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Endogenous Long-Term Productivity Performance in Advanced Countries: A Novel Two-Dimensional Fuzzy-Monte Carlo Approach

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Abstract: Long-term productivity performance at the country level has been a research object under different theoretical lenses, scrutinized by different modelling approaches. This paper revisits the dataset used in Bergeaud et al. (2016) and investigates the endogenous sources of distinct productivity performance in different advanced countries. Country-level data series with more than one hundred year time-span were collected for each one of the following attributes: Labor productivity (LP), Total Factor Productivity (TFP), Capital Intensity (KI), Gross Domestic Product per capita (GDP pc), Average age of equipment capital stock (Age K), and Human Capital intensity (Human K).Differently from previous studies, a Two-Dimensional Fuzzy-Monte Carlo Analysis (2DFMC) approach is proposed here to decompose the sources of long-term productivity performance. In the first dimension, a novel multi-attribute decision-making (MADM) model based on Type-2 Fuzzy Sets (T2FS) is developed to compute and rank long-term productivity performance of each county using Unbiased-Power functions for Ideal Solutions (UP-IS). Next, in the second dimension, a Stochastic Structural Relationship Programming (SSRP) Model based on neural networks is proposed to evaluate the endogenous feedbacks among the aforementioned productivity attributes and overall productivity performance. Results suggest that the UP-IS presented higher cross-performance scores relative to the TOPSIS base-case. Norway is the best performing country with a positive performance score of 0.854 while Portugal is the worst with a score of 0.347. In terms of ordinal ranking of long-term productivity performance, UP-IS ranked first, followed by TPF and next LP and GDPpc. Further, Human K and Age K have positive and negative impact respectively on long-term productivity performance in advanced countries. On the other hand, productivity performance has positive impact on KI and TFP but negative impact on LP and GDPpc.

Keywords: endogeneity; type-2 fuzzy sets; 2DFMC; stochastic performance; long-term productivity; advanced countries.

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1. Introduction

The quest to understand and explain the productivity performance of economies has grown since the seminal work of Solow (1957), which accorded the Total Factor Productivity (TFP) a crucial role in the generation and prediction of economic growth. TFP is an important tool for policy analysis as it measures the performance of economies. Specifically, it is the unexplained or residual portion of output and gives an idea of how efficiently and intensely inputs are used in the production process (Şeker and Saliola, 2018).

The debate about the causes of widespread productivity slowdown within and across countries is still ongoing (Murray, 2017; Tang and Wang, 2020). There are some arguments in favour of both supply and demand factors. The supply factors identified in the literature include slowdown in important innovation, strong decline of entrepreneurship, the waning of the ICT-related productivity boom (Gordon, 2012; Decker et al., 2016; Mckinsey Global Institute, 2018 ; Cette et al., 2015). From the demand side, weak aggregate demand, great uncertainty and financial market disruption could be critical factors explaining productivity slow down (McKinsey Global Institute, 2018). However, Blanchard et al. (2017) noted that the expectation of lower future productivity growth might cause weak demand, that is, reversal causality (Blanchard et al., 2017).

Empirical evidence shows that differences in growth patterns and income levels of countries are associated with differences in the productivity levels and often times TFP accounts for greater portion of these cross-country differences than physical and intangible capital (Hsieh and Klenow, 2010; Jones and Romer 2010; Prescott, 1998). Nonetheless, the process of economic growth is also based on technology and human capital. Thus, the intensity of productivity differentials depends on the interaction of forces acting in the opposite direction, some increasing and others limiting productivity differentials (Tamberi, 2020).

The need to monitor productivity performance cannot be overstressed as this ensures that appropriate and timely decisions, plans and policies are put in place. Therefore, the use of suitable performance measures can assist a country to adopt a long-term perspective and allocate its resources more efficiently. The path for country development has been studied under different theoretical perspectives. One of the rare consensuses is that gains in productivity are the key for promoting economic progress and social welfare. This research digs into historically related productivity data to explore the underlying endogenous relationships that may exist among different, but related, metrics for wealth, physical and human capital accumulation over the course of time.

Given the intrinsic epistemic uncertainty of dealing with so long time series, compiled in a decentralized fashion in several countries with distinct cultural and technological backgrounds, this research aims at employing fuzzy reasoning techniques to investigate the impacts of such underlying vagueness on long-term productive performance. Vague or unprecise pieces of information increase as much as time series go back into the past. Hence, this paper addresses a literature gap, by developing a novel 2DFMC analysis based on the Two-Dimensional Fuzzy-Monte Carlo Analysis (2DFMC) approach was firstly proposed by Kentel and Aral (2005). This approach combines the probability and possibility theory together, bridging gaps between both imprecise and probabilistic pieces of information, which remote origins may be questionable. We are

the first piece of research applying this method in the area of productivity performance analysis at the country-level.

In the first dimension of the 2DFMC approach proposed here, a novel MADM based on T2FS is developed for computing and ranking long-term productivity performance of advanced countries. This model (UP-IS) innovates by using a power tower function of order 2 as cornerstone for building type-2 hesitant fuzzy sets representing the membership degrees of productivity attributes and their distances to ideal solutions. Also, differently from previous studies, this novel model enables the computation of positive and negative decision-making bias that eventually arise from the inherent trade-offs of representing vagueness by T2FS. In the second dimension, an additional contribution of this study is related to the development of a novel Stochastic Structural Relationship Programming (SSRP) Model based on neural networks to evaluate the endogenous feedbacks among long-term productivity performance and its different attributes related to wealth accumulation and physical and human capital intensities, by means of maximal entropy functions. This model allows the cause-effect direction mapping between distinct productivity attributes and respective long-term productivity performance levels in advanced countries, computed at different threshold quantiles. Results indicated that UP-IS performance scores presented better discriminatory power than the TOPSIS base-case. Further, long-term productivity performance in advanced countries is positive impacted by Human K but negatively by Age K. On the other hand, KI and TFP are positively impacted on by productivity performance while LP and PDPpc are negatively impacted by same.

The remainder of this paper is organized as follows. Section 2 presents the literature review and indicates the gap found. Section 3 describes the dataset and the novel methodology developed. Results are analyzed and discussed in Section 4. Section 5 concludes the discussion and shows the limitations of the research while giving suggestions for future studies.

2. Literature Review

The economic and/or productivity performance of countries has been evaluated from both microeconomic and macroeconomic framework using different quantitative techniques including Multi Criteria Decision Making (MCDM) methods. Moreover, drivers of productivity differences including the conventional factors of production and non-conventional demand and supply variables have been analysed in the literature. This section provides an overview of these studies. For instance, Färe et al. (1994) use Malmquist productivity index and DEA to analyse the productivity growth in 17 OECD countries from 1979 to 1988. Decomposing productivity growth into changes in technical efficiency (catching up) and shifts in technology (innovation) and using GDP as the measure of aggregate output, and capital stock and employment as the aggregate inputs, they find that TFP growth driven mainly by innovation while, by contrast, technical efficiency slightly deteriorated over time.[1]

[1] Škare and Rabar (2015) and Rabar (2017) provided a review of studies that specifically employ DEA in analyzing macroeconomic performance of countries.

Gouyette and Perelman (1997) estimate the productivity performances and convergence in service and manufacturing industries by 13 OECD countries over the period 1970-1987 alternative frontier analysis and Divisia index approaches. They find that, productivity levels converge in services in spite of very low growth rates contrary to the manufacturing sector. Further, new investments in capital had a negative effect on total factor productivity growth in service activities, while the effect is positive in manufacturing industries. Golany and Li (1999) applied rank statistics based on DEA to evaluate the macro-economic performance of 17 OECD nations from 1979 to 1988.

Using data on panel of 21 OECD countries for the period 1971–1998 and pooled mean group estimator, Bassanini and Scarpetta (2001) find that, years of schooling, industry R&D and total R&D expenditure have a significant positive effect on GDP per capita growth rate, while public R&D has a negative effect. Guellec and Pottelsberghe (2001) analysed the long term relationship between multifactor productivity growth (MFP) and various types of R&D for 16 OECD countries from 1980 to1998 using an error correction model and instrumental variables. Their results show that whereas the defence-related part of public funding has a negative and significant effect, business R&D and foreign R&D have significant positive effect while on MFP.

Naastepad and Kleinknecht (2004) analysed two aspects of the performance of the Dutch economy from 1982 to 2001 namely rapid employment growth and a significant slowdown in labour productivity growth. Based on their growth accounting analysis, large part of the Dutch labour productivity growth slowdown is due to the wage growth slowdown. Broadberry and Ghosal (2005) compared productivity performance of the United States and Britain while illustrating the importance of development in services. They conclude among others that adjustment in technology-using sectors may be made difficult by technological change if such is not suited to the society's capabilities and also reversal of technological trends could lead to reversal of comparative productivity performance.

Aghion and Cohen (2004) examine the effect of years of schooling and countries labor productivity backwardness relative to USA on total productivity growth for 20 OECD countries. They estimated threshold of 24% under the frontier and find that the more a country is near the technology frontier, an additional year of schooling in primary or secondary level makes the marginal return to decrease. Further, an additional year in higher education entails 8% effect on total factors productivity. This supports the finding by Krueger and Lindahl (2001) that education is statistically significantly and positively correlated. It is also consistent with Vandenbussche et al. (2006) who used data on 19 OECD countries to show empirical evidence of TFP growth by using the world productivity as frontier and also several measures of human capital.

Ball et al. (2005) constructed an alternative productivity growth, Malmquist cost productivity (MCP) index which integrates the externality/social output into a generalized productivity measure reflecting social responsibility. Applying this to the US agriculture they show that conventional measures of productivity are biased upward (download) when production of negative externalities (or bad) outputs is increasing (decreasing). Amendola et al. (2005) argue that "productivity is the outcome of an out-of-equilibrium process triggered by a technological shock and that the potential gains of a superior

technology may only be appropriated if agents succeed in reshaping the productive capacity, and in recovering the inter temporal coordination disrupted by the introduction of the new technique. Physical, human, and financial capital are complementary in this process of reshaping, and may constrain each other. The outcome of the disequilibrium process depends then on the interaction of accumulation choices, learning, and money supply rules". They further argue that the different performances of the US and Europe in the last two decades may be explained along these lines.

Eleren and Karagül (2008) examine the economic performance of Turkey between 1986 and 2006 using TOPSIS and seven macroeconomic indicators namely economic growth rate, current account deficit, total national debt, consumer price index, current account balance, sovereign spread and unemployment rate. Whereas 1986 was found to be the best year with respect to economic performance, 1999, 2000, 2001 and 2006 were among the worst years due to both national and global crisis. Palazuelos and Fernández (2009) in their analysis of the causes of the slowdown in growth in labour productivity in the European economies found that weak domestic demand and the features of the labour markets in European countries are the main determinants. Maroto-Sánchez and Cuadrado-Roura (2009) investigated the role of the service sector, specifically tertiary activities in productivity growth using data on 37 OECD countries from 1980 to 2005. Their results show that the contribution of the service sector to productivity growth has increased in contrast to historic trends.

Mangır and Erdoğan (2011) investigate the effects of global financial crisis on Italy, Greece, Spanish, Portugal, Ireland and Turkey using data from 2002 to 2009 and a number of macroeconomic variables including economic growth rate and unemployment rate. Results based on Fuzzy TOPSIS show that Turkey overcame global financial crisis with lower losses relatively to the other five countries. Urfalioğlu and Genç (2013) use 2010 cross-section data and ELECTRE, TOPSIS and PROMETHEE approaches to evaluate and compare the economic performance of European Union countries and Turkey. Results based on different indicators of macroeconomic performance: GDP per capita, economic growth rate, export, import, employment and inflation rate show that countries with best performances are similar according to all models. Turkey ranked 31st.

Azomahou et al. (2013) analyse the productivity growth for a panel of developed and developing countries from 1998 to 2008 using three measurements of frontier: the economy with the highest level of productivity growth, the world productivity growth and the productivity growth of the USA. Results based on a semi-parametric generalized additive model show a high degree of nonlinearity between productivity growth and its determinants (human capital, R&D expenditure, international trade) including a U-shape, inverted U-shape and W-shape depending on the determinant and the frontier in question. Önder et al. (2015) evaluate the performance of five fragile large and fast growing economies (F5): Brazil, Turkey, India, Indonesia and South Africa from 2001 to 2013 using several macroeconomic indicators including gross domestic product, total investment and unemployment rate among others. Analysis based on Analytical Network Process (ANP) and show that GDP and Unemployment rate were among the four most important macroeconomic parameters for the economic performance of the F5 countries. Their Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) results show that although Turkey has the most fragile economy during great recession period

(2008-2009), its performance became is relatively high afterwards. India has stable economy and generally it has a rank of 1 and 2 while Indonesia is the best performing country in 2013.

Wu et al. (2014) evaluate the performance efficiency of 21 OECD countries and assess whether the undesirable outputs are over-produced relative to desirable outputs using study four data envelopment analysis (DEA) models. Their results support the above arguments and also show that knowledge capital, proxied by R&D improves countries' efficiency scores thereby supporting endogenous growth theory in OECD countries. Eyüboğlu (2015) evaluates the macro performances of developing countries using TOPSIS-AHP couple and data on economic growth rate, inflation rate, unemployment rate and the current account balance from 2003 and 2013. They find that Malaysia and China are the highest performance countries.

Karabiyik and Kutlu (2016) evaluate the international trade performance of OECD countries from 1999 to 2014 using TOPSIS and AHP methods. Analysis based three foreign trade performance indicators, namely; Volume of Exports Per Capita, Normalized Trade Balance and Terms of Trade show that Norway, Ireland and Germany are ranked among the top three countries while Turkey, USA and the Greece are the bottom three. Pan and Ngo (2016) analyse the regional performance of 64 Vietnamese provinces using panel dataset. Their results show that FDI, openness, and capital investment have positive impacts on GDP growth although no support for the endogenous growth model was found in cases in which regional per capita income tended to converge across different regions. They also find that for provinces that have established special economic zones through liberal state regulation, internationalization activities have positive effect on regional performance.

Millemaci and Ofria (2016) reported a positive effect of labour costs, R&D and railway infrastructure on the Italian productivity growth based on results from pooled cross-section OLS and time series LIML estimators using data from 1964–2009.However, Padilla-Pérez and Villarreal (2017) show that highly qualified production factors (both labour and capital) have not showed a significant contribution to value added growth in Mexico. The long run relationship between GDP growth and the job "required" by such a growth is examined by Compagnucci et al. (2018) using data from 1970 to 2015. They find that the break in the relationship between GDP per capita growth and employment is due to the decoupling of productivity, labour compensation and utilisation from each other. They also find that the technological and knowledge intensity of different economic sectors played important role in the productivity change. Fernándeza and Palazuelos (2018) could neither find an empirical support of the thesis that contribution of manufacturing to aggregate productivity in European countries is high but lower than that of the service sector, nor that the growth pattern of technology-intensive branches of the manufacturing sector is more oriented towards productivity.

Tasnim and Afzal (2018) analyse the effects of national systems of entrepreneurship and other macro factors on the efficiency of 59 countries using DEA and Tobit regression. Their results show that inefficiency varied widely across countries and the most efficient ones are the innovation-driven economies. In addition, GDP per capita and social capital were found to be important in efficiency improvement. Using the

subsystems approach and Chinese data from 1995 to 2009, Brondino (2019) finds that the major source of productivity growth across subsystems was direct labor savings and that the best performing subsystems where targeted by industrial policy for promotion. Based on the tradable-nontradable framework, Friesenbichler and Glocker (2019) find that increases in overall productivity among EU Member States are mainly due to the tradable and not the nontradable sectors of production. They find that productivity growth differentials could be explained by differences the legal systems and the quality of public institutions among others.

A decomposition analysis by Moussir and Chatri (2020) show that the intrasectoral component account for much of the labour productivity growth in Morocco. Their results show that an increase in income, education and human capital determinants of labour productivity growth and hence structural transformation while the labour market flexibility, inflation and the financial system had adverse effect on the competitiveness of the economy. Tang and Wang (2020) using micro data decomposed productivity in Canada into technological frontier and technical efficiency. They find that Canada's productivity was mainly due to the retreat of aggregate technological frontier, driven by large and high-productivity firms rather than factors such as R&D, ICTs and other intangibles. Rouyendegh et al. (2020) use an integrated intuitionistic fuzzy Technique for Order of Preference by Similarity to Ideal Solution (IF-TOPSIS) and data envelopment analysis (DEA) to evaluate the performance of the retail industry. They show that combination of the two methods is suitable for any number of DMUs.

Yapa et al. (2020) use the Best Worst Method (BWM), an MCDM method and several macroeconomic indicators: GDP per capita, unemployment, inflation, real interest rates, and growth rates to compare the performance of EU countries and Turkey. They find that Luxembourg ranks first, Denmark second, and Sweden third while Portugal, Croatia, and Greece are the last three by rank. Turkey ranks 24th. Azenui and Rada (2021) studying thirty sub-Sahara African LDCs over 1991–2018 period find that manufacturing and FDI contribute positively towards labour productivity growth with exception of mineral exporting countries while FDI has negative effect whereas the effect of global integration is weak.

In this study we contribute to the literature on country-level productivity performance by developing a novel 2DFMC analysis based on the Two-Dimensional Fuzzy-Monte Carlo Analysis (2DFMC) approach whose merits have been stated earlier.

3. Methodology

3.1. The Data

We use annual time series data from 1890 to 2018 for twenty four (24) countries (Australia, Austria, Belgium, Canada, Chile, Denmark, Euro Area, Finland, France, Germany, Greece, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States). The data are obtained from the long term productivity project

(http://www.longtermproductivity.com/), a database developed and regularly updated by Bergeaud et al. (2016). Table 1 presents the summary statistics for the data. Total factor productivity (TFP) is the Solow residual from a constant return to scale Cobb-Douglas production function with capital stock and hours worked as input. Labor productivity is the ratio of GDP over total hours worked. Capital intensity (KI) is the ratio of total capital stock over total hours worked. Age K is the average age of equipment capital stock in years. Human capital intensity (Human K) is proxied by education attainment. GDP per capita is the Gross Domestic Product per capita. All variables are calculated using GDP and capital stock series converted in US dollars of 2010 ppp. All the variables have positive mean with moderate variation. All the variables are positively skewed with the exception of human capital intensity and average age of equipment capital stock which are negatively skewed.

Table 1. Descriptive statistics on long-term productivity of advanced countries.

Variable	Type	Min	Max	Mean	SD	CV	Skewness	Kurtosis
GDP per capita	Positive Criteria	1496.44	74972.89	16016.38	13788.82	0.86	1.27	1.26
Human K per capita	Positive Criteria	582.42	2726.59	1483.22	362.54	0.24	-0.07	-0.28
Labor Productivity	Positive Criteria	0.93	103.95	20.36	19.14	0.94	1.25	1.17
TFP	Positive Criteria	0.96	18.63	5.47	3.56	0.65	0.82	-0.15
Age K	Negative Criteria	-10.72	11.91	6.46	1.14	0.18	-0.69	17.12
KI	Positive Criteria	0.24	380.37	63.94	65.73	1.03	1.33	1.25

3.2. Performance Measurement and Ideal Solutions (IS)

Performance measurement refers to a broader benchmarking concept that can be structured by using either scalar or ratio variables, or even a mix of them. It is usually employed when there are difficulties in comparing with peers – they may not be homogeneous - and/or, - although not necessarily -, in quantifying monetary or physical values for inputs and outputs, namely, the negative and positive attributes, respectively, for decision-making (Mihaiu et al., 2010). Performance scores are often assessed by MADM matrix-based methods like TOPSIS, VIKOR, or COPRAS, for instance, where specific functions are assumed (e.g. ideal solutions, compromise solutions, utility solutions, etc) (Behzadian et al., 2012). More specifically with respect to ideal solutions, TOPSIS is a well-known MADM model that develops cardinal or scale metrics within the range delimited by positive and negative ideal solutions through a linear combination of attributes. The performance distance in TOPSIS is cardinal, consisting of a Euclidean

distance (Olson, 2004). Putting into other words, TOPSIS computes cardinal distances (scores) from ideal positive solutions while simultaneously presents an ordinal ranking of them (Behzadian et al., 2012). The positive ideal solution has the best level for all attributes considered, while the negative ideal is the one with the worst values (Wanke et al., 2016a). While the determination of the weights of the relative importance of each attribute is exogenously defined in TOPSIS and might be considered a drawback, on the other hand. TOPSIS is computationally simpler because there are virtually no constraints with respect to the number of attributes that can be assessed (Wanke et al., 2016a).

3.3. Type-2 Fuzzy Sets (T2FS)

Treating normalized productivity attributes and its distances to ideal solutions by means ofT2FSyields more freedom for capturing inherent vagueness related to data collection and measurement when compared to Type 1 fuzzy sets (Mendel and John, 2002). This happens because T2FS can reflect the uncertainty of inaccurate data by means of primary and secondary membership functions related, respectively, to normalized attributes and their distances to ideal solutions (Turk et al., 2014). As regards the application on advanced countries, while distances to ideal solutions are usually computed at each normalized attribute level, very often identifying an unbiased global ideal solution is difficult due to multicollinearity issues among attributes. This being the case, extensive pairwise comparisons may assist in developing unbiased evaluation alternatives (Zavadskas et al., 2014).

T2FS are defined by primary and secondary membership functions to represent uncertain information more effectively (Hu et al., 2015). In this section, basic concepts and arithmetic operations of T2FS are summarized from Lee and Chen (2008), Chen and Lee (2010a, 2010b), Mendel et al. (2006), Hu et al. (2015), and are given as it follows.

Definition 1. A T2FS$\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type-2 membership function $\mu\tilde{\tilde{A}}$, as shown next (Mendel et al., 2006):

$$\tilde{\tilde{A}} = \left\{\left((x,u), \mu_{\tilde{\tilde{A}}}(x,u)\right) | \forall_X \in X, \forall_u \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1\right\} \quad (1)$$

Where J_x denotes an interval in $[0,1]$ which represents the support for the secondary membership function. Moreover, the T2FS$\tilde{\tilde{A}}$ also can be represented as follows (Mendel et al., 2006):

$$\tilde{\tilde{A}} = \iint_{x \in X\ u \in J_x} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u), \quad (2)$$

where $J_x \subseteq [0,1]$ and the double integral denotes union over all admissible *x* and *u*.

Definition 2. The concept of hesitant T2FS (HT2FS) is introduced to represent a particular subset of the original T2FS and can be used to explore alternative membership possibilities with respect to *x* and *u* (Hu et al., 2015).

Definition 3. Let $\tilde{\tilde{A}}$ be a fixed set. An HT2FS on $\tilde{\tilde{A}}$ is expressed in terms of a function that returns a type-2 fuzzy subset when applied to each (*x*,*u*) pair in $\tilde{\tilde{A}}$(Xia and Xu, 2011). To make it easily understood, this HT2FS is represented by $E = \{< (x,u), \tilde{h}_E(x,u) >$

$|x \in X, u \in J_x\}$,where $\tilde{h}_E(x,u)$ denotes the possible membership degrees for each pair (x, u) with respect to the subset E.

3.4. Unbiased-Power functions for Ideal Solutions (UP-IS) based on T2FS

Let's consider a set of d advanced countries sections, each one of them formed by I positive attributes to productivity performance -$pos_{d,i}$- and o negative attributes to productivity performance -$neg_{d,o}$-, where $d = \{1..n\}$, $i = \{1..m\}$, $o = \{1..s\}$, **pos** and **neg** are, respectively, positive and negative attribute matrices with dimensions nxm and nxs. The negative ideal solution for all d countries for each negative attribute o is given by max(neg_o) while the positive ideal solution for each positive attribute i is given by max(pos_i), for all DMU – Decision Making Unit, i.e., country, d. These ideal solutions are the cornerstones for computing the normalized values for each productivity performance positive attribute i and for each productivity performance negative attribute o at each country level, with respect to their ideal solutions, such as:

$x_{d,i} = (pos_{d,i} - \min(pos_i))/(\max(pos_i) - \min(pos_i))$, $x_{d,I}$ ranges between 0 and 1 for all i and d (3)

$x_{d,o} = (\max(neg_o) - neg_{d,o})/(\max(neg_o) - \min(neg_o))$, $x_{d,o}$ranges between 0 and 1 for all o and d (4)

where $x_{d,i}$ is the normalized positive attribute i at each country d, while $x_{d,o}$ is the normalized negative attribute o at each country section d. One can see that maximal values for positive attributes would correspond to a normalized value of 1. Conversely, maximal values for negative attributes would correspond to a normalized value of zero, thus allowing to treat both sets of attributes simultaneously in a matrix **x** with dimensions nx(m+s). At last, **u** represents the nx(m+s) distance matrix to each ideal solution. For positive attributes, it follows that $u_{d,i} = 1 - x_{d,i}$ and, for negative attributes, $u_{d,o} = x_{d,o}$. Analogously to normalized attributes, distances to ideal solutions range between zero to one. The UP-IS computational steps are detailed next.

Suppose $\mu_{\tilde{\tilde{A}}}(x)$denotes the partial membership function for the elements of the normalized attribute matrix **x**, expressed as a power tower function of order (or height) 2. Precisely, power tower functions denote interactive exponentiation (tetration).

$$\mu_{\tilde{\tilde{A}}}(x) = \frac{(1-x^x)}{k_1}, \quad (5)$$

where all x range between 0 and 1,$k_1$is a constant given by $\left(1 - e^{(-e^{-1})}\right) \approx 0.3078$, $\mu_{\tilde{\tilde{A}}}(0) = \mu_{\tilde{\tilde{A}}}(1) = 0$, and $\mu_{\tilde{\tilde{A}}}(x) = 1$ for $x = e^{-1} \approx 0.3679$.

The partial membership function for any given country productivity attribute distance to the ideal solution, $\mu_{\tilde{\tilde{A}}}(u)$, can be defined by solving $\mu_{\tilde{\tilde{A}}}(x) = u$ in terms of x, thus expressing x as a function of u. This way, J_x, a wavy slice representation of $\mu_{\tilde{\tilde{A}}}(u)$ is obtained as an embedded set within the type 1 fuzzy set X. This wavy slice is also known as the Mendel-John representation theorem. This theorem makes operational the creation of a 3D membership function, $\mu_{\tilde{\tilde{A}}}(x,u)$, based on a bidimensional space (x, u) defined by the union of two membership functions of type 1,$\mu_{\tilde{\tilde{A}}}(x)and\mu_{\tilde{\tilde{A}}}(u)$,both ranging over the interval between 0 and 1.

$$\mu_{\tilde{\tilde{A}}}(u) = \frac{\beta}{eW(\beta/e)}, \tag{6}$$

where β is a function of u given by $\beta = [\ln(u(1 - e^{-e}) + e^{-e})\, e - 1]$, W is the Lambert function, all u range between 0 and 1, and $\mu_{\tilde{\tilde{A}}}(u) = 1$ for $u = 0$.W is the transcendent function that satisfies $W(z)e^{W(z)} = z$ and that can be numerically evaluated using MAPLE functions or R libraries (codes are available to readers upon request). W is also related to the interated exponentiaton problem, or power tower functions, namely $f(z)$, such that $f(z) = -W(-\ln(z))/\ln(z)$, converging for real values of z in the range from $e^{-e} \approx 0.066$ to $e^{1/e} \approx 1.44$ (Corless et al., 1996; Lynch, 2017).

Fig. 1depicts the type 1 representations of partial membership functions $\mu_{\tilde{\tilde{A}}}(x)and\mu_{\tilde{\tilde{A}}}(u)$ and the type 2 representation of $\mu_{\tilde{\tilde{A}}}(x,u)$ – recall that $\mu_{\tilde{\tilde{A}}}(x,u) = \mu_{\tilde{\tilde{A}}}(x) \cap \mu_{\tilde{\tilde{A}}}(u)$. Fig. 2 depicts the *x* and *u* axis-slices representing the hesitant membership degrees for $\tilde{h}_E(x,u)$. One can note from Fig. 1 that higher membership grades are assigned to smaller distances towards the ideal solutions ($\mu_{\tilde{\tilde{A}}}(u)$), although lower membership grades are assigned to extreme measurement of normalized attributes ($\mu_{\tilde{\tilde{A}}}(x)$). This being the case, this T2FS - for assessing each country productivity performance - counterbalances the vagueness, inherent to extreme measurements of normalized attributes, with their distances to ideal solutions. These characteristics can also be inferred from the hesitant memberships depicted in Fig. 2. Shorter distances to ideal solutions (lower values of *u*) imply higher membership grades for normalized attributes (especially for those mid-valued in the *x*-axis).

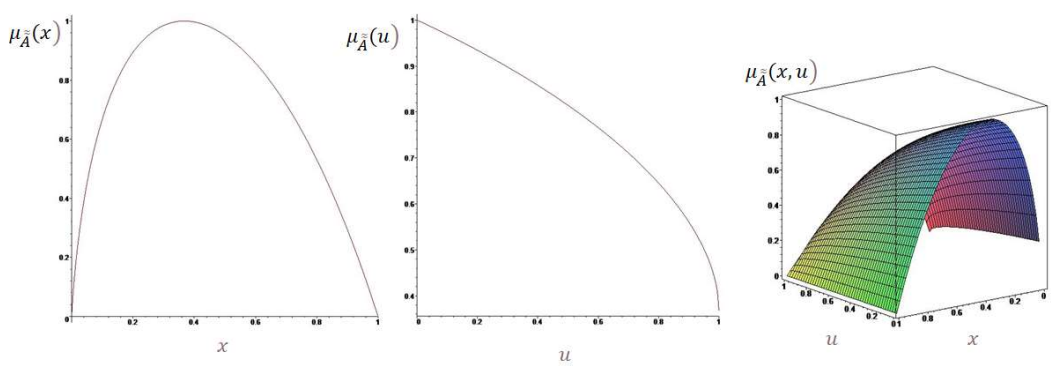


Fig. 1. Membership functions.

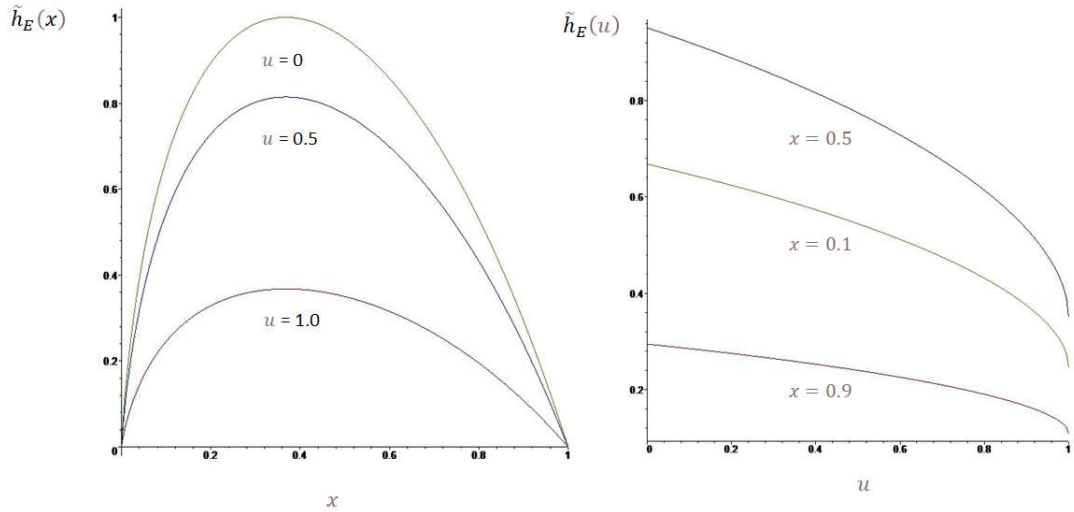


Fig. 2. Hesitant fuzzy sets.

This intrinsic counterbalance or trade-off between the partial membership grades assigned to each (x, u) pair yields a measurement bias that should be accounted for when computing productivity performance for country section based on their normalized productivity attributes, their distances to ideal solutions and their consequent weights.

<u>Positive bias</u>

Consider $\mathbf{x}^+$,$\mathbf{u}^+$, and $\mathbf{h}^+$three nx(m+s) positive bias matrices for normalized attributes, distances to ideal solutions, and type-2 membership functions. The positive bias integration departs from **Definition 3**. As regards normalized attributes and distances to ideal solutions it is computed as the average positive deviation weighted by the respective membership degrees for each element x and u with respect to the subset E, as defined by the integral limits. As regards type-2 membership functions, it is the average difference in membership degree for each pair (x, u) with respect to the subset E.

$$\mathbf{x}^+ = \int_{x=\mathbf{x}}^{1} \frac{\mu_{\tilde{\tilde{A}}}(x)(x-\mathbf{x})}{(1-\mathbf{x})} dx \tag{7}$$

$$\mathbf{u}^+ = \int_{u=\mathbf{u}}^{1} \frac{\mu_{\tilde{\tilde{A}}}(u)(u-\mathbf{u})}{(1-\mathbf{u})} du \tag{8}$$

$$\mathbf{h}^+ = \iint_{x=\mathbf{x};u=\mathbf{u}}^{1;1} \frac{\mu_{\tilde{\tilde{A}}}(x,u)}{(1-\mathbf{x})(1-\mathbf{u})} dudx - \mu_{\tilde{\tilde{A}}}(\mathbf{x},\mathbf{u}) \tag{9}$$

<u>Negative bias</u>

Negative bias computation is defined analogously to positive bias.

$$\mathbf{x}^- = \int_{0}^{x=\mathbf{x}} \frac{\mu_{\tilde{\tilde{A}}}(x)(\mathbf{x}-x)}{(\mathbf{x})} dx \tag{10}$$

$$\mathbf{u}^- = \int_{0}^{u=\mathbf{u}} \frac{\mu_{\tilde{\tilde{A}}}(u)(\mathbf{u}-u)}{(\mathbf{u})} du \tag{11}$$

$$\mathbf{h}^- = \iint_{0;0}^{x=\mathbf{x};u=\mathbf{u}} \frac{\mu_{\tilde{\tilde{A}}}(x,u)}{(\mathbf{x})(\mathbf{u})} dudx - \mu_{\tilde{\tilde{A}}}(\mathbf{x},\mathbf{u}) \tag{12}$$

<u>Unbiased or bias-corrected T2FS</u>

While the computation of unbiased normalized attributes, distance to ideal solutions, and membership functions is straightforward, readers should note that two sets of resulting

matrices are produced in accordance to the bias removal: positive (denoted by extension_**pos**) and negative (denoted by extension_**neg**).

$$\mathbf{x\_pos} = \mathbf{x} + \mathbf{x}^{+} \quad (13)$$
$$\mathbf{x\_neg} = \mathbf{x} - \mathbf{x}^{-} \quad (14)$$
$$\mathbf{u\_pos} = \mathbf{u} + \mathbf{u}^{+} \quad (15)$$
$$\mathbf{u\_neg} = \mathbf{u} - \mathbf{u}^{-} \quad (16)$$
$$\mathbf{h\_pos} = \mu_{\tilde{\tilde{A}}}(\mathbf{x}, \mathbf{u}) + \mathbf{h}^{+} \quad (17)$$
$$\mathbf{h\_neg} = \mu_{\tilde{\tilde{A}}}(\mathbf{x}, \mathbf{u}) + \mathbf{h}^{-} \quad (18)$$

Normalized attribute weights

Positive (w_k^+) and negative (w_k^-) weights for each normalized attribute$k$ ($k = i \cup o$) can be defined based on the following equations:

$$w_k^+ = \frac{\sum_{d=1}^{n} u\_pos_{d,k} * h\_pos_{d,k}}{\sum_{k=1}^{m+s} \sum_{d=1}^{n} u\_pos_{d,k} * h\_pos_{d,k}} \text{ for all } k \quad (19)$$

$$w_k^- = \frac{\sum_{d=1}^{n} u\_neg_{d,k} * h\_neg_{d,k}}{\sum_{k=1}^{m+s} \sum_{d=1}^{n} u\_neg_{d,k} * h\_heg_{d,k}} \text{ for all } k \quad (20)$$

In what follows, normalized attribute weights are given by the pounded unbiased distances to ideal solutions in terms of the unbiased respective membership degrees.

Performance scores for each country *d*

Positive (s_d^+) and negative (s_d^-)productivity performance scores for each country$d$ is computed based on the following equations. Readers should note that positive and negative cross-performance scores for each country *d* may also be computed (cs_d^+ and cs_d^-, respectively).

$$s_d^+ = \sum_{k=1}^{m+s} x\_pos_{d,k} * w_k^+ \text{ for all } d \quad (21)$$

$$s_d^- = \sum_{k=1}^{m+s} x\_neg_{d,k} * w_k^- \text{ for all } d \quad (22)$$

$$cs_d^+ = \sum_{k=1}^{m+s} x\_pos_{d,k} * w_k^- \text{ for all } d \quad (23)$$

$$cs_d^- = \sum_{k=1}^{m+s} x\_neg_{d,k} * w_k^+ \text{ for all } d \quad (24)$$

3.5. Stochastic Structural Relationship Programming (SSRP) Model

This paper proposes a novel SSRP model based on the neural network architecture to unveil the existing endogenous relationships among productivity performance relative to ideal solutions and countries´ normalized productivity attributes, while identifying the relevant structural cause-effect relationships that may exist. Precisely, neural networks are employed on unveiling endogeneity among a given country productivity performance and its normalized productivity attributes, whether positive or negative, in terms of the residuals produced by the following models.

Model 1: Performance ~ f(GDP; Human K; Labor Prod.; TFP; Age K; KI)

Model 2: GDP ~ f(Performance; Human K; Labor Prod.; TFP; Age K; KI)

Model 3: Human K ~ f(GDP; Performance; Labor Prod.; TFP; Age K; KI)

Model 4: Labor Prod ~ f(GDP; Human K; Performance.; TFP; Age K; KI)

Model 5: TFP ~ f(GDP; Human K; Labor Prod.; Performance; Age K; KI)

Model 6: Age K ~ f(GDP; Human K; Labor Prod.; TFP; Performance; KI)

Model 7: KI ~ f(GDP; Human K; Labor Prod.; TFP; Age K; Performance)

These residuals are subsequently used to generate a full set of conditional residual distributions between the respective dependent variable pairs identified in each model. These conditional residual distributions allow exploring the directional relationships that may exist among variables. The novel SSRP model is structured in two consecutive steps that allow unveiling endogeneity while identifying relevant cause-effect relationships among the reminder variables. These steps are described next.

Step 1: Minimal Endogenous Relationship Variance

The relative importance of models (1)-(7) in explaining the feedback process between normalized attributes and long-term productivity performance levels in advance countries, besides the endogenous nature of these variables, were explored, respectively, by the variances of each model and the covariances between models. Variances and covariances of the residuals (R_i) of these 7models are simultaneously minimized by a non-linear stochastic optimization problem, as presented in Model (25), where w_i stands for the weights – which range from 0 to 1 - assigned, respectively, to the residual vectors of each one of the 7 models previously described. The values of *w* are optimized so that the variance (*Var*) and covariance (*Covar*) of the pooled residuals is minimal. Model (25) was solved by means of differential evolution (DE). DE is a research stream of genetic algorithms, also emulating the natural selection and evolution.

$$\min\left[Var\left(\sum_{i=1}^{7} w_i * R_i\right) + \left(2 * \sum_{i,j=1}^{7} Covar\,(w_i * w_j * R_i * R_j), i \neq j, j < i\right)\right]$$
$$subject\ to$$
$$\sum_{i=1}^{7} w_i = 1 \qquad (25)$$
$$0 \leq w_i \leq 1 \quad \forall i$$

Residuals of the MLP models were bootstrapped 100 times allowing the collection of a distributional profile of *w* for the most accurate prediction of productivity performance scores and contextual variables.

Step 2: Maximal Information Entropy for Directional Weighted Residuals

The principle of maximum entropy states that the probability distribution which best represents the current state of knowledge is the one with largest entropy, in the context of precisely testable information. Subsequently, in the second step, a full combinatorial set of conditional distributions of residuals (CR_k) was computed. The previous 100 bootstrapped replications for the unconditional individual residuals (R_i) of each model served as cornerstones for this computation, where $CR_k \sim f(R_i/R_j)$ for all i and j, $i \neq j$, and $K = i * j - i = 7 * 7 - 7 = 42$. Similarly, DE was employed for solving the

following non-linear integer programing model to diagnose whether conditional distributions of for each residual pair presented significant differences in terms of directions. For instance, weights assigned to $f(R_i/R_j)$ could yield higher entropy those assigned to $f(R_j/R_i)$ levels in comparison to the unconditional residuals analyzed in Step 1 to unveil endogeneity. This non-linear integer programming problem is depicted in model (26).

$$\max\left[\left(\sum_i\sum_j H\left(f\left(\frac{R_i}{R_j}\right) * w_i * w_j\right)\right) OR \left(\sum_i\sum_j H\big(g(R_i,R_j) * w_i * w_j\big)\right), i \neq j\right]$$

$$subject\ to$$
$$\sum_{i=1}^{7} w_i = 1 \ (26)$$
$$0 \leq w_i \leq 1, \qquad \forall i$$

Where:
$H(.)$ denotes the information entropy function,
$g(R_i, R_j)$denotes the unconditional marginals of the residuals from models $(1) - (7)$, $\forall i, j,\ i \neq j$,
$f(R_i/R_j)$ denotes the conditional distribution of the residuals from models (1) - (7), $\forall i, j,\ i \neq j$.

This non-linear integer programming model returns the structural relationship among dependent variables defined in models (1)-(7) for which information entropy is maximal. This assures the uniqueness and consistency of the probabilistic weight profile computed in Step 1 for which the overall residual variance is also minimal. Hence, weights computed in Step 1 were used as starting values for Step 2 optimization. Again, differential evolution was employed for finding optimal solutions as regards maximal entropy for each *ij* pair. This model returns, for a given *ij* pair, whether its relationship is endogenous or whether *i* causes *j* (or the other way around). Table 2 presents the pseudo code used for computing *f*(.) and *g*(.) estimates used in Step 2 optimization model.

Table 2. Pseudo code used for computing *f*(.) and *g*(.) estimates used in Step 2

1	Fit the statistical distributions for each model residuals
2	Calculate the correlation matrix between all model residuals
3	Fit the best distribution of residuals for each model (cf. Table 3)
4	Generate multivariate Copulas preserving the correlation and distribution structure for all model residuals
5	Select percentile thresholds to stress directional relationships under extreme Copulas distribution
a	for p in percentiles do
b	for i in 1 to models do
c	for j in i to models do
d	Evaluate the conditional distribution of copula ij
e	Evaluate the unconditional marginal distributions of copula ij

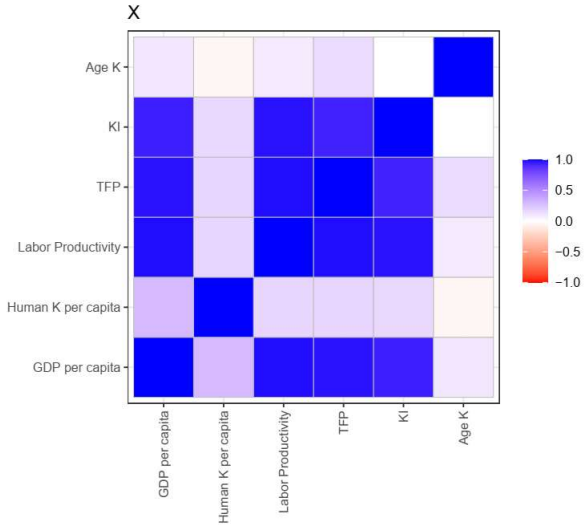
f	Solve model (26) using Differential Evolution Approach

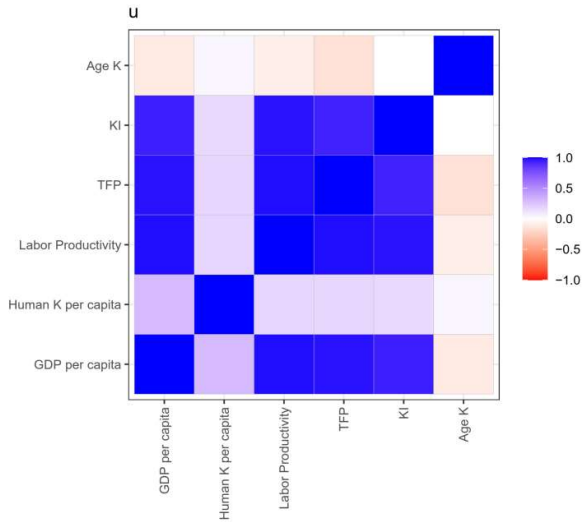
Table 3. Distributional fit for residuals obtained in models (1)-(7).

Model	Distribution	Parameters	
		meanlog	sdlog
SD+	lnorm	-2.03	0.04
		shape1	shape2
GDP per capita	beta	21.62	15.82
		mean	sd
Human K per capita	norm	-0.36	0.19
		shape1	shape2
Labor Productivity	beta	19.68	10.50
		shape1	shape2
TFP	beta	15.46	12.98
		shape1	shape2
KI	beta	24.93	25.25
		shape1	shape2
Age K	beta	17.27	6.49

4. Analysis and Discussion of Results

Correlograms for the normalized attributes (x), their respective distances to ideal solutions (u), and their resulting hesitant membership function values (h) are depicted in Fig. 3 (top-left, top-right, and bottom). With the exception of Age K, all other productivity attributes tend to be strongly correlated to one another. However, as regards the distances to ideal solutions, Age K presents negative, albeit weakly, correlations to the remainder productivity attributes. Being the only negative attribute, this means that, the higher the age of capital stock, the farther from their ideal solutions are the other productivity attributes. Yet, an analogous interpretation can be considered for Human K as regards membership functions. Lower productivity attributes such as GDP pc, LP, TFP can be compensated by higher levels of human capital as pertaining a higher membership grade of long-term productive performance in advanced countries. Hence, Age K and Human K appears to be the two cornerstones for long-term productivity performance in advanced countries. Age K as a necessary condition, since a newer stock of capital raises all other productivity attributes towards their ideal solutions. Human K as a sufficient condition, since it is capable of compensating deficiencies in other productivity attributes. These issues are further discussed throughout the text.


X
Age K
KI
TFP
Labor Productivity
Human K per capita
GDP per capita
1.0
0.5
0.0
−0.5
−1.0


u
Age K
KI
TFP
Labor Productivity
Human K per capita
GDP per capita
1.0
0.5
0.0
−0.5
−1.0

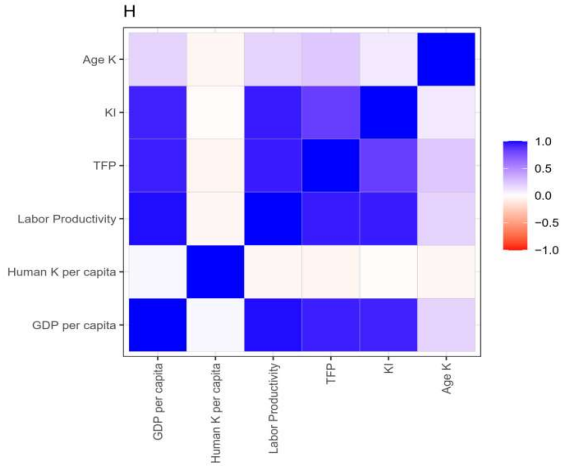

H
Age K
KI
TFP
Labor Productivity
Human K per capita
GDP per capita
1.0
0.5
0.0
−0.5
−1.0

Fig. 3. Correlograms for the performance attributes.

Fig. 4 reports on the normalized attribute weights obtained after removing the positive and negative bias that are inherent to the measurements in **x**, **u**, and **h**. Readers should recall that these weights are obtained as the pounded averages of the unbiased distances to ideal solutions in terms of the respective unbiased membership degrees. Readers may note that normalized attribute weights obtained after positive bias removal are more balanced than those obtained after negative bias removal. This suggest that the key for long-term productivity performance in advanced countries relies on a balance among capital intensity, labor productivity, total factor productivity, and GDP per capita, productivity attributes that presented practically the same positive weights. Less important for a better long-term productivity performance are Human K and Age K, when all other attributes are balanced. On the other hand, worse levels of longer-term productivity performance depend on a conjunction of attributes that reinforces lower levels of human capital, higher capital stock age, and lower TFP. This worse productivity performance seems to be created by an unfortunate combination between the two cornerstones productivity attributes identified in the correlation analysis: Human K and Age K.

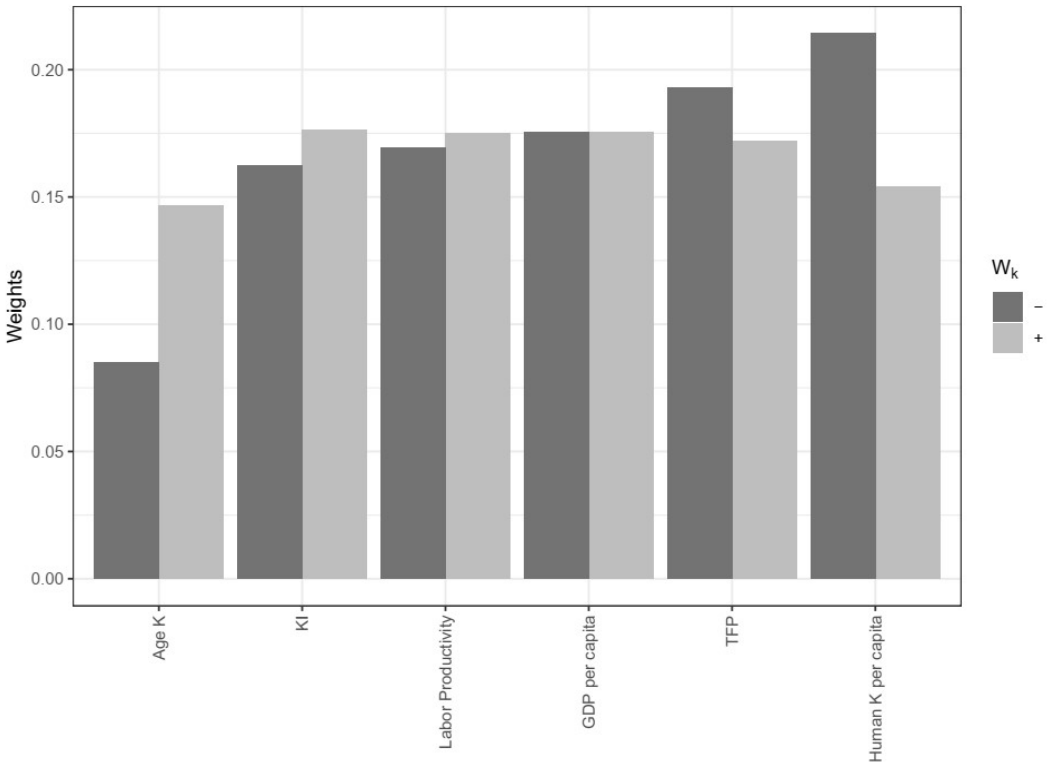


Fig. 4. Positive and negative weights for each normalized attribute.

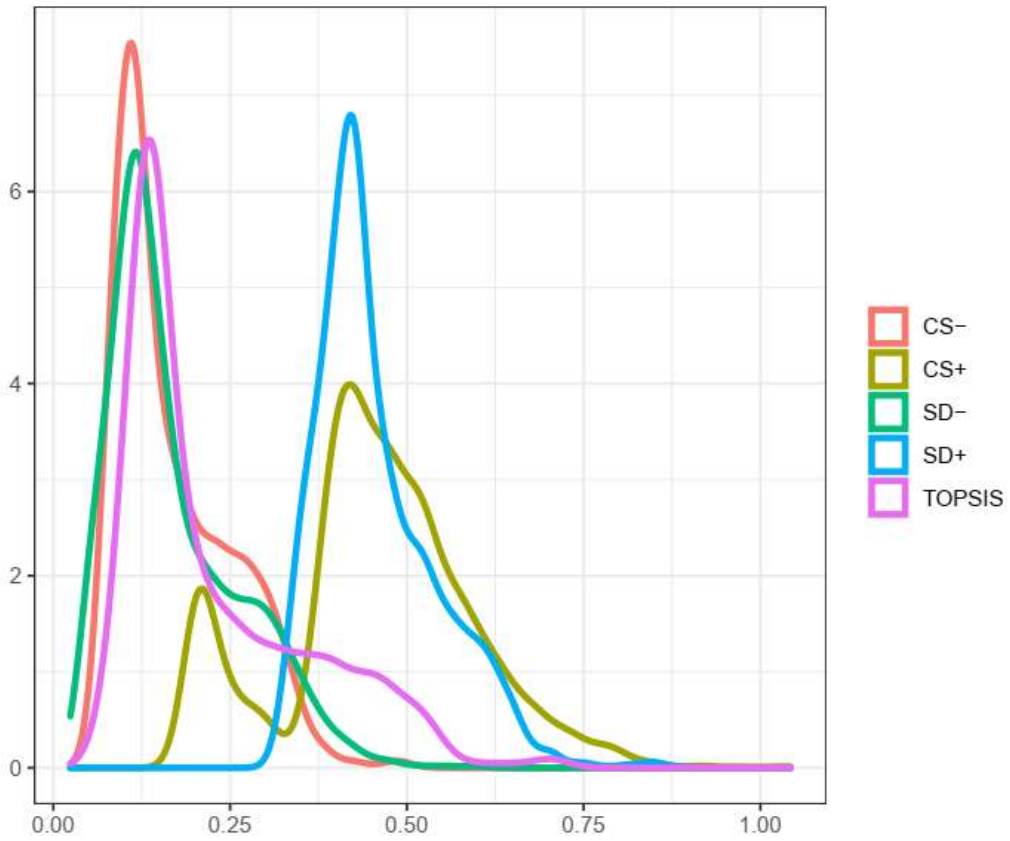


Fig. 5. Density plot comparison for UP-IS performance under alternative weighting schemes in light of basic TOPSIS results.

Density plots for the four alternative UP-IS performance scores obtained using weights from eqs. (21)-(24) are given in Fig. 5, altogether with the TOPSIS base-case comparison for equal weights. As can be seen, UP-IS performance scores presented better discriminatory power – widespread shape although still apart from 0 and 1 boundaries - in comparison to the TOPSIS base-case. As expected, performance and cross-performance scores computed with the weight set obtained after the positive bias removal presented median values higher than the TOPSIS base-case. The contrary was verified for performance and cross-performance UP-IS scores computed with the weight set determined after negative bias removal. Table 4 presents the descriptive statistics altogether with information entropy computation for each one of the distributions depicted in Fig. 5. According to the maximal entropy principle, whenever deciding in favor of a given distribution to the detriment of the other without any prior information, maximal entropy distributions should be preferable for the sake of robustness, as long as they are not impacted, in subsequent analysis, by any modelling bias that eventually have escaped the decision makers.

Table 4. Descriptive statistics for alternative performance distributions

Variables	Mean	SD	CV	Kurtosis	Skewness	IE
TOPSIS	0.228	0.130	0.569	0.555	1.160	0.493
SD+	0.458	0.086	0.187	1.315	1.073	0.494
SD-	0.170	0.092	0.542	0.588	1.002	0.497
CS+	0.457	0.136	0.298	0.260	0.045	0.496
CS-	0.170	0.082	0.480	0.196	0.916	0.502

The performance scores of each of the countries with respect to their best and worst ranks are presented in Table 5. The results show that most of the countries had their productivity performance in 2018 while the worst performance occurred around late 1800s to mid 1900s. Norway is the best performing country with a score of 0.854 while Portugal is the least with a score of 0.347.

Table 5: Positive productivity performance scores by country

S/N	Country	Best Rank Order (Year)	Worst Rank Order (Year)	Score for Best Rank	Score for Worst Rank
1	Norway	2018	1905	0.854	0.397
2	Ireland	2018	1921	0.751	0.371
3	Switzerland	2018	1891	0.722	0.424
4	United States	2016	1894	0.676	0.377
5	Australia	2018	1897	0.652	0.389
6	Netherlands	2018	1945	0.644	0.379
7	Denmark	2017	1891	0.641	0.394
8	Belgium	2016	1918	0.634	0.366
9	Germany	2016	1947	0.633	0.385
10	Sweden	2018	1921	0.616	0.381
11	Canada	2017	1890	0.612	0.391
12	France	2018	1944	0.611	0.377
13	Austria	2017	1919	0.609	0.366
14	Finland	2018	1918	0.601	0.361
15	Euro Area	2018	1946	0.596	0.383
16	New Zealand	2018	1890	0.590	0.397
17	Japan	2017	1949	0.585	0.386
18	United Kingdom	2018	1893	0.575	0.388
19	Italy	2007	1945	0.575	0.372
20	Spain	2017	1896	0.557	0.364
21	Chile	2018	1944	0.510	0.366
22	Portugal	2018	1919	0.499	0.347
23	Greece	2018	1945	0.494	0.348
24	Mexico	2016	1932	0.463	0.355

Endogeneity performance results using the novel SSRP model are now discussed considering the UP-IS performance set computed using the positive weight set.

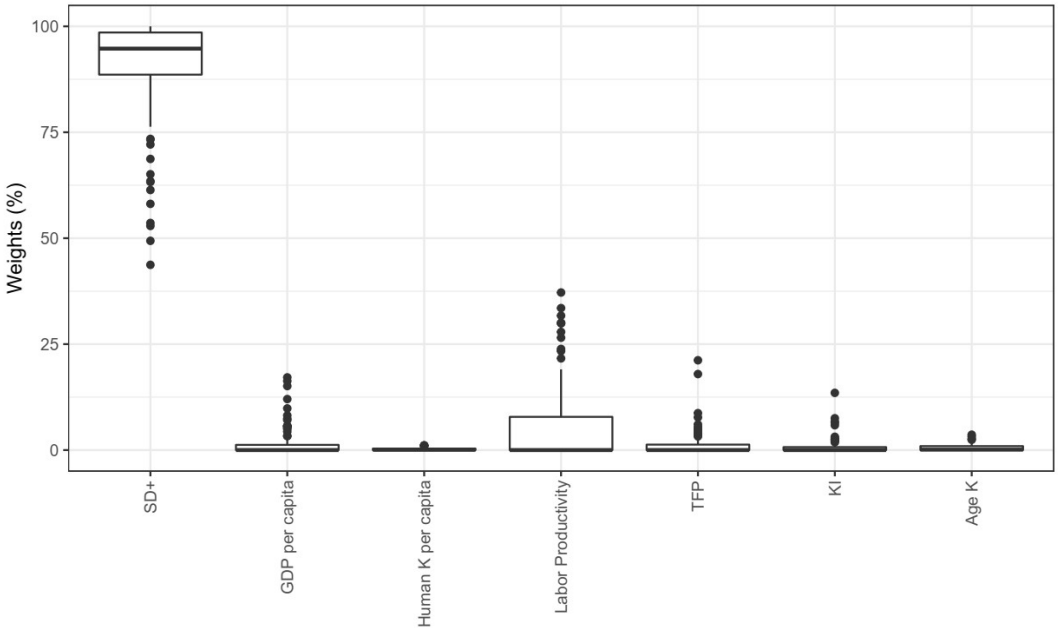


Fig. 6. Relative importance of models (1)-(7).

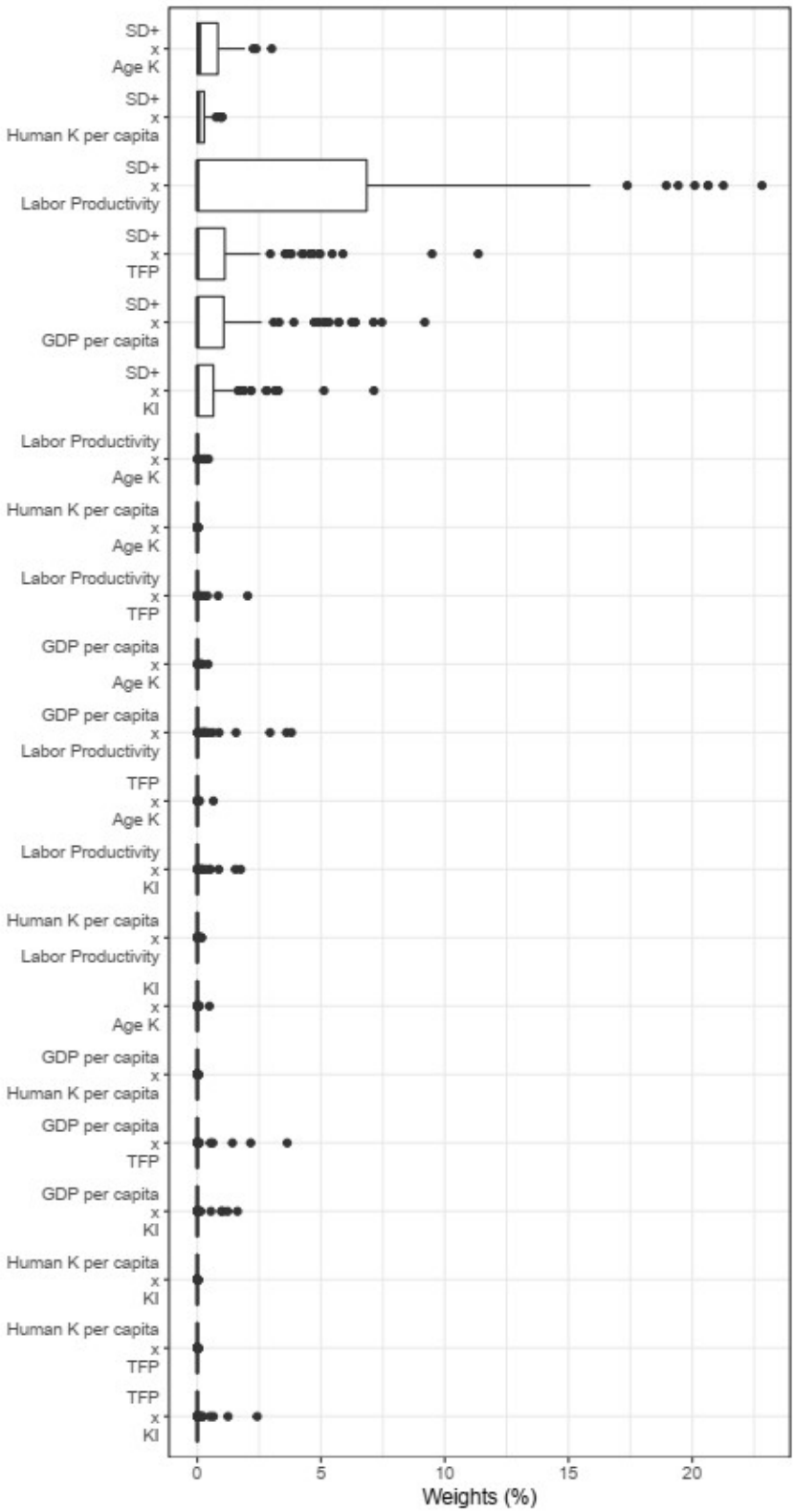


Fig 7. Endogeneity weights for pairs of models (1)-(7): major combinations.

Association among performance and key attributes. The relative importance of models (1)-(7) for achieving minimal residual variance is heavily concentrated in resulting productivity performance (cf. Fig. 6), accounting for almost the total median weights. Yet, long-term productivity performance appears to be associated with feedback or cause-effect processes that may occur with the other productivity attributes, particularly with respect to labor productivity. Notwithstanding, results for the relative importance of the major interaction pairs in explaining overall residual variance are presented in Fig. 7 and also confirm these issues, where productivity performance also appears associated with the six productivity attributes detailed in Table 1.

Endogeneity. The joint feedback effect of all model pairs on overall residual variance is around20% (cf. Fig.7), pretty low when compared to the maximal possible endogeneity effect, achievable when all 7 models would account individually for 14.29% of the total residual variation. Under these equal weights case, the maximal joint effect totals85.76% = 2*42.88% = 2*14.29%*14.29%*21, where 21 is the number of combinations taken two by two, as obtained from joint variations of models (1)-(7). This first screening on weights computed in Step 1 suggests that endogeneity appears to be not as relevant as directional cause-effect for apprehending the relationships among productivity attributes and productivity performance.

Table 5. Information Entropy of conditional distribution of [ROW] given that [COLUMN] for 0.975 percentile.

Models	SD+	GDP per capita	Human K per capita	Labor Productivity	TFP	KI	Age K
SD+		0.669876	0.668941	0.669588	0.669308	0.669085	0.667061
GDP per capita	0.652034		0.655528	0.657469	0.653186	0.652839	0.653042
Human K per capita	0.670181	0.671262		0.670389	0.670138	0.669353	0.668754
Labor Productivity	0.646793	0.655154	0.650398		0.65079	0.65472	0.648201
TFP	0.660281	0.660774	0.660747	0.661244		0.659537	0.659065
KI	0.640757	0.636911	0.638873	0.639065	0.640965		0.637307
Age K	0.671979	0.672483	0.672182	0.671076	0.672242	0.671298	

Table 6. Information Entropy of unconditional distributions for 0.975 percentile. (*)

Models	SD+	GDP per capita	Human K per capita	Labor Productivity	TFP	KI	Age K
SD+		0.510284	0.496786	0.534606	0.498858	0.504953	0.550925
GDP per capita	0.510284		0.547034	0.575171	0.532416	0.539707	0.558466
Human K per capita	0.496786	0.547034		0.550645	0.573801	0.539375	0.55573
Labor Productivity	0.534606	0.575171	0.550645		0.59235	0.573361	0.559225
TFP	0.498858	0.532416	0.573801	0.59235		0.573366	0.582221
KI	0.504953	0.539707	0.539375	0.573361	0.573366		0.492459
Age K	0.550925	0.558466	0.55573	0.559225	0.582221	0.492459	

(*) Differently for the conditional distribution matrix, the unconditional distribution is a symmetric one.

Cause-Effect Relationships. Tables 6 and 7 report on the conditional and unconditional distribution results used in model (26), obtained for the 0.975 percentile after bootstrapping the fitted individual residual distributions as displayed in Table 3.Figs. 8 and 9 present the directional relationships among variables, and their respective signs, based on optimal weights computed in Step 2 for 0.975 percentile threshold. Readers should observe that the median expected weight for each conditional distribution would be 0.0238 (1/42), considering a balanced bi-directional relationship among variables.

Considering the percentile threshold of 0.975, where relationships are stressed to a 2.5% false positive rate, long-term productivity performance in advanced countries is both impacted by Human K (positively) and Age K (negatively). It is interesting to note that Age K and Human K are endogenously related, in a sense that newer capital stock and highly skilled labor force present a feedback process relevant for achieving higher productivity performance. Age K and Human K also exert cause-effect relationships with other productivity attributes, either acting as countervailing forces with respect to labor productivity[2], or acting in the same direction with respect to KI and GDP pc (negatively) and TFP (positively). Readers should note that Human K can lead to KI destruction due to the movement towards economies based on services and less dependent on heavy industries. This being case, however, GDP pc is also put into jeopardy, as long as heavy industries are more value added intensive and scalable even in comparison to high-tech sophisticated services such as IT. On the other hand, the interpretation for Age K is more straightforward, given that older capital stocks may be even physically depreciated or obsolete, without being able to catch-up with newer technological advances in production of goods and services.

However, productivity performance itself impacts on the other productivity attributes: KI and TFP (positively) and LP and GDP pc (negatively). To interpret the signs these effects, productivity performance scores should be apprehended as the joint effect between Human K and Age K. Positive impacts on KI and TFP suggest that increased productivity performance simultaneously opens room for capital renovation and adoption of newer production technologies (higher KI), but also contributes to a better balance between capital and labor production factors (higher TFP). This productivity performance spillover is achieved when Human K is high and Age K is low. On the contrary, a negative influence of higher productivity performance on LP and GDP pc may result from third party contractors and, more recently, from the "uberization" phenomenon in different economic sectors. Wealth generation per worker under these circumstances has lowered dramatically in the last decades also contributing for a lost of impetus in GDP pc increases. These results also suggest a dichotomous development path that should be follow by advanced countries: high tech versus "uberized" economies.

It is interesting to note that KI is the ultimate consequence of the countervailing forces that co-exist in these dichotomous development path that commences with Human K and Age K. All remainder variables – productivity performance, TFP, LP, and GDP pc, either act as causes or consequences. It is also worth noting that productivity performance scores computed by UP-IS and TFP, LP, and GDP pc attributes consist of different expressions or metrics of the amount of wealth generated given the amount of

[2]As expected, higher levels of Human K positively impact labor productivity, while higher levels of Age K present a negative impact, thus suggesting capital obsolescence may limit productivity gains due to better schooling, for instance.

resources (either capital or labor) used. Taking a closer look among them, another endogenous relationship is found between labor productivity and GDP pc, and its interpretation is straightforward: per capita welfare is built upon higher levels of labor productivity and per capita welfare engenders the means of becoming more productive. As regards their ordinal rank, as expected, UP-IS productivity performance is the most important, impacting LP, GDP pc, and TFP. TFP is the second in ranking, impacting on GDP pc and LP, while being impacted on by productivity performance. LP and GDP pc are the third in ranking, being impacted on by productivity performance and TFP. This analysis embeds a rationale for attribute hierarchy:

- **Human K and Age K**: are the descriptors of labor and capital quality.
- **UP-IS productivity performance**: is the synthetic measure derived by both resource quality descriptors, reflecting the countervailing forces of a dichotomous development path based on high tech industries versus third party contractors.
- **TFP**:is the measure that captures the resource allocation between capital and labor, which should reflect to some extent this dichotomous path.
- **GDP pc and LP**: are the measures of wealth generation at the individual level, which is "supposed" to have opportunities in both sides of this dichotomous path.
- **KI**: is the ultimate result of the developing process, which importance has dramatically changed due to transformations in society with respect to IT, labor relations, environmental pressures, emergence of China as novel superpower etc.

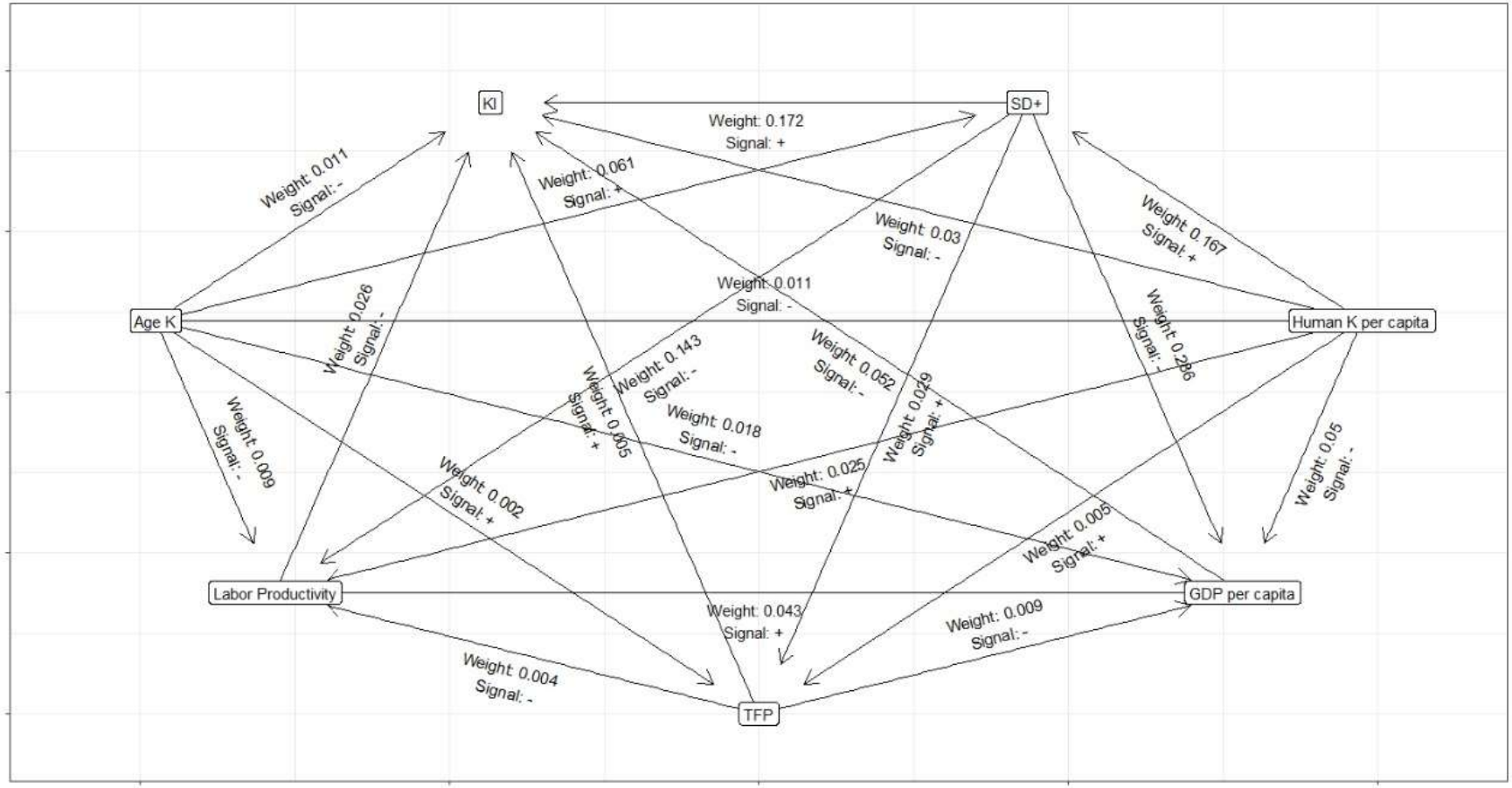


Fig. 8 Cause and effect framework for selected attributes at 0.975 percentile (weights are derived from Step 2, signs are derived from Olden´s (2002) sensitivity analyses).

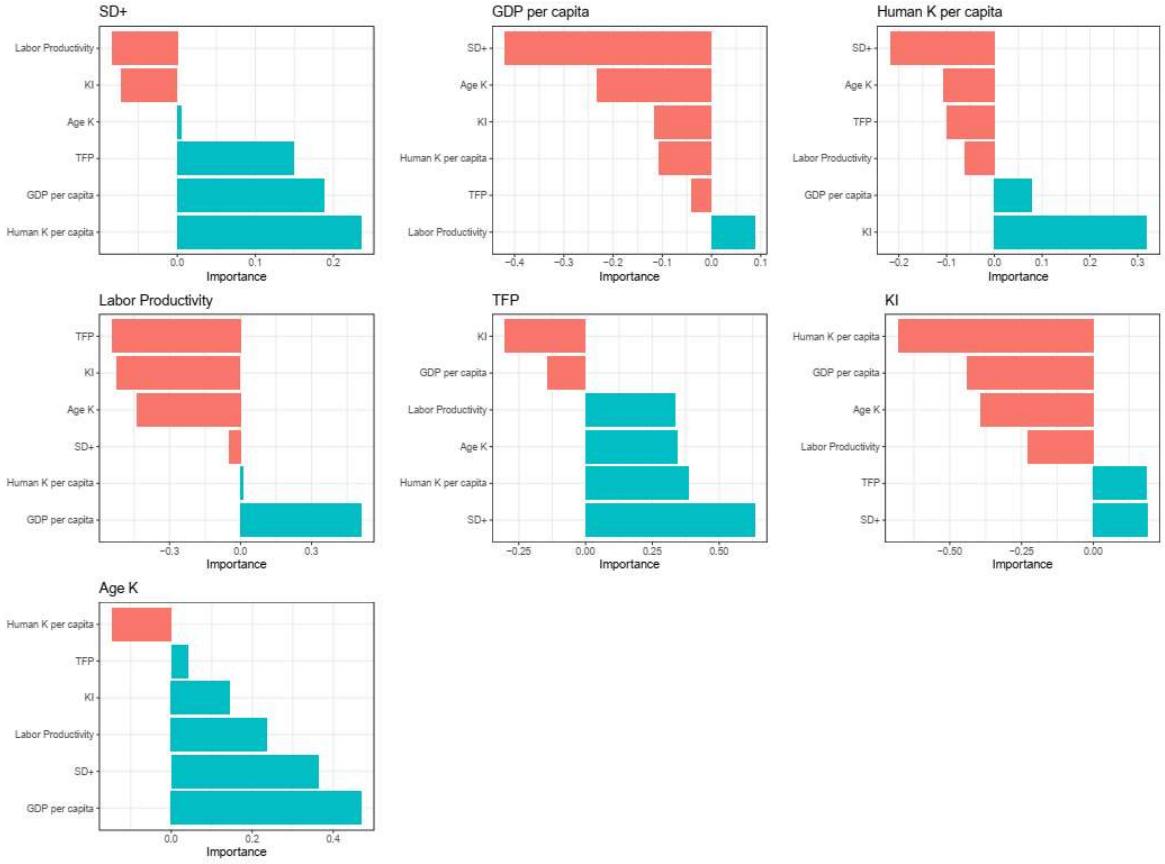


Fig. 9. Results for Olden´s sensitivity analysis in terms of signs and relative importance for models (1)-(7).

5. Conclusions

This paper decomposes the endogenous sources of long-term productivity performance in different advanced countries using a Two-Dimensional Fuzzy-Monte Carlo Analysis (2DFMC) approach. We find that the UP-IS presented higher cross-performance scores than the TOPSIS base-case. Further, Norway is the best performing country with a positive performance score of 0.854 in 2018 while Portugal is the worst with a score of 0.347 in 1919. In terms of ordinal ranking of long-term productivity performance, UP-IS ranked first, followed by TPF and next LP and GDPpc. More so, average age of equipment capital stock and human capital intensity have positive and negative impact respectively on long-term productivity performance in advanced countries. Conversely, productivity performance has positive impact on KI and TFP but negative impact on LP and PDPpc. These results suggest a dichotomous development path that should be followed by advanced countries: high tech versus “uberized” economies.

Policy implications for both types of economies should focus on, as a starting point, on improvement of physical and human capital levels, which involves massive investments in logistics and energy infrastructure in parallel to integral elementary and fundamental schooling. However, "uberized" countries will still need increased levels on capital intensity and renewal, which requires WTO tariff incentives and waivers for (re)industrialization to some extent, aligned to the needs of industry 4.0 and their embeddedness with technological start-ups.

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