Is there a National Housing Market Bubble Brewing in the United States?
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Is there a National Housing Market Bubble Brewing in the United States?*

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Abstract

We use a time-varying parameter dynamic factor model with stochastic volatility (DFM-TV-SV) estimated using Bayesian methods to disentangle the relative importance of the common component in FHFA house price movements from state-specific shocks, over the quarterly period of 1975Q2 to 2017Q4. We find that the contribution of the national factor in explaining fluctuations in house prices is not only critical, but also has been increasing and has become more important than the local factors since around 1990. We then use a Bayesian change-point vector autoregressive (VAR) model, that allows for different regimes throughout the sample period, to study the impact of aggregate supply, aggregate demand, (conventional) monetary policy, and term-spread shocks, identified based on sign-restrictions, on the national component of house price movements. We detect three regimes corresponding to the periods of “Great Inflation”, “Great Moderation”, and the zero lower bound (ZLB). While the conventional monetary policy is found to have played an important role in the historical evolution of the national factor in the first-regime, other shocks are found to be quite dominant as well especially during the second-regime, with monetary policy shocks playing virtually no role during this period. In the third-regime, unconventional monetary policy shock is found to have led to a (delayed) recovery in the housing market. But more
importantly, we find evidence that the national housing factor has been detached from the identified macroeconomic shocks (fundamentals) since 2014, thus suggesting that a “national bubble” might be brewing again in the US housing market. Understandably, our results have important policy implications.

Keywords: House Prices, Time-Varying Dynamic Factor Model, Change-Point Vector Autoregressive Model, Macroeconomic Shocks, Bayesian Analysis

JEL classification: C11, C32, E31, E32, E43, E52, R31
1 Introduction

In a seminal contribution related to the (regional and national) housing market of the United States (US), Del Negro and Otrok (2007) used a Bayesian dynamic factor model (DFM) to deduce the importance of the common component in the Office of Federal Housing Enterprise Oversight’s (OFHEO’s), now the Federal Housing Finance Agency’s (FHFA’s), house price movements relative to state- or region-specific shocks, estimated on quarterly state-level data from 1986 to 2005. The authors found that, while movements in house prices have been mainly driven by the local component, the period of 2001 to 2005 was different in the sense that the overall increase in house prices was a national phenomenon, though “local bubbles” were important in some states. As a next step, Del Negro and Otrok (2007) used a (constant parameter) vector autoregressive (VAR) to investigate the role of monetary policy in explaining the movements of the the common component of house price. The authors concluded that the impact of monetary policy shocks, identified based on sign-restrictions, on the national house price factor was marginal. Within the context of trying to explain the movement in overall US house prices based on macroeconomic shocks, a recent study by Plakandaras et al. (2018) employed a Bayesian time-varying parameter VAR (TVP-VAR) covering the period of 1830 to 2016. This is undoubtedly an important question since according to recent financial accounts data of the US, residential real estate represents about 83.7% of total household non-financial assets, 28.3% of total household net worth and 24.6% of household total asset. Based on a model which identified (permanent) technology, price and financial (money) shocks, and (temporary) housing market-related demand/supply shocks, these authors found that technology shocks dominate in driving the US housing market. This finding further corroborates the analysis of conditional volatilities and correlations with macroeconomic shocks. Interestingly, these results are in line with those obtained earlier by Iacoviello and Neri (2010) from a micro-founded dynamic stochastic general equilibrium (DSGE) model of the US economy, which incorporated an explicit housing sector.

Motivated by the findings of the two above-mentioned VAR-based studies, i.e., there are possibly more important other shocks than just monetary policy surprises that drives house prices in the US, we aim to revisit the work of Del Negro and Otrok (2007), based on updated data covering the quarterly period of 1975 to 2017. Understandably, recent data, allows us to include the tumultuous episodes of the “Great Recession”, the global financial crisis (GFC) (as well as the European sovereign debt crisis) that followed thereafter, and unconventional monetary policy decisions in the wake of the zero lower bound (ZLB) of monetary policy rates, with the roots of all these events associated with the bursting of the

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1See: https://www.federalreserve.gov/releases/z1/20190920/html/b101h.htm

2This result is actually in line with Gupta, Lv and Wong (2019), who made similar observations for the the Real Estate Investment Trusts (REITs) sector of the US economy. Reverting back to the paper by Plakandaras et al. (2018), when the authors also conducted a comparative analysis for the United Kingdom (UK) over the period of 1845 to 1846, interestingly they found that monetary policy is the most important driver of house price.

3In fact Del Negro and Otrok (2007) clearly pointed out that there are indeed many other potential causes that led to the booming housing market before its collapse.
US housing bubble and the subprime mortgage crisis (Leamer 2015). Besides, using updated data, we extend the work of Del Negro and Otrok (2007), and the general literature which concentrates primarily the role of (conventional and more recently, unconventional) monetary policy shocks (rather than other macroeconomic surprises) in driving the US housing market (see, Rahal (2016), Simo-Kengne et al. (2016), Huber and Punzi (2018), Caraiani et al. (2020), and Fischer et al. (2019) for detailed reviews in the regard). In the following ways: (i) Instead of the constant-parameter DFM originally used by Del Negro and Otrok (2007), we estimate a extended version of the traditional DFM with time-varying loadings and stochastic volatility (DFM-TV-SV, henceforth), as developed by Del Negro and Otrok (2008), to obtain the national and local factors associated with US state-level house price movements. As pointed out by these authors, an assumption of most DFMs is that both the stochastic process driving volatility and the nature of comovement among variables has not changed over time, but large amount of recent empirical work has shown that the assumption of structural stability is invalid for many macroeconomic aggregate and regional datasets of the US (Gupta et al., 2018), including house prices (Canarella, Miller and Pollard, 2012; Karoglou, Morley and Thomas, 2013; Huang, 2019). Naturally, a DFM model with fixed parameters is less likely to do well at describing house price data. As such, the generalized DFM-TV-SV not only captures changing comovements among the house prices of the 50 states and the District of Columbia by allowing for their dependence on common factors to evolve over time, but also allows for stochastic volatility in the innovations to the processes followed by the factors and the idiosyncratic components; (ii) Unlike Del Negro and Otrok (2007), and inspired by Plakandaras et al. (2018), we identify not only monetary policy shocks, but also aggregate supply, aggregate demand and term-spread shocks based on sign-restrictions, to analyze the impact of these shocks on the national component of house price movements. It must be realized that the spread shock is important for us since the time period of our analysis involves the period of ZLB and hence, that of unconventional monetary policy, which in turn involved compression of the long-term yield spread; (iii) Furthermore, differently from the constant parameter VAR model used by Del Negro and Otrok (2007), we estimate changes in macroeconomic dynamics by using an innovative change-point VAR model, proposed by Liu et al. (2018), that allows for different regimes throughout the sample period while studying the impact of the various shocks on the common component of the state-level housing prices. This approach enables the VAR model to endogenously identify changes to the structure of the US economy as well as variations to the properties of the exogenous shocks during the sample period. Consistent with evidence of time-varying effects of macroeconomic variables on the (regional) housing market of the US (Bork and Møller, 2015; Li et al., 2015; Nyakabawo et al., 2015; Bork, Møller and Pedersen, 2019; Christou, Gupta and Nyakabawo, 2019), the change-point VAR model with nonrecurrent states offers a novel way to estimate changes in the transmission mechanism of a variety of shocks over an extensive period, and; (iv) Since we estimate a DFM-TV-SV model, we are also able to recover the the stochastic volatility of the national factor, which we also incorporate

4More recently, some studies have also emphasized the role of fiscal policy shocks in driving the US housing market in the wake of the ZLB (see, ? and El Montasser et al. (2020) for further details).
into our change-point VAR model. This in turn, even though our primary focus is on house prices, allow us to simultaneously, as an aside, analyze the impact of the various identified shocks on the (common) housing market volatility, which have been shown to be also driven by macroeconomic variables \cite{Miller2006,Fairchild2015,Andre2017,Plakandaras2018}, besides housing returns \cite{Miles2008}. As pointed out by \cite{Segnon2002} with housing serving the dual role of investment and consumption, the effects of housing on savings and portfolio choices are extremely important questions, and hence, understanding the drivers of the house price volatility cannot be ignored because it has individual portfolio implications, as it affects households’ investment decisions regarding tenure choice and housing quantity. But more importantly, including the national factor of the stochastic volatility of the states in our change-point VAR, allows us to control for the possible effect of real estate uncertainty on the corresponding national component of house prices (over and above the identified macro shocks), as has been shown to play an important role in driving the US housing market \cite{Christidou2018,Nguyen2018}.

At this stage, it must be pointed out that there is widespread worry among academicians and policy authorities alike that, the ultra-low interest rate environment, along with the rise in liquidity caused by unconventional monetary policy measures that followed in the wake of the GFC, is inflating new housing bubbles \cite{Jorda2015a,Jorda2015b,Blot2018,Hubert2018,Alpert2019,Rosenberg2019}. Hence, distinguishing the national factor from local factors in the housing market, and determining what fraction of the variation in house prices across the states is explained by the common component, remain important questions, since answering them allow us to deduce whether the US economy is facing a “national bubble” or “local bubbles”. While “local bubbles” are attributable to circumstances that are specific to each geographic market given the widespread acceptance that housing markets are partially segmented \cite{Apergis2012,Montanes2013,Barros2013,Miles2015}, by linking the national price factor to (conventional and unconventional) monetary policy and other macroeconomic shocks, we will be able to gauge the part of common regional housing market movement attributable to changes in fundamentals (wider-array of macroeconomic shocks, besides the monetary policy shock) and the portion that could be due to speculation or pricing errors. Naturally, our analysis has tremendous significance from the policy perspective, if indeed the national factor dominates the local factors in explaining state-level housing price movements, and monetary policy shocks have had a role to play in driving the common component. These findings would in turn also align our study to the large existing literature \cite{Gali2015,Gambetti2015,Caraiani2018} on the relationship between monetary policy and bubbles in asset (housing) markets.

To the best of our knowledge, this is the first paper to use a Bayesian DFM-TV-SV model to first decompose the state-level house price movements of the US into a national and local factors, and then use a Bayesia change-point VAR to analyze the impact of aggregate

\footnote{In this regard, the role of monetary policy in producing second moment macroeconomic effects for the US economy, including the equity market, has also been recently depicted by \cite{Mumtaz2019}.}
supply, aggregate demand, monetary policy and term-spread shocks (identified based on sign-restrictions) on the common component. The remainder of the paper is organized as follows: Section 2 discusses the data used and the two methodologies associated with the DFM-TV-SV and the change-point VAR; Section 3 presents the empirical results from these two models and Section 4 concludes.

2 Data and Methodologies

2.1 Data

To be consistent with Del Negro and Otrok (2007), we use the FHFA (then OFHEO) seasonally-adjusted house price indexes for the 50 US states and District of Columbia over the quarterly period of 1975Q1 to 2017Q4, with the start date driven by the availability of the house price data, and the end of the sample corresponding to the latest data at the time of writing this paper. The FHFA house price indexes provide a broad measure of the movement of single-family house prices. The FHFA indexes are weighted, repeat-sales data, i.e., they measure average price changes in repeat sales or refinancings on the same properties. This information is obtained by reviewing repeat mortgage transactions on single-family properties whose mortgages have been purchased or securitized by Fannie Mae or Freddie Mac since January 1975. In particular, we use the quarterly “All-Transactions Indexes. 6 To create a real version of house price, we deflate the indexes by the (seasonally-adjusted) Consumer Price Index (CPI) of the US, derived from the FRED database of the Federal Reserve Bank of St. Louis. We work with the quarter-on-quarter (QoQ) version of the real house price indexes to obtain the national and local factors from the DFM-TV-SV model for both real housing returns and the corresponding stochastic volatilities.

As far as the data used in the change-point VAR is concerned, besides the two national factors of real housing returns and stochastic volatility, we include data on the federal funds rate (FFR), QoQ growth rate of seasonally-adjusted real Gross Domestic Product (GDP), QoQ growth of the CPI measuring the inflation rate, and the term-spread, which was defined as the difference between 10-year government bond yield and the FFR. Data on FFR, real GDP, and the long-term government bond yield is again sourced from the FRED database. The transformations of the data implies that our effective sample covers the period of 1975Q2 to 2017Q4.

2.2 The Generalized Dynamic Factor Model

In this section we present a generalized dynamic factor model (DFM) that is employed to decompose the real housing returns in all states into a common (or national) factor and an idiosyncratic (or state-specific) factor. The DFM is often used to tease out the common movements among multiple time series, and has become a standard tool since the work by

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6The data can be downloaded from: https://www.fhfa.gov/DataTools/Downloads/Pages/House-Price-Index.aspx
Stock and Watson (1989). We generalize the standard DFM with constant parameters to one that allows for time-varying loading parameters and the stochastic volatility (DFM-TV-SV, henceforth). As such, the generalized DFM-TV-SV captures important time-varying comovements among multiple time series. Formally, our model specification closely follows Del Negro and Otrok (2008), and is specified as follows:

\[ r_{i,t} = \beta_{i,t} \cdot f_t + e_{i,t} \]  

(1)

Here, \( r_{i,t} \) is the first-difference of the natural log of the real house price for state \( i \) at time \( t \). \( f_t \) is the national factor that affects all house prices at time \( t \), and \( \beta_{i,t} \) is the time-varying loading parameter of this national factor. \( e_{i,t} \) is the idiosyncratic factor.

The common factor and the idiosyncratic factors are assumed to be independent from each other. Therefore, the variance decomposition of our model is given by:

\[ Var(r_{i,t}) = \beta_{i,t}^2 \cdot Var(f_t) + Var(e_{i,t}) \]  

(2)

Note that either the time-varying loading parameters or the stochastic volatility of the factors enables the factors contributions to the total variations of each variable to vary over time.

Following the standard practice in this literature, we model the common factor \( f_t \) using a stationary AR(\( p \)) process:

\[ f_t = \phi_1^f f_{t-1} + \phi_2^f f_{t-2} + \cdots + \phi_p^f f_{t-p} + \exp(h_t^f) \cdot \varepsilon_t^f \]  

(3)

where \( \varepsilon_t^f \sim i.i.d.N(0,\sigma_f^2) \). Therefore, the shock to the factor has a stochastic volatility, and its time-varying volatility is governed by \( \exp(h_t^f) \).

To keep the model parsimonious, we employ a driftless random walk process to capture the time variation of the volatility:

\[ h_t^f = h_{t-1}^f + \sigma_f^h \cdot \xi_t^f, \quad \xi_t^f \sim i.i.d.N(0,1) \]  

(4)

The factor loading \( \beta_{i,t} \) varies over time, and is also assumed to follow a random walk process:

\[ \beta_{i,t} = \beta_{i,t-1} + \sigma_i^\beta \cdot \eta_{i,t}, \quad \eta_{i,t} \sim i.i.d.N(0,1) \]  

(5)

Here shocks to the loading parameters in different series are assumed to be orthogonal to each other.

The idiosyncratic factor follows a stationary AR(\( q \)) process:

\[ e_{i,t} = \phi_{i,1} e_{i,t-1} + \phi_{i,2} e_{i,t-2} + \cdots + \phi_{i,q} e_{i,t-q} + \exp(h_{i,t}) \cdot \varepsilon_{i,t} \]  

(6)

where \( \varepsilon_{i,t} \sim i.i.d.N(0,\sigma_i^2) \). The stochastic volatility of the idiosyncratic factor follows a random walk process:

\[ h_{i,t} = h_{i,t-1} + \sigma_i^h \cdot \xi_{i,t}, \quad \xi_{i,t} \sim i.i.d.N(0,1) \]  

(7)

\(^7\)It is straightforward to see that potential comovements in the factor loadings across all series can be captured by the common factor volatility. This was pointed out by Del Negro and Otrok (2008).
Here we assume that the shocks to the stochastic volatility in different factors are independent from each other. This assumption simplifies the estimation algorithm.

As usual, some normalizations of the factor rotations are needed before the model can be identified and estimated. The loading parameters and the variance of the shock to the common factor are not separately identifiable. We choose to set $\sigma_f^2 = 1$ to achieve the identification. Following Del Negro and Otrok (2008) we also restrict that the time-varying volatility all start from zero for the same identification purpose. We demean each series before the estimation since the means of factors are not separately identifiable. Finally, following works such as Neely and Rapach (2011) and Bhatt, Kishor and Ma (2017), we set $p = q = 2$ to keep the model parsimonious.

### 2.3 Estimation Procedure

We estimate this DFM-TV-SV model using the Monte Carlo Markov Chain (MCMC) Bayesian estimation method. Specifically, we employ the well-established Gibbs-Sampling algorithm by breaking the model into several blocks and sampling sequentially from posterior conditional densities. The idea of the Gibbs-Sampling algorithm is that when the algorithm converges after the initial burn-in draws, these random draws from the conditional densities altogether constitute a good approximation of the underlying joint densities. Applying the law of large numbers, the numerical integration can be easily taken to obtain the marginal densities of the parameters and the state variables of our interest. Most blocks in the model are linear and Gaussian, and as a result the standard algorithms in Kim and Nelson (1999a) are readily applicable. The stochastic volatility introduces a non-Gaussian feature into the model. We apply the procedure proposed in Kim, Shephard and Chib (1998) that utilizes a mixture of normal densities to approximate the underlying non-Gaussian distribution in order to simulate the stochastic volatility. This procedure has been widely used in the literature, see e.g., Stock and Watson (2007) and Primiceri (2005).

We briefly outline the Gibbs-Sampling estimation algorithm below. Further details are given in the Appendix A.2.

1. Cast the model into its state-space form as given in the Appendix A.2 and draw the national factor $\{f_t\}_{t=1}^T$ from the conditional density:

   $$ f \left( \{f_t\}_{t=1}^T \left| \{\beta_{i,t}\}_{i=1}^n, \phi_f, \{\phi_{i,e}\}_{i=1}^n, \{\sigma_i^2\}_{i=1}^n, \{h_i^f\}_{t=1}^T, \{h_i^{e}\}_{t=1}^T \right. \right) $$

   where $\phi_f = (\phi_1^f, \phi_2^f, ..., \phi_p^f)'$, and $\phi_{i,e} = (\phi_{i,1}^e, \phi_{i,2}^e, ..., \phi_{i,q}^e)$ for $i = 1, 2, ..., n$.

2. Draw the AR parameters of the common factor from the conditional density:

   $$ f \left( \phi_f \left| \{f_t\}_{t=1}^T, \{h_i^f\}_{t=1}^T \right. \right) $$

   Draws of the AR parameters outside the unity circle are discarded to ensure the stationarity.
3. Sample the AR and variance parameters for each idiosyncratic factor from the conditional density:

\[ f \left( \phi_{i,e}, \sigma_i^2 \mid \{ f_t \}_{t=1}^{T}, \{ \beta_{i,t} \}_{t=1}^{T}, \{ h_{i,t} \}_{t=1}^{T}, \{ r_{i,t} \}_{t=1}^{T} \right) \]

Because the idiosyncratic factors are orthogonal to each other, these draws can be made one by one for each \( i = 1, 2, \ldots, n \). Again, to ensure the stationarity any draw of the AR parameters outside the unity circle is discarded.

4. Draw the loading parameters and the shock variance parameters from the conditional density:

\[ f \left( \{ \beta_{i,t} \}_{t=1}^{T}, (\sigma_{i_t}^2) \mid \{ f_t \}_{t=1}^{T}, \phi_{i,e}, \sigma_i^2, \{ h_{i,t} \}_{t=1}^{T} \right) \]

Again, due to the orthogonality condition, these draws can be made one by one for each \( i = 1, 2, \ldots, n \).

5. Draw the stochastic volatility of the common factor from the conditional density:

\[ f \left( \{ h_f \}_{t=1}^{T}, \sigma_f^h \mid \{ f_t \}_{t=1}^{T}, \phi_f \right) \]

And draw the stochastic volatility of the idiosyncratic factor from the conditional density:

\[ f \left( \{ h_i \}_{t=1}^{T}, \sigma_i^h \mid \{ f_t \}_{t=1}^{T}, \{ \beta_{i,t} \}_{t=1}^{T}, \phi_{i,e}, \{ r_{i,t} \}_{t=1}^{T} \right) \]

Starting with initial values, we repeat steps (1) through (5) for \((D + S)\) number of times. Here \( D \) is the initial burn-in draws needed for the algorithm to converge, and the results are based on the saved \( S \) number of draws. We set \( D \) to 2000 and \( S \) to 8000.

### 2.4 Change-Point VAR Model

This section reviews the empirical model used for structural analysis. It also discusses how the marginal likelihood of the model is applied to determine: i) the number of regimes and ii) the order of lags. A similar framework has been applied by Kapetanios et al. (2012) and Liu et al. (2018).

To assess whether agents’ responses to macroeconomic shocks vary across regimes, the following VAR model is estimated

\[ Z_t = c_S + \sum_{j=1}^{K} B_S Z_{t-j} + \varepsilon_t, \tag{8} \]
where $\varepsilon_t \sim N(0, \Omega_S)$ and the data matrix $Z_t$ contains quarterly data on the FFR, real GDP growth, inflation, term-spread, and the national factors of real housing returns and volatility derived from the DFM-TV-SV model. $B_S$ and $\Omega_S$ denote the VAR coefficient and covariance matrix, respectively, which vary across regimes.

The empirical model permits $M$ breaks to take place at unknown dates and as in Chib (1998) the evolution of these breaks is prescribed by the latent state variable, $S_t$. The latter state variable is follows an $M$ state Markov chain with restricted transition probabilities, $p_{ij} = p(S_t = j | S_{t-1} = i)$, given by

$$p_{ij} > 0 \text{ if } i = j$$

$$p_{ij} > 0 \text{ if } j = i + 1$$

$$p_{MM} = 1$$

$$p_{ij} = 0 \text{ otherwise.}$$

For example, if $M = 3$, the transition matrix is defined as

$$\tilde{P} = \begin{pmatrix}
    p_{11} & 0 & 0 \\
    1 - p_{11} & p_{22} & 0 \\
    0 & 1 - p_{22} & 1 
\end{pmatrix}.$$  

Equations (8) and (9) describe a Markov switching VAR with non-recurrent states where transitions from one regime are restricted to happen sequentially. For example, to move from Regime 1 to Regime 3, the process has to visit Regime 2. The transition matrix also precludes transitions to past regimes. As discussed in Sims, Waggoner and Zha (2008), this is a Markov Switching model where structural breaks are modelled as multiple change points. We believe that this approach is advantageous over standard Markov switching models as it permits the user to associate changes in the macroeconomic dynamics with structural breaks in the economy. For instance, it is shown below that the marginal likelihood metric selects as the best “fitting” model the one with three regimes. Furthermore, these regimes seem to coincide with the Great Inflation, Great Moderation and Great Recession-Zero Lower Bound (ZLB) periods.

### 2.5 Estimation and Selection of the Number of Change Points and Lags

We follow Chib (1998) and adopt a Bayesian Gibbs sampling approach to the estimation of the change-point VAR models. Appendix A provides a detailed description of the prior and Appendix B describes the main steps of the algorithm. The only feature that is perhaps important to be mentioned here is that during the last regime, the policy rate does not respond to any variable into the system (to proxy for the ZLB). This characteristic is imposed via tight priors (see the discussion in the Appendix A).

The choice of the number of breakpoints is a crucial specification issue. The number of regimes is selected by comparing the marginal likelihood across models (i.e. different number
of regimes or/and lags). The maximum number of regimes has been set equal to three, while the maximum number of lags equal to four. Both choices are driven by concerns regarding the limited number of observations per regime.

Given $m$ and $l$ the marginal likelihood is estimated based on Chib (1998) and Bauwens and Rombouts (2012):

$$\ln G(Z_t | m, k) = \ln f(Z_t | m, k, \Theta, \tilde{P}) + \ln p(\Theta, \tilde{P} | m, k) - \ln g(\Theta, \tilde{P} | Z_t, m, k)$$ (10)

where $\ln G(Z_t | m, k)$ denotes the marginal likelihood, $\ln f(Z_t | m, k, \Theta, \tilde{P})$ is the likelihood, while $\ln p(\Theta, \tilde{P} | m, k)$ and $\ln g(\Theta, \tilde{P} | Z_t, m, k)$ are the prior and posterior distribution of the VAR parameter vector, respectively. Note that as $\ln G(Z_t | m)$ does not depend on the parameters of the model and in theory it can be evaluated at any value of the parameters. Following standard practices, we evaluate the marginal likelihood at the posterior mean. The first two terms on the right-hand side of equation (10) are easily evaluated whereas the calculation of the normalizing constant, $\ln g(\Theta, \tilde{P} | Z_t, m)$ requires some work. As described in detail in Bauwens and Rombouts (2012), this term can be evaluated by considering reduced Gibbs runs on an appropriate factorization of $g(\Theta, \tilde{P} | Z_t, m)$. We use 10,000 additional Gibbs replications to evaluate $g(\Theta, \tilde{P} | Z_t, m)$ at the posterior mean.

2.6 Shock Identification

This section explains briefly the identification scheme employed in this paper, and it is motivated by the work of Uhlig (2004), Mountford and Uhlig (2009), Barsky and Sims (2011) and Mumtaz, Pinter and Theodoridis (2018). The identified shocks maximise their contribution on selected variables and also satisfy the sign restrictions described in Table 1, which are imposed for four periods.

The mapping between reduced and structural errors is given by

$$\varepsilon_t = A_{0,S}v_t$$ (11)

For any orthogonal matrix $D$ ($DD' = I$, where $I$ is the identity matrix) the above mapping can be written as

$$\varepsilon_t = A_{0,S}D_Sv_t$$ (12)

Since

$$\Omega_S = A_{0,S}D_SD_S'\Lambda_{0,S} = A_{0,S}A_{0,S}'$$ (13)

*Allowing for a larger number of breakpoints and lags turns out to be a feasible task as there are not enough observations per regime.
Table 1: Sign Restrictions

<table>
<thead>
<tr>
<th>Variables</th>
<th>Demand</th>
<th>Supply</th>
<th>Monetary Policy</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Policy Rate</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>GDP Growth</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Inflation</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Term-Spread</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>+</td>
</tr>
<tr>
<td>House Prices Factor</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>House Prices Volatility Factor</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

Notes: All sign restrictions have been imposed for 4 periods. During the ZLB regime, the policy rate does not respond to the demand and supply shock as well (i.e. additional zero restrictions).

Using the companion form of the VAR(p) model, the impulse of variable \( j \) and the impulse of shock \( i \) in the period \( h \) can be expressed as

\[
IRF_{i,j,S}(h) = J_j B_{S}^{h-1} A_{0,S} D_S J_i^t
\]

where \( J_i \) and \( J_h \) are selection matrices of zeros and ones.

In Uhlig (2005) the matrix \( D_S \) results from the following minimisation problem

\[
D_S^* = \arg \max_{D_S} e \left[ \sum_{h=0}^{H-1} \sum_{j=0}^{h-1} B_{S}^{h-j} A_{0,S} D_S D_S' A_{0,S} (B_{S}^{h-j})' \right] e
\]

s.t.

\[
D_S D_S' = I
\]

\[
IRF_{i,j,S}(h) \in \mathcal{I}_+ \cup \mathcal{I}_-
\]

where \( e \) is a selection matrix of ones and zeros, \( \mathcal{I}_+ \) is the index set of variables, for which the identification of a given shock restricts the impulse response to be positive and \( \mathcal{I}_- \) is the index set of variables, for which the identification restricts the impulse response to be negative.

The use of this scheme is to identify “meaningful” macroeconomic shocks and to study how these disturbances affect house prices.

Although the identification scheme employed shares many similarities with the one developed by Mountford and Uhlig (2009), Barsky and Sims (2011) and Mumtaz, Pinter and Theodoridis (2018), two features of the proposed methodology worth to be mentioned explicitly. The first point is that the matrix \( D_S \) is regime depended, meaning that the above maximisation problem (expression 15) needs to be performed for each regime. More importantly, no restriction is placed on the house price level and volatility factors, along the lines of Del Negro and Otrok (2007). In other words, our approach about the effects of “standard”
macroeconomic shocks on the series that describe the evolution of the common components associated with real housing returns and stochastic volatility is agnostic.

Table 1 reports the sign restrictions employed to identify a demand, supply, monetary policy and term-spread shock. The restrictions for the first three shocks are uncontroversial, and they have been used in the literature extensively. Although the last shock has been studied in a number of papers Kapetanios et al. (2012), Baumeister and Benati (2013) and Liu et al. (2018) (among others), it is less common and aims to capture movements at the long-end of the yield curve that are not induced by variations in the policy rate. During the Great Moderation period, this shock could reflect the foreign capital inflows to the US capturing the so-called savings glut phenomenon (Sá, Towbin and Wieladek, 2014; Sá and Wieladek, 2015; Cesa-Bianchi, Ferrero and Rebucci, 2018). While during the ZLB period, this shock could proxy the Federal Reserve’s unconventional policies.

3 Empirical Results

In this section we present the estimation results of the DFM-TV-SV model as described in Section ?? and when applied to the quarterly real housing returns in 50 states and the District of Columbia. We first present the national factor of the real housing returns together with its time-varying stochastic volatility, and then discuss its time-varying contributions to real housing returns in all states, followed by discussions of the implied time-varying cross-state correlation and the cross-state volatility dispersion from the DFM-TV-SV model. Finally, we turn our attention to analyzing the impact of sign restrictions-based identified aggregate demand, aggregate supply, monetary policy and credit shocks on the evolution of the national factor using a change-point VAR model.

3.1 The National Factor of the Real Housing Returns

Figure 1 plots the national factor (in a solid line), together with the 90% probability intervals (in dotted lines). One important advantage of this generalized DFM is that it allows exposures of the real housing returns in all states to the national factor to vary over time, and thus permits a time-varying integration of the local housing market with the national market. The national factor started to increase in 1975, but then declined in the late 1970s through the early 1980s. Since around the mid-1980s, the national factor had steadily risen for an extended period of time until about 2006, when the national factor plunged leading to a severe financial crisis and recession in 2007-2009. The national real housing returns factor leveled off around 2010, and has rebounded sharply since 2011. The dynamics of the national factor of real housing returns estimated from our DFM-TV-SV model is in general consistent with those findings documented over the common period of the past literature (see for example, Del Negro and Otrok (2007) and Fairchild, Wu and Ma (2015)). We explain this pattern below by identifying various shocks in the context of a change-point VAR framework.

Figures 2 and 3 plot the time-varying loading parameters of the national factor, together
with the 90% probability intervals, for the growth of real house price in each state. Overall, the exposures of the real housing returns in all states to the national factor vary substantially over time. These loading parameters appear to be positive for all states at all time periods, indicating that there is no identification issue. The dynamic patterns of these time variations also display substantial heterogeneity across states. A number of states, including Alaska, California, Delaware, Florida, Hawai‘i, Idaho, Illinois, Maryland, Nevada, New York, Oregon, Virginia, Vermont, Washington, and Wisconsin, all witnessed a steady increase in the exposures of their real housing returns from the early 1990s to the pre-crisis period. Interestingly, many other states, including Arkansas, Colorado, Connecticut, DC, Iowa, Indiana, Kansas, Kentucky, Louisiana, Massachusetts, Maine, Mississippi, North Dakota, Nebraska, New Hampshire, Ohio, Oklahoma, Rhode Island, South Dakota, Texas, Utah, West Virginia, and Wyoming, all experienced a steady decline in the exposures of their real house price growth in the same period.

Figure 4 shows the stochastic volatility of the national factor with its 90% probability interval. There is a rapid and substantial increase in the stochastic volatility of the national factor from around 1998 to around 2011, followed by a large decline afterwards. Figures 5 and 6 plot the stochastic volatility of each state-specific factor. Overall, there is a substantial time variation in the stochastic volatility for all idiosyncratic factors, and there is a substantial heterogeneity in the dynamic patterns across states.

Recall, from equation (2), that both the time-varying loading parameters and the stochastic volatility of the national and the state-specific factors jointly determine the time-varying contributions of the national factor to the total variations of the growth in real house price in the states. Figures 7 and 8 show the dynamic evolution of the relative contribution of the national factor to the real housing returns in all states. The relative contribution of the national factor has increased since the mid-1980s for most states. These include states that have experienced an overall steady increase in its loading of the national factor, such as California and New York, and states that have witnessed a decline in its loading of the national factor, such as Connecticut and Massachusetts. For the latter group of states, it seems that the decline in the loading of the national factor is more than offset by a large increase in the stochastic volatility of the national factor and a large decline in the stochastic volatility of the idiosyncratic factor, resulting in an increasing relative contribution of the national factor. To quantify the relative contribution of the national factor to the total variations, we note that the contribution of the national factor for the full sample period in all states is 44.85% on average. This contribution is only about 28.85% on average during the period from 1975Q1 to 1989Q4, rises to as much as 52.12% during the period from 1990Q1 to 2006Q4, and remains as high as 55.08% during the period from 2007Q1 to 2017Q4. Although the contribution of the national factor has declined somewhat after the financial crisis in some states, we conclude that overall the role of the national factor in explaining the house prices in all states is not only critical but also has been increasing to become more important than the local factors since around 1990. These findings are broadly in line with previous works such as Del Negro and Otrok (2007), but recall, the sample period of that study ended in 2005, and hence did not cover the most recent periods, including that of the
GFC which corresponded with a tumultuous period of the US housing market. Given the dominance of the national factor in explaining state-level house price movements, especially since 1990, identifying the role of various macroeconomic shocks in driving this common real housing returns component in a regime-specific context, is clearly of paramount importance, and this is what we focus on in detail below in a short while.

3.2 Cross-State Time Varying Correlation

The generalized DFM as employed here can capture potentially time-varying comovements among multiple series. To this end, we compute the implied correlation for each pair of states and present the average of these pairwise correlations at each time point in Figure 9. The average cross-state correlations increased from the mid-1980s till around 2011, and then declined until the end of the sample. The increase in this correlation was more rapid in 1985-1995 than in 1996-2005. This correlation increased more rapidly again between 2005 and 2011, which may have been driven by the financial crisis.

3.3 Cross-Sectional Dispersion in Volatility

Another metric that can be computed based on the DFM-TV-SV model to usefully summarize the dynamic patterns of these growth in real house prices is the volatility dispersion, which is the standard deviation of the implied volatility of all states, as shown in Del Negro and Otrok (2008). In Figure 10, we present this time-varying volatility dispersion and the decomposition of it into the component driven by the national factor and that by the state-specific factor. We find that there is a large increase in the volatility dispersion at the beginning of the sample period till around the mid-1980s, followed by a large decline. The third panel in this figure indicates that this rise and decline in the total volatility dispersion is primarily attributed to the idiosyncratic factor. The total volatility dispersion increased between 2000 and 2009 again, and then declined until the end of the sample. The second panel suggests that this is primarily due to the national factor.

3.4 Change-Point VAR Model Specification and Evolution of Regimes

Next, we focus our attention on the change-point VAR, using which we analyze the impact of various identified macroeconomic shocks on the national factors of real housing returns and stochastic volatility. We start with the discussion of the dynamic specification of the model. Table 2 reveals that the model that gets the most support from the data is the most flexible one. In words, the data prefer a model with 3 regimes and 4 lags.

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9 An interesting area of future research in this regard would be to determine what state-specific characteristics drive the differences in exposures to the common factor.

10 Extending the search for a larger number of regimes or/and lags is not possible given the scale of the empirical model and the quarterly nature of the data. There are no enough observations per-regime to ensure meaningful estimates.
Figure 11 helps us to understand the model selection implied by the marginal likelihood statistic. The evolution of the regimes coincides with the three phases of inflation and, consequently, the policy rate during the time interval considered in this study. Namely, the first regime is associated with high inflation and policy rate (“Great Inflation”). In contrast, the second regime overlaps (mostly) with the Great Moderation (stable low inflation and policy rate) and the final regime with the ZLB period. Interestingly, the second regime more or less corresponds to sample period used by Del Negro and Otrok (2007) under the rationale that this sample corresponds to a single monetary policy-regime and thus helps in correctly identifying the monetary policy shocks. The “good-luck” versus “good-policy” and ZLB literature document that the dynamics across these different regimes are dramatically different (see Sims and Zha (2006), Cogley and Sargent (2005) and Liu et al. (2018) among others). In addition, when we look at these three-regimes, we observe the first-regime being characterized by the declining national factor of real housing returns and then recovery of the same with low volatility, followed by the rapid increases with eventual collapse and highly volatile national factors in the second regime, and then finally the recovery of the common component with declining variability in the wake of expansionary unconventional monetary policies. As a result, a model with sufficient flexibility is required to capture accurately the extensive non-linearities in the data. The results displayed by Table 2 are inline with this rationale.

3.5 Shocks

The evident time-varying importance of the national real housing returns factor in explaining movements at the state-level especially post-1990 tends to suggest that the housing market boom before the collapse in 2007, and then again the recovery after that is not necessarily purely driven by local factors (“local bubbles”). Hence, this section is devoted to studying the responses of the economy including the national housing market factors of returns and volatility, to each identified shocks and whether these responses are regime-dependent. The importance of the shock across all three regimes is also assessed in this section.

3.5.1 Demand Shock

Figure 12 illustrates responses to a demand shock. To remind the reader, the effects of the shock on the term-spread, the real housing returns factor and the house prices volatility factor are left unrestricted. Figure 12 makes apparent that the transmission of the shock to the macroeconomy varies dramatically across the three regimes. The shocks seems to have a larger effect on the economy during the “Great Moderation” (regime 2), and the forecast variance decomposition also confirms this (Figure 13 and Table 3). Although the effect on the real GDP growth is comparable across the three regimes, inflation rises substantially more in the second regime, and this leads to a protracted increase in the interest rate. Despite the long-lasting policy rate increase, the term-spread falls indicating that the long-term interest

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11 This period is known to be associated with large structural changes in the credit market culminating into the end of regulation Q.
rate rises by less than the short-end of the yield curve; perhaps due to well-anchored inflation expectation during the Great Moderation. The housing factor increase initially, but this is only a short-lasting effect, since as monetary authorities start ‘fighting’ higher inflation and the policy rate increases, the real housing returns factor falls.

During the Great Moderation, the demand shock explains 50% of the variability of the policy rate, indicating that the Federal Reserve’s intense effort to mitigate the inflationary consequences of the shock. This elevated impact is also reflected on GDP (20%), inflation (20%), term-spread (35%), real housing returns factor (30%) and real house price volatility factor (20%).

3.5.2 Supply Shock

The responses of output growth, inflation and the policy rate are stronger in the first than in the other two regimes (Figure 14). The stimulative monetary policy needed for the (negative) output-gap to be closed, leads to the higher housing returns factor in regimes one and two. This effect is supported further by the strong (income) growth in the economy, which also leads to lower volatility of the national factor of housing returns for regimes one and two.

The forecast variance contribution of the shock does not seem to vary across the 1st and 2nd regimes and it fluctuates between 10% and 20% (Figure 13 and Table 3). The supply shock plays a more important role during the 3rd regime, with its contribution to GDP growth, and the factors of real housing returns and stochastic volatility rising above 20%.

3.5.3 Policy Shock

A policy ‘cut’ increases output growth persistently during the second regime (Figure 15). Interestingly, the term-spread rises approximately by 200 basis points suggesting that long-term interest rates do not decrease as much as the policy rate. The stimulative environment created by the Federal Reserve leads to higher values of the real housing returns factor during the Great Moderation, while this effect is insignificant in the first regime. This highly accommodative policy results in higher volatility of the national housing returns factor, although this effect is not precisely estimated.

The forecast variance contribution to GDP growth and inflation is admittedly quite small (Figure 13 and Table 3), which is in line with a number of existing studies (see Bernanke, Boivin and Eliasz (2005), Smets and Wouters (2007) and Justiniano, Primiceri and Tambalotti (2010) among others). On the other hand, the contribution of the policy shock on the term-spread and national housing returns factor rises to (almost) 30%, indicating the strong link between policy actions and decision of agents to invest in either long-term or/and housing debt.

12This is actually more than twice the figure of 13% detected by Del Negro and Otrok (2007), and could be an indication of our model picking up the existing nonlinearity in the relationships among the variables of the change-point VAR.
3.5.4 Slope Shock

The slope shock is a perturbation that lowers the long-end of the yield curve, while the policy rate remains constant contemporaneously. In the first regime, the shock stimulates demand and inflation. As the authorities start increasing the policy rate to restore price stability, the national house price factor starts falling with (about) a year delay, while the corresponding volatility increases contemporaneously. These effects carry over to the next regime. However, the response of volatility of the national housing returns factor is not precisely estimated. As the economy moves into the ZLB regime, the responses of the level and volatility of common factors change sign. In this regime, real housing returns factor increases and volatility falls (but again, the latter response is not precisely estimated). Recall that, the slope shock in the last regime proxies the Federal Reserve’s unconventional policies adopted to repair macroeconomic stability (something which we will return to below) in the wake of the Great Recession.

The shock seems to have a limited effect on the macroeconomic variables including the term-spread (Figure 13 and Table 3). An exception is the contribution of the GDP on growth during the Great Moderation, especially at short-horizons such as one quarter and one year, whereby the shock explains 35% and 30% of the variability of growth. The contribution of the shock to the slope of the yield curve rises as the forecast horizon increases. However, the magnitude of this effect is (approximately) 15%. Interestingly, the contribution of the term-spread shocks is much higher for the remaining two variables (between 20% and 30%). What seems to be more noteworthy is the fact that the contribution of the shock peaks in the third regime.

3.6 Sensitivity Analysis

It is illustrated in the Appendix that the results are robust: i) when the value of the hyper-parameter that controls the tightness of the VAR coefficients is increased (looser priors), ii) when the unemployment (instead of the GDP growth) series, derived from the FRED database, is used to proxy the real sector of the economy, and iii) when the dynamic order to the VAR is reduced to 3, 2 and 1 lags (per-regime).

3.7 The National Housing Returns Factor and Identified Shocks

The discussion in this section is concentrated on the national real housing returns factor in relation to not only conventional and unconventional monetary policy shocks, but also aggregate demand and aggregate supply innovations. Table 3 illustrates that the identified macroeconomic economic shocks explain between 50% and 70% of the variability of the national housing returns factor (depending on the horizon and the regime). This message is further reinforced by Figure 17 where the identified shocks account for (almost) all the historical evolution of the common component of state-level growth in real house prices during the first and second regimes. The explanatory power of the identified shocks collapses

13For all these exercises the number of regimes is set equal to 3 as in the benchmark model.
during the Great Recession, while it improves in the period between 2011Q1 and 2014Q2, but breaks down again from 2014Q3 till the end of the sample (2017Q4).

Several interesting facts emerge from both Table 3 and Figure 17. The first one is that, unlike in the high inflation regime, conventional monetary policy played almost no role in the growth of the national factor related to the housing market during the Great Moderation – a result in line with Del Negro and Otrok (2007); if anything, its contribution is rather negative (Nelson, Pinter and Theodoridis (2018)). This finding is consistent with the work of Justiniano, Primiceri and Tambalotti (2017), Justiniano, Primiceri and Tambalotti (2019), and Cox and Ludvigson (2019) where the authors explain that factors related to the credit supply and demand, and not monetary policy, are behind the increase of house prices (and housing debt/leverage). In this study, credit supply and demand shocks are not identified explicitly, but are probably captured by the demand, supply and slope shocks in our model and, interestingly, the contribution of all these three types of shocks to the national factor of real housing returns is positive during this period, which in turn, corroborates the findings of Plakandaras et al. (2018), especially in terms of the importance of the aggregate supply shocks.

The national real house price growth factor collapsed during the Great Recession, with one-third of this fall not being explained by the shocks identified. During this period, the (conventional) monetary policy is constrained by the ZLB, which started binding at the beginning of 2009. Interestingly, substantial negative policy contribution started cumulating a few quarters before the ZLB, and these adverse effects picked up during the ZLB period (persistence effect). This anecdotal evidence could suggest that the inability of monetary authorities to lower the policy rate, which cannot be modelled as a shock, could explain a large part of the unexplained wedge.

Although quantitative easing (QE) was introduced in the last quarter of 2008, the model suggests that the slope (or QE) shock started contributing positively to the economy around 2012. This period coincides with the introduction of the open-ended QE3 and the Forward Guidance (FG) unconventional monetary policy. This does not mean that the first two QE programs had no positive effects on recovering the housing market, as this question cannot be answered from the historical decomposition, and in turn requires the knowledge of the counterfactual profile of the national factor in the absence of the QE1 and QE2 programs. However, the assumption that the large positive contributions by the slope shock during the third regime are associated with explicit guidance about future policy rate would be consistent with the point made in the previous paragraph. In other words, what matters most for agents when it comes to house prices is the systematic part of the monetary policy described by the policy rate.

Finally, the real housing returns factor post-2014 has decoupled again from the identified macroeconomic shocks. This perhaps reflects the concerns, as also as outlined above in the introduction, from important policy institutions and policy-makers (see for example, Borio (2019) and Carstens (2019), among others) that low interest rates led many investors to search for higher returns that are also subject to elevated risks.
4 Conclusions

In this paper, we use a time-varying parameter dynamic factor model with stochastic volatility (DFM-TV-SVP) estimated using Bayesian methods to disentangle the relative importance of the common component in FHFA house price movements from state-specific shocks, over the quarterly period of 1975Q2 to 2017Q4. We find that the contribution of the national factor in explaining fluctuations in house prices has declined somewhat after the financial crisis in some states, but overall the role of the national factor in all states is not only critical but also has been increasing and has become more important than the local factors since around 1990. This result suggests that, while “local bubbles” have been important in some states, but that overall the increase in house prices is a national phenomenon. We then use a Bayesian change-point vector autoregressive (VAR) model, that allows for different regimes throughout the sample period, to study the impact of not only (conventional) monetary policy shocks, but also aggregate supply, aggregate demand and term-spread shocks, identified based on sign-restrictions, on the national component of house price movements, with the term-spread surprises measuring unconventional monetary policy decisions. We detect three regimes corresponding to the periods of “Great Inflation”, “Great Moderation”, and the episodes of the “Great Recession” and the global financial crisis thereafter, associated with the zero lower bound (ZLB). While conventional monetary policy is found to have played an important role in the historical evolution of the national factor of real housing returns of the US in the first-regime, other shocks are found to be quite dominant as well, especially during the second-regime of the “Great Moderation”, with monetary policy shocks playing virtually no role in explaining the national housing market boom during this period. As far as the third-regime is concerned, unconventional monetary policy shocks, associated with the phase 3 of the quantitative easing (QE3), is found to have led to a (delayed) recovery in the housing market. Since the DFM-TV-SV model also allows us to recover the national factor of housing market volatility, we could incorporate it into our change-point VAR to analyze what shocks play a role in driving this factor. In this context, again the role of monetary policy is limited, with dominant effect coming from the term-spread shock, followed by the aggregate supply and aggregate demand shocks. But perhaps more importantly, we find evidence of the national real housing returns factor to have got detached from the identified macroeconomic shocks, i.e., fundamentals since 2014 – somewhat similar to what was observed in terms of the low explanatory power of the shocks during the Great Recession. This result seems to suggest that a “national bubble” is brewing again in the US housing market, resulting from the prolonged period of loose unconventional monetary policies following the recent financial crisis. Naturally, our findings call for careful monitoring of the behavior of house prices, in order for the policy authorities to decide whether or not to “lean against the wind”, by raising policy rates. Besides the cost of producing a future recession, whether such policies can in fact affect the housing market also remains debatable, given our finding of a limited role for conventional monetary policy in driving the US housing market historically. Perhaps, in this regard the role of macro-prudential tools become important, which are often considered as the best instruments to prevent the build up of credit-driven bubbles, notably because they can be tailored to address specific market failures. 

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Figure 1: The National Factor

Notes: The solid black line is the median of the posterior distribution, while the dotted lines represent the 5% – 95% percentiles.

Table 2: Marginal Likelihood Comparison

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Figure 2: Time Varying Loading Parameters of the National Factor

Notes: The solid black line is the median of the posterior distribution, while the dotted lines represent the 5% – 95% percentiles.
Figure 3: Time Varying Loading Parameters of the National Factor - Continued

Notes: The solid black line is the median of the posterior distribution, while the dotted lines represent the 5% – 95% percentiles.
Figure 4: The Stochastic Volatility of the National Factor

Notes: The solid black line is the median of the posterior distribution, while the dotted lines represent the 5% – 95% percentiles.
Figure 5: The Stochastic Volatility of the Individual Factor

Notes: The solid black line is the median of the posterior distribution, while the dotted lines represent the 5% – 95% percentiles.
Figure 6: The Stochastic Volatility of the Individual Factor - Continued

Notes: The solid black line is the median of the posterior distribution, while the dotted lines represent the 5% – 95% percentiles.
Figure 7: Time Varying Variance Contributions of the National Factor

Notes: The solid black line is the median of the posterior distribution, while the dotted lines represent the 5% – 95% percentiles.
Figure 8: Time Varying Variance Contributions of the National Factor - Continued

Notes: The solid black line is the median of the posterior distribution, while the dotted lines represent the 5% – 95% percentiles.
Figure 9: The Time Varying Average of Cross-States Correlations

Notes: The solid black line is the median of the posterior distribution, while the dotted lines represent the 5% – 95% percentiles.
Figure 10: The Total Volatility Dispersion and Its Decomposition into the National and the Individual Components

Notes: The solid black line is the median of the posterior distribution, while the dotted lines represent the 5% – 95% percentiles.
Figure 11: Evolution of Regimes

Notes: Observed data (solid blue line), regime 1 (red shaded area) spans between 1975Q1-1984Q4, regime 2 (blue shaded area) between 1985Q1 and 2008Q4 and regime 3 (yellow shaded area) between 2009Q1 and 2017Q4.
Figure 12: Impulse Responses: Demand Shock

Notes: The solid (black) line represent the (pointwise) median, while the shaded are captures the 16% – 84% percentiles of the posterior distribution. The shock has been normalised to increase GDP growth by 1 percentage point in the second quarter.
Figure 13: Forecast Variance Decomposition
Table 3: Forecast Variance Contributions

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**Notes:** The table reports the posterior mean forecast variance shares. $R = 1$, $R = 2$ and $R = 3$ indicate regimes 1, 2 and 3, respectively. While $H = 1Q$, $H = 4Q$, $H = 12Q$ and $H = 40Q$ refer to forecast horizons, 1 quarter, 4 quarter, 1 year and 10 years, respectively.
Figure 14: Impulse Responses: Supply Shock

Notes: The solid (black) line represent the (pointwise) median, while the shaded are captures the 16% – 84% percentiles of the posterior distribution. The shock has been normalised to increase GDP growth by 1 percentage point in the second quarter.
Figure 15: Impulse Responses: Policy Shock

Notes: The solid (black) line represent the (pointwise) median, while the shaded are captures the 16% – 84% percentiles of the posterior distribution. The shock has been normalised to increase GDP growth by 1 percentage point in the second quarter.
Figure 16: Impulse Responses: Slope Shock

Notes: The solid (black) line represent the (pointwise) median, while the shaded are captures the 16% – 84% percentiles of the posterior distribution. The shock has been normalised to increase GDP growth by 1 percentage point in the second quarter.
Notes: The historical decomposition is calculated for each posterior draw. The (posterior) mean of these calculations is reported here. The quarter on quarter contributions have been aggregated to annual changes contributions. The same transformation is applied for the data series.
A Bayesian MCMC Estimation Algorithm

In this section we provide further details of each step in the Gibbs-Sampling estimation algorithm.

A.1 Draws of the National Factor

We follow a procedure as laid out in Kim and Nelson (1999) to reduce the dimensionality of the resulting state-space model so as to facilitate the estimation. Specifically, substitute equation (1) into equation (6) to yield the following state-space representation:

**Measurement Equation:**

\[ r^*_{i,t} = H_t \cdot Z_t + \exp(h_{i,t}) \cdot \varepsilon_{i,t} \]  

\[ (16) \]

**Transition equation:**

\[ Z_t = F \cdot Z_{t-1} + \zeta_t \]  

\[ (17) \]

where in the measurement equation, \( r^*_{i,t} = (1 - \phi_{i,1} L - \phi_{i,2} L^2) r_{i,t} \), \( H_t = (\beta_{1,t}, -\phi_{i,1} \beta_{i,t-1}, -\phi_{i,2} \beta_{i,t-2}) \).

In the transition equation, the state vector \( Z_t = (f_t, f_{t-1}, f_{t-2})' \) and the matrix \( F \) is:

\[ F = \begin{pmatrix} \phi_1^f & \phi_2^f & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]  

\[ (18) \]

The shock vector \( \zeta_t = \left( \exp(h_{f_t}) \cdot \varepsilon_{f_t}, 0, 0 \right)' \), and its variance matrix is denoted by \( Q_t \).

Conditioning on the previous draws of \( \{H_t\}_{t=1}^T \), \( \{h_{i,t}\}_{t=1}^n \), \( \{\sigma_i^2\}_{i=1}^n \), \( F \), and \( \{Q_t\}_{t=1}^T \), we rely on the above state-space representation to take random draws of the national factor \( \{f_t\}_{t=1}^T \). Formally, we employ the Kalman filter and the ”filter forward and sample backwards” algorithm as in Carter and Kohn (1994). See Kim and Nelson (1999) for details of this standard algorithm.

A.2 Draws of the Model Parameters in the Factor Dynamics

Given previous draws of the common factor \( \{f_t\}_{t=1}^T \) and its stochastic volatility \( \{h_f^t\}_{t=1}^T \), the AR parameter of the common factor dynamics is sampled from the linear regression (3). The conjugate prior for \( \phi_f \) is a Gaussian distribution with a zero mean and a variance that is an identify matrix.

Conditional on the previous draws of the common factor \( \{f_t\}_{t=1}^T \) and the time-varying loading \( \{\beta_{i,t}\}_{t=1}^T \), the idiosyncratic factor \( e_{i,t} \) can computed from equation (1). Given the idiosyncratic factor together with its stochastic volatility \( \{h_{i,t}\}_{t=1}^T \), the AR and the variance parameters are sampled from the linear regression (3). Again the conjugate prior for \( \phi_{i,e} \) is a Gaussian distribution with a zero mean and a variance that is an identity matrix. In
addition, the conjugate prior for the variance parameter is $\sigma^2_i \sim IG(0, 0)$, where $IG$ denotes the Inverted-Gamma distribution. This specification ensures a diffuse prior for the variance parameters.

Since in both cases, the factor volatility is time varying, the regression errors are heteroskedastic. We re-scale the variables in each equation to make the errors homoskedastic, essentially doing a WLS (Weighted-Least-Squares). We sample these parameters from the transformed regression.

A.3 Draws of the Loading Parameters and the Shock Variance

Applying the same procedure as in section (A.2) to reduce the dimensionality, we can cast the model into its following state-space representation:

Measurement Equation:

$$ \mathbf{r}^*_{i,t} = \mathbf{X}_{i,t} \cdot \mathbf{B}_{i,t} + \exp(\mathbf{h}_{i,t}) \cdot \epsilon_{i,t} $$  \hspace{1cm} (19)

Transition equation:

$$ \mathbf{B}_{i,t} = \mathbf{G} \cdot \mathbf{B}_{i,t-1} + \xi_{i,t} $$  \hspace{1cm} (20)

where $\mathbf{r}^*_{i,t} = (1-\phi_{i,1}L-\phi_{i,2}L^2)r_{i,t}$, $\mathbf{X}_{i,t} = (f_{t,1}, -\phi_{i,1}f_{t-1,1}, -\phi_{i,2}f_{t-2,1})$, $\mathbf{B}_{i,t} = (\beta_{i,t}, \beta_{i,t-1}, \beta_{i,t-2})'$, $\xi_{i,t} = (\sigma^\beta_i \cdot \eta_{i,t}, 0, 0)'$, and the matrix $\mathbf{G}$ in the transition equation is:

$$ \mathbf{G} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

Because conditional on $\mathbf{r}^*_{i,t}, \mathbf{X}_{i,t}, \mathbf{h}_{i,t}, \sigma_i, \sigma^\beta_i$, the loading parameters are independent across series, this step can be conducted for each series $i$. The algorithm "filter forward, sample backwards" in Carter and Kohn (1994) is employed to draw the latent factors $\mathbf{B}_{i,t}$. Conditional on $\mathbf{B}_{i,t}$, the variance parameter $\sigma^\beta_i$ is sampled from the linear regression (5). For the variance parameter $\sigma^\beta_i$, we employ the conjugate prior: $(\sigma^\beta_i)^2 \sim IG(0.002, 2)$ with a relatively diffuse prior.

A.4 Draws of the Stochastic Volatility

Conditional on factors and corresponding parameters in the stochastic volatility process, the stochastic volatility of factors are independent from each other. As a result, we explain the sampling algorithm for the stochastic volatility of the common factor, and the same procedure applies to each idiosyncratic factor.

Given a draw of the common factor and other parameters, compute the random shock in equation (3):

$$ f^*_t = f_t - \phi^f_{t,1}f_{t-1} - \phi^f_{t,2}f_{t-2} = \exp(h^f_t) \cdot \epsilon^f_t $$  \hspace{1cm} (21)
Square and take a natural logarithm of both sides to obtain the following state-space representation:

Measurement Equation:

\[ f_t^{**} = 2h^g f_t + \zeta_t^f \]  \hspace{1cm} (22)

Transition Equation:

\[ h_t^f = h_{t-1}^f + \sigma_f^h \cdot \xi_t^f, \quad \xi_t^f \sim i.i.d.N(0, 1) \]  \hspace{1cm} (23)

where \( f_t^{**} = \ln(f_t^*)^2 \), \( \zeta_t^f = \ln(\varepsilon_t^f)^2 \).

First note that the shocks \( \zeta_t^f \) and \( \xi_t^f \) are independent. However, the shock \( \zeta_t^f \) in the measurement equation is not normally distributed, and its distribution is \( \ln\chi^2(1) \). Kim, Shephard and Chib (1998) propose an approach based on a mixture of normal densities to approximate the underlying non-normal distribution when utilizing the Kalman Filter to draw the stochastic volatility in this context. Specifically, they suggest using seven normal densities with different means \( m_k - 1.2704 \) and variances \( \tau_k^2 \), for \( k = 1, 2, ..., 7 \), with the component probabilities being \( \theta_k \). They carefully choose these values to closely replicate the exact density of \( \ln\chi^2(1) \). Table 1 below is taken from Kim, Shephard and Chib (1998) and reports these values.

Table 4: Selection of the mixing distribution to be

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Conditional on knowing \( f^{**} \) and the component probabilities of the seven normal densities, the above state space model is approximately linear and Gaussian. Therefore, the standard sampling algorithm in Carter and Kohn (1994) can be employed again to draw the stochastic volatility. Given a sample of the stochastic volatility, the component probabilities is then updated using the Bayes’ rule. The specific sampling algorithm follows those in Primiceri (2005), Del Negro and Primiceri (2015), and Koop and Korobilis (2010). For the shock variance to the volatility process, we use the conjugate with a relatively diffuse prior: \( IG(0.002, 2) \).
A Description of the Priors

The priors for the VAR($P$) coefficients and the error covariance matrices are set via dummy observations. The normal inverse Wishart prior and is defined as

$$Y_D = \begin{pmatrix}
\text{diag}(\gamma_1\sigma_1\ldots\gamma_N\sigma_N) \\
0_{N\times(P-1)\times N} \\
\ldots \\
\text{diag}(\sigma_1\ldots\sigma_N) \\
0_{1\times N}
\end{pmatrix},$$

and

$$X_D = \begin{pmatrix}
J_P \otimes \text{diag}(\sigma_1\ldots\sigma_N) \\
0_{N\times NP} \\
0_{NP \times 1} \\
0_{NP \times 1} \\
0_{1\times NP} \\
c
\end{pmatrix},$$

where $\sigma_i$ for $i = 1, 2, \ldots N$ represents scaling factors, $\gamma_i$ denotes the prior mean for the coefficients on the first lag, $\tau$ is the tightness of the prior on the VAR coefficients, $c$ is the tightness of the prior on the constant terms. In order to obtain a value for $\gamma_i, \sigma_i,$ we estimate an AR(1) model via OLS for each endogenous variable. $\gamma_i$ is set equal to OLS estimate of the AR(1) coefficient, while $\sigma_i$ is the standard deviation of the residual. The matrix $J_P$ is defined as $\text{diag}(1, 2, \ldots P)$. We set $\tau = 0.1$ and $c = 1$ in our implementation. The value for $\tau$ implies a relatively high degree of shrinkage, however, the model is estimated using quarterly macroeconomic data and a larger number of both regimes and lags is considered in order to be confident that the dynamics of the data captured properly. A tight prior and a low number of observations per regime should bias the results against state variation. However, we know from the discussion in the text that this is not the case. Despite the tight priors the estimation of the model reveals significantly different dynamics across regimes. Note that in the final regime covering the unconventional monetary policy period, we introduce an additional prior on the VAR coefficients that ensures that lagged coefficients on the non-dependent variables in the interest rate equation are close to zero. This prior is implemented via a prior covariance matrix with the diagonal elements corresponding to the coefficients of interest in the interest rate equation set to small values ($1e-12$). The remaining diagonal elements are set to $1e12$.

The prior for the non zero elements of the transition probability matrix $p_{ij}$ is of the following form

$$p_{ij}^0 = D(u_{ij}),$$

where $D(.)$ denotes the Dirichlet distribution and $u_{ij} = 15$ if $i = j$ and $u_{ij} = 1$ if $i \neq j$. This choice of $u_{ij}$ implies that the regimes are fairly persistent. The posterior distribution is:

$$p_{ij} = D(u_{ij} + \eta_{ij}),$$

where $\eta_{ij}$ denotes the number of times regime $i$ is followed by regime $j$.

B Description of the Gibbs sampling algorithm

The Gibbs sampling algorithm proceeds in the following steps:
1. **Sampling $S_t$**

   Following [Kim and Nelson, 1999](#), Chapter 9, we use Multi-Move Gibbs sampling to draw $S_t$ from the joint conditional density, $f(S_t|Z_t, c_S, B_{1:S}, \ldots, B_{K:S}, \Omega_s, \tilde{P})$. Note that we impose the restriction that each regime must have at least $N \times K + 2$ observations, where $N$ denotes the number of endogenous variables in the VAR, to ensure sufficient degrees of freedom for each regime.

2. **Sampling $c_S, B_{1:S}, \ldots, B_{K:S}, \Omega_S$**

   Conditional on a draw for $S_t$, the model is simply a sequence of Bayesian VAR models. The regime-specific VAR coefficients are sampled from a Normal distribution and the covariances are drawn from an inverted Wishart distribution. For the first $M$ regimes, we use a Normal Inverse Wishart prior (see Kadiyala and Karlsson (1997)). However, as described in detail below, we employ a (Normal diffuse) prior distribution for the VAR coefficients to the final regime, which is compatible with the identification of the shock to the government bond spread. In our sample, the recent financial crisis coincides with the final regime of the estimated VAR model. The prior on the VAR coefficients in this regime implies that the policy rate does not respond to lagged changes in other endogenous variables. This assumption is compatible with restrictions used to identify the shock to the bond-yield spread and reflects the fact that policy rates have reached the ZLB.

3. **Sampling $\tilde{P}$**

   Given the state variables $S_t$, the non-zero elements of the transition probability matrix are independent of $Z_t$ and the other parameters of the model, and they are drawn from a Dirichlet posterior.

### C Robustness Analysis

#### C.1 Looser Priors

This section illustrates that agents’ responses to identified macroeconomic shocks are (almost) unchanged when the hyper-parameter that controls the tightness of the VAR coefficients is increased to 2.
Figure 18: Impulse Responses: Demand Shock

Notes: The solid (black) line represent the (pointwise) median, while the shaded are captures the 16% – 84% percentiles of the posterior distribution. The shock has been normalised to increase GDP growth by 1 percentage point in the second quarter.
Figure 19: Impulse Responses: Supply Shock

Notes: The solid (black) line represents the (pointwise) median, while the shaded area captures the 16% – 84% percentiles of the posterior distribution. The shock has been normalized to increase GDP growth by 1 percentage point in the second quarter.
Figure 20: Impulse Responses: Policy Shock

Notes: The solid (black) line represents the (pointwise) median, while the shaded area captures the 16% - 84% percentiles of the posterior distribution. The shock has been normalised to increase GDP growth by 1 percentage point in the second quarter.
Figure 21: Impulse Responses: Slope Shock

Notes: The solid (black) line represent the (pointwise) median, while the shaded are captures the 16% – 84% percentiles of the posterior distribution. The shock has been normalised to increase GDP growth by 1 percentage point in the second quarter.
C.2 Unemployment

This section illustrates that agents’ responses to identified macroeconomic shocks are again (almost) unchanged when the unemployment (instead of GDP growth) is used.

Figure 22: Impulse Responses: Demand Shock

Notes: The solid (black) line represents the (pointwise) median, while the shaded area captures the 16% – 84% percentiles of the posterior distribution. The shock has been normalised to increase GDP growth by 1 percentage point in the second quarter.
Figure 23: Impulse Responses: Supply Shock

Notes: The solid (black) line represent the (pointwise) median, while the shaded are captures the 16% – 84% percentiles of the posterior distribution. The shock has been normalised to increase GDP growth by 1 percentage point in the second quarter.
Figure 24: Impulse Responses: Policy Shock

Notes: The solid (black) line represents the (pointwise) median, while the shaded area captures the 16% – 84% percentiles of the posterior distribution. The shock has been normalised to increase GDP growth by 1 percentage point in the second quarter.
Figure 25: Impulse Responses: Slope Shock

Notes: The solid (black) line represent the (pointwise) median, while the shaded area captures the 16% – 84% percentiles of the posterior distribution. The shock has been normalised to increase GDP growth by 1 percentage point in the second quarter.