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Abstract

We study whether level of risk aversion can be used to predict Bitcoin returns. Using a copula-quantile approach, we find evidence of predictability for the lower and upper quantiles of the conditional distribution of returns (i.e., in bull and bear markets). To reveal the sign of the predictability, we apply the cross-quantilogram approach and find that the cross-quantilogram is similar when risk aversion is at the low or medium level for various quantiles of Bitcoin returns. In particular, we find positive predictability when the risk aversion is very low and at the medium level. However, the predictability becomes negative when both the risk aversion and Bitcoin returns are very high, suggesting that very high levels of risk aversion are likely to drive down Bitcoin returns in a bull market.

Keywords: Risk-aversion; Bitcoin returns; price predictability; copulas; quantiles.

JEL code: C22; G10

1. Introduction

Risk aversion measures the willingness of investors to take financial risks, and thereby hold risky assets. Economic intuition suggests that the level of risk aversion affects portfolio payoff and its distribution (Dybvig and Wang, 2012). The level of risk aversion is cited by market participants and policy-makers as a key driver of the trend dynamics in risk premiums (Londono and Wilson, 2018), reinforcing its ability to affect the return dynamics in financial markets. However, the ability of risk aversion to drive Bitcoin returns remains unexplored, although Bitcoin is often seen as a shelter from global market uncertainty (Bouri et al., 2017b).

With the growing popularity of Bitcoin as an alternative digital asset, numerous studies examine the safe-haven role of this protocol-governed currency for equities (e.g., Bouri, 2017a; Baur et al., 2018), embracing its detachment from the global financial system (Corbet et al., 2018; Ji et al., 2018), mostly due its independence from any third-party control such as a government or a central bank. Other studies (e.g., Klein et al., 2018; Smales, 2018) provide an opposing view, pointing to Bitcoin's high volatility, illiquidity, and high transaction costs. Interestingly, the hedging ability of Bitcoin is affected by the level of global EPU (Fang et al., 2019), and Bitcoin shows some ability to hedge financial market uncertainty (Bouri et al., 2017b), and economic policy uncertainty (EPU) (Demir et al., 2018; Wang et al., 2018). However, risk aversion is different from economic uncertainty, with the latter representing the amount of risk, and the former representing the price of risk (Xu, 2017).

Given the ability of risk aversion to drive the trend dynamics in financial markets and the mixed views on the role of Bitcoin as a safe-haven, one wonders about the ability of the level of risk aversion to drive Bitcoin returns. This is where we aim to contribute. Specifically, we use the time-varying risk aversion measure recently developed by Bekaert et al. (2017) and assess its ability to predict Bitcoin returns under various market conditions. Our assessment is conducted via a copula-based causality in quantiles approach (Lee and Yang, 2014), which allows us not only to differentiate bear, normal, and bull market conditions in Bitcoin but also consider various copula functions for testing Granger-causality in distribution and in quantiles. We apply the cross-quantilogram approach of Han et al. (2016) to reveal the sign of predictability.

Our empirical analyses establish, for the first time, a direct link between global risk aversion and Bitcoin returns. They show that the risk aversion index is useful for predicting Bitcoin returns in bearish (lower quantiles) and bullish (upper quantiles) market periods. Further evidence indicates a positive predictability when the risk aversion is very low or at the medium level. However, the predictability becomes negative when both the risk aversion and Bitcoin returns are very high, suggesting that very high levels of risk aversion are likely to drive down Bitcoin returns in a bull market.

2. Data

The data used are the daily Bitcoin price (https://coinmarketcap.com/) and the risk aversion index of Bekaert et al. (2017) (available at https://www.nancyxu.net/). Bekaert et al. (2017) develop a new measure of time-varying risk aversion based on observable financial information (involving the term spread, credit spread, a detrended dividend yield, realized and risk-neutral equity return variance and realized corporate bond return variance) at daily frequencies. An important feature of this measure is that it distinguishes time variation in economic uncertainty (the amount of risk) from time variation in risk aversion (the price of risk), and thus provides an unbiased representation for time-varying risk aversion based on a utility function in the hyperbolic absolute risk aversion (HARA) class. Our analysis covers the period 30th April, 2013 to 30th December, 2016, a total of 919 observations, with the start and end dates based on data availability of Bitcoin prices and the risk aversion index. While the risk aversion index is stationary by design, we compute the log-returns of the Bitcoin prices for our analysis, given that the econometric approach used requires mean-reverting data. The data is summarized in Table A1 and plotted in Figure A1 in Appendix 1. As can be seen, the results of the Augmented Dickey and Fuller (1981) (ADF) test confirm the stationarity of the two variables under study, and the strong evidence of non-normality warrants a quantiles-based approach.

3. Methodology

We use the GCD test of Lee and Yang (2014) to examine the ability of the level of risk aversion to predict Bitcoin returns in low, medium and upper quantiles. We then apply the cross-quantilogram approach of Han et al. (2016) to reveal the sign of predictability.

3.1. Granger causality in distribution (GCD) test of Lee and Yang (2014)

We examine the dynamic causality between the returns of Bitcoin (RBC) and the risk aversion index (RAI) through quantile forecasts which are largely reliant on inversion concerning parametric conditional copula distribution from Lee and Yang's (2014) model. It is realistic for market practitioners to view causality anticipated at high RBC and RAI quantiles, since RBC can be a safe haven in case of panic in the global financial market (i.e. risk aversion is high). Firstly, we test the null hypothesis in which Y_t in the distribution is not Granger caused by X_t: H0: c(u, v) = 1. Where c(u, v) represents conditional copula density function and u and v represent X_t (i.e. $RAI_{t)}$ and Y_t (i.e. RBC_t) conditional probability integral transforms respectively. The following formulas are used to compute the proposed conditional variance for $\{X_t\}$ and $\{Y_t\}$, $\hat{h}_{x,t+1}$ and $\hat{h}_{y,t+1}$:

$$\hat{h}_{x,t+1} = \hat{\beta}_{x0} + \hat{\beta}_{x1}x_t^2 + \hat{\beta}_{x2}\hat{h}_{t,x}$$

$$\hat{h}_{y,t+1} = \hat{\beta}_{y0} + \hat{\beta}_{y1}y_t^2 + \hat{\beta}_{y2}\hat{h}_{t,x} \tag{1}$$

The empirical distribution function (EDF) is used to calculate the cumulative distribution function (CDF) values of $\hat{u}_{t+1}$ and $\hat{v}_{t+1}$ for y_{t+1} and x_{t+1}. On the other hand, pared EDF values $\{\hat{u}_{t+1}, \hat{v}_{t+1}\}_{t=R}^{T-1}$ are used to estimate a nonparametric copula function through the use of a quartic kernel function:

$$k(u) = \frac{15}{16}(1 - u^2)^2 I(|u| \leq 1) \tag{2}$$

The GCD result of test statistics established by Hong and Li (2005) for $\{x_{t+1}, y_{t+1}\}_{t=R}^{T-1}$ is 36.158, which is significant at the level of 1%, showing that there exists an important GCD between RBC and RAI. On the other hand, the GCD test evidence is not a reflection of Granger causality in every conditional quantile. Our empirical study focuses on 3 main regions of distribution i.e. the right tail (99% quantile, 95% quantile, and 90% quantile), central region (60% quantile, median, and

40% quantile), and the left tail (10% quantile, 5% quantile, and 1% quantile), which is similar to Lee and Yang (2014)[1].

[Insert Table 1 about here]

Table 1 shows the outcomes of the analysis of GCQ in p-values. The small p-value of the authenticity check indicates the denial of the worthless supposition, demonstrating that there exists a copular task to model GCQ and, in return, yield a good quintile predictor of the RBC based on the RAI. We observe that a quantile forecasting model with no Granger causality in the quantile is rejected in many quantiles, but not the quantiles at 50% and 60%, with evidence at 1% significance level. This result shows that the risk aversion index strongly Granger-causes the Bitcoin returns at the left tail (poor performance) and right tail (superior performance) but not at the centre (usual performance) of the distribution of the Bitcoin return, conditional on the risk aversion index. In other words, risk aversion is important primarily at the tails of the conditional distribution, i.e. when the Bitcoin market is in bearish (lower quantiles) or bullish (upper quantiles) phases, dependent on the information content of our measure of risk aversion. When the conditional distribution is around the median, i.e. when the market is in normal mode, the predictive content of risk aversion is not significant, which makes sense, since agents operating in the market are likely to be averse to heightened risk when the market is performing poorly or well, rather than when it is in its normal phase.[2]

3.2. Directional predictability test of Han et al. (2016)

Han et al.'s (2016) directional predictability test is used in this study to complement the GCD test. Investors may prefer to use RAI for predicting RBC movement. As such, access to the RBC forecasting performance using RAI as a predictor is

[1] Details of the method of Lee and Yang (2014) are given in Appendix 2.

[2] Following Pástor and Veronesi (2018), who relate risk-aversion with Democratic governments in the US, we use a dummy variable taking the value of 1 under Democratic presidents and zero otherwise (i.e., Republican president), and apply the quantiles-based Granger causality test of Jeong et al. (2012) as a robustness check on our analysis. Based on a data sample covering 30th April, 2013 to 1st January, 2019, we obtain qualitatively similar results (i.e., stronger causality at the tails of the conditional distribution (though weaker predictability is observed at the median)) to those reported in Table 1. This is not surprising since, from 2017, the US did have a Republican government, resulting in our risk aversion dummy being zero. Complete details of these results are available upon request from the authors. Note that since the cross-quantilogram requires two continuous variables, we could not use the democratic dummy variable further for the analysis of directional predictability.

necessary. There is no directional expectedness for RAI in any other time series. This is the null hypothesis. The cross-quantilogram's ability to detect duration, magnitude and spontaneous relationship direction for RAI and RBC is an added advantage over GCD, which cannot. However, this is not the only advantage. The model allows us to choose arbitrary quantiles for both RBC and RAI. There are no pre-set quantiles as far as GCD is concerned. Using the bootstrap technique enhances the large lag in the directional predictability test.

Han et al. (2016) propose that the cross-quantilogram technique may show a quantile-to-quantile relationship between RBC and RAI. The following equation specifies linear quantile regression:

$$q_{\alpha(\tau_{t+1}|\mathcal{F}_t)} = \beta_{0,\alpha} + \beta_{1,\alpha}x_t + \beta_{2,\alpha}x_t q_\alpha(\tau_t|\mathcal{F}_{t-1}) + \beta_{3,\alpha}|\tau_t| \tag{3}$$

where τ_t and x_t represent RBC and RAI, and $q_{\alpha(\tau_{t+1}|\mathcal{F}_t)}$ denotes RBC conditional quantile information $\mathcal{F}_t$ at given time t. The cross-quantilogram $\hat{p}_\alpha(k)$ is shown in the figures below, so as identify directional predictability from RAI to RBC. We take a range of α_1 into consideration, 0.05, 0.1, 0.2, 0.3, 0.5, 0.7, 0.8, 0.9 and 0.95 for RBC $q_1(\alpha_1)$ quantiles. We take a range of α_2 into consideration, 0.1, 0.5 and 0.9 for RAI $q_2(\alpha_2)$. Every graph in Figure 1 to Figure 3 shows bootstrap confidence intervals of 95% for no predictability on the basis of 1,000 bootstrapped replicates. A 60-day maximum lag is taken (k=60). Estimating critical values from limiting distribution requires nonparametric estimation in line with Politis and Romano's (1994) stationary bootstrap (SB), a block bootstrap technique containing random length blocks with the resample being stationary and conditional on the first sample.

Han et al.'s (2016) directional predictability approach has a number of advantages over GCD, the first being the ability to detect the duration, magnitude and spontaneous directional relationship between RBC and RAI. This information can be used by investors to make decisions on trading strategies. The second advantage is that the directional predictability test helps researchers choose arbitrary RBC and RAI quantiles instead of pre-set quantiles. Another advantage is that the bootstrap method enhances the use of large lags in the directional predictability test. The results of the directional predictability test are presented in Figures 1 to 3. Fig. 1(a) shows the case where the aversion index is in the lower quantile, i.e. q2(α2) for α2=0.1. Directional predictability can be detected to RBC from RAI using he sample cross-quantilogram

$\hat{p}_{\alpha}(k)$. Sample cross-quantilograms are denoted by bar graphs while the 95 percent bootstrap confidence intervals are depicted by lines. The result shows cases where risk aversion lies within the lower quantile, i.e. $q2(\alpha_2)$ where $\alpha_2=0.1$. For $\alpha_1=0.2$, the cross-quantilogram $\hat{p}_{\alpha}(k)$ is positive and becomes essential after week 2, implying that the probability of huge negative Bitcoin loss is minimal when both Bitcoin return and risk aversion are at their lowest levels. When $\alpha_1=0.7$, 0.8, 0.9 and 0.95, the cross-quantilogram $\hat{p}_{\alpha}(k)$ for most lags is negative and significant, showing that the likelihood of Bitcoin having a huge positive gain during a low risk aversion period is low, given that the Bitcoin return is relatively high.

[Insert Figure 1a about here]

Figure 1(b) shows cases where risk aversion lies within the middle quantile, i.e. q2 (α_2) where $\alpha_2=0.5$. The cross-quantilogram $\hat{p}_{\alpha}(k)$ for $\alpha 1=$ (0.1, 0.2, 0.3, and 0.5) is positive and significant for a long period of 80 days, provided that Bitcoin return is in a bear market or a normal time and risk aversion is normal. However, when Bitcoin return becomes relatively high ($\alpha 1=0.7$, 0.8, $\alpha 1=0.9$, $\alpha 1=0.95$) the cross-quantilogram is only positive and significant for 60 lags and turns negative thereafter. This finding means that when the risk aversion of investors is normal ($\alpha 2=0.5$), there is likely to be a gain for Bitcoin in the bear market but not necessary in the bull market.

[Insert Figure 1b about here]

Figure 1(c) shows cases where risk aversion lies within the high quantile i.e. $q2(\alpha 2)$ where $\alpha 2=0.9$. The cross-quantilogram $\hat{p}_{\alpha}(k)$ for $\alpha 1=$ (0.05, 0.1, 0.2, 0.3, and 0.5) is positive and significant, which implies risk aversion can provide positive predictability for Bitcoin return only when risk aversion is high and in a bear market. However, when the Bitcoin return becomes relatively high ($\alpha 1=0.8$, $\alpha 1=0.9$, $\alpha 1=0.95$) the cross-quantilogram becomes negative and significant, which suggests that when the risk aversion of investors is high ($\alpha 2=0.9$), there is likely to be a loss for Bitcoin during a bull market, and this is exactly what happens after 2018.

[Insert Figure 1c about here]

4. Conclusion

We have provided insight into the investment role of Bitcoin by showing the ability of the risk aversion index to predict Bitcoin returns in bull and bear markets. Interestingly, predictability is positive when the risk aversion is very low or at a medium level. However, the predictability becomes negative when both the risk aversion and Bitcoin returns are very high, suggesting that very high levels of risk aversion are likely to drive down Bitcoin returns at the time of a bull market. Overall, this result is not surprising and supports earlier findings (e.g., Bouri et al., 2017a, b) that Bitcoin can act as a safe-haven in case of panic in the global financial market (i.e., when the risk aversion is high). In fact, Bitcoin plays a key role in an alternative economy and its price formation depends upon unique non-financial and non-economic factors such as attractiveness, transaction anonymity, and computer-programming enthusiasm (Ciaian et al., 2016; Böhme et al., 2015; Kristoufek, 2015; Yelowitz and Wilson, 2015), making it detached from the global financial system. For further studies, it is necessary to consider whether risk aversion can generate correlation among the returns of otherwise unrelated assets.

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Tables

Table 1. Testing for CGQ

	1%	5%	10%	20%	30%	40%	50%	60%	70%	80%	90%	95%	99%
Bitcoin	0	0	0	0	0	0.001	0.627	0.325	0.048	0.002	0	0	0

Notes: We compute quantile forecasts by inverting the parametric conditional copula distribution. We use six copulas (Gaussian, Frank, Clayton, Clayton Survival, Gumbel and Gumbel Survival copulas). The check loss functions are compared to evaluate the predictive ability of different quantile forecasting using different copula models. The benchmark quantile forecasts are computed using the independent copula, so there is no GCQ. The bootstrap p-values for testing the null hypothesis show that none of the six copula models (which model GCQ) makes a better quantile forecast than the independent copula (which gives no GCQ). The small p values of the reality check indicate rejection of the null hypothesis, indicating that there exists a copula function to model GCQ that makes a better quantile forecast.

Figures

Figure 1. The sample cross quantilogram

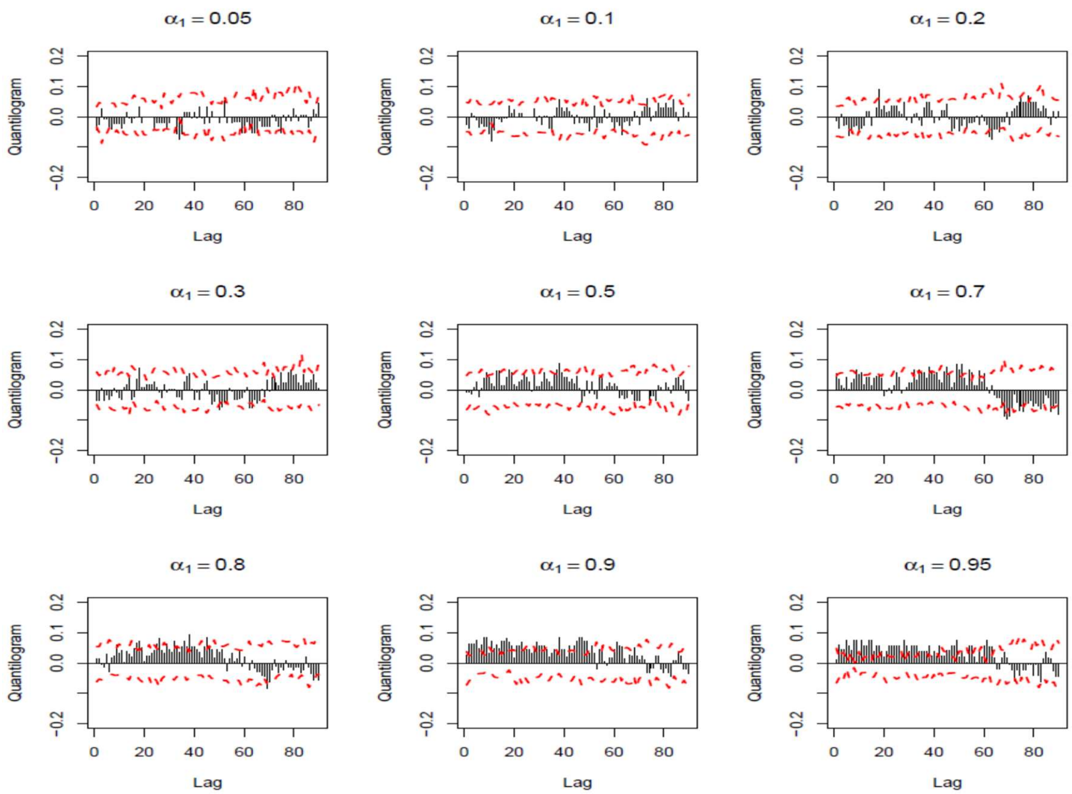


Fig. 1(a). The sample cross quantilogram $\hat{\rho}_{\alpha}(k)$ for α2=0.1 to detect directional predictability from RAI to RBC. Bar graphs describe sample cross quantilograms, and lines are the 95% bootstrap confidence intervals.

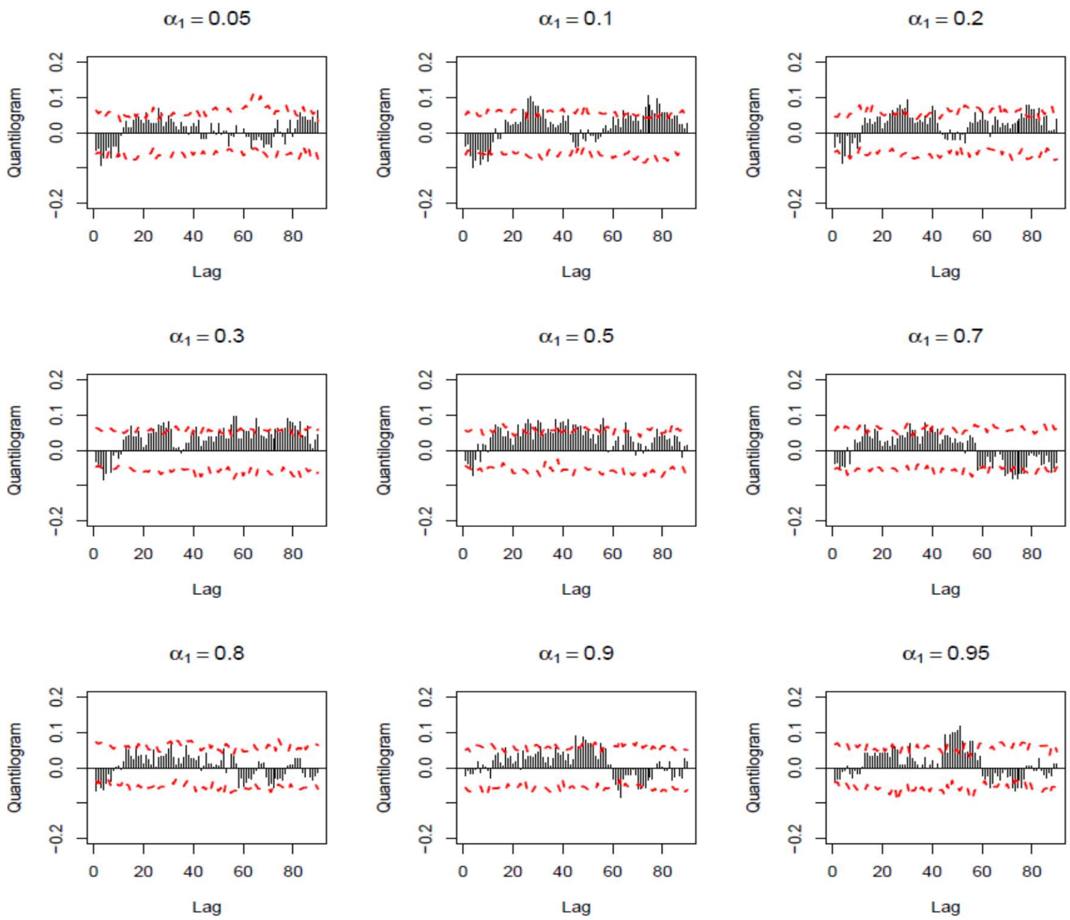


Fig. 1(b). The sample cross quantilogram $\hat{\rho}_{\alpha}(k)$ for α2=0.5 to detect directional predictability from RAI to RBC. Bar graphs describe sample cross quantilograms, and lines are the 95% bootstrap confidence intervals.

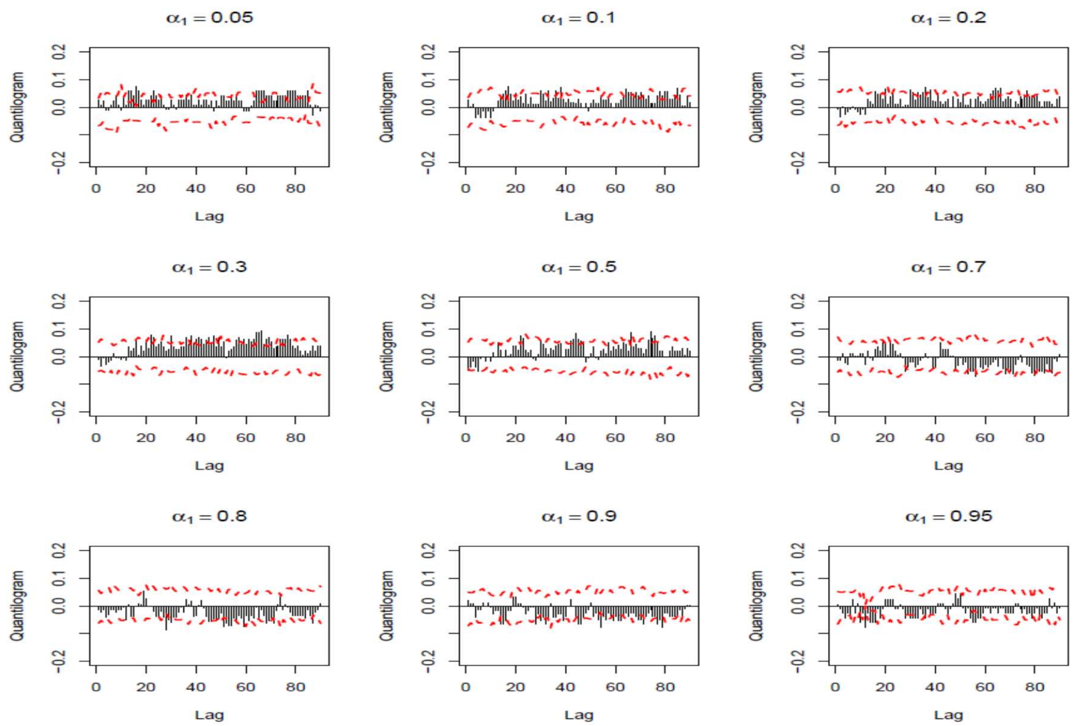


Fig. 1(c). The sample cross quantilogram $\hat{\rho}_{\alpha}(k)$ for α2=0.9 to detect directional predictability from RAI to RBC. Bar graphs describe sample cross quantilograms, and lines are the 95% bootstrap confidence intervals.

APPENDIX

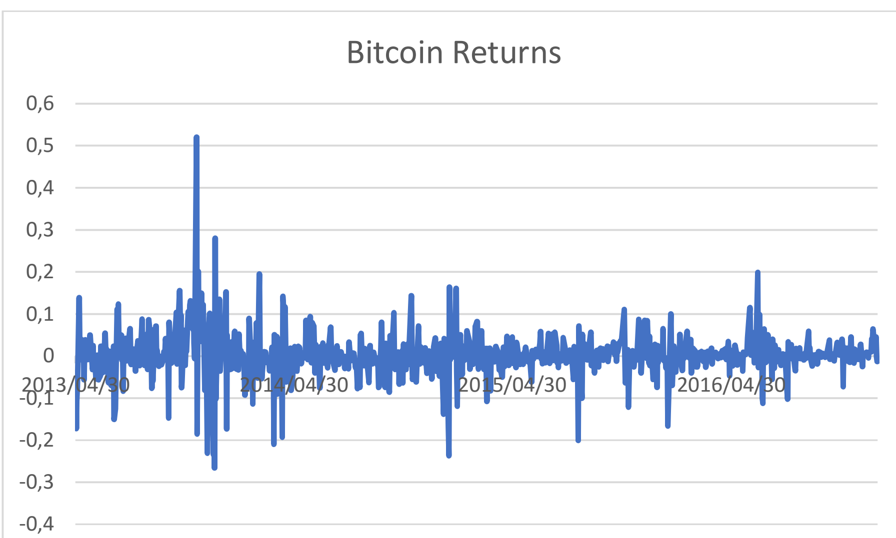
Appendix 1.

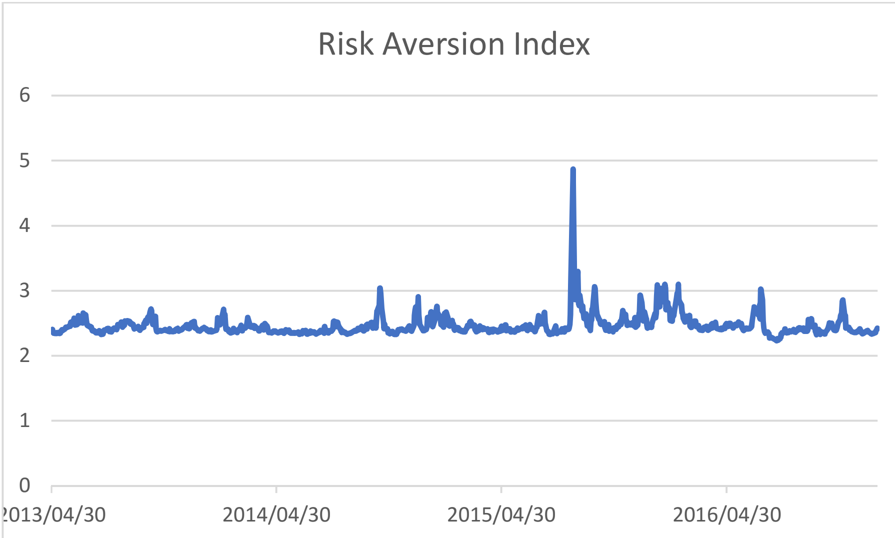
Table A1. Summary Statistics

	Bitcoin Returns	Risk Aversion Index
Mean	0,0021	24,669
Median	0,0011	24,212
Maximum	0,5208	48,708
Minimum	-0,2662	22,322
Std. Dev.	0,0511	0,1676
Skewness	0,7842	54,119
Kurtosis	192,565	594,517
Jarque-Bera	10213,6700***	126513,8000***
ADF Test	-31,5013***	-9,8573***
Observations	919	

Notes: This table provides summary statistics and results of the augmented Dickey-Fuller (ADF) test of stationarity. *** indicates the rejection of the null of normality and unit root under the Jarque-Bera test of normality and the ADF test of stationarity.

Figure A1. Data Plots





Appendix 2. Detailed Method of Lee and Yang (2014)

This method aims to forecast the conditional quantile $q_\alpha(Y_t|\mathcal{F}_t)$ in which α denotes the left tail probability. The conditional quantile $q_\alpha(Y_t|\mathcal{F}_t)$ can be calculated using a conditional distribution function's inverse function:

$$q_\alpha(Y_t|\mathcal{F}_t) = F_Y^{-1}(\alpha|\mathcal{F}_t \quad \text{(A1)}$$

where $F_Y(Y_t|\mathcal{F}_t)$ denotes the projected Y_t conditional distribution function whose inverse is to calculate $q_\alpha(Y_t|\mathcal{F}_t)$ from:

$$\int_{-\infty}^{q_\alpha(Y_t|\mathcal{F}_t)} f\gamma(y|\,\mathcal{F}_t)dy = \quad \text{(A2)}$$

where; $f\gamma(y|\mathcal{F}_t)$ denotes the projected conditional distribution function. One can compute quantile forecasting models $q_\alpha(Y_t|\mathcal{F}_t)$ by solving the equation:

$$C_u(F_X(x_{t+1}), F_Y\left(q_\alpha(Y_t|\mathcal{F}_t)\right) = \quad \text{(A3)}$$

Koenker and Bassett's (1978) check loss function is used to evaluate the quantile forecasting models $q_\alpha(Y_t|\mathcal{F}_t)$ predictive ability, derived from the 7 copula functions concerning C(u; v). The projected quantile forecast $q_\alpha(Y_t|\mathcal{F}_t)$ check loss at a specific α is:

$$Q(\alpha) = E[\alpha - I(Y_t - q_\alpha(Y_t|\mathcal{F}_t) < 0)](Y_t - q_\alpha(Y_t|\mathcal{F}_t) \qquad \text{(A4)}$$

The k'th copula function type is denoted by $C_k(u;v)$ (k =1,…,l =7). The resulting quantile forecast for every copula distribution function $C_k(u;v)$ is denoted by $q_{\alpha,k}(Y_t|\mathcal{F}_t)$. The expected check loss is denoted by $Q_k(\alpha)$. The following corresponding check loss-differentials are considered in order to compare copula model k (= 2,…,l) and model 1 (benchmark):

$$D_k = Q_1(\alpha) - Q_k(\alpha) \qquad \text{(A5)}$$

D_k can be estimated by:

$$\widehat{D}_{k,p} = \widehat{Q}_{1,p}(\alpha) - \widehat{Q}_{k,p}(\alpha) \qquad \text{(A6)}$$

where $\widehat{Q}_{k,p}(\alpha) = \frac{1}{p}\sum_{t=R}^{T-1}[\alpha - I(Y_t - q_\alpha(Y_t|\mathcal{F}_t) < 0)]\big(Y_t - q_\alpha(Y_t|\mathcal{F}_t)\big),\ k = 1, \dots, l$

Conditional quantile forecasts from the use of copula distribution function C_k (k = 2,…) that have the highest value $\widehat{D}_{k,p}$ are preferable. P is the size of the out-of-sample period for forecast evaluation.