiating (43) and using (40) we see that the resulting impact of the policy changes on the mean growth rate is given by

$$\begin{aligned} d\gamma = -\tfrac{d\tau_y}{1-\tau_y} - \left(1 + \tfrac{(1-\theta)(1-\rho)x}{2(1+x)}\left(\tfrac{e^{\sigma_k^2}-1}{e^{\sigma_k^2}}\right)\right)\tfrac{d\lambda}{\lambda} - \left(\tfrac{1-\theta}{1+x}\right)\tfrac{d\varphi}{\varphi} \\ +\tfrac{1}{2}\left(\left(\tfrac{1-\theta-\rho}{1-\theta}\right) + \tfrac{(1-\theta)(\rho-1)x}{2(1+x)}\tfrac{e^{\sigma_y^2}}{e^{\sigma_k^2}}\right)d\sigma_y^2 \end{aligned} \tag{48}$$

From (48) we see that tax and expenditure policies impact the mean growth rate both directly and indirectly through a range of channels. We may summarize these in

Proposition 4: The direct effect of an increase in the income tax rate is to reduce the growth rate. In addition, to the extent that the income tax rate and transfer increase the income effect, λ, and the substitution effect, φ, the growth rate is further reduced. These effects are further compounded by their impact on income inequality.

4.4 Cobb-Douglas technology

By applying L'Hôpital's rule to (29) - (32), we can derive the distribution of capital and income, respectively, in the Cobb-Douglas case ($\rho = 0$),

$$\sigma_{k,t+1}^2 = \ln\left(\lambda^2\left(e^{((1-\alpha)(1-\theta))^2\sigma_{k,t}^2+\upsilon^2} - 1\right) + 1\right) \tag{49}$$

and

$$\sigma_{y,t}^2 = ((1-\alpha)(1-\theta))^2 \sigma_{k,t}^2 + \upsilon^2 \tag{50}$$

where $(1-\alpha)(1-\theta)$ is the factor share of $k_{i,t}$ in this case. We see from (50) that the substitution effects of inequality have disappeared and there is only one channel through which policy could affect inequality, namely the income effect, λ. In the Cobb-Douglas case, factor shares are constant, and hence independent of the factor's proportion, leading to the distributional neutrality of policy.

Aggregate income in the Cobb-Douglas case is given by,

$$y_t = a_1 \varphi^{-\alpha(1-\theta)} k_t e^{0.5\sigma_{k,t}^2(1-\alpha)(1-\theta)((1-\alpha)(1-\theta)-1)}$$

from which we derive the growth rate of per capita labor income,

$$\gamma_t^y = \ln\left((1-\tau_y+m)\,\nu\theta a_1 \varphi^{\alpha(1-\theta)}\right) - \frac{1}{2}\frac{1-(1-\alpha)(1-\theta)}{(1-\alpha)(1-\theta)}\left(\sigma_{y,t}^2 - \upsilon^2\right) \tag{51}$$

The steady state inequality and growth are given by, respectively,

$$e^{\sigma_k^2} - 1 = \lambda^2 (e^{((1-\alpha)(1-\theta))^2\sigma_k^2+\upsilon^2} - 1) \tag{52}$$

and,

$$\gamma^y = ln((1-\tau_y+m)\nu\theta a_1\varphi^{\alpha(1-\theta)}) - 1/2(1-(1-\alpha)(1-\theta))(1-\alpha)(1-\theta)\sigma_k^2 \tag{53}$$

where σ_k^2 is implicitly defined in (52).

5 Imperfect capital market ($\chi > \phi > 0$)

In the more general case of restricted borrowing, $\chi > \phi > 0$, the ith household's private capital at time $t+1$, are given by (13) and (7). Recalling that the return to capital is, $R_{i,t+1} = (1-\theta)y_{i,t+1}$, this can be rewritten as

$$k_{i,t+1} = a\left(y_{i,t} + \overline{m}y_t\right) - b\left(k_{i,t+1} + \overline{m}y_{t+1}\frac{k_{i,t+1}}{y_{i,t+1}}\right) \tag{54}$$

where[21]

$$a \equiv \theta(1-\phi)\,\nu\,(1-\tau_y)\,; b \equiv \frac{\theta}{1-\theta}\phi\,(1-\tau_y)\,; \overline{m} \equiv \frac{m}{1-\tau_y}$$

The first term on the right hand side of (54) is the net saving of the adult household member, while the second term represents the consumption of the young.

[21] Recall that the labor share received by household i is $w_{i,t} = \theta y_{i,t}$, implying that the share of capital earned by the household is $R_{i,t+1}k_{t+1} = (1-\theta)\,y_{i,t+1}$.

Young age, adulthood, and old age consumptions ($c_{i,t}^y$, $c_{i,t}^a$ and $c_{i,t}^o$, respectively), given by (9) - (11) may be rewritten as follows:

$$(1+\tau_c)\, c_{i,t}^y = s_{i,t}^a - k_{i,t+1} = b\left(k_{i,t+1} + \overline{m} y_{t+1} \frac{k_{i,t+1}}{y_{i,t+1}}\right) \tag{55}$$

$$(1+\tau_c)\, c_{i,t}^a = (1-\nu)\,(1-\phi)\,\theta\,[(1-\tau_y) y_{i,t} + m y_t] = \frac{1}{\beta} s_{i,t+1}^a \tag{56}$$

$$(1+\tau_c)\, c_{i,t}^o = R_{i,t} s_{i,t-1}^a = \left((1-\theta)\,\frac{y_{i,t}}{k_{i,t}}\left(s_{i,t-1} + (1+\tau_c) c_{i,t-1}^y\right)\right) = p y_{i,t} + \phi\theta m y_t \tag{57}$$

where $p \equiv (1-\theta+\phi\theta\,(1-\tau_y))$. Thus, the young member of the household consumes effectively the household's current savings net of investment. The adult consumes a fraction of his current income, which equals β^{-1} times the young agent's future savings. Such an intergenerational resource transfer has implications for both the individual and aggregate dynamics in the economy. Finally, the old household member consumes the household's savings in the previous period inclusive of its return.

5.1 Aggregate capital and inequality dynamics

Aggregating (54) yields the economy-wide rate of private capital accumulation,

$$k_{t+1}\left(1 + b + e^{\upsilon^2} s_{t+1} E h_{i,t+1}\right) = a y_t\,(1+\overline{m}) \tag{58}$$

where

$$h_{i,t+1} \equiv \left(a_2 \varphi_{t+1}^{\theta}\right)^{-1}\left(1 + \frac{1-\alpha}{\alpha}\left(\varphi_{i+1,t}\right)^{\rho}\right)^{\frac{\theta-1}{\rho}} \varphi_{i,t+1} \tag{59}$$

$$s_{t+1} \equiv b\overline{m}\frac{y_{t+1}}{g_{t+1}} \tag{60}$$

Recall that $\varepsilon_{i,t+1}$ is iid and independent of the distribution of $k_{i,t+1}$. Given that $\varphi_{i,t+1}$ is lognormally distributed, we can aggregate $Eh_{i,t+1}$ numerically. Dividing both sides of (58) by k_t, the growth rate of aggregate capital is given by

$$\gamma_{t+1} = \ln\frac{y_t}{k_t} - \ln\varpi_{t+1} \tag{61}$$

where

$$\varpi_{t+1} \equiv \frac{y_t}{k_{t+1}} = \left(\frac{1 + b + e^{\upsilon^2} s_{t+1} E h_{i,t+1}}{a\,(1+\overline{m})}\right) \tag{62}$$

The dynamics of inequality under imperfect credit market are also derived in Appendix B:

$$Var\left(\Phi_{i,t+1}\right) = E\Phi_{i,t+1}^2 - \left(E\Phi_{i,t+1}\right)^2 = a^2\left(\varpi_{t+1}\varphi_{t+1}\right)^2\left(e^{\sigma_{t,y}^2} - 1\right) \tag{63}$$

where

$$E\Phi_{i,t+1}^2 = (1+b)^2 \varphi_{t+1}^2 e^{\sigma_{k,t+1}^2} + 2s_{t+1}(1+b)e^{\upsilon^2}E\left[h_{i,t+1}\varphi_{i,t+1}\right] + e^{3\upsilon^2}s_{t+1}^2 Eh_{i,t+1}^2 \tag{64}$$

$$E\Phi_{i,t+1} = \varphi_{t+1} + b\varphi_{t+1} + e^{\upsilon^2}s_{t+1}Eh_{i,t+1} \tag{65}$$

It should be noted that if $\phi = 0$, (34) reduces to (29)-(32). Thus, the role of φ in inequality depends not only on ρ but also on the severity of the liquidity constraint, as represented by ϕ.

Comparing the equilibrium dynamics of the economy, summarized in equations (33) and (34) with those reported previously in (17), highlights how the introduction of the (limited) ability to borrow introduces a fundamental difference into the structure of the equilibrium dynamics. In contrast to the previous equilibrium, the dynamic relation between inequality and other macroeconomic variables is bidirectional. While if $\phi = 0$ inequality is the driving force behind the evolution of the aggregate quantities, if $\phi > 0$ the latter also influence the evolution of inequality. This is a result of intergenerational resource transfer at the individual level combined with the credit market imperfection, enabled by the limited ability to borrow.

Consumption aggregates are derived from (55) - (57):

$$(1+\tau_c)\,c_t^y = s_t^a - k_{t+1} = ay_t\,(1+\overline{m}) - k_{t+1} \tag{66}$$

$$(1+\tau_c)\,c_t^a = (1-\nu)(1-\phi)\,\theta\,(1-\tau_y+m)\,y_t = \frac{a}{\beta}(1+\overline{m})\,y_t \tag{67}$$

$$(1+\tau_c)c_t^o = (1-\theta+\phi\theta\,(1-\tau_y+m))\,y_t \tag{68}$$

Comparing (35) to the case with missing credit market, now there is a positive young age consumption in the economy $c_t^y > 0$. Aggregate young age consumption increases in ϕ with credit availability but later age consumption decreases. The reason for that is young age consumption is suboptimal with credit constraints. In contrast to the case without credit market, old age consumption is a function after tax income. Fiscal policy becomes more involved as individuals transfer resources between early and later stages of their life.

Combining (66) to (68) with (14) we obtain the dynamics for public investment:

$$g_{t+1} = \frac{1}{1+\tau_c}\left(\tau_c + \theta\,(\tau_y - m)\right)y_t - \frac{\tau_c}{1+\tau_c}k_{t+1} \tag{69}$$

The first term shows the share of consumption and income taxes (net of transfer) that goes to public investment. But, of course, this time public investment includes taxes from young age consumption, which is financed through borrowing and paid back at a later stage. The last term captures part of the consumption tax raised during young age and repaid at the later stage. From (54) and (69), we get the private to public capital ratio under imperfect capital market:

$$\frac{k_{t+1}}{g_{t+1}} = (1+\tau_c)\left(\left(\tau_c + \theta\,(\tau_y - m)\right)\varpi_{t+1} - \tau_c\right)^{-1} \tag{70}$$

Thus, in contrast to the case of missing credit market, the private capital to public capital ratio is not constant (φ_t), but evolves with inequality. This is most apparent by applying L'Hôpital's rule to $h_{i,t+1}$ (when $\rho = 0$ but $\phi > 0$), which gives a closed form solution: $h_{i,t+1} = \left(a_1 \varphi_{t+1}^{\theta}\right)^{-1} \varphi_{i,t+1}^{\zeta}$ where $\zeta \equiv \alpha + \theta(1-\alpha)$ and $Eh_{i,t+1} = (a_1)^{-1} \varphi_{t+1}^{\zeta-\theta} e^{0.5(\zeta-1)\zeta\sigma_{k,t+1}^2}$. Substituting the latter in to (62) and then in to (70), we see the capital ratio displays transitional dynamics, $\varphi_{t+1} = z\left(\sigma_{k,t+1}^2\right)$.[22] Equations (34) and (70) represent the dynamic system that characterizes the economy with the imperfect credit market.

5.2 Steady State inequality, growth, and welfare

Assuming convergence, steady-state inequality is easily derived from (63), and is given by the following implicit function,

$$E\Phi^2 - (E\Phi)^2 = a^2 (\varpi\varphi)^2 \left(e^{\sigma_y^2} - 1\right) \tag{71}$$

whereas the steady state growth rate is,

$$\gamma = \ln\frac{y}{k} - \ln\varpi \tag{72}$$

where

$$\varpi \equiv \left(\frac{1 + b + e^{\upsilon^2} sEh_i}{a(1+\overline{m})}\right)$$

$$E\Phi^2 = (1+b)^2 \varphi^2 e^{\sigma^2} + 2s(1+b) e^{\upsilon^2} E[h_i \varphi_i] + e^{3\upsilon^2} s^2 Eh_i^2$$

$$E\Phi_i = \varphi + b\varphi + e^{\upsilon^2} sEh_i$$

$$s \equiv b\overline{m}\frac{y}{g}$$

$$\frac{y}{k} = a_2 \varphi^{\theta-1} (x+1)^{(1-\theta)/\rho} z^{(1-\theta)(1-\theta-\rho)/\left(2\rho^2\right)}$$

$$z \equiv \left(x^2 \left(e^{\rho^2\sigma_k^2} - 1\right)(x+1)^{-2} + 1\right)$$

$$x \equiv (1-\alpha)\alpha^{-1} \varphi^{\rho} e^{\sigma_k^2 0.5\rho(\rho-1)}$$

The steady-state private-public capital ratio, under imperfect credit market, is given by:

$$\varphi = (1+\tau_c)\left((\tau_c + \theta(\tau_y - m))\varpi - \tau_c\right)^{-1} \tag{73}$$

Aggregate welfare of the family at time t is similarly derived as (see Appendix C for details):

$$W_t = \ln\left(a(1+\overline{m}) - \varpi_{t+1}\right) + \Xi + 3\ln y_t - \frac{1}{2}\left(\sigma_{c^y,t}^2 + \sigma_{c^a,t}^2 + \sigma_{c^o,t}^2\right) \tag{74}$$

[22] We also obtain solutions for inequality and capital ratio dynamics, by applying second-order Taylor series expansion on the general equation (??) ($\rho \neq 0$ and $\phi >$) around *the expected values*, $Ek_{i,t} = k_t$ and $E\varepsilon_{i,t} = 1$, that show the bidirectional dynamics, omitted for brevity.

where

$$\Xi \equiv \ln\left(\frac{a}{\beta}\left(1+\overline{m}\right)\right) + \ln\left(\left(1-\theta+\phi\theta(1-\tau_y)\left(1+\overline{m}\right)\right)\right) - 3\ln(1+\tau_c) \quad (75)$$

From (74), it is interesting to note that aggregate welfare at time t is also affected by the net income distribution in the next period. Such a relationship is a result of existence of an inter-temporal imperfect credit market.

As shown in Appendix C, the steady-state welfare that represent all generations is given by

$$W^s = \frac{1+R}{R}\left(\ln\left(a\left(1+\overline{m}\right)-\varpi\right) + \Xi + 3\ln y_0 + \frac{3\gamma}{R} + \gamma - \frac{1}{2}\left(\sigma^2_{c^y} + \sigma^2_{c^a} + \sigma^2_{c^o}\right)\right) \quad (76)$$

Compared to the case of missing credit market and no borrowing, steady-state welfare increases by young age consumption but decreases by its distribution (inequality).

We study the macroeconomic effects of fiscal policy and the role of precautionary motives in the economy numerically in the next.

6 Numerical analysis

In this section, we supplement our previous analytical results by studying the quantitative effects of consumption and income taxes and transfers on steady-state market and net income distributions, growth, and welfare. As indicated, our motivation is the South African economy, and we evaluate the effectiveness of alternative public policy choices *vis-a-vis* the NDP 2030 targets. We start by calibrating a benchmark economy, choosing parameter values most of which are standard and reasonably reflect the current South African economy. The chosen benchmark parameters are summarized in Table 1.

6.1 Calibration

Row 1 of Table 1 summarizes the preference parameters. We assume an annual discount factor of 0.96, which matches a 4.17 percent rate of time preference in infinite lived agent models. Assuming that a generation is approximately 25 years, this implies $\beta = 0.96^{25} \approx 0.35$, as the private generational discount factor. The social planner's discount factor $(1+R)^{-1}$ measures the relative weight he assigns to the utility of successive generations and has no particular relationship to β. Setting $R = 0.3$ implies a generational discount factor of 0.77, while seemingly plausible is purely illustrative. Furthermore, the only role that Rplays is in assessing the welfare associated with any equilibrium.[23] There are no available estimates for the value of ϕ. Accordingly, our approach is to experiment for different values, $\phi = \{0, 0.20, 0.40\}$, with increasing ϕ

[23] Welfare is computed by calculating (**??**) for the equilibrium values.

representing a weakening of the liquidity constraint, and is associated with a weakening of the precautionary motive.

Turning to the production parameters, we set the benchmark value for the labor elasticity, θ at 0.53, which is somewhat lower than its typical international value (around 0.60), but nevertheless reflects the South African economy. Using data from Statistics South Africa, Burger (2015) estimated labor's share for South Africa 56 percent in 1993, which subsequently declined to 48 percent in 2008 and then increased to 52% in 2014. Getachew and Turnovsky (2015) estimated the elasticity of substitution between public and private capital for a set of Sub Saharan African countries to range between 0.466 and 0.564. In view of this, we set the baseline elasticity of substitution at 0.54 (or $\rho \approx -0.85$), which characterizes strong complementarity between the two types of capital.[24] This matches closely their calibration of the elasticity of substitution that is associated with the share of public capital $\alpha = 0.3$ and the public capital elasticity of output 0.2. The latter is consistent with Bom and Ligthart (2014) whose comprehensive study summarizes 578 estimates and found the average productivity elasticity of public capital to be 0.19. The value for the share of public capital is also within the ranges of the plausible values parametrized by Eden and Kraay (2014). Finally, we set the variance for the idiosyncratic shocks, $\upsilon^2 = 0.16$ (Bartelsman et al., 2013).

The policy parameters provided in the bottom line of the table are justified as follows. First, we set $m = 0.0412$, following the social benefit expense in South Africa, which averaged about 4.12 percent between 1996 and 2016 for general government transfers, based on the IMF database.[25] From Africa Development Indicators, the World Bank, government public investment spending and private investment in South Africa are about 4.26 and 13.16 percent respectively, between 1988 and 2011.[26] We set the income tax rate (τ_w) associated with financing public capital at 11.96%.[27] We set the consumption tax rate in the benchmark economy at $\tau_c = 0$; however, we also experiment with positive values of the consumption tax that we substitute for the income tax. While we calibrate a_1 to match the South African long run growth rate of about 1.3%, we set the capital-output ratio to reflect the relatively lower saving rate, of less than 0.13. In the standard representative agent model that assume a unit period of one year, a stylized value for capital-output ratio is about 2.5. In our case, with the time unit being of the order of 25 years, this would correspond to a ratio of about 0.1.

[24] We also perform sensitivity analysis for a different value of ρ

[25] IMF data available online at: http://data.imf.org/?sk=a0867067-d23c-4ebc-ad23-d3b015045405&sId=1544448157598.

[26] https://databank.worldbank.org/reports.aspx?source=africa-development-indicators

[27] We should note that the average income tax rate in South Africa over this period was around 42%. However, most of this was devoted to financing a public consumption good, which is excluded from in our analysis.

6.2 Growth, inequality, and welfare effects of taxes and transfer

Table 2 presents the steady state macroeconomic effects of increased income and consumption taxes and transfers, for ϕ ranging from 0 to 0.40.[28] An increase in either tax rate of 2 percentage points increases the growth rate, and hence welfare. Taxes stimulate growth by boosting public investment, which is initially set at a sub-optimally low level. This reduces the private to public capital ratio (k/g) and triggers the substitution effect. The consumption tax has a much stronger positive impact on growth, and hence welfare, due to its benign impact on private investment. As public investment is complementary to private investment, the reduction in the k/g ratio increases gross income inequality, which in turn leads to higher net income inequality. But the increase in income tax, unlike the consumption tax, also has a direct positive effect on net inequality (as reflected by a decrease in λ). This offsets some of the adverse substitution effects of the increased income tax on inequality. Accordingly, a two percentage point increase in the consumption tax will increase net inequality between about $1.70 - 2.07\%$, whereas a similar increase in the income tax would reduce the increase to $1.05 - 1.22\%$.

We also see in Table 2 that an increase in the transfer alone, at the expense of public investment, may favor income distribution. But by putting downward pressure on the already sub-optimally low public investment, it leads to a negative growth rate, with a significant adverse consequence for welfare. Although the effect on net inequality is strong, decreasing it between 3.65-3.74%, it reduces welfare by 4.2-2.6% for $\phi = 0.20$ to $\phi = 0.40$.[29] The increase in the private to public capital ratio ($> 18\%$) also contributes to a significant decrease in market inequality.

If the increase in transfer is financed by an identical increase in the income tax rate ($\Delta m = \Delta\tau_w$), public investment is barely affected, resulting in a negligible impact on the growth rate. The net effect on growth is positive, but quite marginal, this resulting from the decrease in inequality. But with the tax rate and transfer having offsetting effects, the private-public capital ratio is hardly affected, so that this policy change will have a relatively small effect on market inequality, decreasing it by between 0.2%.[30] In contrast, net inequality decreases significantly (by 2.8%) reflecting the significant decrease in λ (by 1.414%).

Table 2.A-2.C also capture individuals' precautionary motives. Savings rates decline as the liquidity constraint is relaxed (ϕ increases), whereas consumptions increase. Individual responses in their savings and consumption decisions to

[28] While eqs. (23) and (C.9b) represent steady-state market (σ_y^2) and net ($\sigma_{\tilde{y}}^2$) inequality, capital inequality is implicitly defined in (??). Steady state growth, private-public capital ratio ($\varphi \equiv k/g$) and welfare (W) are given by (??), (??) and (??) respectively. Steady-state aggregate per capita consumption can easily be obtained from (35), whereas the per capita saving rate is defined by ϑ.

[29] With a missing credit market, young age consumption and inequality is zero, therefore welfare values cannot be precisely computed.

[30] The previous case of increased transfer (with no change in income tax) leads to a lower market inequality more than 5.8 times of the present case.

policy change also reflect their precautionary motives. With less severe credit constraints, individuals spend more of every dollar of the additional transfer they receive due to their motive of self-insuring.

7 National Development Plan 2030

South Africa is unenviably leading the world in being the most unequal country, with a gross income inequality Gini coefficient of 69. The South African National Development Plan 2030's (NDP 2030) has two targets. The first is to reduce the Gini coefficient for gross income inequality by 9 points to 60. The second is to increase the GDP per capita by 140 percent between 2010 and 2030, from Rand 50000 ($\$ 3,344$) to Rand 120000 ($\$ 8,026$). The current long run growth rate of the country is about 1.3%, which the calibration we have described yields as an equilibrium. The required average growth rate to achieve the target level of per capita income over a 20 year period is 4.47 percent.

From Table 2 it is clear that it is in general impossible to achieve both targets simultaneously with a single policy tool. Increasing consumption or income tax may increase growth but likely aggravates inequality. Whereas increasing the transfer may decrease inequality, the impacts on growth and welfare are mainly negative. Indeed, as Tinbergen (1952) demonstrated many years ago, it is in general necessary to introduce as many linearly independent policy instruments as there are objectives.[31] Accordingly, in Tables 3-6 below, we study the growth, inequality and welfare impacts of different policy combinations, and the extent to which they may move the South African economy in the direction of the NDP's 2030 targets.[32]

7.1 Substituting income tax by consumption tax

With a higher consumption tax enhancing growth but having an adverse effect on inequality, and with a higher income tax having the opposite impact on inequality, a natural starting point is to combine the two in some appropriate way. Thus, Table 3 shows the macroeconomic effects of decreasing the income tax by $2.12\%pts$ and introducing an identical consumption tax of 2.12%. While tax substitution of this type is not in the NDP's 2030 proposal, the desirable impacts on growth and hence welfare suggest that it merits serious consideration. The strong impact of the consumption tax on public investment (which is below the optimum at the current rate), causes the growth rate to increase. The reduction in the income tax increases savings, while the increase in the consumption tax barely affects it. The higher consumption tax leads to a reduction in the aggregate consumption to GDP ratio. However, both gross (market) inequality and net inequality increase. The lower private to public capital ratio

[31] For an extensive discussion of this issue and the related topics of static and dynamic controllability of deterministic and stochastic systems see Turnovsky (1977, Chapter 13).

[32] In evaluating them, we should bear in mind the earlier comments regarding the scale of the national development plan, we constrain ourselves to only small policy changes.

(k/g) means higher gross inequality, while the lower income tax means higher λ and hence higher net inequality. The rise in inequality tends to reduce the growth rate, but this is easily offset by the strong positive effects of increased public and private savings.

Table 4 modifies the changes reported in Table 3 by reducing the income tax by 1.12%*pts* and increasing the transfer by 1%*pt*, with the resulting 2.12%*pts* deficit being financed by the consumption tax. Compared to the results in Table 3, growth is slightly improved despite the small decrease in the savings rates. The reason for this is that there is now an improvement in terms of income distribution. As in the previous case, market inequality rises due to the decrease in the private-public capital ratio *but* at a lesser rate. In fact there is a much more dramatic contrast in the impact on net income inequality. The policy specified in Table 4 leads to a decrease in λ, which triggers the income effect, and hence leads to a decline in net inequality, which clearly helps to mitigate some of the negative effect of the policy on market inequality.[33]

7.2 Increased transfer and consumption tax

With a higher consumption tax enhancing growth but having an adverse effect on inequality, and with a transfer having the opposite effects, a more satisfactory outcome could be expected by combining the two. Thus, Table 5 shows the effects of a 3.18% pts increase in transfers, financed by a 2.12% pts increase in the consumption tax, leaving the income tax unchanged at its benchmark rate of 11.96%. The net effect of this composite policy is for gross inequality to decline by slightly less than 0.50%, but for net inequality to decline sharply by more than 6.63%. The long run growth rate jumps to between $4.17 - 4.38\%$ rates, which if sustained would accumulate to between 226-235% increase in GDP per capita at the end of the NDP's 2030 planning period. The aggregate consumption to GDP ratio decreases, due to the increase in the consumption tax, but because of the growth, the net welfare effect is positive, increasing significantly by about $5.4 - 5.2\%$, (for $\phi = 0.20$ to $\phi = 0.40$).

The increase in consumption tax increases public investment (with a trivial direct impact on private investment) and the impact is to decrease the private-public capital ratio (k/g). But the impact of the increase in transfer is to increase the k/g ratio, by increasing private investment and decreasing public investment. The net impact depends on the degree of the precautionary motive, parameterized by ϕ. As ϕ increases from 0 to 0.40, the saving rate decreases uniformly, as the individual precautionary motive declines with more credit availability. Therefore, the net impact of the policy on the capital ratio (k/g) is to increase it when the precautionary motive is strong, but to decrease it when it is weaker. For instance, for $\phi = 0 \quad k/g$ increases; but for $\phi = 0.20$ or $\phi = 0.40$, it decreases as a result of changes in the policy. Accordingly, in the former case we see a decrease in market inequality, despite an increase in the private to

[33] The decrease in k/g are more or less similar in Tables 4 and 5. Therefore, the contrasting difference in market inequality should come from the indirect effect of net inequality on market inequality.

public capital ratio. This is due to the relatively stronger effect of net inequality on market inequality. For changes in the capital ratio to have a dominant effect on market inequality the magnitude of the change should be relatively large. The decrease in net inequality comes from a decline in λ and market inequality although the former channel is much more important. Overall, this shows that the income effect on inequality is stronger than the substitution effect.

7.3 The nature of public investment: complementary vs substitute public investment?

The previous analysis provides a perspective on the implication of changes in policies and how that affects inequality, growth, and welfare for a given type of public investment. In the examples reported in Tables 3-5, we have assumed that the elasticity of substitution of 0.54 ($\rho \approx -0.85$) consistent with the evidence for Sub Saharan Africa. As noted previously this corresponds to public and private capital being complements in production and favors wealthier individuals, who own disproportionately more of the capital. Accordingly, more government investment, by favoring the wealthy will increase gross income inequality, and this may well account for the increase in inequality that has characterized South Africa in the years following the Apartheid period. This also suggests that one effective way to reduce gross income inequality would be to change the nature of public investment to a form that is more substitutable for private capital, and will therefore favor the poor.

Tables 6 shows the different impacts of a revenue neutral policy initiatives, while changing the nature of public investment to two different types of public capital, one is highly complementary (ρ = -1.35) while the other is more of a substitute (ρ = 0.35), and assuming that credit is partially available ($\phi = 0.20$). The second rows in Table 6.A and 6.B compare the macroeconomic impacts of a 5% pts increase in consumption tax with a similar decrease in income tax. In all cases both private and public investment increase, but the net effect is to reduce the private public capital ratio (k/g) and increase the saving rate. Growth and welfare increase significantly in both cases, but the impact is stronger when public investment is complementary (with about 2.75% pts differences in growth rate). But the impact on inequality contrasts quite sharply in the two cases. Market and net inequalities increase by 2.7% and 3.13%, respectively, when private and public capital are complements, but decrease by 0.61% and 0.17% when they are substitutes (ρ = 0.35). The decrease in the private-public capital ratio is associated with a higher market inequality in the former case, but with lower inequality in the latter, through the substitution effect. The lower income tax increases λ, and as a result increases net inequality through the income effect (as reflected in the case ρ = -1.35). In the case ρ = 0.35, this effect is offset by the indirect effect of market inequality in net inequality.

The second rows compare 5% pts increases in the consumption tax and transfers, while leaving the income tax unaffected. Such a policy mix is seen to have a significant positive growth effect and to boost welfare (but to a lesser degree) in both cases. Again, the private-public capital ratio declines and the saving rate increases. Market inequality rises by 1.67% when $\rho = -1.35$, but decreases by 1.6% when $\rho = 0.35$. Net inequality decreases in both cases, sharply when the two capital; goods are substitutes (by more than 3.26% pts). The third rows report the impacts of increasing both the income tax and transfer by 5%pts, leaving the consumption tax at its baseline value. The impact on growth is quite marginal, while by decreasing the saving rate it decreases welfare. As most of the tax increase is used to finance the transfers, public investment rises only marginally, which is reflected in its relatively lower impact on market inequality. In the last rows, the burden of financing the increase in transfers is shared between both the consumption and income taxes. The saving rate increases and the private-public capital ratio decreases leading to a significant increase in growth rate and welfare (though lower than the first two cases, but much higher than in the preceding case). Net inequality decreases significantly in both cases when $\rho = 0.35$ and -1.35, whereas market inequality rises in the latter.

In general, comparing the four policy changes across the two types of public investment, we see that policies related to complementary public investment are more associated with enhanced efficiency (and in many cases aggravating market inequality) while policies that relate to substitutable public investment have the tendency to both reduce inequality and increase efficiency (though moderately).

8 Conclusion

Despite the hope that government spending may have curative inequality effects, and that such spending has undergone a marked increase in the past few decades, inequality has risen in most countries in the world during these same periods. This has been a subject of concern, particularly for developing countries, because it could impact growth sustainability and poverty reduction strategies with potentially important adverse consequences for welfare. This paper has examined different channels through which taxes and government expenditure may impact the inequality–growth nexus. In particular, we have focused on the roles of income and consumption taxes, transfers and public investment on gross and net inequality, growth, and welfare. For this purpose we have developed a three-period overlapping generations model in which individuals are heterogeneous in terms of their initial endowments of capital and face credit constraints.

The credit constraint causes initial individual productivity differences to persist. When this is coupled with diminishing returns to investment, the poor will have a higher marginal product than do the rich and this leads to the rise of inequality dynamics that drives the dynamics of aggregate capital. But in addition, aggregate dynamics impinges directly on the dynamics of inequality. Such a bidirectional casualty between inequality and aggregate capital, which appears to us to be both novel and plausible, is a direct consequence of individuals' limited ability to borrow from their future income for their young age consumptions.

Several channels have been identified through which fiscal policy may influence income inequality. A policy change could affect net income inequality directly, as well as indirectly by effecting structural changes in the private-public capital ratio. This in turn influences market inequality, and determines the distribution of the next period's investment and net income. Income tax and transfers have both a direct income effect and an indirect substitution effect, whereas the consumption tax has only the latter.

The calibration of the model to reflect key features of the South African economy suggests that the sharp decline of the country's private-public ratio after the introduction of democracy in 1994, coupled with the high degree of complementarity between public and private capital, could be behind the persistence of increased market inequality during the last two decades. South Africa's National Development Plan 2030 targets to reduce inequality by 9 Gini points and increase GDP per capita by 240 percent over a twenty year horizon. Policies that simultaneously expand social grants and increase the consumption tax may help achieve these targets, together with the extensive social policies that have been proposed. Carefully balanced, such economic policies have a net positive impact on growth, inequality, and welfare, together although individually they work in opposite directions. The consumption tax could increase growth through boosting public investment, which is currently sub-optimally low, while it has a benign impact on private investment. But, as public investment has been complementary to private capital, increasing it will exacerbate market inequality. Accordingly, consideration could be given to changing the nature of public investment to areas that are more in the nature of substitutes to private investment and will therefore favor the less affluent. In doing this, the government should coordinate its expenditure policy with its transfer policy, which while favorable from the standpoint of income distribution, has an adverse impact on the growth rate and welfare as it competes with public investment for the economy's resources.

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Appendix

A. Aggregation and distribution in absence of credit market $\phi = 0$

The computation of aggregate and distributional variables is simplified due to the assumption of a lognormal distribution of wealth. We use the expectation and variance notations E and var to denote aggregation and variance of a variable, respectively. Thus, for example, aggregate capital at time *t* is defined as $k_t \equiv \mathrm{E}[k_{i,t}] = \int k_{i,t} di$. The distribution of wealth $k_{i,t}$ is represented by its log variance, $\sigma_{k,t}^2 \equiv \mathrm{var}[\ln k_{i,t}]$.

This Appendix derives the basic dynamic equations generating the degree of inequality, aggregate capital, and output, using key relations pertaining to the normal and lognormal distributions. These relationships imply that a lognormal random variable preserves its property under multiplication and addition by constant where the latter leads to a special case of a *shifted* lognormal distribution. Specifically, we use the following result stated by Kleiber and Kotz (2003, p.121): "If there exists $\lambda \in R$ such that $Z \sim \ln(X - \lambda)$ follows a normal distribution, then X is said to follow a three- parameter [shifted] lognormal distribution." Thus letting $Y \equiv \exp(Z)$, then $Y + \lambda = X$. This implies that if Y is lognormal then $Y + \lambda$ is a shifted lognormal. For this to be the case, the probability of X of taking any value below λ must be zero. But X could take any value greater than λ. This is further discussed by Aitchison and Brown (1957, p.14), who point out that as long as λ is given exogenously, the variate X has all the properties of the two parameter lognormal variate.

Using these results, suppose $u_{i,t}$ is a log-normal random variable, then $u_{i,t} + 1$ is a *shifted* log-normal where $u_{i,t} + 1 > 0$. Suppose

$$\ln u_i \sim N(\mu_1, \sigma_1^2) \text{ and } \ln(u_i + 1) \sim N(\mu_2, \sigma_2^2).$$

The mean and the variance of u_i are given by the following relations:

$$\mathrm{E}[u_{i]} = e^{\mu_2 + 0.5\sigma_2^2} - 1 = e^{\mu_1 + 0.5\sigma_1^2} \quad \text{(A.1)}$$

$$\mathrm{var} u_i = e^{2\mu_2 + \sigma_2^2}\left(e^{\sigma_2^2} - 1\right) = e^{2\mu_1 + \sigma_1^2}\left(e^{\sigma_1^2} - 1\right) \quad \text{(A.2)}$$

From (A.1) and (A.2), one may solve for σ_2^2 and μ_2 in terms of $u \equiv Eu_i$ and σ_1^2 to obtain

$$\sigma_2^2 = \ln\left(u^2\left(e^{\sigma_1^2} - 1\right)/(z+1)^2 + 1\right) \quad \text{(A.3)}$$

$$\mu_2 = \ln\left[(u+1)\left(\frac{z^2}{(z+1)^2}\left(e^{\sigma_1^2}-1\right)+1\right)^{-0.5}\right] \tag{A.4}$$

To aggregate individual output, (3), we rewrite it in logarithmic form, as

$$\ln y_{i,t} = \ln\left(a_1\alpha^{\frac{1-\theta}{\rho}}\varphi^\theta g_t\,\varepsilon_{i,t}\right) + ((1-\theta)/\rho)\ln\left(1+x_{i,t}\right) \tag{A.5}$$

where

$$x_{i,t} \equiv (1-\alpha)\alpha^{-1}\left(k_{i,t}/g_t\right)^\rho;\ \varphi_{i,t} \equiv k_{i,t}/g_t \text{ and } \mathrm{E}x_{i,t} \equiv x_t$$

Note $k_{i,t}$ being log-normal implies that $x_{i,t}$ is also log-normal.

The next step is to use (A.3) and (A.4) to compute the mean and variance of (A.5), yielding

$$\mathrm{E}(\ln y_{i,t}) = \ln\left(\alpha^{\frac{1-\theta}{\rho}}\varphi^\theta g_t\right) - 0.5\upsilon^2$$
$$+ ((1-\theta)/\rho)\ln\left((x_t+1)\left(\frac{x_t^2}{(x_t+1)^2}\left(e^{\rho^2\sigma_{k,t}^2}-1\right)+1\right)^{-0.5}\right)$$

and

$$\sigma_{y,t}^2 \equiv \mathrm{var}\left[\ln y_{i,t}\right] = ((1-\theta)/\rho)^2\ln\left(\frac{x_t^2}{(x_t+1)^2}\left(e^{\rho^2\sigma_{k,t}^2}-1\right)+1\right)+\upsilon^2$$

where

$$x_t \equiv (1-\alpha)\alpha^{-1}\varphi^\rho e^{\sigma_{k,t}^2 0.5\rho(\rho-1)}$$

Since $\ln y_t = \mathrm{E}\left[\ln y_{i,t}\right] + 0.5\mathrm{var}\left[\ln y_{i,t}\right]$, aggregate income ($y_t$) is given by

$$y_t = a_2\varphi^\theta g_t(x_t+1)^{(1-\theta)/\rho}\left(x_t^2\left(e^{\rho^2\sigma_{k,t}^2}-1\right)(x_t+1)^{-2}+1\right)^{(1-\theta)(1-\theta-\rho)/(2\rho^2)} \tag{A.6}$$

where $a_2 \equiv a_1\alpha^{\frac{1-\theta}{\rho}}$.

In deriving (17a)-(17d), first note that from (4), (11a)-(11d), we have,

$$\mathrm{var}(\ln k_{i,t+1}) = \mathrm{var}(\ln c_{i,t}^a) = \mathrm{var}\left[\ln\left(\left(1-\tau_y\right)y_{i,t}+my_t\right)\right] \tag{A.7}$$

That is the distribution of current consumption, after tax income. and next period investment are equal. Then, the last term can be solved following similar procedures as above:

$$\sigma_{k,t+1}^2 = \sigma_{c^a,t}^2 = \ln\left(\lambda^2\left(e^{\sigma_{y,t}^2}-1\right)+1\right) \tag{A.8}$$

B. Aggregating with Imperfect Credit Market

In this section we derive the inequality dynamics for capital investment and young age consumption for the case, $\chi > \phi > 0$.

B.1 Capital inequality dynamics

In deriving the inequality dynamics for the case, $\chi > \phi > 0$, first rewrite (30) as

$$\Phi_{i,t+1} = \frac{a}{g_{t+1}}\left(y_{i,t} + \overline{m}y_t\right) \qquad \text{(B.1.1)}$$

where

$$\Phi_{i,t+1} = \varphi_{i,t+1}\left(1 + b + \left(\varphi_{i,t+1}\varepsilon_{i,t+1}\right)^{-1}s_{t+1}h_{i,t+1}\right) \text{ and } s_{t+1} \equiv b\overline{m}\frac{y_{t+1}}{g_{t+1}} \qquad \text{(B.1.2)}$$

Taking the variance from both sides of (B.1.1) gives

$$V\Phi_{i,t+1} = \frac{a^2}{g_{t+1}^2}Vy_{i,t} \qquad \text{(B.1.3)}$$

The right hand side of this is given by

$$\frac{a^2}{g_{t+1}^2}Vy_{i,t} = a^2(\varpi_{t+1}\varphi_{t+1})^2\left(e^{\sigma_{y,t}^2} - 1\right) \qquad \text{(B.1.4)}$$

where income inequality is given by

$$\sigma_{y,t}^2 = \left((1-\theta)/\rho\right)^2 \ln\left(\frac{x_t^2}{(x_t+1)^2}\left(e^{\rho^2\sigma_{k,t}^2} - 1\right) + 1\right) + \upsilon^2$$

Also, we can write the left hand side of (B.1.1) as

$$V\Phi_{i,t+1} = E\Phi_{i,t+1}^2 - \left(E\Phi_{i,t+1}\right)^2 \qquad \text{(B.1.5)}$$

Then given (B.1.2) the first term of (B.1.5) is

$$E\Phi_{i,t+1}^2 = E\left(\varphi_{i,t+1}^2\left(1 + b + \left(\varphi_{i,t+1}\varepsilon_{i,t+1}\right)^{-1}s_{t+1}h_{i,t+1}\right)^2\right)$$

or

$$E\Phi_{i,t+1}^2 = (1+b)^2\varphi_{t+1}^2e^{\sigma_{k,t+1}^2} + 2s_{t+1}(1+b)e^{\upsilon^2}E\left[h_{i,t+1}\varphi_{i,t+1}\right] + e^{3\upsilon^2}s_{t+1}^2Eh_{i,t+1}^2 \qquad \text{(B.1.6)}$$

given that $\varepsilon_{i,t+1}$ is iid and independent of $\varphi_{i,t+1}$. There are no analytical solutions for the above, but we can derive a numerical solution by computing $Eh_{i,t+1}^2$ and $E\left[h_{i,t+1}\varphi_{i,t+1}\right]$ numerically. The second term of (B.1.5) is simply derived:

$$\left(E\Phi_{i,t+1}\right)^2 = \left(E\left(\varphi_{i,t+1} + b\varphi_{i,t+1} + \left(\varepsilon_{i,t+1}\right)^{-1}s_{t+1}h_{i,t+1}\right)\right)^2, \text{ or}$$

$$\left(E\Phi_{i,t+1}\right)^2 = \left(\varphi_{t+1} + b\varphi_{t+1} + e^{\upsilon^2}s_{t+1}Eh_{i,t+1}\right)^2 \qquad \text{(B.1.7)}$$

Then, combining (B.1.4), (B.1.6) and (B.1.7) leads to equation (34).

B.2 Variance of young age consumption

We can use a similar procedure to derive the dynamics of young age consumption. First, divide both sides of (31a) by g_{t+1} to get

$$(1+\tau_c)\frac{c_{i,t}^{y}}{g_{t+1}} = b\varphi_{i,t+1} + s_{t+1}\left(\varepsilon_{i,t+1}\right)^{-1}h_{i,t+1} \equiv \Omega_{i,t+1} \qquad \text{(B.2.1)}$$

Note, $\Omega_{i,t+1} = \Phi_{i,t+1} - \varphi_{i,t+1}$. Then, calculating the variance from both sides of (B.2.1):

$$(1+\tau_c)^2\left(\frac{c_t^{y}}{g_{t+1}}\right)^2\left(e^{\sigma_{t,c^y}^2}-1\right) = V\Omega_{i,t+1} \qquad \text{(B.2.2)}$$

The right hand side is given by

$$V\Omega_{i,t+1} = E\Omega_{i,t+1}^2 - \left(E\Omega_{i,t+1}\right)^2 \qquad \text{(B.2.3)}$$

where

$$E\Omega_{i,t+1}^2 = b^2\varphi_{t+1}^2 e^{\sigma_{k,t+1}^2} + 2e^{\upsilon^2}s_{t+1}(1+b)E\left[h_{i,t+1}\varphi_{i,t+1}\right] + e^{3\upsilon^2}(s_{t+1})^2 Eh_{i,t+1}^2 \qquad \text{(B.2.4)}$$

$$\left(E\Omega_{i,t+1}\right)^2 = \left(b\varphi_{t+1} + e^{\upsilon^2}s_{t+1}Eh_{i,t+1}\right)^2 \qquad \text{(B.2.5)}$$

considering (B.1.6) and (B.1.7). Combing (B.2.2), (B.2.4), and (B.2.5), we have

$$\sigma_{c^y,t}^2 = \ln\left(\frac{E\Omega_{i,t+1}^2 - \left(E\Omega_{i,t+1}\right)^2}{\varphi_{t+1}^2(\varpi_{t+1}a(1+\overline{m})-1)^2} + 1\right) \qquad \text{(B.2.6)}$$

C. Derivation of welfare

Aggregate welfare at time t is the average welfare of all individuals alive at that time:

$$W_t = \int_0^1\left(\ln c_{i,t}^{y} + \beta\ln c_{i,t}^{a} + \beta^2\ln c_{i,t}^{o}\right)di \qquad \text{(C.1)}$$

where $c_{i,t}^{y}$, $c_{i,t}^{a}$ and $c_{i,t}^{o}$ are given in (31a) to (31c). Aggregating (C.1), using similar procedures to those used in Appendix A:

$$W_t = \ln c_t^{y} + \ln c_t^{a} + \ln c_t^{o} - \frac{1}{2}\left(\sigma_{c^y,t}^2 + \sigma_{c^a,t}^2 + \sigma_{c^o,t}^2\right) \qquad \text{(C.2)}$$

where c_t^{y}, c_t^{a} and c_t^{o} are given by (35a) to (35c). With respect to the variance terms, note:

$$\sigma_{c^y,t}^2 \equiv \mathrm{var}[\ln c_{i,t}^{y}] \qquad \text{(C.3a)}$$

$$\sigma_{c^a,t}^2 \equiv \mathrm{var}[\ln c_{i,t}^{a}] = \ln\left(\lambda^2\left(e^{\sigma_{y,t}^2}-1\right)+1\right) \qquad \text{(C.3b)}$$

$$\sigma_{c^o,t}^2 \equiv \mathrm{var}[\ln c_{i,t}^{o}] = \ln\left(q^2\left(e^{\sigma_{y,t}^2}-1\right)/(q+1)^2+1\right);\; q = \frac{p}{\phi\theta m} \qquad \text{(C.3c)}$$

First, young age and adulthood consumption variances, $\sigma_{c^y,t}^2$ and $\sigma_{c^a,t}^2$, are given in (B.2.6) and (A.8) respectively. Second, to derive (C.3c), first rewrite $\mathrm{var}(\ln c_{it}^{o}) = \mathrm{var}(\ln\left(q_{i,t}+1\right))$

where $q_{i,t} \equiv \frac{py_{i,t}}{\phi\theta m y_t}$ and follow a similar approach as in Appendix A. Now substitute (35a) to (35c) and (C3a) to (C3c) into (C.2) to get:

$$W_t = \ln(a(1+\overline{m}) - \varpi_{t+1}) + \Xi + 3\ln y_t - \frac{1}{2}\left(\sigma^2_{c^y,t} + \sigma^2_{c^a,t} + \sigma^2_{c^o,t}\right) \qquad \text{(C.7)}$$

where $\Xi \equiv \ln\left(\frac{a}{\beta}(1+\overline{m})\right) + \ln\left(\left(1-\theta+\phi\theta(1-\tau_y)(1+\overline{m})\right)\right) - 3\ln(1+\tau_c)$

In the steady state, consumptions inequality are constant and the economy grows at a constant rate γ. Given initial level of capital k_0, we have $\ln y_t \approx \ln y_0 + t\gamma$. Then, the steady-state welfare at date t (W^s_t):

$$W^s_t = \ln(a(1+\overline{m}) - \varpi) + \Xi + 3\ln y_0 + 3t\gamma - \frac{1}{2}\left(\sigma^2_{c^y} + \sigma^2_{c^a} + \sigma^2_{c^o}\right) \qquad \text{(C.8)}$$

where

$$\sigma^2_{c^y} = \ln\left(\frac{E\Omega_i^2 - (E\Omega_i)^2}{\varphi^2\ (\varpi\ \ a(1+\overline{m})-1)^2} + 1\right) \qquad \text{(C.9a)}$$

$$E\Omega_i^2 = b^2\varphi^2\, e^{\sigma^2} + 2e^{\upsilon^2}s(1+b)E[h_i\varphi_i] + e^{3\upsilon^2}s^2Eh_i^2 \qquad \text{(C.9b)}$$

$$(E\Omega_i)^2 = \left(b\varphi + e^{\upsilon^2}sEh_i\right)^2 \qquad \text{(C.9c)}$$

$$\sigma^2_{c^a} = \ln\left(\lambda^2\left(e^{\sigma^2_y} - 1\right) + 1\right) \qquad \text{(C.9d)}$$

$$\sigma^2_{c^o} = \ln\left(q^2\left(e^{\sigma^2_y} - 1\right)/(q+1)^2 + 1\right) \qquad \text{(C.9e)}$$

σ^2_k is implicitly defined in (38). Aggregating (C.8) over all generations at discount rate R

$$W^s = \sum_{t=0}^{\infty} W^s_t\,(1+R)^{-t}$$

yields

$$W^s = \frac{1+R}{R}\left(\ln(a(1+\overline{m}) - \varpi) + \Xi + 3\ln y_0 + \frac{3\gamma}{R} - \frac{1}{2}\left(\sigma^2_{c^y} + \sigma^2_{c^a} + \sigma^2_{c^o}\right)\right) \qquad \text{(C.10)}$$

Table 1: Baseline calibration

Preference parameters	β = 0.35, R = 0.3
Production	ρ = -0.85, α = 0.3, θ = 0.53
Policy	τ_y = 0.1196, τ_c = 0, m = 0.0412
Inequality	υ^2 = 0.16

Table 2: Impacts of increased transfer, income and consumption taxes

A. ϕ = 0.0

	Υ	σ_y^2	$\sigma_{\tilde{y}}^2$	**λ**	**s/y**	**k/g**	**c/y**	***W***
Baseline	**1.3003**	**0.1692**	**0.1555**	**0.9553**	**0.1266**	**3.0476**	**0.8318**	**-**
$\Delta\tau_y = 0.02$	4.8426 (3.542%pts)	0.1713 (1.237%)	0.1571 (1.053%)	0.9543 (-0.104%)	0.1239 (-2.194%)	2.3755 (-24.916%)	0.8240 (-0.948%)	-
$\Delta\tau_c = 0.02$	8.7873 (7.487%pts)	0.1720 (1.688%)	0.1581 (1.700%)	0.9553 (0.000%)	0.1266 (0.000%)	2.1886 (-33.112%)	0.8155 (-1.980%)	-
$\Delta m = 0.0124$	-1.9725 (-3.273%)	0.1675 (-0.988%)	0.1501 (-3.474%)	0.9426 (-1.337%)	0.1283 (1.337%)	3.6689 (18.553%)	0.8367 (0.584%)	-
$\Delta m = \Delta\tau_y = 0.0124$	1.3534 (0.053%pts)	0.1689 (-0.173%)	0.1512 (-2.796%)	0.9419 (-1.414%)	0.1266 (0.000%)	3.0476 (-0.000%)	0.8318 (0.000%)	-

B. ϕ = 0.20

	ϒ	σ_y^2	$\sigma_{\tilde{y}}^2$	**λ**	**s/y**	**k/g**	**c/y**	***W***
Baseline	**1.3004**	**0.1730**	**0.1591**	**0.9553**	**0.0838**	**2.0171**	**0.8746**	**-26.1487**
$\Delta\tau_y = 0.02$	4.1675 (2.867%pts)	0.1754 (1.380%)	0.1610 (1.198%)	0.9543 (-0.104%)	0.0823 (-1.820%)	1.5782 (-24.542%)	0.8655 (-1.045%)	-25.3427 (3.131%)
$\Delta\tau_c = 0.02$	7.7338 (6.433%pts)	0.1765 (1.983%)	0.1623 (1.997%)	0.9553 (0.000%)	0.0838 (-0.000%)	1.4278 (-34.552%)	0.8575 (-1.980%)	-23.8146 (9.350%)
$\Delta m = 0.0124$	-1.4356 (-2.736%pts)	0.1710 (-1.161%)	0.1534 (-3.645%)	0.9426 (-1.337%)	0.0847 (1.083%)	2.4222 (18.300%)	0.8803 (0.645%)	-27.6969 (-4.189%)
$\Delta m = \Delta\tau_y = 0.0124$	1.3309 (0.030%pts)	0.1727 (-0.197%)	0.1546 (-2.816%)	0.9419 (-1.414%)	0.0838 (-0.023%)	2.0167 (-0.023%)	0.8747 (0.002%)	-26.6984 (-0.517%)

C. ϕ = 0.40

	ϒ	σ_y^2	$\sigma_{\tilde{y}}^2$	**λ**	**s/y**	**k/g**	**c/y**	***W***
Baseline	**1.3001**	**0.1778**	**0.1635**	**0.9553**	**0.0536**	**1.2901**	**0.9048**	**-25.7221**
$\Delta\tau_y = 0.02$	3.4118 (2.112%pts)	0.1803 (1.405%)	0.1655 (1.223%)	0.9543 (-0.104%)	0.0528 (-1.556%)	1.0120 (-24.278%)	0.8951 (-1.086%)	-25.2780 (1.742%)
$\Delta\tau_c = 0.02$	6.4985 (5.198%pts)	0.1815 (2.082%)	0.1669 (2.096%)	0.9553 (0.000%)	0.0536 (-0.001%)	0.9041 (-35.557%)	0.8871 (-1.980%)	-23.9838 (6.997%)
$\Delta m = 0.0124$	-0.8236 (-2.124%pts)	0.1756 (-1.260%)	0.1575 (-3.739%)	0.9426 (-1.337%)	0.0541 (0.905%)	1.5464 (18.122%)	0.9109 (0.670%)	-26.4012 (-2.606%)
$\Delta m = \Delta\tau_y = 0.0124$	1.3127 (0.013%pts)	0.1774 (-0.223%)	0.1589 (-2.836%)	0.9419 (-1.414%)	0.0536 (-0.038%)	1.2896 (-0.038%)	0.9049 (0.002%)	-25.7856 (-0.247%)

Table 3: Effects of substituting consumption tax for income tax

($\Delta\tau_c = +0.0212$ and $\Delta\tau_y = -0.0212$)

	Υ	σ_y^2	$\sigma_{\tilde{y}}^2$	λ	s/y	k/g	c/y	*W*
Baseline (ϕ = 0.0)	**1.3003**	**0.1692**	**0.1555**	**0.9553**	**0.1266**	**3.0476**	**0.8318**	**-**
	6.3239 (5.024%pts)	0.1701 (0.585%)	0.1567 (0.784%)	0.9563 (0.105%)	0.1295 (2.274%)	2.7126 (-11.644%)	0.8227 (-1.102%)	- -
Baseline (ϕ = 0.20)	**1.3004**	**0.1730**	**0.1591**	**0.9553**	**0.0838**	**2.0171**	**0.8746**	**-26.2531**
	5.9045 (4.604%pts)	0.1744 (0.808%)	0.1607 (1.008%)	0.9563 (0.105%)	0.0854 (1.880%)	1.7547 (-13.939%)	0.8659 (-1.001%)	-24.3124 (7.680%)
Baseline (ϕ = 0.40)	**1.3001**	**0.1778**	**0.1635**	**0.9553**	**0.0536**	**1.2901**	**0.9048**	**-25.7221**
	5.2980 (3.998%pts)	0.1795 (0.941%)	0.1653 (1.142%)	0.9563 (0.105%)	0.0545 (1.602%)	1.1046 (-15.528%)	0.8962 (-0.958%)	-24.0519 (6.714%)

Table 4: Effects of decreased income tax, increased consumption tax and transfer

($\Delta\tau_c = +0.0212$, $\Delta\tau_y = -0.0112$ and $\Delta m = +0.010$)

	Υ	σ_y^2	$\sigma_{\tilde{y}}^2$	λ	s/y	k/g	c/y	*W*
Baseline (ϕ = 0.0)	**1.3003**	**0.1692**	**0.1555**	**0.9553**	**0.1266**	**3.0476**	**0.8318**	**-**
	6.3672 (5.067%pts)	0.1699 (0.435%)	0.1532 (-1.434%)	0.9457 (-1.010%)	0.1295 (2.274%)	2.7126 (-11.644%)	0.8227 (-1.102%)	- -
Baseline (ϕ = 0.20)	**1.3004**	**0.1730**	**0.1591**	**0.9553**	**0.0838**	**2.0171**	**0.8746**	**-26.2531**
	5.9290 (4.629%pts)	0.1742 (0.637%)	0.1571 (-1.227%)	0.9457 (-1.010%)	0.0854 (1.862%)	1.7543 (-13.958%)	0.8659 (-0.999%)	-24.3875 (7.371%)
Baseline (ϕ = 0.40)	**1.3001**	**0.1778**	**0.1635**	**0.9553**	**0.0536**	**1.2901**	**0.9048**	**-25.7221**
	5.3078 (4.008%pts)	0.1791 (0.752%)	0.1617 (-1.107%)	0.9457 (-1.010%)	0.0545 (1.571%)	1.1042 (-15.559%)	0.8962 (-0.956%)	-24.1206 (6.429%)

Table 5: Effects of increased transfer and consumption tax *vis-à-vis* NDP's 2030 targets

($\Delta\tau_c = 0.0212$ and $\Delta m \quad = 0.0318$)

	Υ	σ_y^2	$\sigma_{\tilde{y}}^2$	**λ**	**s/y**	**k/g**	**c/y**	***W***
Baseline (ϕ = 0.0)	**1.3003**	**0.1692**	**0.1555**	**0.9553**	**0.1266**	**3.0476**	**0.8318**	**-**
	4.3839 (3.084%pts)	0.1683 (-0.483%)	0.1453 (-6.787%)	0.9234 (-3.392%)	0.1310 (3.392%)	3.1025 (1.784%)	0.8268 (-0.608%)	- -
Baseline (ϕ = 0.20)	**1.3004**	**0.1730**	**0.1591**	**0.9553**	**0.0838**	**2.0171**	**0.8746**	**-26.1487**
	4.3671 (3.067%pts)	0.1723 (-0.407%)	0.1488 (-6.699%)	0.9234 (-3.392%)	0.0861 (2.742%)	1.9961 (-1.047%)	0.8707 (-0.451%)	-24.7766 (5.390%)
Baseline (ϕ = 0.40)	**1.3001**	**0.1778**	**0.1635**	**0.9553**	**0.0536**	**1.2901**	**0.9048**	**-25.6275**
	4.1704 (2.870%pts)	0.1772 (-0.351%)	0.1530 (-6.630%)	0.9234 (-3.392%)	0.0548 (2.287%)	1.2520 (-2.996%)	0.9013 (-0.387%)	-24.3297 (5.197%)

Table 6: Macroeconomic effects of a budget-neutral policy changes with different types of public investment (ϕ = 0.20)

A. ρ = -1.35

	Υ	σ_y^2	$\sigma_{\tilde{y}}^2$	**λ**	**s/y**	**k/g**	**c/y**	***W***
Baseline	**1.3000**	**0.1713**	**0.1575**	**0.9553**	**0.0838**	**2.0172**	**0.8746**	**-26.2422**
$\Delta\tau_c = +0.05, \Delta\tau_y = -0.05,\ \Delta m = 0.0$	11.9500 (10.650%pts)	0.1760 (2.669%)	0.1625 (3.131%)	0.9576 (0.241%)	0.0875 (4.354%)	1.5150 (-28.625%)	0.8547 (-2.309%)	-21.7450 (18.799%)
$\Delta\tau_c = +0.05, \Delta\tau_y = 0.0,\ \Delta m = +0.05$	12.0484 (10.748%pts)	0.1742 (1.665%)	0.1452 (-8.124%)	0.9061 (-5.283%)	0.0875 (4.262%)	1.5135 (-28.724%)	0.8547 (-2.300%)	-22.2208 (16.634%)
$\Delta\tau_c = +0.0, \Delta\tau_y = 0.05,\ \Delta m = +0.05$	1.4050 (0.105%pts)	0.1700 (-0.791%)	0.1401 (-11.666%)	0.9010 (-5.847%)	0.0837 (-0.094%)	2.0153 (-0.094%)	0.8747 (0.009%)	-26.5565 (-1.190%)
$\Delta\tau_c = +0.03, \Delta\tau_y = +0.02,\ \Delta m = +0.05$	8.3330 (7.033%pts)	0.1727 (0.792%)	0.1433 (-9.415%)	0.9042 (-5.501%)	0.0860 (2.552%)	1.6687 (-18.963%)	0.8625 (-1.398%)	-23.7385 (10.027%)

B. ρ = +0.35

	Υ	σ_y^2	$\sigma_{\tilde{y}}^2$	**λ**	**s/y**	**k/g**	**c/y**	***W***
Baseline	**1.3004**	**0.1808**	**0.1662**	**0.9553**	**0.0838**	**2.0170**	**0.8746**	**-26.3034**
$\Delta\tau_c = +0.05, \Delta\tau_y = -0.05,\ \Delta m = 0.0$	9.2014 (7.901%pts)	0.1797 (-0.613%)	0.1660 (-0.174%)	0.9576 (0.241%)	0.0875 (4.355%)	1.5149 (-28.625%)	0.8547 (-2.309%)	-22.9624 (13.584%)
$\Delta\tau_c = +0.05, \Delta\tau_y = 0.0,\ \Delta m = +0.05$	9.2580 (7.958%pts)	0.1780 (-1.589%)	0.1484 (-11.385%)	0.9061 (-5.283%)	0.0875 (4.254%)	1.5133 (-28.733%)	0.8548 (-2.299%)	-23.4563 (11.456%)
$\Delta\tau_c = +0.0, \Delta\tau_y = 0.05,\ \Delta m = +0.05$	1.3676 (0.067%pts)	0.1788 (-1.099%)	0.1475 (-11.935%)	0.9010 (-5.847%)	0.0837 (-0.103%)	2.0149 (-0.103%)	0.8747 (0.010%)	-26.6253 (-1.216%)
$\Delta\tau_c = +0.03, \Delta\tau_y = +0.02,\ \Delta m = +0.05$	6.3142 (5.014%pts)	0.1783 (-1.425%)	0.1480 (-11.623%)	0.9042 (-5.501%)	0.0860 (2.543%)	1.6684 (-18.973%)	0.8625 (-1.397%)	-24.6491 (6.496%)

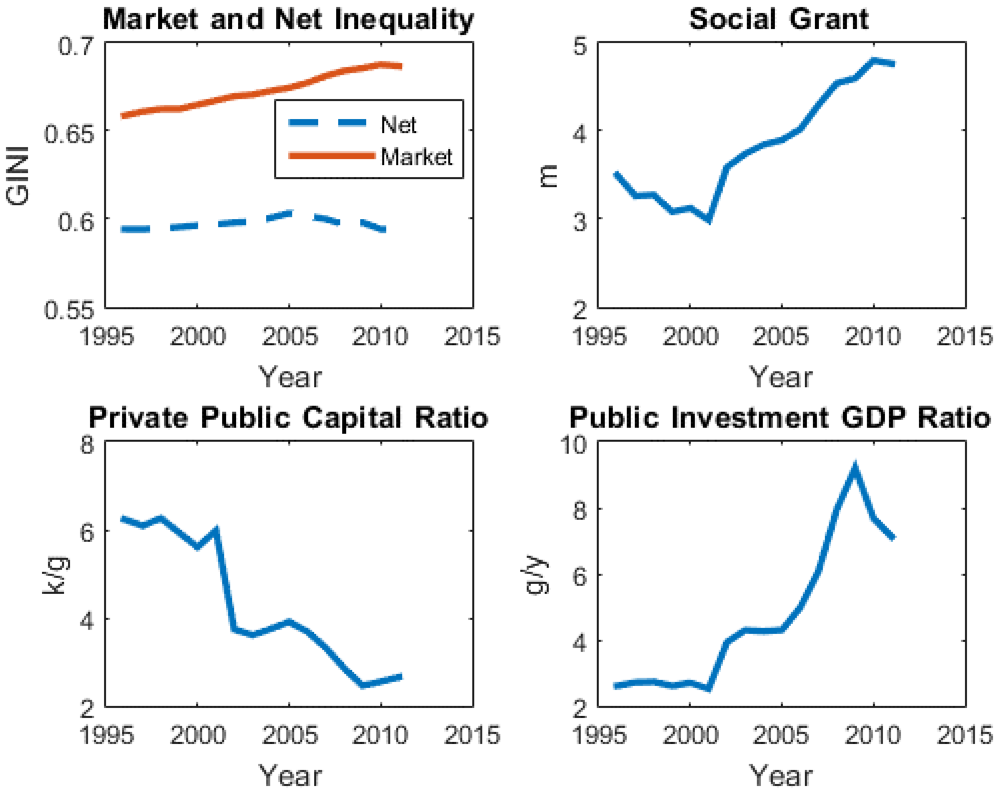

Figure 1. Trends in Inequality, Public and Private Investment and Social Grant in South Africa