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Abstract

This paper investigates potential portfolio diversification between Bitcoin, bonds, equities and the US dollar. We make use of two approaches for constructing the portfolio. The first is the standard minimum variance approach, and the alternative is based on combining risk and return when the portfolio is constructed. The portfolio based on the minimum variance approach does not result in increasing the return per unit risk compared to the corresponding value for the best single asset, in this case Bitcoin. However, the portfolio based on the approach that combines risk and return in the optimization problem does show a return per unit risk higher than the corresponding value for any of the four assets. Thus, the portfolio diversification benefit with respect to these four assets, in terms of return per unit risk, exists only if the portfolio is constructed via the new approach.

Keywords: Portfolio Diversification; Bitcoin; Equity; Bond; US Dollar; Risk and Return

JEL Classifications: C6, G10, G12

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1. Introduction

Cryptocurrencies have become an area of interest for the media, regulators, and investors. Some of their valuable features are blockchain technology and decentralization, which make cryptocurrencies self-governing and thus independent of governmental authorities (Bouri et al., 2017; Corbet et al., 2019). The leading and dominant cryptocurrency, Bitcoin, is generally found to have its price driven by a set of unique factors such as attractiveness and attention (Kristoufek, 2015; Bouoiyour et al., 2016), energy prices (Li and Wang, 2017), user anonymity (Ober et al., 2013), and computer programming enthusiasts (Yelowitz and Wilson, 2015). Bitcoin is somewhat detached from the global financial system and thus from conventional assets such as equities, bonds, and currencies, offering diversification benefits (Baur et al., 2018; Bouri et al., 2017). The positive aspects of the diversification benefit that Bitcoin offers have recently become an issue debated by financial reporters, scholars, and investors who are always in quest of an ultimate alternative investment asset capable of enhancing the portfolio return while reducing, or at least maintaining, the risk level of the portfolio.

Studies such as Klein et al. (2018) provide evidence against the hedging ability of Bitcoin, arguing that Bitcoin is positively correlated with downward equity markets. Other studies (Baur et al., 2018; Bouri et al., 2017; Guesmi et al., 2019; Corbet et al., 2018) argue that Bitcoin offers diversification benefits due to its very weak correlation with equity markets. Interestingly, the existing literature generally relies on optimal portfolio design and hedging ratios to make inferences about diversification (Charfeddine et al., 2019; Guesmi et al., 2019) and, importantly, does not include either risk or return directly in the portfolio diversification problem for optimization (e.g., Brière et al., 2015; Bouri et al., 2019; Kajtazi and Moro, 2019). Nevertheless, it is widely agreed in the literature that investors care about returns and risk combined and not separately as in the traditional approach of Markowitz (1952, 1970). Recently, alternative methods have been developed for this purpose that include both risk and return directly in the portfolio diversification problem for optimization (e.g., Hatemi-J and El-Khatib, 2015).

To address this shortcoming in the Bitcoin literature, the authors of this paper make use of both a traditional approach and an alternative approach in order to construct a portfolio with respect to the Bitcoin index, bond index, world equity index and dollar index over the period 2011-2019, using daily data.

The results show that the diversification benefits of Bitcoin, measured by the amount of return per unit of risk, are offered to investors if the underlying portfolio is constructed combining

risk and return, along the lines of Hatemi-J and El-Khatib (2015). This finding is important for investors and portfolio managers and suggests the need to combine risk and return directly in the portfolio diversification problem for optimization, otherwise the valuable virtue of Bitcoin may not be uncovered.

The rest of the paper is structured as follows. Section 2 describes the methodology. The estimation results are presented in Section 3. The last section offers the conclusions.

2. Methodology

Portfolio diversification is commonly used by investors in order to deal with the unsystematic risk of their investments. The original and well-known approach for this purpose, introduced by Markowitz (1952), results in finding the optimal weights as budget shares that minimize the variance of the underlying portfolio subject to the budget constraint. Let r_i represent the return of asset i, which is normally distributed such that $r_i \sim N(\bar{r}_i, \sigma_i^2)$. The variance-covariance matrix for n assets is defined as $\Omega = (\sigma_{i,j})_{1 \le i,j \le n}$, where σ_{ij} is the covariance between the returns of assets i and j. Let w_i represent the budget share of asset i. Thus, the average return the portfolio can be defined as $F(w) = \sum_{i=1}^{n} \bar{r}_i w_i$ and its variance as $G(w) = w'\Omega w$. That is, the minimization problem of Markowitz (1952) is:

$$Minimize\ G[w] = w'\Omega w. \tag{1}$$

subject to the following budget restriction:

$$C(w) = \sum_{i=1}^{n} w_i = 1 \tag{2}$$

The solutions for w_i of this problem are as follows, when n=4:

$$w_1 = \frac{-\begin{vmatrix} D_{1,2} & D_{1,3} & D_{1,4} \\ D_{2,2} & D_{2,3} & D_{2,4} \\ D_{3,2} & D_{3,3} & D_{3,4} \end{vmatrix}}{|E|} \tag{3}$$

$$w_2 = \frac{\begin{vmatrix} D_{1,1} & D_{1,3} & D_{1,4} \\ D_{2,1} & D_{2,3} & D_{2,4} \\ D_{3,1} & D_{3,3} & D_{3,4} \end{vmatrix}}{|E|} \tag{4}$$

$$w_3 = \frac{-\begin{vmatrix} D_{1,1} & D_{1,2} & D_{1,4} \\ D_{2,1} & D_{2,2} & D_{2,4} \\ D_{3,1} & D_{3,2} & D_{3,4} \end{vmatrix}}{|E|} \tag{5}$$

$$w_4 = \frac{\begin{vmatrix} D_{1,1} & D_{1,2} & D_{1,3} \\ D_{2,1} & D_{2,2} & D_{2,3} \\ D_{3,1} & D_{3,2} & D_{3,3} \end{vmatrix}}{|E|} \tag{6}$$

where

$$E = \begin{pmatrix} D_{1,1} & D_{1,2} & D_{1,3} & D_{1,4} \\ D_{2,1} & D_{2,2} & D_{2,3} & D_{2,4} \\ D_{3,1} & D_{3,2} & D_{3,3} & D_{3,4} \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Note that $|X|$ represents the determinant of the matrix X. It should be mentioned that D is an $n \times n$ matrix with the following properties:

$$D_{i,j} = \left(\sigma_{i+1,j} + \sigma_{j,i+1}\right) - \left(\sigma_{i,j} + \sigma_{j,i}\right), \qquad for \qquad 1 \le i \le n-1 \qquad and \qquad 1 \le j \le n.$$

The Markowitz method that is frequently applied in the literature results in the construction of a portfolio that has the minimum possible risk. However, it is widely agreed that a rational investor should pay attention not only to risk but also to return. Thus, Hatemi-J and El-Khatib (2015) suggest combining the return and risk in the optimization problem when a portfolio is constructed. That is, the optimization problem should, according the authors, be:

$$Maximize \ \frac{F(w)}{\sqrt{G[w]}} = \frac{F(w)}{\sqrt{w'\Omega w}}. \tag{7}$$

subject to

$$C(w) = \sum_{i=1}^{n} w_i = 1 \tag{8}$$

By using Theorem 1 in Hatemi-J, Hajji and El-Khatib (2019), the solutions for optimal budget shares of this risk adjusted return problem are as follows, when n=4:

$$w_1 = \frac{-\begin{vmatrix} H_{1,2} & H_{1,3} & H_{1,4} \\ H_{2,2} & H_{2,3} & H_{2,4} \\ H_{3,2} & H_{3,3} & H_{3,4} \end{vmatrix}}{|K|} \tag{9}$$

$$w_2 = \frac{\begin{vmatrix} H_{1,1} & H_{1,3} & H_{1,4} \\ H_{2,1} & H_{2,3} & H_{2,4} \\ H_{3,1} & H_{3,3} & H_{3,4} \end{vmatrix}}{|K|} \tag{10}$$

$$w_3 = \frac{-\begin{vmatrix} H_{1,1} & H_{1,2} & H_{1,4} \\ H_{2,1} & H_{2,2} & H_{2,4} \\ H_{3,1} & H_{3,2} & H_{3,4} \end{vmatrix}}{|K|} \tag{11}$$

$$w_4 = \frac{\begin{vmatrix} H_{1,1} & H_{1,2} & H_{1,3} \\ H_{2,1} & H_{2,2} & H_{2,3} \\ H_{3,1} & H_{3,2} & H_{3,3} \end{vmatrix}}{|K|} \tag{12}$$

where

$$K = \begin{pmatrix} H_{1,1} & H_{1,2} & H_{1,3} & H_{1,4} \\ H_{2,1} & H_{2,2} & H_{2,3} & H_{2,4} \\ H_{3,1} & H_{3,2} & H_{3,3} & H_{3,4} \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Note that H is an $n \times n$ matrix defined as:

$$H_{i,j} = \bar{r}_i(\sigma_{i+1,j} + \sigma_{j,i+1}) - \bar{r}_{i+1}(\sigma_{i,j} + \sigma_{j,i}), \quad for \quad 1 \le i \le n-1 \quad and \quad 1 \le j \le n.$$

Thus, this method combines risk and return in the optimization problem.

3. Data and results

3.1. The dataset

Our daily dataset is from 18 August 2011 to 14 March 2019, with the starting date determined by the availability of Bitcoin prices from Bitstamp (Guesmi et al., 2019)[1]. It covers daily US

[1] Bitstamp (https://www.bitstamp.net/) is the world's longest standing cryptocurrency exchange.

dollar values of Bitcoin prices, the World MSCI equity index, the PIMCO Investment Grade Corporate Bond Index, and the US Dollar index (DXY), extracted from the DataStream database. The empirical analysis is conducted with log-returns, calculated as the first-differences of the natural logarithms of the prices (indices).

3.2. The estimation results

The estimation results via both methods are presented in Table 1. The results show that Bitcoin has the highest return, but also has the highest risk measured by standard deviation. Combining risk and return, Bitcoin has the highest value of the four assets. This implies that, if an investor is interested in selecting only one asset for investment, Bitcoin should be chosen because it gives the highest return per unit risk, i.e. the risk adjusted return is around 8.184%. The next question is whether the investor can increase the amount of the return per unit of risk by constructing a portfolio that includes all four of the assets. For this purpose, we construct two portfolios. The first is constructed by calculating the weights via equations (3)-(6). The estimation results for this portfolio, which is based on the minimum variance approach, provides a risk adjusted return equal to 4.518 %. This value is, of course, less than the risk adjusted return for Bitcoin. Thus, this portfolio does not increase the return per unit risk even though it provides the lowest possible risk. Next, we construct a portfolio using equations (9)-(12). The estimation results for this second portfolio give a risk adjusted return equal to 10.266%, which is higher than the risk adjusted return of the best single asset, in this case Bitcoin. This means that portfolio diversification benefits, in terms of higher return per unit risk, are available to investors only if the portfolio is constructed based on weights that combine risk and return.

Table 1. The Estimation Results for a Portfolio Consisting of Bitcoin, Bonds, the World Equity Index (WEI) and Dollars, Using Two Methods

Statistics		***Minimum Variance Approach***	***Risk Adjusted Return Approach***
Average return for the Bitcoin	0.004940		
Average return for Bonds	1.682E-06		
Average return for WEI	0.00035		
Average return for Dollars	0.00015		
Standard Deviation for Bitcoin	0.06035		
Standard Deviation for Bonds	0.00296		
Standard Deviation for WEI	0.00797		
Standard Deviation for Dollars	0.00444		
Risk Adjusted Return for Bitcoin	0.08185		
Risk Adjusted Return for Bonds	0.00057		
Risk Adjusted Return for WEI	0.04415		
Risk Adjusted Return for Dollars	0.03395		
Weight for the Bitcoin in the Portfolio		0.00008	0.05989
Weight for Bonds in the Portfolio		0.55876	0.12628
Weight for WEI in the Portfolio		0.12477	0.30248
Weight for Dollars in the Portfolio		0.31639	0.51135
Average Return of the Portfolio		0.00009	0.00048
Standard Deviation of the Portfolio		0.00206	0.00467
Risk Adjusted Return of the Portfolio		0.04518	0.10266

Notes: the estimations are implemented by making use of a code written in Mathematica, which is available on request from the authors.

4. Concluding remarks

Investors regularly make use of portfolios in order to tackle the unsystematic risk of their investments. The current paper explores the potential portfolio diversification benefits of Bitcoin, bonds, equities and the US dollar. Two alternative approaches are used to construct conditional optimal portfolios. Firstly, we find the weights as budget shares that are based on the minimum variance approach, and the findings from the resulting portfolio indicate that the return per unit risk of this portfolio is lower than the return per unit risk of the best single asset, which is Bitcoin. Secondly, we construct an alternative portfolio by estimating the underlying budget shares via maximization of the return per unit risk subject to the budget constraint. The results of this alternative portfolio show that the rerun per unit risk is higher than the corresponding value for the best single asset option. Our findings imply that the diversification benefits measured by the amount of return per unit of risk are available to the investor only if the underlying portfolio is constructed by the new (second) approach, i.e. by combining risk and return when the portfolio is constructed (Hatemi-J and El-Khatib, 2015). In fact, the portfolio based on the standard approach results in the lowest possible risk but not in the highest return per unit of risk, compared to investing in only one asset. It should be pointed out that the weights for all four assets are positive, indicating a long position for each asset in the portfolio. Our findings concord with Hatemi-J and El-Khatib (2015) who show the importance in equity markets of constructing portfolios by combining risk and return. Therefore, our findings provide new insights for the Bitcoin literature and nicely complement previous studies that highlight the risk-reduction benefits of Bitcoin investment (e.g., Brière et al., 2015; Corbet et al., 2018; Bouri et al., 2019; Guesmi et al., 2019; Kajtazi and Moro, 2019). Specifically, our findings help investors optimize their decision-making, using the maximization of the return per unit of risk subject to the budget constraint, rather than considering return and risk separately as in the traditional approach of Markowitz (1952, 1970). In this sense, our findings are useful for making more refined investment decisions in the controversial Bitcoin market.

While we rely on Bitcoin prices against the US dollar, there is a possibility that the diversification potential of Bitcoin is affected by the currency used to trade Bitcoin. Further research is needed to fruitfully address this issue. Future research could incorporate changes in market conditions, as in Nystrup et al. (2018), while constructing portfolios.

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