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Abstract: We employ the nonlinear unit-root test recently developed by Omay et al. (2018), as well as other linear and nonlinear tests, to examine the stationarity of five multi-century historical U.K. series of real output compiled by the Bank of England (Thomas and Dimsdale, 2017). Three series span 1270 to 2016 and two series span 1700 to 2016. These datasets represent the longest span of historical real output data available and, thus, provide the environment for which unit-root tests are most powerful. A key feature of the Omay et al. (2018) test is its simulataneous allowance for two types of nonlinearity: time-dependent (structural breaks) nonlinearity and state-dependent (asymmetric adjustment) nonlinearity. The key finding of the test, contrary to what other more popular nonlinear unit-root tests suggest, provides strong evidence that the main structure of the five series is stationary with a sharp trend break and an asymmetric nonlinear adjustment. This finding is highly significant from the perspective of current macroeconomic debate because it refutes, for the historical U.K. series at least, the most stylized fact that real output follows a non-stationary process.

Keywords: Unit Root; Structural Break; Smooth Transition; Fourier Approximation; State-Dependent Nonlinearity

JEL Classification: C12, C22, O05

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1. Introduction

In their seminal paper, Nelson and Plosser (1982) claimed that U.S. real GDP contains a unit root, that is, stochastic trend dominates its movements. This finding has important macroeconomic implications, as it proves inconsistent with the traditional view of the business cycle. Most importantly, it suggests that real factors such as technology shocks play an important role in economic fluctuations, supporting the hypotheses of the real business cycle theory (Christopoulos, 2006).

Prior to Nelson and Plosser (1982), the prevailing view argued that real GDP exhibited a stationary process around a deterministic trend (Barro, 1976; Blanchard, 1981; Kydland and Prescott, 1980). Distinct practical differences exist between trend-stationary and unit-root processes. First, the deterministic trend provides the optimal forecast for a trend stationary process, while the current value provides the optimal forecast for a unit-root process. Second, a finite zone bounds the MSE of a trend stationary forecast whereas the MSE of a unit-root forecast grows linearly and, thus, becomes less precise the longer the forecast horizon. Third, the effect of a shock to a trend stationary process will eventually disappear, or put differently, a trend stationary process exhibits only a limited memory of its past behavior, whereas the effect of a shock on a unit-root process does not decay over time, implying a permanent effect.

Almost four decades since Nelson and Plosser (1982), the question of deterministic versus stochastic trend in real GDP remains unresolved. Following Nelson and Plosser (1982), a large body of empirical work failed to reject the hypothesis of unit root for real GNP, leading Stock and Watson (1999) to conclude that the unit-root econometric literature supports the contention of Nelson and Plosser (1982). Empirical studies by Wasserfallen (1986), Perron and Phillips (1987), Campbell and Mankiw (1987), Evans (1989), Papell and Prodan (2004), Ben-David and Papell (1995), Cheung and Chinn (1996), and most recently, Murray and Nelson (2000), among many others, reach the conclusion that U.S. real GDP is nonstationary. That is, no evidence exists that the economy self-corrects, in the sense that output never returns to its previous trend. Walton (1988) reaches a similar conclusion for the United Kingdom. Moreover, Kormendi and Meguire (1990), Cogley (1990), Fleissing and Strauss (1999), and Rapach (2002) provide international evidence supporting the null of a unit root in real GDP for OECD economies.

These results, however, are far from conclusive. Other papers since Stock and Watson (1999), using several modifications and extensions, reject the unit-root hypothesis. Ben-David et al. (2003), Papell and Prodan (2004), Vougas (2007), Beechey and Österholm (2008), Cook (2008), and Shelley and Wallace (2011) find empirical evidence to reject the unit-root hypothesis in real GDP.

The power of the tests lies at the heart of the issue. The power of the standard unit-root tests depends on the specification of the alternative hypothesis. Structural breaks and nonlinearities cause undersized standard unit-root tests, resulting in a reduction of statistical power (Habimana et al., 2018). Perron (1989) first notes that stationary processes with structural breaks are too often mistakenly interpreted as a unit-root processes. Perron (1989) suggests that standard unit-root tests such as the standard ADF test probably cannot distinguish the behavior of a unit-root process from that of a stationary process with structural breaks. Kapetanios et al. (2003), in turn, maintain that the standard unit-root tests suffer from a power problem when applied to data characterized by a nonlinear DGP.

A growing literature, starting with Enders and Granger (1998), relaxes the assumption of linearity implicit in the standard unit-root tests and develops tests that can distinguish linear nonstationary processes from nonlinear stationary processes. These tests examine the unit-root hypothesis against the alternative of a nonlinear stationary process. In this context, the literature analyzes two sources of nonlinearity: state-dependent (regime-wise) nonlinearity (i.e., nonlinearity in the speed of mean reversion) and time-dependent (structural breaks) nonlinearity (i.e., nonlinearity in the deterministic components). Kapetanios et al. (2003) and Sollis (2009) implement state-dependent nonlinear tests. The two tests differ on the dynamics of the speed of adjustment towards equilibrium. Kapetanios et al. (2003) employ the exponential smooth transition autoregressive (ESTAR) model while Sollis (2009) employs the asymmetric exponential smooth transition autoregressive (AESTAR) model. Leybourne et al. (1998), Omay (2015), and Çorakcı et al. (2017) develop nonlinear structural break unit-root tests. Leybourne et al. (1998) consider a single permanent sharp break; Omay (2015) considers multiple smooth breaks; and Çorakcı et al. (2017) consider a single temporary break. Christopoulos and Leon-Ledesma (2010), Omay and Yıldırım (2014), and Omay et al. (2018) implement tests that incorporate both structural break(s) and state-dependent nonlinearity simultaneously. The Omay et al. (2018) test provides the most comprehensive nonlinear unit-root test as it combines the time-dependent nonlinearity of the unit-root test of Leybourne et al. (1998) with the state-dependent nonlinearity of the unit-root test of Sollis (2009). Thus, while the Christopulous and Leon-Ledesma (2010) and Omay and Yildirim (2014) tests impose a symmetric (ESTAR) nonlinear adjustment, the Omay et al. (2018) test allows for asymmetric (AESTAR) nonlinear adjustment. The Omay et al. (2018) test considers two alternative specifications of the trend function, the logistic transition function, which models a single smooth break, and the Fourier series (Becker et al., 2004; Becker et al., 2006; Enders and Lee, 2012; Rodrigues and Taylor, 2012), which models multiple smooth breaks. In contrast, the Omay and Yildirim (2014) test can deal only with a single smooth break, and the Christopulous and Leon-Ledesma (2010) test can deal only with multiple smooth breaks.

We employ the Omay et al. (2018) test, as well as a battery of other linear and nonlinear tests, to investigate the stationarity properties of five historical U.K. real output series that the Bank of England recentlty compiled in the database *A Millennium of Macroeconomic Data* maintained at https://www.bankofengland.co.uk/statistics/research-datasets. Three series span

1270 to 2016, and two series span 1700 to 2016. These datasets represent the largest span of historical real output data available and, thus, provide the environment for which unit-root tests are most powerful.

The main result of the Omay et al. (2018) tests are strong and powerful. The tests reject the unit-root hypothesis in each of the five historical U.K. real output series and provides strong evidence that the main structure of the data is stationary with a sharp trend break and an asymmetric nonlinear adjustment. Although we view the findings of the Omay et al. (2018) tests as our main results, for completeness, we also consider several other linear and nonlinear unit-root tests that are popular in the econometric literature. They include two standard linear unit-root tests (the augmented Dickey-Fuller (ADF 1979) and Ng and Perron (2001) tests), and seven nonlinear unit-root tests (the Leybourne et al. (1998), Çorakcı et al. (2017), Kapetanios et al. (2003), Sollis (2009), Omay and Yildirim (2014), Christopoulos and Leon-Ledesma (2010), and Omay (2015) tests). With the exception of Omay (2015), the other nonlinear tests nest in the Omay et al. (2018) tests.

The rest of the paper is organized as follows. Section 2 provides a brief outline of the two versions of Omay et al. (2018) unit-root test. Section 3 presents the findings of the Omay et al. (2018) unit-root tests and the results of nine other tests. Section 4 comments and concludes.

2. The Omay et al. (2018) unit-root test

The Omay et al. (2018) tests are the newest and most comprehensive nonlinear unit-root tests. We offer the second application of the tests. Omay et al. (2018) first applied the procedure to test the purchasing power parity (PPP) hypothesis using both trade-weighted REER series (Bahmani-Oskooee, et al. 2007) and bilateral real exchange rates. The key findings of the tests suggest that the PPP holds in the majority of the countries in the sample, which details the importance of employing highly complex models in the analysis and tests of aggregate data.

The Omay et al. (2018) unit-root tests consider two alternative specifications of the trend function, the logistic transition function and the Fourier function. The former can only model a single smooth break, while the latter accommodates multiple smooth breaks.

The first specification of the test combines the time-dependent (time-varying) nonlinearity of Leybourne et al. (1998) and the state-dependent (regime-wise) nonlinearity of the AESTAR model of Sollis (2009). Omay et al. (2018) utilize the following equation for modeling the deterministic and stochastic components of an observed time series y_t:

$$y_t = \phi(t) + u_t, \tag{1}$$

where $\phi(t)$ is the deterministic nonlinear trend function and u_t is the stochastic deviation from the trend. Omay et al. (2018) consider two approaches to model the nonlinear deterministic trend function of Eq. (1). The first version of the test uses the logistic smooth transition function under three alternative models:

$$\text{Model A: } y_t = \alpha_1 + \alpha_2 S_t(\gamma, \tau) + \varepsilon_t, \tag{2a}$$

$$\text{Model B: } y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + \varepsilon_t, \text{and} \tag{2b}$$

$$\text{Model C: } y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t(\gamma, \tau) + \beta_2 S_t(\gamma, \tau) t + \varepsilon_t, \tag{2c}$$

where t=1,2,....,T, ε_t is a zero mean process, and $S_t(\gamma, \tau)$ is the logistic smooth transition function with a sample size of T. That is,

$$S_t(\gamma, \tau) = [1 + exp\{-\gamma(t - \tau T)\}]^{-1}. \tag{3}$$

In this framework, a smooth transition process between different regimes governs structural change as in Leybourne et al. (1998), rather than an instantaneous structural break as in Lumdaine and Papell (1997) and Lee and Strazicich (2003). This reflects the now prevailing view in the current literature that the cyclical behavior of real GDP is best represented by a nonlinear model rather than a linear model with structural breaks (Beechey and Österholm, 2008). That is, real GDP movements between peaks and troughs occur gradually and not instantaneously. The model in Eqs. (1)-(3) captures a regime-switching model with two regimes associated with the extreme values of the transition function $S_t(\gamma,\tau)=0$ and $S_t(\gamma,\tau)$=1, where the the transition from one regime to the other occurs gradually. $S_t(\gamma,\tau)$ is a continuous function, and the parameters γ and τ determine the smoothness or speed of transition and location between the two regimes, respectively. Since the value of $S_t(\gamma,\tau)$ depends on the value of the parameter γ, the transition between the two regimes occurs slowly for small values of γ whereas the transition between the regimes becomes almost instantaneous at time $t=\tau T$ for large values of γ. When $\gamma=0$, then $S_t(\gamma,\tau)=0.5$ for all values of *t*. Therefore, in Model A [Eq. (2a)], y_t is stationary around a mean that changes from α_1 to $\alpha_1+\alpha_2$. Model B [Eq. (2b)] allows for a fixed slope term β_1, whereas the intercept term changes from α_1 to $\alpha_1+\alpha_2$. Model C [Eq. (2c)] allows, in addition to the similar changes in the intercept, the slope changes from β_1 to $\beta_1+\beta_2$ at the same time. See, for further details, Leybourne, et al. (1998). The logistic smooth transition function given in Eq. (3), however, can capture only one gradual structural break.

The second specification of the test utilizes the Fourier series (Enders and Lee, 2012; Omay, 2015) to approximate multiple smooth structural breaks:

$$\phi(t)=\alpha_0+\delta t+\sum_{k=1}^{n}a_k\sin(\tfrac{2\pi kt}{T})+\sum_{k=1}^{n}b_k\cos(\tfrac{2\pi kt}{T})+u_t, \quad (4)$$

where $n\leq T/2$ represents the number of frequencies, k is the selected frequency in the approximation process, and a_i and b_i are the measurements for the amplitude and displacement of the sinusodial components of the function. As stated in Omay et al. (2018), the Fourier series with an appropriate lag order in most cases can approximate any function with unknown numbers of breaks of unknown forms. Under the assumption of $a_i=b_i=0$ for all *i*, the Fourier function becomes a linear model without a structural break. As a result, rejecting the null of $a_i=b_i=0$ implies a structural break in the series. If Eq. (4) allows for a structural break, the minimum frequency component must equal at least one.

To model the stochastic component, Omay et al. (2018) utilize the asymmetric exponential smooth transition autoregressive (AESTAR) model of Sollis (2009), which captures the nonlinear asymmetric adjustment process toward equilibrium. The AESTAR model considers both a logistic function and an exponential function as follows:

$$\Delta u_t = G_t(\theta_1, u_{t-1})\{F_t(\theta_2, u_{t-1})\rho_1 + \left(1 - F_t(\theta_2, u_{t-1})\right)\rho_2\}u_{t-1} + \epsilon_t, \quad (5)$$

$$G_t(\theta_1, u_{t-1}) = 1 - \exp\left(-\theta_1(u_{t-1}^2)\right), \qquad \theta_1 > 0, \text{ and} \quad (6)$$

$$F_t(\theta_2, u_{t-1}) = \left[1 + \exp\left(-\theta_2(u_{t-1})\right)\ \right]^{-1}, \qquad \theta_2 > 0, \quad (7)$$

where $\epsilon_t \sim iid(0, \sigma^2)$. $F_t(\theta_2, u_{t-1})$ is the logistic transition function for two regimes, determined by the positive and negative deviations from the equilibrium of u_t (i.e., the sign of disequilibrium). $G_t(\theta_1, u_{t-1})$ is the U-shaped symmetric exponential transition function, defined over the range from 0 and 1, determined by the small and large deviations from the equilibrium in absolute terms.

The AESTAR function implies a globally stationary process, which requires $\theta_1 > 0$, $\rho_1 < 0$, and $\rho_2 < 0$ as stated in Sollis (2009). If $\rho_1 \neq \rho_2$, the adjustment process captures not only sign but also size adjustment to the equilibrium. On the other hand, if $\rho_1 = \rho_2$, the adjustment to the equilibrium becomes a symmetric exponential smooth transition autoregressive (ESTAR) process.

We can test the null hypothesis of a linear unit root against the alternative hypothesis of a globally stationary AESTAR process. The hypotheses are as follows:

$$H_0 = \theta_1 = 0 \quad \text{(unit root)}, \quad (8)$$

$$H_1 = \theta_1 > 0. \quad (9)$$

Testing the null hypothesis proves problematic, since ρ_1, ρ_2, and θ_2 are unidentified nuisance parameters under the null. To overcome this problem, Sollis (2009) applies a first-order Taylor expansion and derives the following auxiliary equation:

$$\Delta u_t = \varphi_1 u_{t-1}^3 + \varphi_2 u_{t-1}^4 + \omega_t. \quad (10)$$

Under Eq. (10), the null hypothesis in Eq. (8) becomes $H_0 : \varphi_1 = \varphi_2 = 0$. Eq. (5) assumes a serially uncorrelated error term. To allow for serial correlation, we augment the regression equation as follows:

$$\Delta u_t = G_t(\theta_1, u_{t-1})\{F_t(\theta_2, u_{t-1})\rho_1 + \left(1 - F_t(\theta_2, u_{t-1})\right)\rho_2\}u_{t-1}$$
$$+ \sum_{j=1}^{p} \delta_j \Delta u_{t-j} + \epsilon_t, \quad (11)$$

where $\epsilon_t \sim iid(0, \sigma^2)$. Therefore, we use the following auxiliary regression to test the null hypothesis $H_0: \varphi_1 = \varphi_2 = 0$:

$$\Delta u_t = \varphi_1 u_{t-1}^3 + \varphi_2 u_{t-1}^4 + \sum_{j=1}^{p} \delta_j \Delta u_{t-j} + \vartheta_t. \quad (12)$$

The testing procedure in Omay et al. (2018) consists of two steps (see, also, Kapitanios et al., 2003; Leybourne et al., 1998; Sollis, 2009). First, Omay et al. (2018) estimate (a) the Fourier model (by OLS) for the frequency k over the range $1 \leq k \leq k_{max}$ and obtain the optimal k that minimizes RSS through a grid search over the interval $1 \leq k \leq k_{max}$ or (b) the logistic model (by NLS). Second, using the residuals from (a) or (b), Omay et al (2018) estimate Eq. (12) (by OLS) and test the null hypothesis $H_0: \varphi_1 = \varphi_2 = 0$, using a conventional F-test. The F-test statistic is denoted by F_{LBAE}, if the logistic transition function is utilized to model a single gradual break, and by F_{FSAE}, if the Fourier function is employed to model multiple smooth breaks. Omay et al. (2018) obtain the critical values of F_{LBAE} and F_{FSAE} via stochastic simulation and show that the tests possess satisfactory size and power small-sample properties.

3. Empirical results

The dataset contains five historical annual output series compiled by the Bank of England. We use Version 3.1 of the dataset, updated to 2016. For detailed information about the historical sources of the data, see Thomas and Dimsdale (2017). The five series are defined as follows: (a) Series 1: Real U.K. GDP at market prices (1700-2016), geographically-consistent estimate based on post-1922 borders, million of British pounds, chained volume measure, 2013 prices; (b) Series 2: Real U.K. GDP at factor cost (1700-2016), geographically-consistent estimate based on post-1922 borders, million of British pounds, chained volume measure, 2013 prices; (c) Series 3: Real GDP of England at market prices (1270-2016), million of British pounds, chained volume measure, 2013 prices; (d) Series 4: Real GDP of England at factor cost (1270-2016), million of British pounds, chained volume measure, 2013 prices; and (e) Series 5: Composite Estimate of English and (geographically-consistent) U.K. real GDP at factor cost (1270-2016), 2013=100.

We report the constant and the constant and trend versions for the ADF, Kapitanios et al. (2003), Sollis (2009), Omay (2015), and the Fourier version of Omay et al. (2018). For economy of space, we only report the constant and trend version of the Ng and Perron (2001) tests. We also report Models A, B, and C for the Leybourne et al. (1998) test and the logistic version of Omay et al. (2018) test. Given the length of the data, however, we find more appropriate to emphasize Model C or the constant and trend versions.

As a preliminary step, we apply two sets of standard linear unit-root tests: The Augmented Dickey-Fuller (ADF 1979) test (constant, and constant and linear trend) and the four versions of the Ng and Perron (2001) test (constant and linear trend). The Ng and Perron (2001) procedure yields substantial power gains over the standard unit-root test. Significant modifications of existing unit-root tests improve their power and size. The MZa and MZt tests modify the Phillips (1987) and Phillips and Perron (1988) Za and Zt tests, respectively; the MSB test relates to the Bhargava (1986) R1 test; and the MPT test modifies the Elliott et al. (1996) point optimal test. Tables 1 and 2 report the results of applying these tests. We choose the proper lag length by the SIC criterion from a maximum of 12 lags. We cannot reject the null hypothesis of a unit root for any of the five series. As mentioned above, linear unit-root tests can suffer from power problems in the presence of nonlinearities in the data leading to a bias towards the non-rejection of the null hypothesis.

We next consider the results of two tests that allow for state-dependent nonlinearity. That is, the tests allow for symmetric nonlinearity (Kapetanios et al., 2003) and asymmetric nonlinearity (Sollis, 2009), but ignore the possibility of structural breaks. Tables 3 and 4 tabulate the results of the Kapetanios et al. (2003) unit-root tests of a symmetric ESTAR model and of the Sollis (2099) tests of an asymmetric ESTAR (AESTAR) model. The Kapetanios et al. (2003) test rejects the null of a unit root in only one series (Series 5), for the constant, and constant and trend cases. Conversely, the Sollis (2009) unit-root test rejects the null of unit root in four series (Series 2, 3, 4, and 5) in the constant case, and in three series (Series 3, 4, and 5) in the constant and trend case. This provides some evidence that asymmetric adjustment proves an important characteristic of state-dependent nonlinearity.[1]

Table 5 presents the Leybourne et al. (1998) unit-root test results. The test results show that allowing for a sharp permanent break causes a more frequent rejection of the null hypothesis than the Omay (2015) and Çorakcı et al. (2017) tests presented in Table 6 and 7. The Omay (2015) test models multiple smooth structural breaks using the fractional version of the Fourier function, while the Çorakcı et al. (2017) test models temporary structural breaks. The Omay (2015) and Çorakcı et al. (2017) tests never reject the null. In contrast, the Leybourne et al. (1998) unit-root test, and in particular Model C, rejects the unit-root hypothesis in four of the five series (Series 2, 3, 4, and 5). Weaker evidence of rejection appears for Model A, which rejects the null only in three of the five series (Series 3, 4, and 5), while Model B cannot reject the unit-root hypothesis in four of the five series (Series 1, 3, 4, and 5). These findings, however, provide some evidence that the structure of the historical series does not include a single temporary break or multiple smooth breaks.

[1] Following Camacho (2011), we also applied the regime-switching in the conditional mean of the unit root model with trend to the five series of real GDP under consideration, but the test could not reject the null of unit root in any of the cases. This result possibly highlights the importance of modelling breaks in not only the mean, but also the trend. Complete details of these results are available upon request from the authors.

Table 8 reports the Omay and Yildirim (2014) test results. The test statistics for Model A reject the null of a unit root at the 1-percent level in Series 1, 2, 4, 5 and at the 5-percent level in Series 3. The test statistics for Model C reject the null of unit root at the 1-percent level in all five series. These results provide some evidence that favours a permanent structural break and of nonlinear symmetric adjustment for the five historical real output series. Some ambivalence exists, however, since the results of Model B indicate failure to reject the null for all series.

Table 9 reports the Christopoulos and Leon-Ledesma (2010) test results. The test statistics for the constant, and constant and trend cannot reject the null of a unit root for all five series. This suggests that a model with multiple smooth breaks and symmetric adjustment does not adequately support the hypothesis of stationarity. The critival values for the constant only test statistic come from Christopoulos and Leon-Ledesma (Table 2). The critical values for the constant and trend test statistic are not available from Christopoulos and Leon Ledesma (2010) and are obtained by our Monte Carlo simulation for T=500 and k = 1.[2]

Finally, Tables 10 and 11 report the results of two versions of the Omay et al. (2018) unit-root tests, the logistic function version with single permanent break and the Fourier function version with multiple smooth breaks. Table 10 reports the F_{LBAE} of the logistic transition function for Model A, B, and C. The test results of single break version of the test are quite similar to those in Model A and Model C of Omay and Yıldırım (2014), reiterating the relevance of the single permanent structural break over the multiple smooth breaks. Table 11 reports the F_{FSAE} test statistic for the constant, and constant and trend. We find that the logistic version of the Omay et al. (2018) unit-root test is also preferred to the Fourier version, which

[2] We also computed the critical values for T = 2500. We report them for additional information. They are, respectively, -4.42, -3.86, and -3.56 for the 1%, 5%, and 10% significance level, respectively.

allows for the possibility of multiple smooth breaks, since the logistic version rejects the null in all five series compared to the rejection of only three series in the Fourier function case.

Figures 1-5 present in panels (a) and (b) the historical GDP series along with estimated nonlinear trend functions and the corresponding detrended data. Visual inspection of the series reveals the importance of taking account of gradual breaks when analysing these series.

4. Conclusions

We employ the nonlinear unit-root test recently developed by Omay et al. (2018), as well as a battery of linear and nonlinear tests, to examine the stationarity of five multi-century historical U.K. series of real output compiled by the Bank of England (Thomas and Dimsdale, 2017). Three series span 1270 to 2016 and two series span 1700 to 2016. These datasets represent the longest span of historical real output data available and, thus, provide the environment for which unit-root tests are most powerful.

Linear unit-root tests, such as the ADF and the Ng and Perron (2001) tests, systematically fail to reject the unit root in all five historical real output series.

Nonlinear unit-root tests exhibit mixed success. Time-dependent tests, such as Leybourne et al. (1998), which impose on the structure of the data one permanent sharp break, reject the unit-root hypothesis in four of the five real output series (Series 2, 3, 4, and 5). Oddly, state-dependent tests, such as Sollis (2009), which impose asymmetric adjustment, reject the null of unit root also in four of the five series (Series 2, 3, 4, and 5). This shows that time-dependent nonlinearity in the form of a single structural break, and state-dependent nonlinearity in the form of asymmetric adjustment can imitate each other. That is, a sharp break in trend and intercept can also be modelled by an AESTAR type nonlinearity. In contrast, time-dependent tests such as Omay (2015) and Çorakcı et al. (2017), which impose multiple smooth breaks and one temporary break, respectively, consistently fail to reject the unit-root hypothesis. The results of the Christpoulos and Leon-Ledesma (2010) tests fail to reject the unit-root hypothesis, confirming that the structure of the series is not inclusive of multiple smooth structural breaks. State-dependent tests with symmetric adjustment, such as Kapetanios et al. (2003), also fail to reject the null of a unit root. Thus, the above-mentioned tests are capable, on their own, of delivering some bits and pieces of empirical information about the structure of the five historical series. Applied to the first series, however, none of these tests provides evidence of stationarity.

In contrast, the key findings of the Omay et al. (2018) unit-root test, Model C, provides strong evidence that the main structure of all the five series is stationary with a sharp trend break and an asymmetric nonlinear adjustment. This finding is highly significant from the perspective of current macroeconomic debate because it refutes, for the historical U.K. series at least, the most stylized fact that real output follows a non-stationary process. This result is highly at odds with the much more popular nonlinear tests that consider only one facet of the nonlinear process, such as the Kapetanios et al. (2003) unit-root test that allows for state-dependent nonlinearity, but ignores structural breaks, or the Christopoulos and Leon-Ledesma (2010) unit-root test that allows for multiple smooth breaks but ignore asymmetric adjustments.

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Table1. ADF unit-root test results

Output Series	Constant	Constant and trend
Series 1	7.791	5.348
Series 2	7.293	4.676
Series 3	11.426	10.927
Series 4	12.197	11.713
Series 5	11.740	11.231
Test critical values:		
1%	-3.438	-3.420
5%	-2.865	-2.910
10%	-2.568	-2.620

* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Table 2. Ng and Perron (2001) unit-root test results

Output Series	MZa	MZt	MSB	MPT
Series 1	2.891	1.957	0.677	136.294
Series 2	2.943	2.061	0.700	145.605
Series 3	-5.623	-1.099	0.195	15.221
Series 4	-6.181	-1.177	0.190	14.597
Series 5	-6.275	-1.196	0.191	14.498
Test critical values:				
1%	-23.800	-3.420	0.143	4.030
5%	-17.300	-2.910	0.168	5.480
10%	-14.200	-2.620	0.185	6.670

* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Table 3. Kapetanios, et al. (2003) unit-root test results

Output Series	Constant	Constant and trend
Series 1	-0.900	-1.513
Series 2	-0.747	-1.379
Series 3	-1.184	-1.758
Series 4	-0.764	-1.306
Series 5	-5.868***	-8.628***
Test critical values:		
1%	-3.480	-3.970
5%	-2.930	-3.400
10%	-2.660	-3.130

* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Table 4. Sollis (2009) unit-root test results

Output series	Constant	Constant and trend
Series 1	1.638	2.412
Series 2	8.292***	2.140
Series 3	29.473***	20.859***
Series 4	25.062***	15.647***
Series 5	17.421***	50.774***
Test critical values:		
1%	6.883	8.531
5%	4.954	6.463
10%	4.157	5.460

* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Table 5. Leybourne et al. (1998) unit-root test results

Output Series	Model A	Model B	Model C
Series 1	-3.023	-3.447	-3.447
Series 2	-3.186	-5.882***	-5.882***
Series 3	-6.699***	-0.579	-7.126***
Series 4	-7.320***	-0.543	-7.697***
Series 5	-7.130***	-0.451	-7.468***
Test critical values:			
1%	-4.882	-5.479	-5.560
5%	-4.232	-4.771	-5.011
10%	-3.909	-4.427	-4.697

* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Table 6. Omay (2015) unit-root test results

Output Series	Constant	Constant and trend
Series 1	1.238	-1.912
Series 2	1.236	-1.945
Series 3	11.005	6.943
Series 4	11.365	7.985
Series 5	10.957	7.661
Test critical values:		
1%	-4.31	-4.94
5%	-3.67	-4.35
10%	-3.33	-4.05

* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Table 7. Çorakcı et al. (2017) unit-root test results

Output Series	Model A	Model B	Model C
Series 1	-1.006	-1.649	-1.687
Series 2	-0.932	-1.481	-2.056
Series 3	-0.164	-0.787	-0.876
Series 4	-0.010	-0.008	-0.698
Series 5	-1.504	-2.997	-1.954
Test critical values:			
1%	-5.017	-5.544	-5.797
5%	-4.374	-4.900	-5.166
10%	-4.051	-4.572	-4.844

* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Table 8. Results of Omay and Yıldırım (2014) unit-root tests

Output Series	Model A	Model B	Model C
Series 1	-4.152**	-3.865	-5.142***
Series 2	-4.506***	-3.701	-5.478***
Series 3	-5.051**	-2.716	-5.174***
Series 4	-5.027***	-2.332	-5.120***
Series 5	-23.873***	-1.540	-26.669***
Test critical values:			
1%	-4.443	-4.777	-5.041
5%	-3.821	-4.202	-4.411
10%	-3.509	-3.889	-4.090

* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Table 9. Results of Christopoulos and Leon-Ledesma (2010) unit-root tests

Output Series	Constant	Constant and trend
Series 1	3.560	-0.442
Series 2	3.712	-0.359
Series 3	6.693	4.045
Series 4	7.188	4.634
Series 5	7.113	6.322
Test critical values:		
1%	-4.41	-4.44
5%	-3.86	-3.86
10%	-3.54	-3.57

* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Table 10. Results of Omay et al. (2018) unit-root tests (logistic trend function version)

Output Series	Model A	Model B	Model C
Series 1	10.720**	9.377*	18.759***
Series 2	12.230***	9.651**	22.811***
Series 3	25.248***	52.249***	28.732***
Series 4	30.734***	9.756**	34.318***
Series 5	386.214***	42.540***	433.129***
Test critical values:			
1%	10.756	12.681	13.621
5%	8.110	9.642	10.617
10%	7.101	8.339	9.209

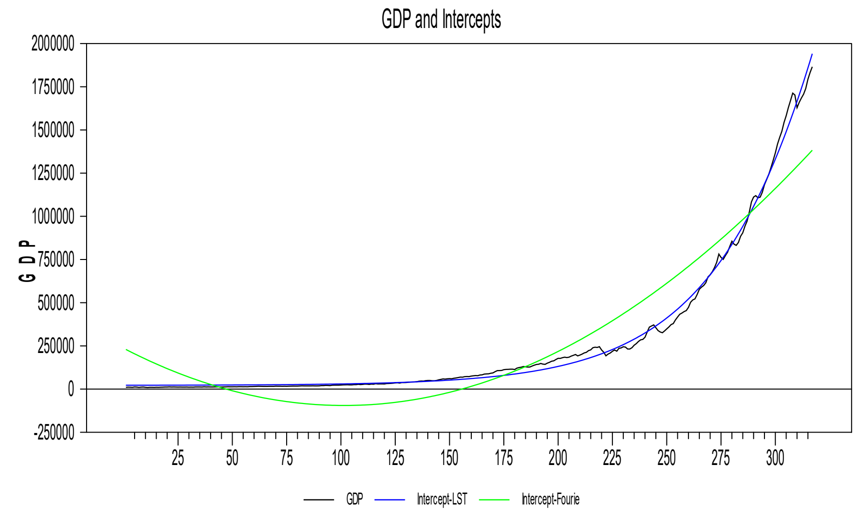
* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Table 11. Results of Omay, et al. (2018) unit-root test (Fourier trend function version)

Output Series	Constant	Constant and trend
Series 1	21.346***	4.867
Series 2	21.805***	4.726
Series 3	73.806***	40.620***
Series 4	70.126***	37.119***
Series 5	47.809***	35.947***
Test critical values:		
1%	8.68	10.61
5%	6.36	7.93
10%	5.31	6.75

* denotes 10% significance level; **denotes 5% significance level; *** denotes 1% significance level.

Figure 1. Series 1: Real U.K. GDP at market prices (1700-2016), geographically-consistent estimate based on post-1922 borders; million of British pounds, chained volume measure, 2013 prices



(a)

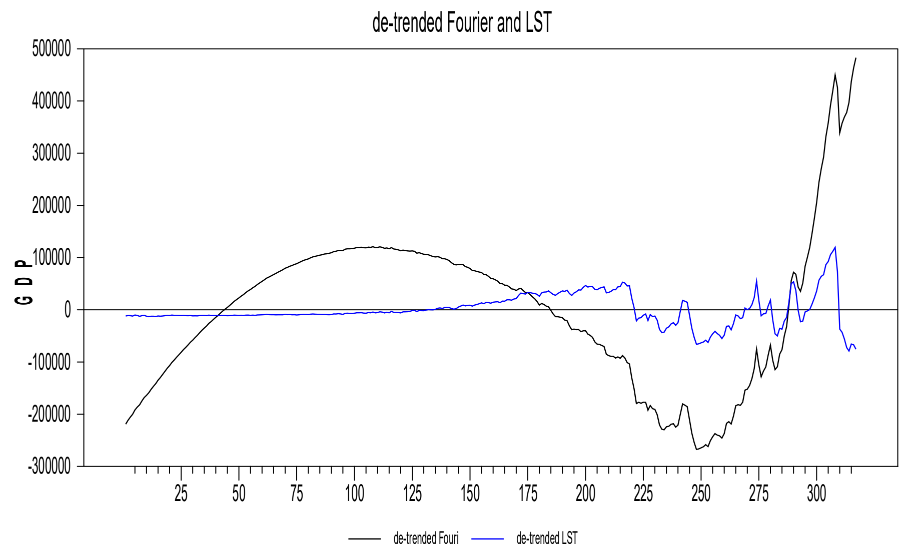


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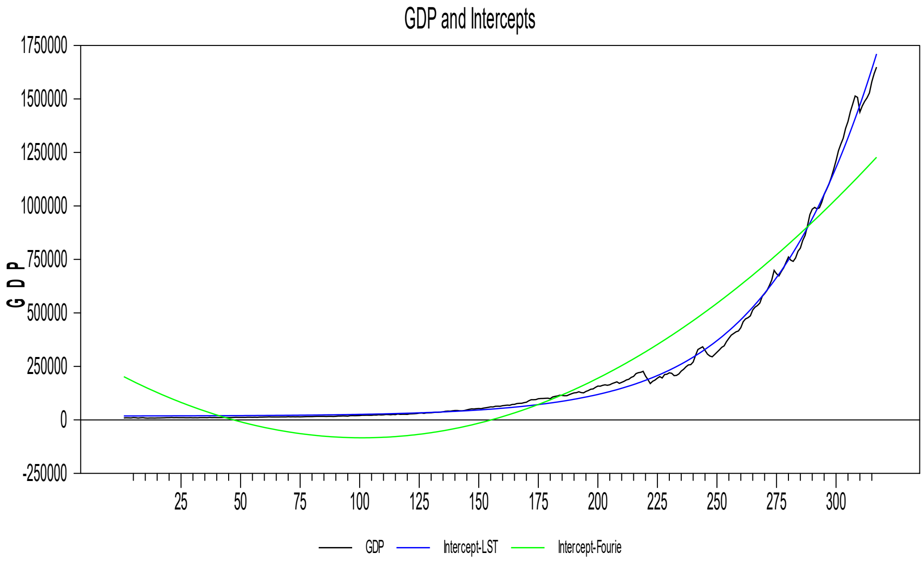
Figure 2. Series 2: Real U.K. GDP at factor cost (1700-2016), geographically-consistent estimate based on post-1922 borders; million of British pounds, chained volume measure, 2013 prices



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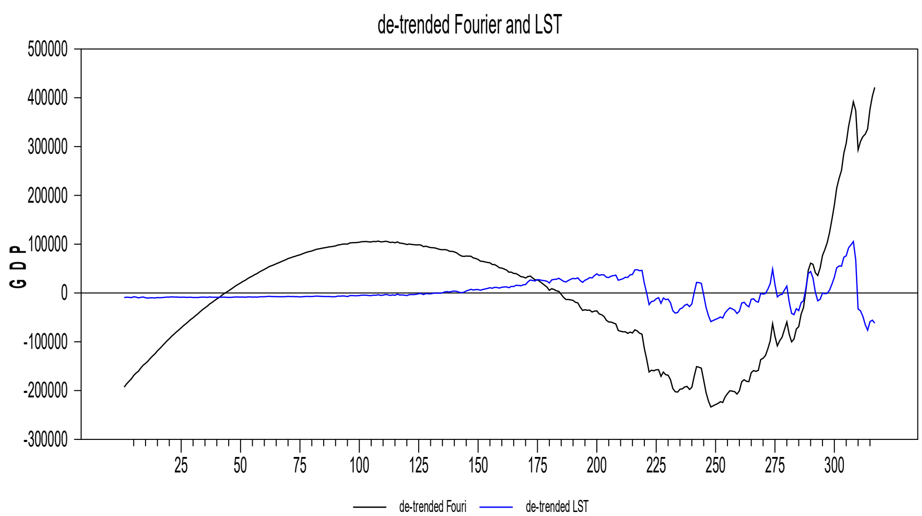

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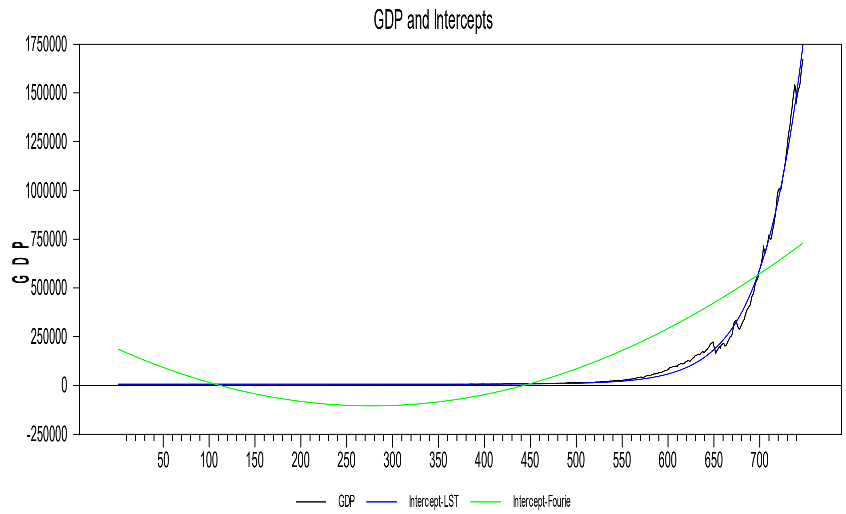
Figure 3. Series 3: Real GDP of England at market prices (1270-2016), million of British pounds, chained volume measure, 2013 prices.



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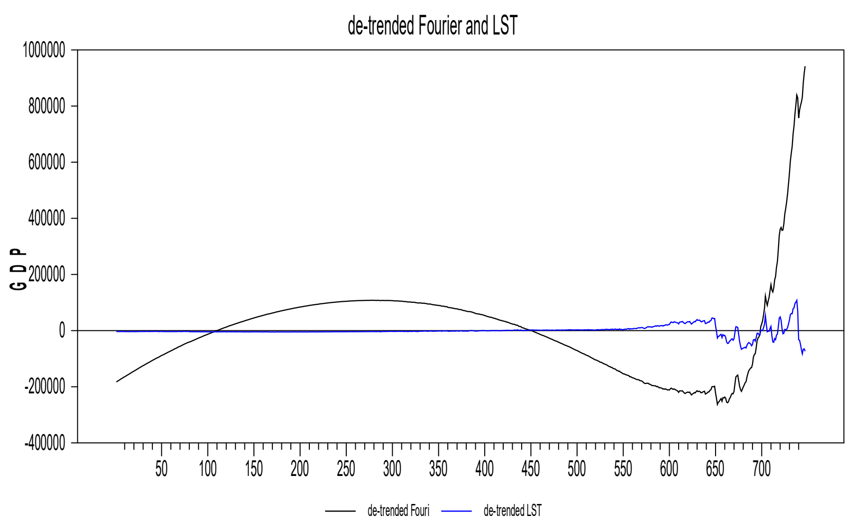

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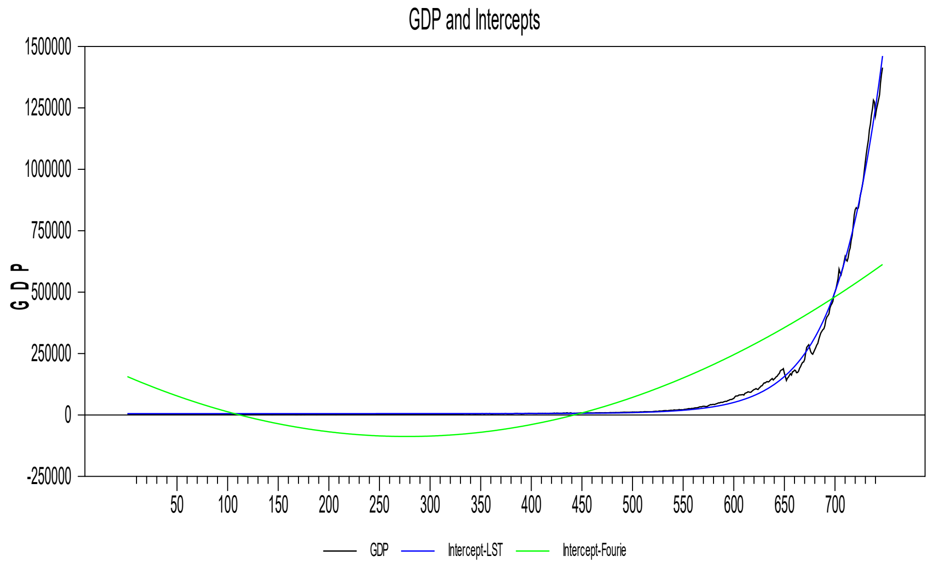
Figure 4 Series 4: Real GDP of England at factor cost (1270-2016), million of British pounds, chained volume measure, 2013 prices.



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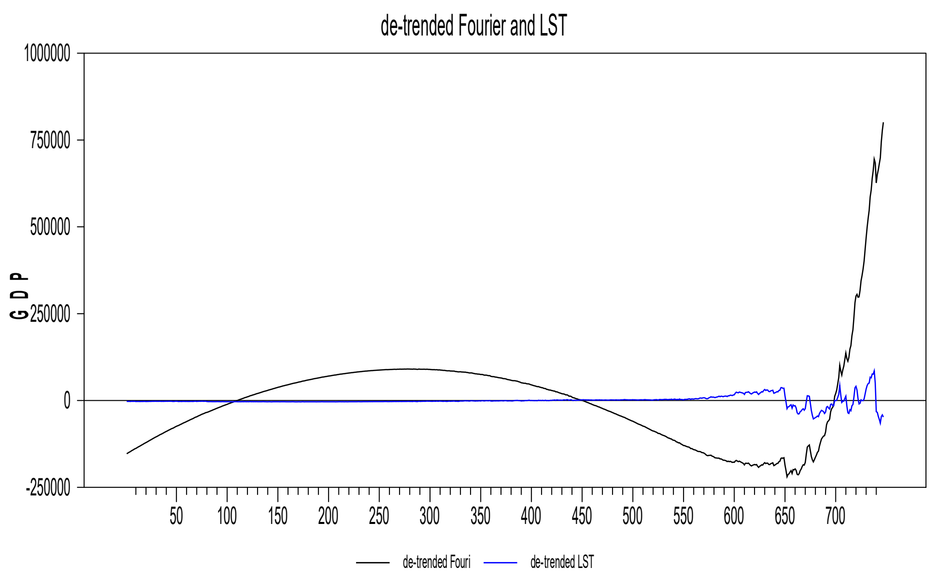

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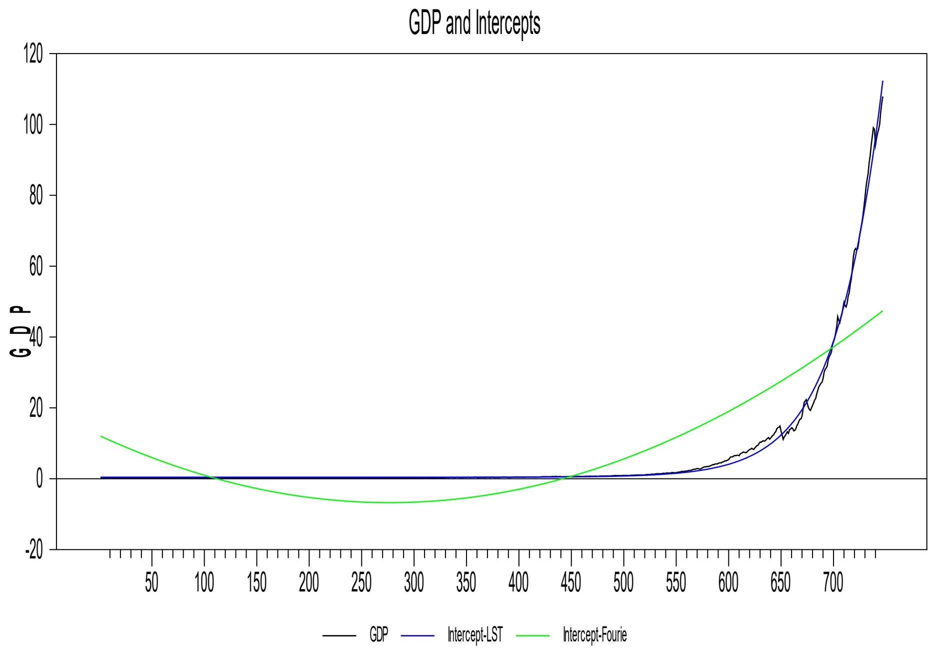
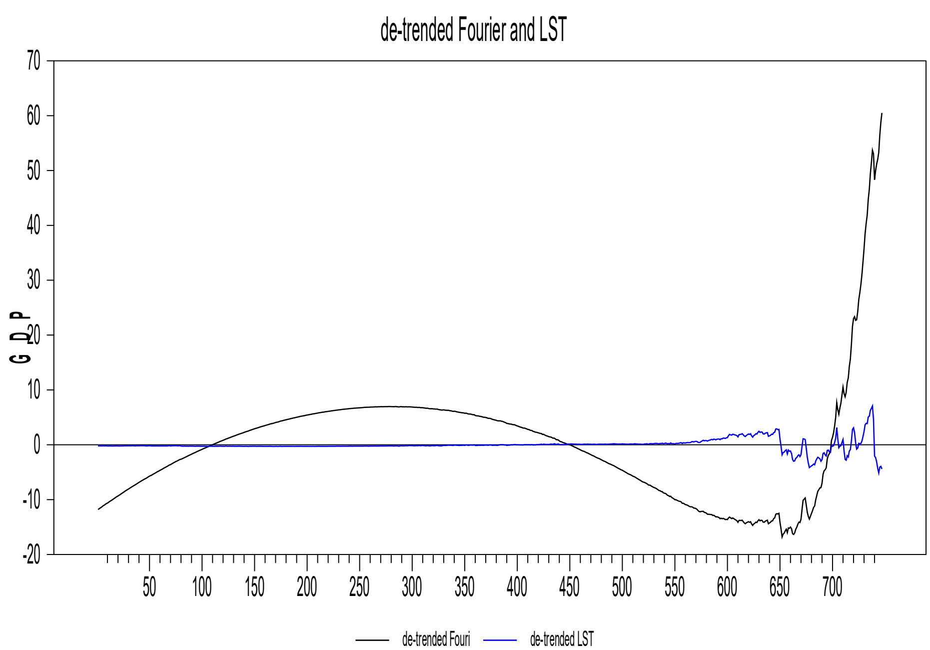
Figure 5. Series 5: Composite Estimate of English and (geographically-consistent) U.K. real GDP at factor cost (1270-2016), 2013=100.



(a)



(b)