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Information Entropy, Continuous Improvement, and US Energy Performance: A Novel Stochastic-Entropic Analysis for Ideal Solutions (SEA-IS)

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Abstract: Previous energy performance studies neglected the role of information entropy in feedback processes between input and output slacks. Superior energy performance may be achieved through the capability of learning from how increased outputs could yield reduced inputs and vice-versa. This paper focus on this gap, by presenting an assessment of US states for a 35-year period in lieu of relevant socio-economic and demographic variables. US is the world largest energy producer and consumer, being well-known not only for innovation in efficient energy use but also for managerial feedback mechanisms in the energy field which ensures continuous improvement in generation and consumption. First, a novel SEA-IS (Stochastic-Entropic Analysis for Ideal Solutions) model is developed to assess the potential information gains that may arise from energy slacks minimization given different optimal reduction quantiles in US states. This non-linear stochastic optimization model not only relies on Beta distributed priors to model the odds-ratio of learning feedback but also takes advantages of numerous strengths present in DEA and TOPSIS approaches for performance management. Machine learning methods are also employed to predict information gains in terms of contextual variables. Results indicate that California is the only U.S. state that has indicate strong mutual information feedback and continuous improvements in efficiency. There is ample scope for harnessing the power of information gains in improving energy efficiency, particularly in 37 U.S. states, which indicates scope for a public-private partnership to achieve this goal.

Keywords: US energy; performance; state-level; stochastic-entropic approach; information gains; slack management; feedback.

1. Introduction

Since the inception of performance/efficiency measurement tools in the context of energy utilization in production processes, almost all studies have neglected the impact of information entropy on score estimates computed using alternative multi-criteria decision-making (MCDM) and productive frontier models (for example, Tsai et al., (2014)). Instead of focusing on feedback learning processes that may arise when reducing inputs or increasing outputs (Kendall et al., (2017); Ashouri et al., (2020)), energy performance studies have either focused on analyzing, at the country level, (i) how physical, human, and energy resources (or inputs) were converted into social-welfare and undesirable pollutants (or outputs) under a "black-box assumption" (Lee and Lu (2010);

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Menezes et al., (2012)) or (ii) how positive and negative criteria (the MCDM counterparts of productive frontier models) might delimit a performance possibility set by establishing boundaries for their positive and negative ideal solutions (Wanke, 2015).

Some studies have incorporated uncertainty in models in terms of random variations supposing a data process such as in stochastic productivity frontiers or, in terms of fuzziness, as in imprecise decision-making environments ((Tsaur et al., 2002). However, little effort has been expended on exploring whether energy performance related inputs and outputs (or negative and positive criteria, respectively) share some level of mutual information (Tsanas and Xifara (2012)), thereby allowing producers to learn simultaneously from each other's distributional behaviors.

Different from correlation coefficients, which are already explored in some stochastic MCDM models (for example, Peng (2015)), mutual information is a concept closely related to information entropy and represents the distance between two marginal distributions (Walters-Williams and Li (2009)), setting the grounds for feedback, learning, or synergistic endogenous processes that may eventually arise within the ambit of continuous improvement initiatives.

Note that the utilization of different energy usage related variables for assessing performance has a more fertile ground under the application of MCDM techniques instead of the application of parametric and non-parametric efficiency measurement techniques, such as SFA (Stochastic Frontier Analysis) and DEA (Data Envelopment Analysis), respectively (Kumar et al. (2017); Kaya et al. (2017)). These variables can include consumption and generation variables, as well as undesirable byproducts, which are often strongly correlated with each other. The advantage of using these variables that MCDM models have over parametric and non-parametric frontier techniques stem from the complexity that often arises in the context of choosing the appropriate subset of energy variables. Parametric frontier techniques are mostly impacted by the curse of collinearity while non-parametric frontier techniques are mostly affected by the curse of dimensionality (Geenens (2011)). In both cases, these limitations may yield performance scores biased towards one, unless an adequate variable reduction criterion is implemented (Wu et al., 2005; Hollingsworth and Smith, 2003; Gonzalez-Bravo, 2007; Despic et al., 2007; Emrouznejad and Amin, 2009). However, putting these issues in comparison to MCDM models, the building of partial ideal functions (TOPSIS – Technique for Order of Preference by Similarity to Ideal Solution; (Wanke et al. (2016)), compromise functions (VIKOR method – Opricovic and Tseng (2007)), relative performance functions (SWARA– Step-wise Weight Assessment Ratio; Yazdani et al., (2016)), or utility functions (COPRAS– Complex Proportional Assessment, Zavadskas and Turskis (2010)), just to mention a few examples, in terms of banks (alternatives) and banking variables (criteria), helps in obfuscating these collinearity and dimensionality issues, which tend to be ignored in MCDM models, despite isolated efforts for incorporating some kind of uncertainty in terms of fuzzy numbers.

On the other hand, MCDM models present some shortcomings when compared to parametric and non-parametric efficiency measurement techniques. These shortcomings are primarily related to the establishment of directional or radial measures for developing an improvement path towards the ideal solution. As a direct consequence of this, their slack measures and consequently, the "elasticity" assessment of their performance, that is, whether more or less than proportional – in an analogous parallel to the increasing and decreasing returns to scale within the ambit of a productive function – is not present at all. Notwithstanding, some of these methods, such as TOPSIS, require exogenously defined criteria/alternative weights, while productive resource and DMU (Decision Making Unit) weights are defined endogenously in DEA models, for instance, as part of the primal/dual optimization process with respect to the data envelop computation. Aside these limitations, both frontier and MCDM models present their respective strengths. A novel performance/efficiency model that encompassed strengths from both research streams should present the following features: (i) performance optimization conducted *n* times at the DMU level (supposing that there are n DMUs in the sample); (ii) endogenously defined weights for inputs and outputs (or negative and positive criteria, respectively); (iii) information on slacks in terms of productive frontiers or ideal solutions; and (iv) unbiased scores with good discriminatory power; besides the proper distributional assumptions, and their respective simplifications, that make the quest for information entropy tractable in terms of capturing the learning/feedback/synergy processes due to mutual information sharing.

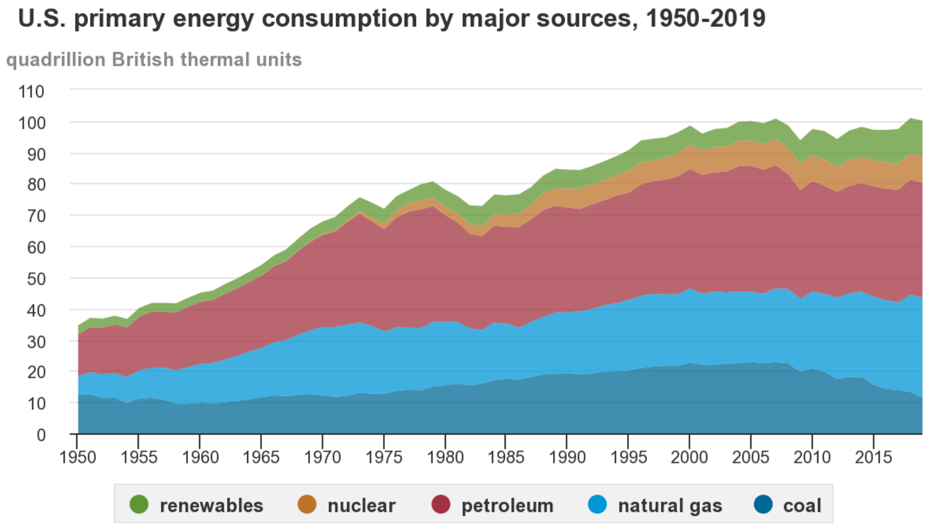
Therefore, this paper aims to fill this literature gap by presenting the novel Stochastic-Entropic Analysis for Ideal Solutions (SEA-IS) model, - a hybrid approach that take advantage of DEA and TOPSIS strengths -, which is applied to a set of energy performance related variables, aiming at assessing its behavior at US state-level from 1980 until 2014. The United States was chosen as the locus of analysis due to its historical leadership on the research, development, and application of alternative energy sources and uses for social-welfare, most of them are based on learning, information sharing, and novel business opportunities in different sectors of economy. Energy performance in the US States appears to be timidly impacted on by mutual information levels between positive (outputs) and negative (inputs) criteria in comparison to the capital stock type existent in each state. This being the case, the impact of these contextual variables on performance and their eventual synergistic effects on US energy performance at the state level due to mutual information are assessed by machine learning techniques. A robustness comparison among SEA-IS, DEA, and TOPSIS scores is also provided in light of information reliability theory.

The remainder of the paper is organized as follows. Section 2 presents the contextual setting of the US energy sector, exploring the specifics and particulars of each state. Section 3 presents the literature review, with a special emphasis on methods for assessing energy performance/efficiency and the US context. Section 4 covers the methodology, focusing on the dataset and SEA-IS modelling. Data analysis and discussion is presented in Section 5. Conclusions and policy implications follow in Section 6.

2. Background on US Energy Sector

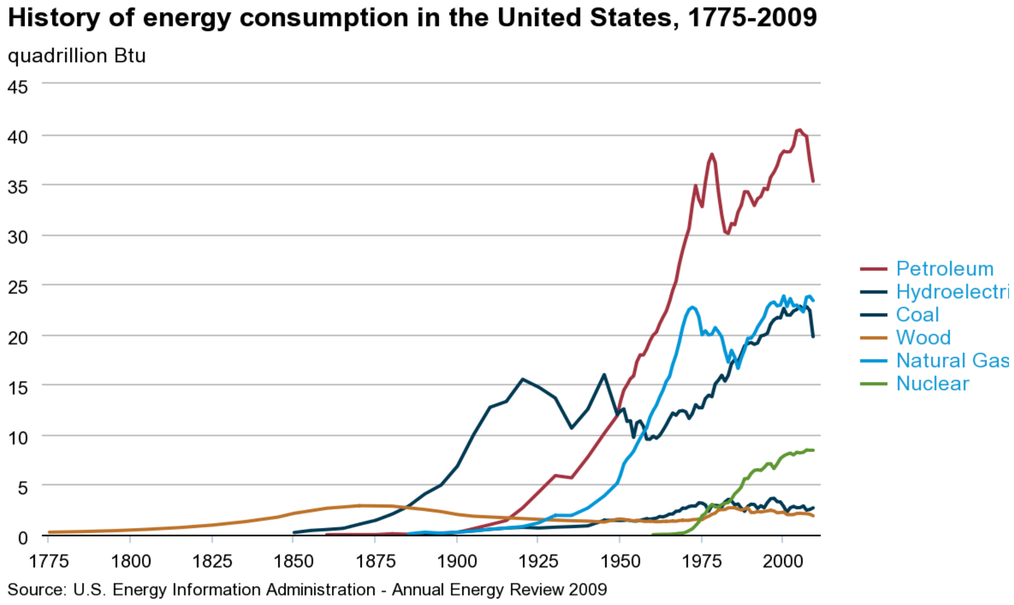
The United States uses produces and uses energy from a range of renewable and non-renewable resources, which includes a combination of energy from primary and secondary energy sources. Primary energy sources that are dominated by fossil fuels such as petroleum, natural gas, and coal. For example, in 2019, 80% or total primary energy production were from three fossil fuel sources. In recent years, other primary energy sources have become available such as wind, hydroelectric, solar, geothermal, and biomass. A secondary source of energy is electricity, which is generated from different primary energy sources.

The U.S. is one of the leading producers and consumer of energy worldwide. For example, in 2017, the total energy consumed from primary energy sources by the U.S. was about 17% of the total energy consumed from similar sources worldwide. The U.S. energy consumption had continued to increase since 1949 with the occasional drop during recessionary periods when the economy slowed down. The graph below shows the U.S. primary energy consumption levels from different sources over the years.



Source: U.S. Energy Information Administration, *Monthly Energy Review*, Table 1.3, April 2020

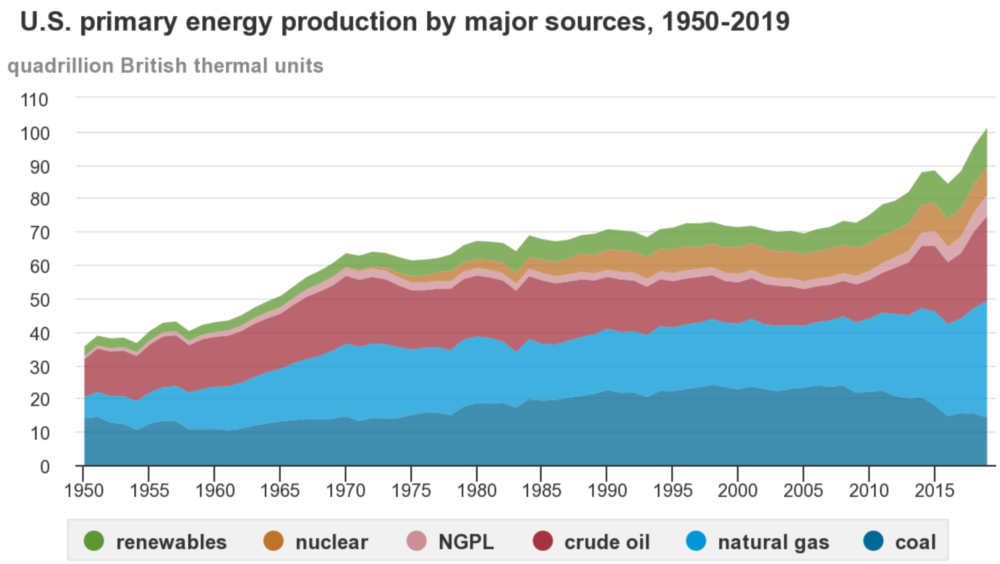
While the total U.S. energy consumption level has increased over the years, the combination of energy from different sources has changed over the years. Coal was the dominant energy source in the late nineteenth century until it was surpassed by petroleum products in the middle of the twentieth century. During this period, natural gas production also steadily increased. The figure below depicts the long-term history of the change in U.S. energy combination mix.



Energy can be used for both resale and end-uses. There are four sectors that utilize energy as end-users, (i) the residential sector including residential houses and apartments, (ii) the industrial sector that includes manufacturing units, agricultural, mining, and construction facilities, (iii) the commercial sector that includes offices, schools, hospitals, shopping malls, restaurants etc., and, finally, (iv) the transportation sector. These end-use sectors consume energy from both primary energy such as coal and buy electricity from the electric power sector, a secondary energy source.

While total energy consumption has increased over the years along with the U.S., the per capita energy consumption has remained relatively flat since the seventies given advancement in energy efficient improving technology and policy interventions such as the CAFÉ standards, among other contributing factors.

After dominating the U.S. energy production for nearly a century, coal production started to decline since the late nineties primarily because of a decline in coal utilization in electricity production. By 2019, U.S. coal production was about 60% of the level produced in 1998. In contrast, natural gas production increased to a peak, as a result of efficient and improved drilling and production techniques. In recent years, both production and consumption of renewable energy have increased reaching a peak in 2019. This rise is attributed to the sharp rise in solar and wind energy. The graph below shows the U.S. primary energy production levels from major sources from 1950 to 2019.



Source: U.S. Energy Information Administration, *Monthly Energy Review*, Table 1.2, April 2020

3. Literature Review

Energy utilization is of paramount importance in defining an economy's growth path because conversion of energy from one form to another is an integral part of every production process. As such, economists have long been interested in analyzing the role of energy in various sectors of the economy. Energy from different sources can vary both in terms of their efficacy in the production process and also the resulting impact of the surrounding physical environment. For e.g., one unit of energy from fossil fuels can have different effects on both the rate of production and the byproducts generated that will determine the net environmental impact.

The variability in the effects of various energy sources on production is often an area of focus for production economists while the environmental effects of different forms of energy consumption continues to garner considerable among environmental economists. One strand of research on energy has focused on the availability of renewable energy options to mitigate the effects of climate change. For example, in a series of three papers, McKendry (2002) examines the background to biomass production, reviews energy conversion technologies, and evaluates the potential of alternative gasification technologies for biomass gasification.

The early research on energy performances primarily focused on the relationship between economic growth and energy use. For example, in an earlier publication exploring the causal relationship between energy use and economic growth in the United States, Stern (1993) found when adjusted for changing fuel composition, final energy use Granger causes gross domestic product (GDP). Asafu-Adjaye (2000) estimated the causal relationships between energy use and income for a selected group of Asian economies. The paper's results showed unidirectional Granger causality from energy to income for India and Indonesia and bidirectional Granger causality for Thailand and the Philippines, indicating the variability in the energy use – income relationship across economies. Paul and Bhattacharya (2004) combined an Engle-Granger cointegration approach with the standard Granger causality test to find a bidirectional causality exists between Indian economic growth and energy consumption for the period 1950 to 1996. For 19 African countries, Wolde-Rufael (2005) estimated the relationship between energy use per capita and per capita real GDP for the years 1971 to 2000. In the African context, the author found a long run relationship between the two series for eight countries and causality for ten out of nineteen countries. This work further highlights the variability in the empirical evidence on the energy and national income relationship across different countries. Chiou-Wei et al., (2008) build on previous research by applying both linear and nonlinear Granger causality tests to estimate the causal relationship between energy consumption and economic growth for a sample of Asian countries and the United States. Apergis and Payne (2010) specifically focus on the relationship between renewable energy consumption and economic growth and provide evidence from a panel of twenty OECD countries. Their Granger-causality results indicate bidirectional causality between renewable energy consumption and economic growth in both the short- and long run.

With an increase in environmental concerns, particularly the impact of fossil fuel consumption on global emissions, a growing body of literature has emerged that investigates linkages between energy consumption, economic growth, and environmental performance. Using data from 1960 to 2007 for China, Zhang and Cheng (2009) found that neither carbon emissions nor energy consumption leads economic growth, which leads them to conclude that a conservative energy policy and a carbon emissions reduction policy will not impede China's economic growth. Ozturk and Acaravci (2010) draw a similar conclusion for Turkey. Similar studies have been done by Menyah and Wolde-Rufael (2010, 2010) for South Africa and the United States, by Pao and Tsai (2010) for the BRICS countries among others. Tahvonen and Salo (2001) analyzed the transitions between nonrenewable and renewable energy at different stages of an economy's growth path and concluded that an inverted-U relation between carbon emissions and income level may be possible under certain conditions even in the absence of environmental policy.

The research presented in this paper is related the role of entropy in energy performance. Even though there is limited economics research in this area, the laws of thermodynamic was introduced to economics almost fifty years ago by Georgescu-Roegen (1971). According to the second law of thermodynamics or the law of entropy, transformation of energy from one form to another will always result in an irreversible loss of some amount of freely available energy. An early publication by Garrison and Paulson (1973) demonstrated that entropy can be a useful index of the geographic concentration of economic activity.

Based on the generalized entropy mobility measures developed in Maasoumi and Zandvakili (1986), Maasoumi and Trede (2001) applied the approach to analyze income mobility in Germany and the United States. Kåberger and Månsson (2001) provided a comparative analysis on the opposing views on the relationship between thermodynamics and economic theory i.e., whether the physical constraints have binding effects on economic growth or not. Antoniou et al., (2002) developed a novel approach to address the problem of efficient resource allocations in different types of economic systems and proposed that entropy should be an indicator of the efficiency of resource distribution.

Zohrabian et al., (2003) applied a maximum entropy method to estimate the success probabilities of research on the economics of crop genetic diversity and gene bank management. The maximum entropy approach allowed the authors to fit a distribution to the data by imposing as little structure as possible and using maximum information from the segment of the distribution that is of most interest.

Within the context of environmental issues, Fernandez (1997) estimated the parameters of the objective function and state equation in an optimal control model of water quality and expenditures for wastewater treatment using a maximum entropy approach. Zhou et al., (2013) first proposed a non-radial data envelopment analysis (DEA) approach by integrating the entropy weight and the SBM model and applied the model to an efficiency analysis of the Chinese power industry. Han et al., (2018) performed a carbon efficiency analysis of industrial departments in China by developing an environmental DEA model based on information entropy. Santos et al., (2019) applied an entropy-TOPSIS-F approach to develop a method for the evaluation and selection of green suppliers for the Brazilian furniture industry. Li et al., (2019) has applied the maximum entropy approach to estimate parameters of probability distributions of demand and supply of water and used an entropy-weight-based TOPSIS method to evaluate agricultural water resources allocation schemes.

This paper advances the literature on the role of information entropy on energy performances by developing a novel SEA-IS (Stochastic-Entropic Analysis for Ideal Solutions) model to assess the potential information gains that may arise from energy slacks minimization given different optimal reduction quantiles in US states. Drawing on the advantages of both DEA and TOPSIS approaches applied to evaluate performance management, the paper assesses the impact of a set of socio-economic and demographic variables and their synergistic effects on the U.S. state level energy performances between the years 1980 and 2014.

4. Methodology

4.1. Data Source

Table 1 presents descriptive statistics of data on US states which was collected from the EIA database and publicly available annual reports from 1980 to 2014. Inputs (negative criteria) and outputs (positive criteria) are variables that are commonly found in the literature review on energy performance papers. Contextual variables were collected from the U.S. Bureau of Economic Analysis database. The state income inequality data were obtained from Mark W. Frank's U.S. State Level Income Inequality website.[5] The state capital stock data were obtained from Yamarik (2013).

The inputs chosen in the model are population, private capital stock, and total energy consumption. We include total population (number of persons in each state) as population size is an indicator of aggregate demand for goods and services, and, also, energy demand. Capital investments are a primary input of all production activities. Energy is

[5] Source: https://www.shsu.edu/eco_mwf/inequality.html

consumption is an indicator of the rate of economic activities in any economy. Some contextual variables were included that can potentially influence energy performance across states. For example, variables such as real personal income and the GINI coefficient are indicators of individual spending power and income inequality, which can be correlated with both the type and level of energy consumption (**Lee and Chien, 2010**). The employment level, indicated by the number of jobs, tend to be correlated with energy consumption though previous authors have found evidence of a bidirectional causal relationship between the two variables (Glasure and Lee, 1995).

Table 1. Descriptive statistics of Inputs (I), Outputs (O) and Contextual variables (C).

	Variable	Unit	Min	Max	Median	Mean	SD	CV
Inputs	Population	Number of persons	490787.00	38680810.00	4089875.00	5845831.11	6530262.66	1.12
	Net Private Capital Stock	Millions of chained 2009 dollars	27349.05	2677884.22	213346.92	336229.98	424584.58	1.26
	Total Energy Consumption	Billion BTU	130720.00	12733681.00	1422363.00	1912461.96	2048468.40	1.07
Outputs	Real GDP	Millions of chained 2009 dollars	20074.00	2113280.00	168218.00	274233.30	331352.56	1.21
	CO2 - Coal	Million metric tons of CO2	0.00	161.91	31.22	39.40	39.28	1.00
	CO2 - Petroleum	Million metric tons of CO2	1.04	346.00	32.35	47.33	56.40	1.19
	CO2 - Natural Gas	Million metric tons of CO2	0.14	240.98	14.26	24.93	34.84	1.40
	CO2 - Total	Million metric tons of CO2	2.65	717.96	80.19	111.66	113.36	1.02
Contextual	Real Personal Income	Millions of dollars	16055.70	1746817.84	141347.75	225729.82	266724.44	1.18
	GINI	-	0.52	0.71	0.59	0.60	0.04	0.06
	Employment NAICS Industry	Number of jobs	310043.00	21997098.00	2288619.00	3377041.64	3646719.04	1.08
	Residential	Billion BTU	30848.00	1745786.00	317981.00	407880.07	373378.34	0.92
	Commercial	Billion BTU	23164.00	1640247.00	244858.00	344087.21	339211.89	0.99
	Industrial	Billion BTU	3507.00	7175851.00	394365.00	631791.21	953276.41	1.51
	Transportation	Billion BTU	18872.00	3354828.00	414292.00	528703.49	576460.78	1.09
	Trend	-	1.00	17.00	9.00	9.00	4.90	0.54
	Trend Squared	-	1.00	289.00	81.00	105.00	90.78	0.86

4.2. Efficiency versus Performance measurement: A focus on DEA and TOPSIS

Broadly speaking, performance is a broader benchmarking concept that can be structured by using either scalar or ratio variables, or even a mix of them. It is usually employed when there are difficulties in comparing with peers – they may not be homogeneous - and/or, - although not necessarily -, in quantifying monetary or physical values for inputs and outputs, namely, the negative and positive criteria, respectively (Mihaiu et al., 2010). Performance scores are often assessed by MCDM matrix-based methods like TOPSIS, VIKOR, or COPRAS, for instance, where specific functions are assumed (e.g. ideal solutions, compromise solutions, utility solutions, etc) (Behzadian et al., 2012). Precisely, TOPSIS develop cardinal or scale metrics within the range delimited by positive and negative ideal solutions through linear combinations of the criteria. The performance distance in TOPSIS is cardinal, consisting of a second power metric in the Euclidean n-space (Olson, 2004). Putting into other words, TOPSIS computes cardinal distances (scores) from ideal positive solutions while simultaneously presents an ordinal ranking of them (Behzadian et al., 2012).

In contrast, although efficiency is another popular stream of performance study, it essentially relies on the assumptions about a productive frontier or a data envelop. Hence, productive efficiency is one particular way for accessing performance (Talley, 2006). The "efficiency" terminology is saved to DEA and SFA, that is, methods that compute performance based on the productive frontiers that envelope a data set. Frontier methods computes the cardinal distances from a data envelope formed by actual observations of the frontier of best practices. Therefore, efficiency methods are capable of indicating how efficiently a bank is in minimizing variables related to decreasing performance and in maximizing other variables related to increasing performance in comparison to other peers (Tsai & Chang, 2010). For the sake of simplicity, TOPSIS measures performance in qualitative terms while DEA measures it quantitatively (Zeydan and Çolpan, 2009). The fine-tuning between efficiency and performance scores is often accomplished by selecting a suitable set of variables/criteria and their expected impacts – whether positive or negative – on banking efficiency/performance, as long as direct analogies can be done between inputs and negative criteria, as well as on outputs and positive criteria.

In a traditional DEA model, performance is calculated using ex-post information collected from historical data with respect to inputs and outputs (Berger & Humphrey, 1997; Charnes, Cooper, & Rhodes, 1978). Battese and Rao (2002) showed that examining performance with DEA presents better discrimination—i.e., efficiency scores that are less biased towards one—if this set of inputs/outputs is considered under a meta-frontier that encompasses several years of observation, similarly to what is emulated within the ambit of MCDMs such as TOPSIS. TOPSIS, in a similar fashion to other MCDMs, is also nonparametric by nature because there are no underlying statistical properties whatsoever. As regards the fundamentals of TOPSIS, this MCDM is based on the concept that the positive ideal solution has the best level for all criteria considered or for the input/output set, while the negative ideal is the one with the worst values for the input/output set (Wanke et al., 2016a). Despite its general resemblance to DEA where outputs may be maximized and/or inputs minimized, the determination of the weights of the relative importance of each criteria is exogenously defined in TOPSIS, whereas in the case of DEA these weights are endogenously calculated within the ambit of the model (Behzadian et al., 2012). Besides, TOPSIS is computationally simpler because there are virtually no constraints with respect to the number of criteria (inputs/outputs) that can be assessed (Wanke et al., 2016a).

4.3. Stochastic-Entropic Analysis for Ideal Solution (SEA-IS)

<u>Stochastic-Entropic Slack Ratios</u>

Let's consider a set of d DMUs, each one of them consuming i inputs $x_{d,i}$ to produce $y_{d,o}$ outputs o, where $d = \{1..n\}$, $i = \{1..m\}$, $o = \{1..s\}$, $\mathbf{x}$ and $\mathbf{y}$ are, respectively, input and output matrices with dimensions nxm and nxs. The positive ideal solution for all DMU d for each output o is given by $\max(y_o)$ while the negative ideal solution for each input i is $\min(x_i)$, for all DMU d. These ideal solutions are the cornerstones for computing relative slacks for each input i and output o at the DMU level, such as:

$\Delta x_{d,i} = (x_{d,i} - \min(x_i))/x_{d,i}$, $\Delta x_{d,i}$ ranges between 0 and 1 for all i and d (1)

$\Delta y_{d,o} = (\max(y_o) - y_{d,o})/\max(y_{d,o})$, $\Delta y_{d,o}$ ranges between 0 and 1 for all o and d (2)

where $\Delta x_{d,i}$ is the relative slack or potential for reducing input i at DMU d, while $\Delta y_{d,o}$ is the potential for increasing output o at DMU d. Weights, $wx_{d,i}$ or $wy_{d,o}$, can be assigned, respectively, to each input or output given that $\sum_i wx_{d,i} = 1$ and $\sum_i wy_{d,o} = 1$ for all d. Therefore, the expected weighted output increasing and input reducing potentials (or relative slack ratios, *EWOS* and *EWIS*, respectively) for each DMU d are given as it follows:

$$EWOS_d = \sum_o wy_{d,o}\Delta y_{d,o} \ \forall\, d \tag{3}$$

$$EWIS_d = \sum_i wx_{d,i}\Delta x_{d,i} \ \ \forall\, d \tag{4}$$

Similarly, information entropy (IE) can also be defined for each DMU d with respect to its inputs (Hx_d) and outputs (Hy_d). Information entropy (Shannon, 1948) is often regarded as a measure of information reliability - the higher the entropy, the lower the reliability - denoting the epistemic uncertainty that surrounds a given phenomenon, measuring the distance between the distribution of its current state and the distribution of the unknown true behavior, which can only be inferred.

$$Hx_d = -\sum_{i=1}^{m} p(x_{d,i}) \ln p(x_{d,i}) \ \forall\, d \tag{5}$$

$$Hy_d = -\sum_{o=1}^{s} p(y_{d,o}) \ln p(y_{d,o}) \ \forall\, d \tag{6}$$

where $0 \leq H_{\cdot d} \leq 1$, $p(.)$ denotes the probability of occurring an specific input/output outcome for DMU d and 1 signifies maximal entropy. The following steps are developed in terms of the i inputs of DMU d, however they are analogous in case of the o outputs. Suppose that the next equivalences between eqs. (3) and (5) hold:

$$p\left(x_{d,i}\right) \sim wx_{d,i} \ \forall\, d, i \tag{7}$$

$$-\ln p\left(x_{d,i}\right) \sim \Delta x_{d,i} \ \ \forall\, d, i \tag{8}$$

While both elements in eq. (7) range between 0 and 1, applying *exp*(.) into both sides of eq. (8) yield the following equivalence:

$$p\left(x_{d,i}\right) \sim exp\ (-\Delta x_{d,i}) \ \forall\, d, i \tag{9}$$

$$f(x_{d,i}; \Delta x_{d,i}) = \Delta x_{d,i}\ exp\ (-\Delta x_{d,i} x_{d,i}\ ), x \geq 0,\ \forall\ d, i \quad (10)$$

That is, the density probability of $x_{d,i}$ can be proxied by an exponential distribution of rate (λ) - scale inverse (1/ λ) - parameter $\Delta x_{d,i}$. Putting it into other words, $X_{d,i}$ can be described as a random variable exponentially distributed - at the DMU level - with rate parameter given by the relative slack or the input reduction potential in terms of the ideal negative solution for input *i* (min(x_i)): $X_{d,i} \sim Exp(\Delta x_{d,i})$. This equivalence is particularly interesting because among all continuous probability distributions with support [0, ∞) and mean $1/\lambda$, the exponential distribution presents the largest differential entropy (Park and Bera, 2009).

Taking DMU *d* into perspective, its Weighted Input Slacks Sum ($WISS_d$) can be expressed as a mixture of m independent exponential distributions with different ratio parameters. X is a Hyperexponential random variable if X is $X_{d,i} \sim Exp(\Delta x_{d,i})$ with weights $wx_{d,i}$, which is an Hyperexponential distribution.

$$WISS_d = \sum_i wx_{d,i} Exp(\Delta x_{d,i})\ \ \forall\ d \quad (11)$$

In order to make this summation analytically tractable, let's assume an average potential for reducing inputs at each DMU d, that is $\overline{\Delta x}_d = \sum_i wx_{d,i} \Delta x_{d,i}/m\ \ \forall\ d$. Plugging this average relative input slack ratio into eq. (11) it becomes a weighted summation of exponential distribution of equal ratio parameters (~Exp($\overline{\Delta x}_d$)) summed *m* times), which results in a Gamma distribution (Crooks, 2019) of shape parameter *m* and scale parameter (inverse ratio) $1/\overline{\Delta x}_d$, such as:

$$WISS_d \sim Gamma(x_d; m, \overline{\Delta x}_d)\ \ \forall\ d \quad (12)$$

Analogous results can be derived observing the same previous steps for a Weighted Outputs Slacks Sum ($WOSS_d$):

$$WOSS_d \sim Gamma(y_d; s, \overline{\Delta y}_d)\ \ \forall\ d \quad (13)$$

Putting eqs. (12) and (13) into perspective, it is equivalent to say that, for each DMU *d*, summed input and output slack ratios observe Gamma distributions with shape parameters that represent the number of inputs and outputs and scale parameters that represent how large the average input or output is greater with respect to its respective ideal solution. Again, it is important to note that the gamma distribution is the maximum entropy probability distribution for a random variable X_d for which E[X_d] = $m/\overline{\Delta x}_d$ is fixed and greater than zero (the same is applied analogously to a random variable Y_d, cf. Park and Bera [2009]).

The equivalence between IE and stochastic slack ratios for inputs and outputs can also be established in terms of distances between two elements of two a set, departing from eqs. (5) and (6) and eqs. (12) and (13). For example, a metric or distance function is a function that defines a distance between each pair of elements of a set. A set with a metric is called a metric space (Cech, 1969). Many information entropy applications require a metric or a distance measure between pairs of points such as:

$$D(x_d, y_d) = Hx_d + Hy_d - 2MI(x_d, y_d)\ \forall\ d \quad (14)$$

where the metric $D(x_d, y_d)$ is particularly known as the variation of information (Melia, 2007) and satisfies the properties of triangle inequality, non-negativity, indiscernibility and symmetry. MI represents the mutual information level of random variables X_d and Y_d and measures the "amount of information" that can be inferred about one random variable by observing the other (Archer et al., 2013). Putting it into other words, MI is intricately linked to the expected "amount of information" held in a random variable, which is not necessarily limited to a linear dependence like the correlation coefficient or other forms of unidirectional causality (Massey, 1990) That is why MI is also known as information gain (Permuter et al., 2009).

Readers should note that, the higher the joint marginal information (MI) between random variables X_d and Y_d, the lower the metric. This means that mutual information bridges the gap between elements in distinct sets (in this research, input and output sets). The information gain for one random variable from learning for the other could even be as high enough to offset their individual entropies yielding, into the limit, a zero distance between X_d and Y_d. Analogously applying these key concepts to eqs. (12) and (13), it would be equivalent to affirm that MI represents the strength or intensity of the feedback processes that exists between the inputs (X_d) and outputs (Y_d) observed at each DMU level, by which information is also gained on how reducing input slack ratios through learning on how increasing output slack ratios (and vice-versa). As long there is a direct relationship between slacks and efficiency levels in production frontier literature, this research posits that the efficiency level of DMU d, or Eff_d, can be proxied by 1 - D(X_d,Y_d), such as:

$$Eff_d = 1 - D(x_d, y_d) = 1 + 2MI(x_d, y_d) - Hx_d - Hy_d \quad \forall\, d \qquad (15)$$

Putting into other words, the efficiency levels of a given DMU d - assessed in terms of how inputs and output slack ratios are stochastically distributed from their negative and positive ideal solutions, respectively - are positively impacted by the feedback learning processes that exists between these very slacks. On the other hand, lower efficiency levels are strongly tied up by higher individual entropy levels for inputs and outputs, from which very little is known to allow information gains on the other variable.

Hutter and Zaffallon (2008) dealt with the issue of determining the posterior distribution of MI for using it in inductive decision-making rather than in descriptive purposes using Bayesian inference. The authors demonstrated the exact analytical expression for the mean, and showed that the analytical approximations for variance, skewness and kurtosis presented an accuracy level of the order O(n^{-3}), where n is the sample size. The derived analytical expressions by the authors allowed the distribution of mutual information to be approximated reliably and quickly, while Beta distribution showed to be one of the most accurate approximations.

One of the advantages of assuming that MI is a function of Beta random variables is the fact that Beta distribution yields the exact solution for the odds-ratio of two Gamma independent random variables (Crooks, 2019). Therefore, assuming that MI is a function Beta distributed odds-ratio can be useful in assessing the likelihood of a joint input/output slack ratio improvement and, hence, of the information gain that helps in achieving higher efficiency levels due to learning about one distribution by another in a feedback processes. Another advantage of assessing MI in terms as a function of Beta distributed odds-ratio (OR) is the establishment of upper boundaries for joint improvement and, therefore, for the "maximal" attainable efficiency level by DMU d. Besides, information gains and efficiency levels can be assessed in probabilistic terms, by determining different quantile thresholds for the joint input/output slack ratio improvement. So, considering that this assumption holds:

$$OR_d \sim Beta(m, s) \sim \frac{\overline{\Delta x}_d Gamma(x_d; m, \overline{\Delta x}_d)}{\overline{\Delta x}_d Gamma(x_d; m, \overline{\Delta x}_d) \ + \overline{\Delta y}_d Gamma(y_d; s, \overline{\Delta y}_d)} \ \forall \ d \tag{16}$$

$$OR_d = Beta(m, s) = \frac{\overline{\Delta x}_d WISS_d}{\overline{\Delta x}_d WISS_d + \overline{\Delta y}_d WOSS_d} \forall \ d \tag{17}$$

$$WOSS_d = WISS_d \frac{\overline{\Delta x}_d (1 - Beta(m,s))}{\overline{\Delta y}_d Beta(m,s)} \ \forall \ d \tag{18}$$

$$MI_d = \frac{\overline{\Delta x}_d (1 - Beta(m,s))}{\overline{\Delta y}_d Beta(m,s)} \ \forall \ d \tag{19}$$

it is possible to express $WOSS_d$ as a stochastic function of $WISS_d$, cf. eqs. (18) and (19), which represents the MI. Next subsection discusses these issues under the paradigm of stochastic optimization, where weights for input/output slack ratios are optimized to determine the respective quantile efficiency thresholds given that the stochastic relationship presented in eq. (18) holds as a robust prior for assessing information gains in joint management of input/output slack ratios. Readers should recall that, to make this stochastic-entropic problem tractable, different assumptions were adopted that made the role of input/output slack ratio weights negligible throughput the distributional assumptions adopted. Therefore, it is deemed necessary to take some steps back in order to assess their relative importance by means of stochastic programming.

Stochastic Optimization of Slack Ratio Weights for Efficiency Quantiles

Let $r.(.)$ denote the weighted Euclidean distance operator. Squared terms are useful in non-linear optimization problems, assuring convexity of the possibility set of solutions while avoiding the trade-off between positive and negative values in the objective function. Applying this operator into eqs. (3) and (4), the weighted Euclidean distance between the inputs and the outputs of DMU d to their respective ideal solutions are given as:

$$ry_d = \left[\sum_o wy_{d,o} \left(\Delta y_{d,o}\right)^2\right]^{1/2} \ \forall \ d \tag{20}$$

$$rx_d = \left[\sum_i wx_{d,i} \left(\Delta x_{d,i}\right)^2\right]^{1/2} \ \forall \ d \tag{21}$$

Besides, eq. (18) can be rewritten as:

$$ry_d = rx_d \frac{[\overline{\Delta x}_d (1 - Beta(m,s))]^2}{[\overline{\Delta y}_d Beta(m,s)]^2} \qquad \forall \ d \tag{22}$$

An analogous efficiency metric can be also defined based on eq. (15), supposing that the equivalence between information entropy and expected weighted slack ratios still holds:

$$Eff_d = 1 + 2 rx_d ry_d - rx_d - ry_d \ \forall \ d \tag{23}$$

Plugging eq. (22) into eq. (23) , differentiating eq. (23) w.r.t. rx_d , and solving it to zero, the optimal weighted Euclidean distance for the inputs of DMU d that yields maximal efficiency - in light of learning about the input distribution through the output distribution - is given by:

$$rx_{d,opt} = \frac{\overline{[\Delta x}_d(1-Beta(m,s))]^2+[\overline{\Delta y}_d Beta(m,s)]^2}{4\overline{[\Delta x}_d(1-Beta(m,s))]^2} \quad \forall\, d \tag{24}$$

Optimal quantiles (q) for the weighted Euclidean input distances (or slack ratios) can be numerically computed[6] observing the following expression with incomplete Beta function terms, $Beta(m, s, q)$ (Bancroft, 1949):

$$rx_{d,opt}(q) = \int_{t=0}^{t=q} \frac{\overline{[\Delta x}_d(1-Beta(m,s,q))]^2+[\overline{\Delta y}_d Beta(m,s,q)]^2}{4\overline{[\Delta x}_d(1-Beta(m,s,q))]^2}\, dt \quad \forall\, d \tag{25}$$

As regards the weighted Euclidean output distances, however, adjustments must be made in integral parameters as long as inputs are random variables defined departing from negative ideal solutions (minimal) while outputs are random variables defined departing from positive ideal solutions (maximal). This being the case, Bancroft (1949) showed that $Beta(m, s, q) = 1\text{-}Beta(s, m, 1 - q)$. Therefore:

$$ry_{d,opt}(1-q) = \int_{t=0}^{t=1-q} \frac{\overline{[\Delta x}_d Beta(s,m,1-q)]^2+[\overline{\Delta y}_d(1- Beta(s,m,1-q))]^2}{4\overline{[\Delta y}_d(1- Beta(s,m,1-q))]^2}\, dt \quad \forall\, d \tag{26}$$

The non-linear programming problem for stochastic-entropic efficiency quantiles, solved for each DMU d at a time, is given in model (27):

Max $Eff_d\,(q)$	(eq. 23)	
s.t.		
$Eff_d\,(q) \geq 0$	(eq. 23)	
$Eff_d\,(q) \leq 1$	(eq. 23)	
$rx_d = rx_{d,opt}(q)$	(eq. 25)	
$ry_d = ry_{d,opt}(1-q)$	(eq. 26)	model (27)
$\sum_o wy_{d,o} = 1$	(from eq. 20)	
$\sum_i wx_{d,i} = 1$	(from eq. 21)	
$wy_{d,o} \leq 1 \;\; \forall\, o$	(from eq. 20)	
$wy_{d,o} \geq 0 \;\; \forall\, o$	(from eq. 20)	
$wx_{d,i} \leq 1 \;\; \forall\, i$	(from eq. 21)	
$wx_{d,i} \geq 0 \;\; \forall\, i$	(from eq. 21)	

5. Analysis and Discussion of Results

Fig. 1 depicts efficiency/performance score densities obtained from traditional DEA models – under variable and constant returns-to-scale assumptions, VRS and CRS,

[6] MAPLE codes were developed for numerically assessing the integral on incomplete Beta functions. These codes are available to readers upon request.

respectively – and from TOPSIS. While DEA scores suffer from the curse of dimensionality, being extremely biased towards one; TOPSIS scores, although more discriminatory, are mostly contained in between the range delimited by 0.55-0.70. On the other hand, Fig. 2 presents SEA-IS scores computed based on the 92.5%, 95%, and 97.5% quantiles for the input/output slack ratios and their mutual feedback. While differences between quantiles appear to be minimal, one can see that the their bi-modal aspect suggests the existence of groups of US states differently impacted by contextual variables. Besides, it is noteworthy the larger range of score fluctuation under SEA-IS computation.

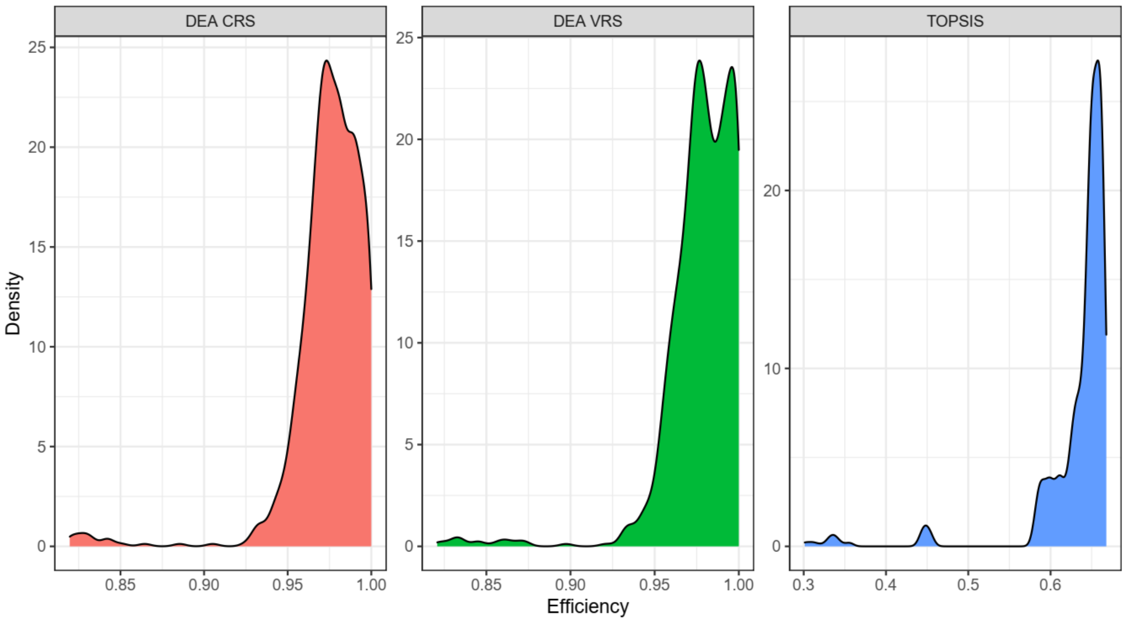


Fig. 1. Density plots for efficiency/performance scores computed under traditional models.

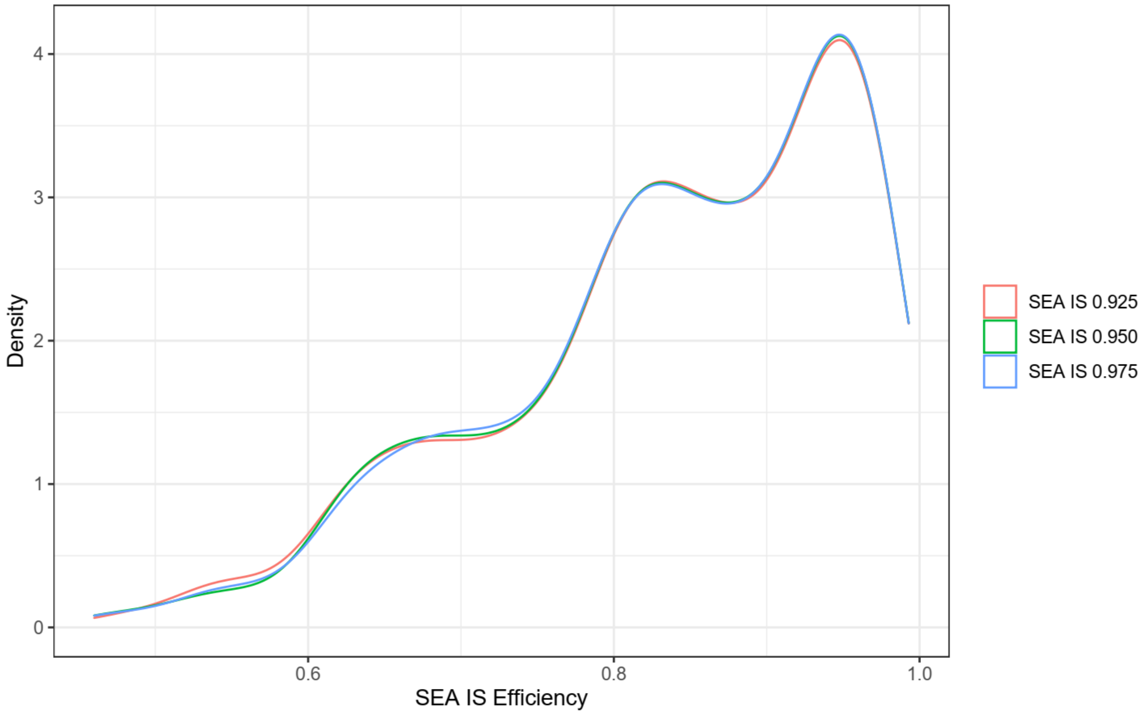


Fig. 2. Density plots for SEA-IS scores computed under different threshold quantiles for input/output slack mutual feedback.

It is interesting to note that, although SEA-IS shares some hybrid features with traditional DEA and TOPSIS methods, as discussed in Section 4, a stronger isotonic relationship only holds between SEA-IS and TOPSIS scores, as suggested by the correlogram presented in Fig. 3. Yet, this isotonic relationship appears to be stable under higher quantile thresholds for input/output mutual feedback. This may suggest that, while tied-up to some extent, mutual information and continuous improvement initiatives may be closely related to higher performance levels, with little space left for disruptive managerial practices in the US energy sector at the state level. One possible reason could be that higher performance levels are characterized by better managerial practices that recognize the value of both mutual information and continuous improvement initiatives in improving and maintaining higher performance levels.

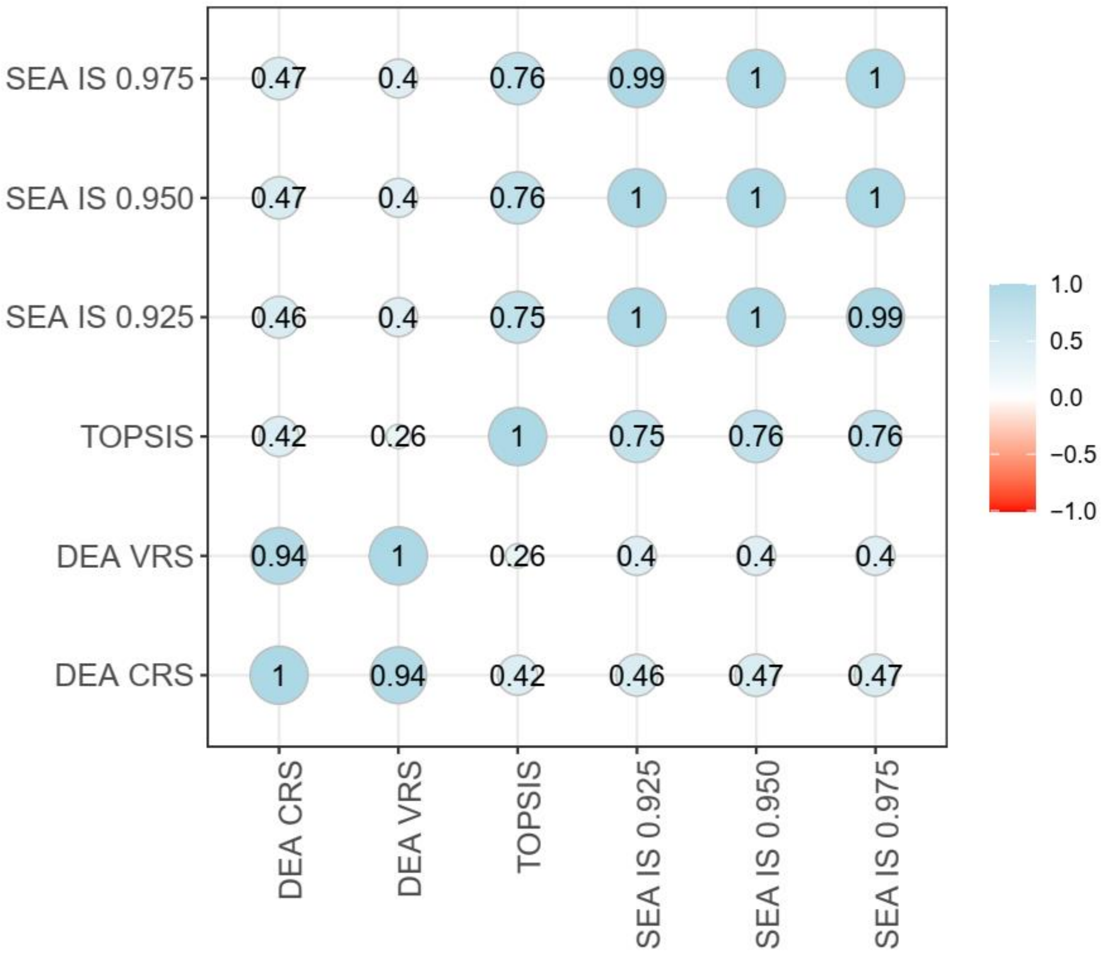


Fig. 3. Alternative score correlogram.

Table 2 helps in putting into perspective this previous discussion. While SEA-IS scores are more dispersed, that is, coefficient of variation (CV) is higher in comparison to traditional models, gains on information reliability – lower information entropy (IE) – cannot be detected under higher quantile thresholds for input/output slack mutual feedback. These information reliability gains in SEA-IS are manifested in bi-modal density scores and loss of isotonicity with traditional models. In fact, when compared to DEA and TOPSIS traditional models, SEA-IS approach provides information on input/output relative weight importance for mutual feedback processes (cf. Fig. 4 top and bottom graphs for mean weights obtained for the 0.95 quantile). While input/output weights are either exogenously defined in TOPSIS at the criteria level or endogenously defined in DEA at the DMU level, SEA-IS differs by ranking input/output relevance in terms of engendering continuous improvement practices.

As regards US energy performance at the state level, net private capital stock is the most relevant input that could be used to learn about outputs, given a current technological frontier on energy usage, generation, and consumption. On the output side, CO2 emissions derived from petroleum and coal, in order, are the more relevant undesirable by-products to learn about input behavior. It is possible to affirm that in US states, common input/output mutual learning practices towards higher efficiency levels commence on basic balance between the capital stock - used as means of production, economic development, and social-welfare – and the undesirable pollutant by-products derived from their use.

The United States is relatively a more capital abundant country, which possibly influences the wage-capital price ratio in a way that makes production processes more capital intensive (vis-à-vis labor intensive). As a result, capital generally plays a dominant role in providing information about the final outputs that are generated. Note, our findings are not industrial sector specific, which indicates the dominant role of capital as an input in the entire economy rather than a specific sector. Similarly, on the output side, we find emissions from coal and petroleum provide more information about input behavior is consistent with the higher percentage use of these fuel sources in the U.S. economy even though natural gas has overtaken coal utilization very recently.

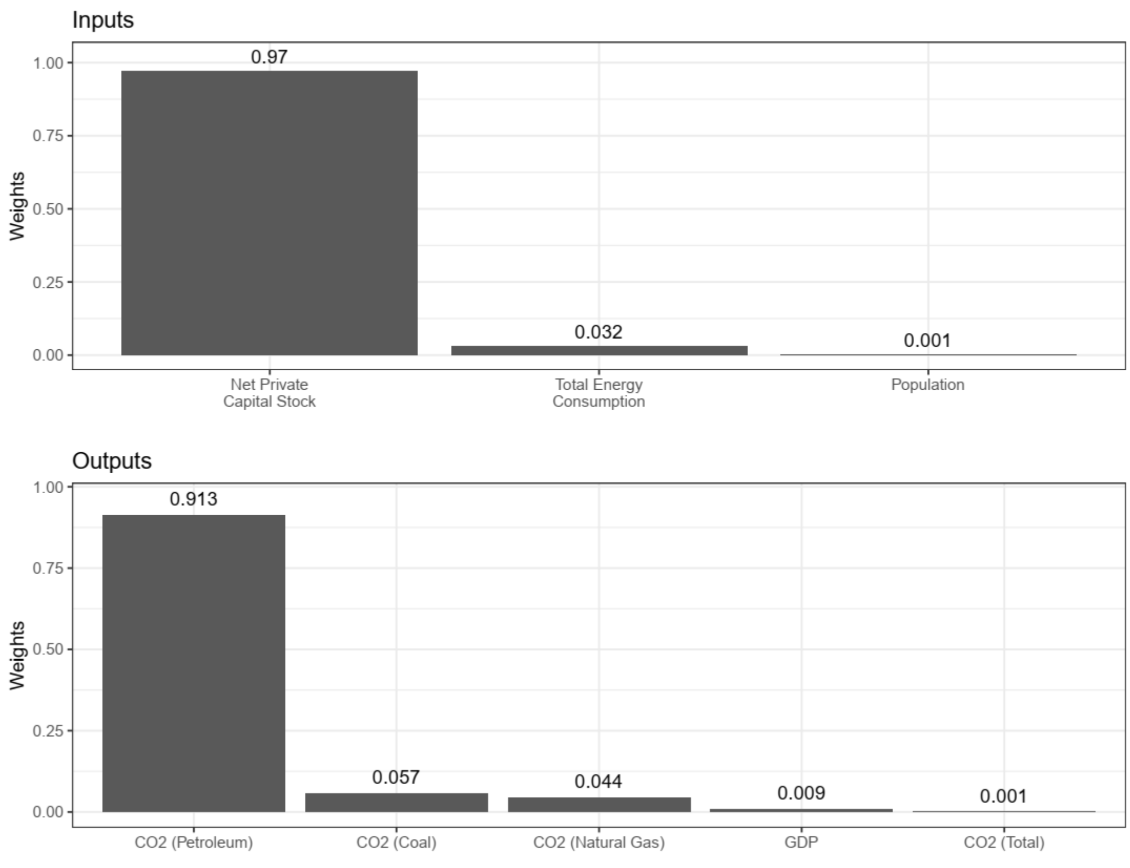


Fig. 4. Most weighted inputs and outputs computed with SEA-IS models for the 95% mutual feedback quantile between input/output slacks.

Fig. 5 illustrates the slight impact of higher mutual feedback quantile thresholds on efficiency levels. It is worth noting that even during the world financial crisis years (2008-2013), learning processes could not have helped in improving median efficiency levels, suggesting that economic dynamics US states were still dependent on petroleum and coal, despite the drop-in overall economic activity. Intuitively, this implies that during the recession years, with higher-than-normal levels of business uncertainty and lower levels of aggregate demand, producers were reluctant to make changes to their input combination (for example, switching to alternative energy sources) and, hence, coal and petroleum remained the primary fuel options in production.

Table 2. Descriptive statistics on alternative scores.

Efficiency	Min	Max	Median	Mean	SD	CV	IE	Kurtosis	Skewness
DEA CRS	0.925	1.000	0.978	0.977	0.015	0.015	1.0000	-0.038	-0.521
DEA VRS	0.932	1.000	0.980	0.979	0.015	0.015	1.0000	-0.189	-0.536
TOPSIS	0.442	0.668	0.649	0.638	0.035	0.055	0.9998	15.281	-3.460
SEA IS 0.925	0.469	0.993	0.854	0.838	0.115	0.138	0.9985	-0.148	-0.770
SEA IS 0.950	0.461	0.993	0.855	0.838	0.115	0.137	0.9985	-0.084	-0.781
SEA IS 0.975	0.460	0.993	0.855	0.839	0.115	0.137	0.9985	-0.062	-0.785

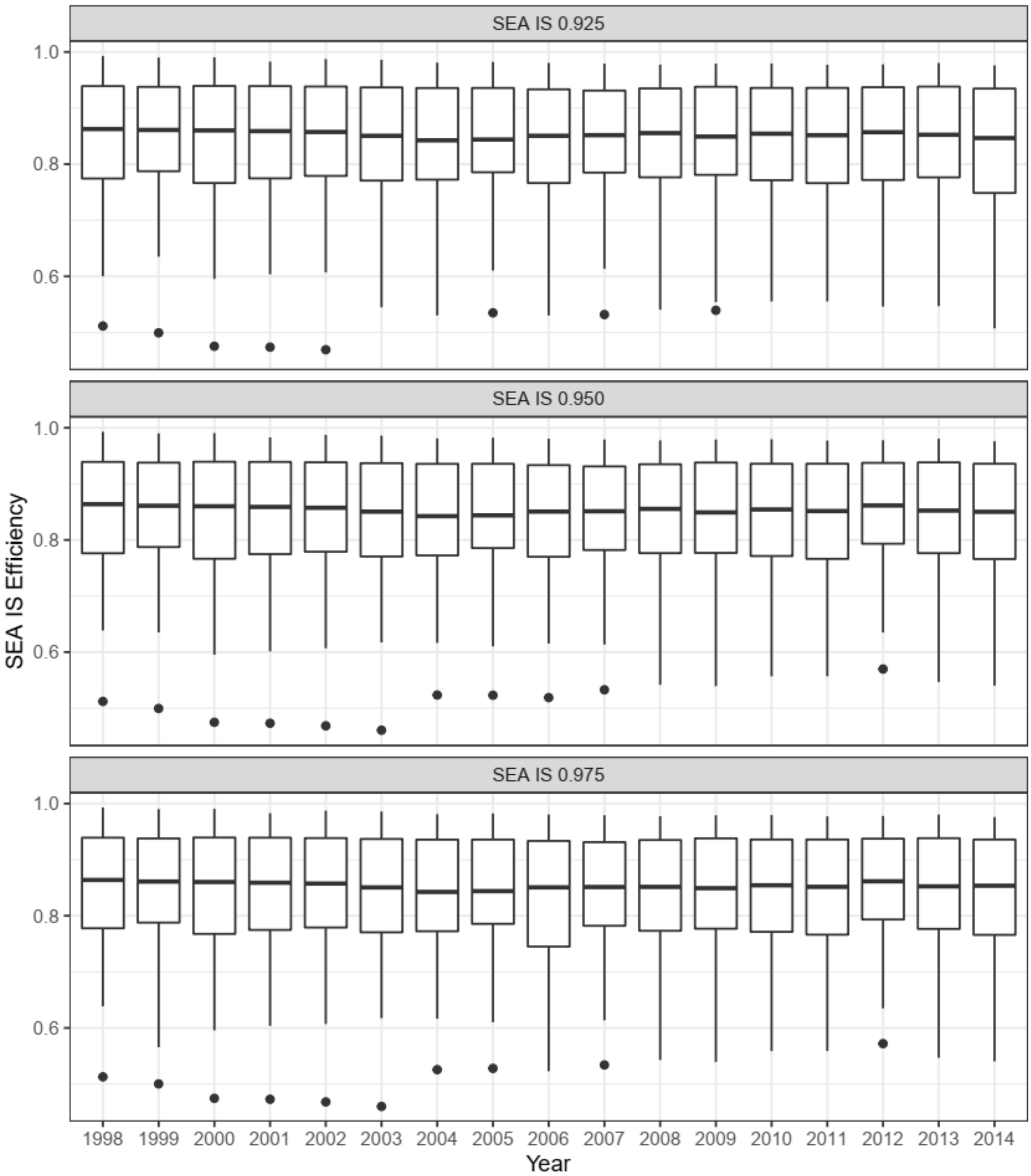


Fig. 5. Yearly comparison for SEA-IS scores under the 92.5%, 95%, and 97.5% mutual feedback quantile between input/output slacks.

To dig into the continuous improvement nature in the US energy sector, although the mutual feedback processes between inputs and outputs appears to be timid when compared to the balance between capital stock and pollutant emissions, it is necessary to map the efficiency behavior at the DMU level beyond quantile thresholds, aiming at what happens in the course of time. Besides, an additional glimpse into the mutual information density, whether below or above median (cf. Fig. 6) is deemed necessary to infer on the very nature of the continuous improvement processes in the US energy sector, even though there are no discernible differences among percentile thresholds or other grouping schemes.

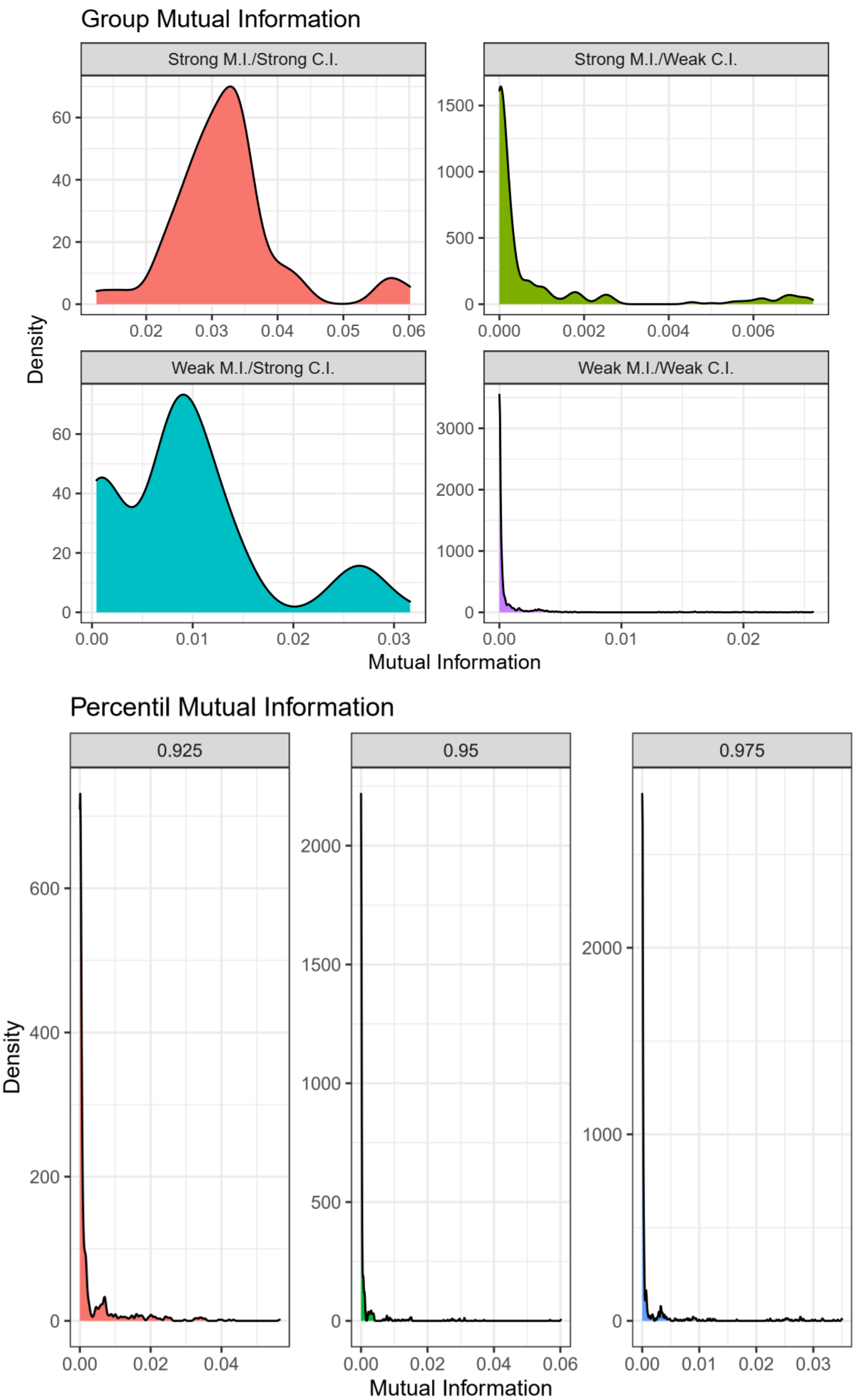


Fig. 6. Mutual information density for US energy performance at the state level.

Let´s consider the following alternative groups of states, which is also reported in Tables 3 and 4:

Group A: Formed by states where mutual information level is above median and SEA-IS efficiency scores both increased from 1998 to 2014 and from the 0.925 to 0.975 threshold quantile. This group is named Strong mutual information/strong continuous improvement management. These states definitely not only present strong mutual feedback between input/output slacks, but are also capable of sustaining efficiency increase over time and over quantile thresholds, which may eventually yield to disruptive energy usage, consumption, and production with technological shift. California is the single state belonging to this group.

It is hard to pinpoint to a single reason behind California's success in consistently improving energy efficiency over time. However, California has long been proactive in experimenting with alternative energy sources with an aim to improve energy efficiency across all sectors. For example, in 2018, California ranked first among all 50 states as a producer of electricity from solar, geothermal, and biomass, and fourth in conventional hydroelectric power generation.[7] This indicates a strategic approach undertaken by the state to improve energy efficiency across the Californian economy, which indicates why they may have been able to take advantage of feedback loops and learning opportunities, which may have risen through initiatives undertaken to achieve continuous improvements.

Group B: Formed by states where mutual information level is above median and SEA-IS efficiency scores just increased from 2002 to 2015 but decreased from the 0.925 to 0.975 threshold quantile. This group is named Strong mutual information/weak continuous improvement management. These states present strong mutual feedback between input/output slacks capable of sustaining efficiency increase over time. However, as regards quantile thresholds, these states fail in producing disruptive improvements or technological changes in energy usage, production, and consumption. Eight states belong to this group: Connecticut, Delaware, Indiana, Maine, Massachusetts, Michigan, Missouri, and West Virginia. The presence of strong mutual feedback indicates that there have been learning opportunities for improving energy efficiency but other existing factors may have prevented the states from making continuous improvements. For example, lack of suitable management practices that would have allowed production units to learn from the mutual information channels.

Group C: Formed by states where mutual information level is below median and SEA-IS efficiency scores decreased from 2002 to 2015 but increased from the 0.925 to 0.975 threshold quantile. This group is named Weak mutual information/strong continuous improvement management. Although these states present a weak mutual feedback between inputs and output slacks and declining efficiency over the course of time, there is still potential, however, for producing disruptive improvements or technological changes in energy production, usage, and generation. Three states belong to this group: Florida, Louisiana, and New York. For these states, while there are limited learning

[7] https://www.eia.gov/state/?sid=CA

opportunities presented through the mutual information pathways, the results indicate there is some evidence of occasional phases of improvements in energy performances, possibly from temporary phases of good managerial practices where units managed to capitalize on intermittent learning opportunities from any feedback process.

Group D: Formed by states where mutual information level is below median and SEA-IS efficiency scores both decreased from 2002 to 2015 and from the 0.925 to 0.975 threshold quantile. This group is named Weak mutual information/weak continuous improvement management. Mutual feedback processes are weak within this group of states, which is incapable of sustaining efficiency growth over time and under higher quantile thresholds. This is the largest group with the 37 states.

Table 3. Distribution of US States among groups.

Group	Frequency	Frequency (%)
(A) Strong M.I./Strong C.I.	1	2.04
(B) Strong M.I./Weak C.I.	8	16.33
(C) Weak M.I./Strong C.I.	3	6.12
(D) Weak M.I./Weak C.I.	37	75.51

Table 4. Listing of US States among groups.[8]

DMU	Group	DMU	Group
California	Strong M.I./Strong C.I.	Maryland	Weak M.I./Weak C.I.
Connecticut	Strong M.I./Weak C.I.	Minnesota	Weak M.I./Weak C.I.
Delaware	Strong M.I./Weak C.I.	Mississippi	Weak M.I./Weak C.I.
Indiana	Strong M.I./Weak C.I.	Montana	Weak M.I./Weak C.I.
Maine	Strong M.I./Weak C.I.	Nebraska	Weak M.I./Weak C.I.
Massachusetts	Strong M.I./Weak C.I.	Nevada	Weak M.I./Weak C.I.
Michigan	Strong M.I./Weak C.I.	New Hampshire	Weak M.I./Weak C.I.
Missouri	Strong M.I./Weak C.I.	New Jersey	Weak M.I./Weak C.I.
West Virginia	Strong M.I./Weak C.I.	New Mexico	Weak M.I./Weak C.I.
Florida	Weak M.I./Strong C.I.	North Carolina	Weak M.I./Weak C.I.
Louisiana	Weak M.I./Strong C.I.	North Dakota	Weak M.I./Weak C.I.
New York	Weak M.I./Strong C.I.	Ohio	Weak M.I./Weak C.I.
Alabama	Weak M.I./Weak C.I.	Oklahoma	Weak M.I./Weak C.I.
Alaska	Weak M.I./Weak C.I.	Oregon	Weak M.I./Weak C.I.
Arizona	Weak M.I./Weak C.I.	Pennsylvania	Weak M.I./Weak C.I.
Arkansas	Weak M.I./Weak C.I.	Rhode Island	Weak M.I./Weak C.I.
Colorado	Weak M.I./Weak C.I.	South Carolina	Weak M.I./Weak C.I.
District of Columbia	Weak M.I./Weak C.I.	South Dakota	Weak M.I./Weak C.I.
Georgia	Weak M.I./Weak C.I.	Tennessee	Weak M.I./Weak C.I.
Hawaii	Weak M.I./Weak C.I.	Utah	Weak M.I./Weak C.I.
Idaho	Weak M.I./Weak C.I.	Virginia	Weak M.I./Weak C.I.
Illinois	Weak M.I./Weak C.I.	Washington	Weak M.I./Weak C.I.

[8] Texas and Vermont were removed due to infeasible solution in SEA-IS optimization.

Iowa	Weak M.I./Weak C.I.	Wisconsin	Weak M.I./Weak C.I.
Kansas	Weak M.I./Weak C.I.	Wyoming	Weak M.I./Weak C.I.
Kentucky	Weak M.I./Weak C.I.		

It is interesting to note that more than 90% of the US states investigated presented weak continuous improvement management practices – California, Florida, Louisiana, and New York are exceptions. Yet, it is still possible to affirm that continuous improvement practices and mutual information levels are attached to some extent, as long as strong continuous improvement practices – as captured by quantile thresholds – are still verified despite weak mutual information levels. Up to some extent, however, this detachment is necessary for the eventual emergence of disruptive practices or technological changes as a consequence of continuous improvement evolution, what appears to be happening in California. In fact, Fig. 7 confirms that weak continuous improvement processes, as captured by quantile thresholds, are more relevant than the mutual information level *per se*, in explaining higher US energy performances at the state level. These states however appear to be stagnant as long as learning opportunities are scarce. Conversely, stronger continuous improvement levels appear to be related to smaller energy efficiency levels, reflecting the plenty of opportunities from learning from feedback processes between input and output slacks. Results indicate that there is plenty of scope for improvements in energy performance across the U.S., given that 37 of 50 states indicated both weak mutual information and weak continuous improvement management practices. However, California's progress is promising and a detailed study might reveal insights that might prove crucial in other states achieving consistent energy efficiency improvements through mutual information feedback loops. This is particular true for the states that fall under the strong mutual information/weak continuous improvements category, where there is already present substantial learning opportunities.

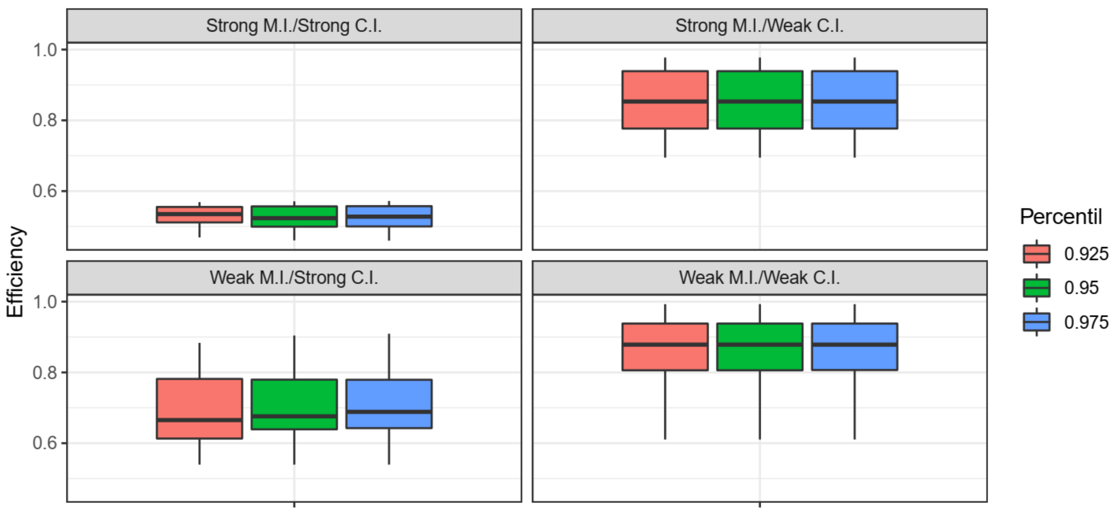


Fig. 7. SEA-IS efficiency density for groups US states.

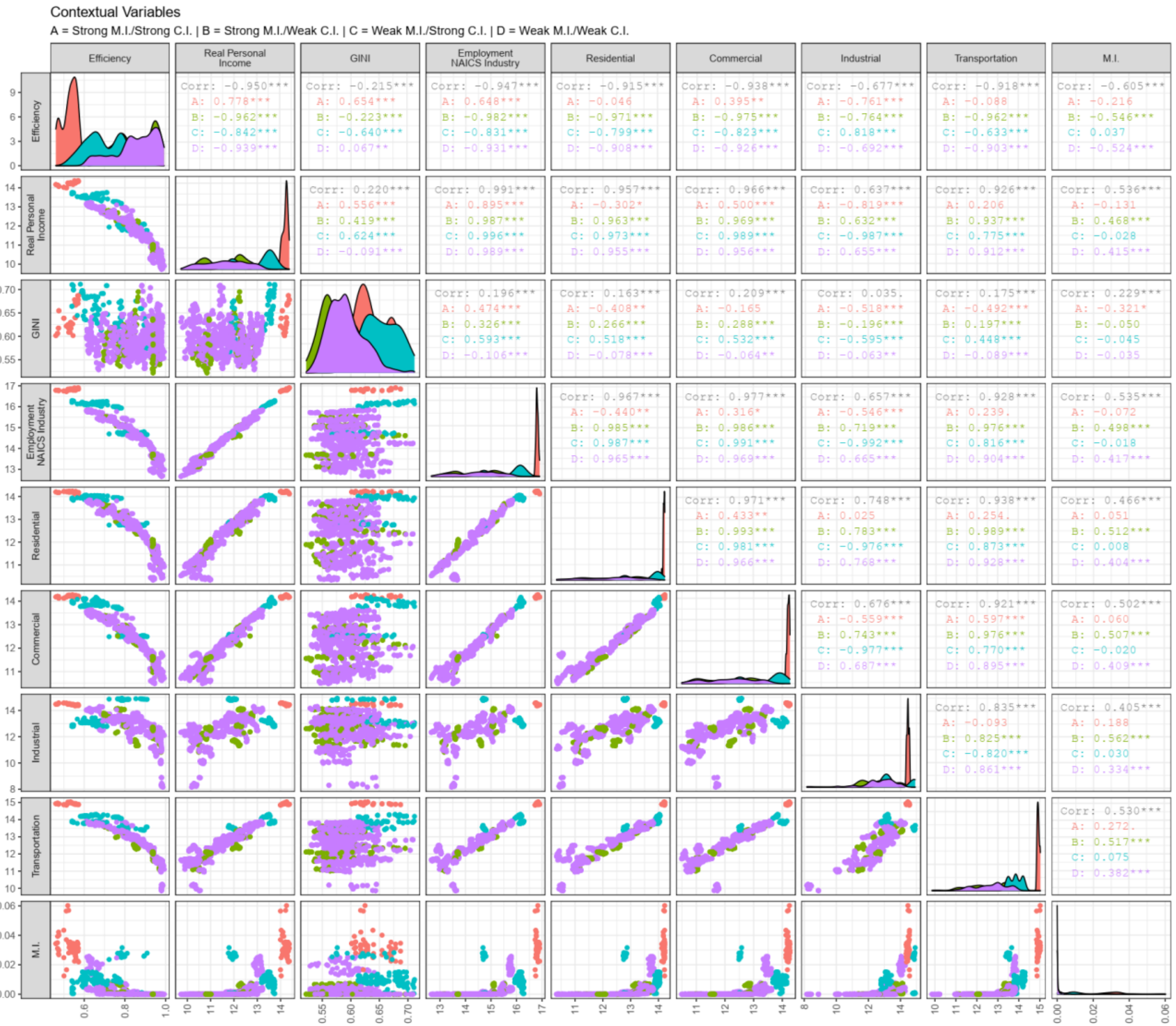


Fig. 8. Contextual variable conditional densities correlogram per group of states.

As regards the impact of contextual variables on SEA-IS efficiency scores and mutual information levels, Fig. 8 suggests strong heterogeneity between each group of US states. Besides, collinearity appears to be present in combinations among sector, consumption, and personal income. An artificial network model, multi-layer perceptron architecture, was developed for classification purposes with respect of each bank group using contextual variables as predictors. The best architecture founding using 10-fold cross-validation was formed by 1 hidden layer with 13 neurons, yielding a median predictive accuracy about 95.3% (cf. Fig. 9). Sensitivity analysis on contextual variable importance for predicting group membership was performed as in Olden et al. (2002) and its results are reported in Fig. 10.

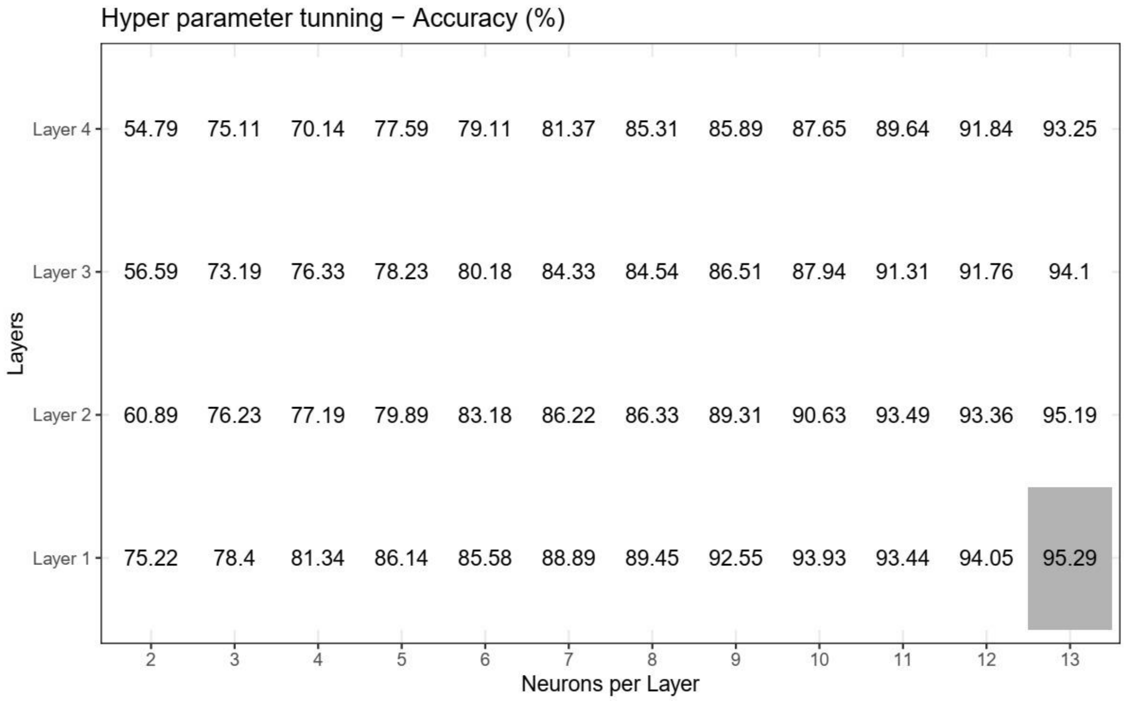


Fig. 9. Parameter tuning for US states group membership neural network.

It is worth noting the close relationship between social welfare – as captured by GINI ad real personal income - and strong mutual information/strong continuous improvement practices in California. Rather than claiming on a cause-and-effect relationship, social-welfare indicators can serve as proxies for relevant mutual feedback between input/output slacks which can eventually yield technological change or business disruption in industry and transportation sectors with respect to energy performance. Interesting to note that, as regards weak mutual information and strong continuous improvement (Florida, Louisiana, and New York), higher welfare inequality as captured by GINI appears to boost continuous **information** in energy residential use, differently from California, where industrial use is more prominent. This could indicate differences in household culture across states. When faced with lower income levels persistently, unlike CA, in these states, residents have developed a cautious approach toward energy use, with an aim to keep energy bills under check. Stagnant lower personal income may result in inhibiting continuous improvement and, eventually technological change, as can be noted in the majority of US states with low mutual information levels. Unlike Florida, Louisiana, and New York, for a large group of states in the weak mutual information/weak continuous improvement category, we find states with low and/or stagnant real income levels are often unable to make continuous energy efficient improvements, which indicates some form of government intervention might be needed to enable these states to leverage the gains from mutual information feedback mechanisms in production systems.

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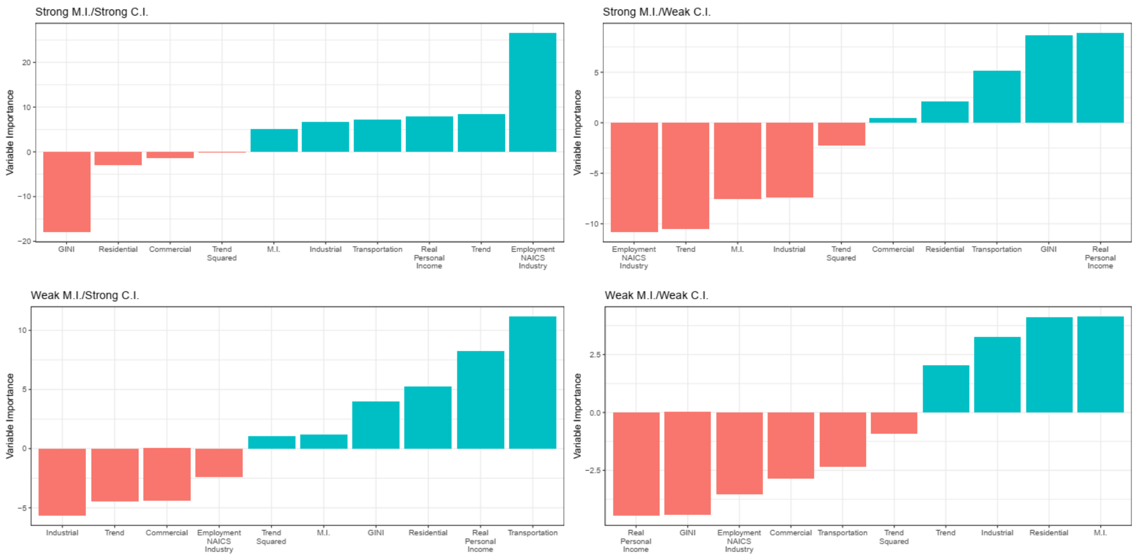

Fig. 10. Contextual variable importance for US states group membership.

5. Conclusions

In production processes, as inputs are transformed into output with the help of technology, learning opportunities may often arise, which can potentially help in improving efficiency over time. However, the extent to which information through feedback loops can be utilized to enhance performance can greatly vary across both industrial sectors and firms within any sector. Information entropy also plays a role in energy performances given that energy is key component of every production process. When energy performance related inputs and outputs share some level of mutual information, there is some potential for producers to learn simultaneously from each other's distributional behaviors. However, to the best of our knowledge, few studies in energy economics, including studies involving MCDM criteria, and both parametric and non-parametric frontier techniques, have explored this area. Thus, to further our understanding of the role played by information entropy in improving energy efficiency in production processes, this paper advances the related literature by presenting a novel stochastic entropy analysis – ideal solution (SEA-IS) model. The model builds on the strengths of traditional DEA and TOPSIS models. The SEA-IS model is used to assess the potential information gains that may arise from energy slacks minimization given different optimal reduction quantiles in all fifty U. S. states. The United States has consistently been one of the highest energy users of the twentieth century with a large and well-developed industrial sector, which allows us to study the feedback mechanisms between various inputs such as capital stock, energy consumption, population and output such as GDP and undesirable by-products such as carbon dioxide emissions from fuel sources such as petroleum, coal, and natural gas. Contextual variables related to social welfare such as real income and GINI coefficient, and the total number of jobs, all of which affect energy utilization levels were also considered. A robustness comparison among SEA-IS, DEA, and TOPSIS scores is provided in light of information reliability theory.

The results of the analysis indicated while DEA scores suffer from the curse of dimensionality and the TOPSIS scores are, mostly, contained in the range 0.55 – 0.70, the SEA-IS model scores were computed based on the 92.5%, 95%, and 97.5% quantiles for the input/output slack ratios and their mutual feedback. The scores indicated the existence of four groups of US states, which varied in terms of the role played by mutual information in improving energy performances and in terms of their scope of making continuous improvements over time. The variation among the four state groups indicate there is ample score for harnessing the power of feedback mechanisms in production process to improve energy efficiency, particularly in the bottom 37 states. There is a role of government intervention and a potential for public-private partnership at state levels for achieving this efficiency goal.

The model presented in this paper can be applied in future studies in studying the role of information entropy in non-renewable vis-à-vis renewable energy sources. Beyond energy performance-based studies, it can be applied to analyze the role of feedback learning mechanisms in improving efficiency in other resource-based sectors such as fisheries.

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