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Forecasting the Term Structure of Interest Rates of the BRICS: Evidence from a Nonparametric Functional Data Analysis

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Abstract

In this paper, we develop a non-parametric functional data analysis (NP-FDA) model to forecast the term-structure of Brazil, Russia, India, China and South Africa (BRICS). We use daily data over the period of January 1, 2010 to December 31, 2016. We find that, while it is in general difficult to beat the random-walk model in the shorter-horizons, at longer-runs our proposed NP-FDA approach outperforms not only the random-walk model, but also other popular competitors used in term-structure forecasting literature. Our results have important implications for both policymakers aiming to stabilize the economy, and for optimal portfolio allocation decisions of financial market agents.

Keywords: Functional data analysis, yield curve forecasting, performance evaluation, BRICS

JEL Codes: C53, E43, G17

1. Introduction

There exists a large number of studies that has highlighted the role played by the term-structure of interest rates as a leading indicator of economic recessions and inflation for both developed and emerging economies (see, Plakandaras *et al.* , 2017b,a; Gupta *et al.* , 2018; Aye *et al.* , 2019; Plakandaras *et al.* , 2019, for detailed reviews in this regard). At the same time, movements in the term-structure of interest rates serve as a valuable input for practioners in finance to carry out bond portfolio management, derivatives pricing, and risk management (Caldeira *et al.* , 2016a). Hence, accurate forecasting of the term-structure of a yield curve is of paramount importance to both policy-makers and financial market agents in general, and have understandably, resulted in a large literature (see, Caldeira *et al.* , 2016b; Byrne *et al.* , 2017; Caldeira *et al.* , 2018, for detailed reviews) .

Although forecasting of the term-structure has received considerable attention in most developed countries, and in particular the United States (US), there is dearth of predictive analysis conducted for emerging countries, barring the works of Luo *et al.* (2012), Vieira *et al.* (2017), Feng & Qian (2018), Shang & Zheng (2018), and Shu & Lo (2018) for

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Brazil, China and South Africa. Against this backdrop, the objective of this paper is to provide a comprehenisive forecasting exercise of the term structure of Brazil, Russia, India, China and South Africa (BRICS). The decision to choose these emerging bloc of countries is motivated by the fact that the BRICS have grown rapidly and have become more integrated with the developed world in terms of trade and investment, with the group accounting for about a quarter of the world's Gross Domestic Product (GDP) (Balcilar *et al.* , 2018; Plakandaras *et al.* , 2018). Understandably, given the trade and financial dependence in the modern globalized world, possible slowdown of growth of the BRICS countries have important implications for the world economy. Hence, accurate prediction of the term-structure, which will in turn contain information about the future path of the GDP of the BRICS, and hence the global world, cannot be underestimated from the perspective of policy decisions. Moreover, post the recent global financial and European sovereign debt crises, emerging market bonds (just like equities) have become an integral component of portfolio decisions of agents in the world financial market searching for better returns (Sowmya *et al.* , 2016; Prasanna & Sowmya, 2017; Ahmad *et al.* , 2018; Stona & Caldeira, 2019). Again, in the process, making the precise forecast of the term-structure of interest rates important from the point of view of the global financial markets.

Much progress has been made in forecasting yields for the advanced economies, as well as emerging markets in the limited number of studies that does exist, based on parametric models following the works of Diebold & Li (2006) and Christensen & Rudebusch (2011). Parametric models are indeed very useful if the "strong" assumptions behind these models hold for the data that is being investigated. But this is not often the case when dealing with high-frequency data from the financial markets, with violations of the assumptions observed, in particular with those involving data generating processes (Doh, 2011; Feldhütter *et al.* , 2018, for detailed discussions). Given this, our aim is to compare, for the first time, a pure nonparametric functional model with parametric factor models and linear models widely considered in the term-structure literature, in forecasting the yield curve of the BRICS. In the process, following the work of Caldeira & Torrent (2017) on the United States (US), we interpret the yields of the BRICS countries as curves, since the term-structure of interest rates defines a relation between the yield of a bond and its maturity, i.e., we conduct a nonparametric functional data analysis (NP-FDA). More importantly, we extend the existing parametric models-based works on Brazil, China and South Africa to the entire BRICS bloc, using a non-parametric approach that is unlikely to be misspecified due to nonlinearity. In this regard, it must be pointed out that the only other study to have used a similar approach for the case of China is by Feng & Qian (2018), who in turn show that a functional principal component analysis model relative to popular alternatives in the out-of-sample forecasting.

Note that, unlike the majority of the existing studies on emerging (and developed) bond markets based on monthly data, we rely on daily data covering the period of January 1, 2010 to December 31, 2016. Further, we use information contained in the term-structure of the interest rates only, rather than macroeconomic variables. With the term-structure being a leading incicator for the macroeconomy, high-frequency (in our case, daily) forecasts are likely to be more valuable to policymakers to predict the future path of lower-frequency macroeconomic variables. In addition, if indeed the term-structure provides leading information for the macroeconomy, then ideally, one should be forecasting the bond yields based on their own (lagged equilibrium) information content, instead of relying on macroeconomic predictors. Hence, we believe our study is more reliable compared to existing work on term-structure forecasting of emerging markets, as we base our forecasts on nonparametric models with high frequency data without macroeconomic information.

The remainder of the paper is organized as follows: Section 2 outlines the basics of the functional data methodology and the associated nonparametric estimation, while Section 3 presents the alternative forecasting models. Section 4 discusses the data and results, with Section 5 concluding the paper.

2. Functional Data Methodology and Nonparametric Estimation

In this section, we present the functional data analysis methodology used in the paper and we explain how that methodology combines with the problem we are interested about.[1]. We start off by defining that a random variable $\mathbf{Y}$ is said to be a functional variable, if it takes values in an infinite dimensional space, say E. An observation of $\mathbf{Y}$ is said to be a functional data and it is denoted here by Y. In this paper, the yield curve is viewed as a function (curve) that links maturities to yields. More precisely, when $\mathbf{Y}$ (respectively Y) denotes a random curve (respectively observation), one would be identifying $\mathbf{Y} = \{\mathbf{y}(\tau); \tau \in M\}$ (respectively $Y = \{y(\tau); \tau \in M\}$), where $M \subset \mathbb{N}$ stands for the set of arbitrary maturities.

Nonparametric Functional Estimation. Now we describe the proposed estimator, and explain how it is used to forecast the US yield for a given maturity. Consider that a researcher is interested in forecasting a dependent scalar variable Y as a function of a functional regressor $\mathbf{Y}$. Let $r(Y) = E(y|\mathbf{Y} = Y)$ be the nonlinear regression operator, where $\mathbf{Y}$ is a functional variable taking values in a semi-metric space (E, d). Suppose y is a real random variable and Y is a fixed element in E. Specific to our problem, the regressand is the yield at some specific maturity, while the regressor is the yield curve. Now, let $(\mathbf{Y}_t)_{t=1,\ldots,T}$ be a sample of T random curves. The kernel estimator (KER) for

[1]For further details about the method see Ferraty & Vieu (2006) and Ramsay & Silverman (2005). And for a more complete explanation on how to adapt this methodology to yield curve forecasting, the reader is referred to Caldeira & Torrent (2017)

$r(Y)$ adapted to our problem is

$$R^{KER}(Y_s) = \frac{\sum_{t=1}^{T-h} y_{t+h}(\tau) K(d_q(Y_s, \mathbf{Y}_t)/b_{opt})}{\sum_{t=1}^{T-h} K(d_q(Y_s, \mathbf{Y}_t)/b_{opt})}, \tag{1}$$

where d is a suitable semi-metric (as described below), K is an asymmetric kernel since d is non-negative, $b_{opt} > 0$ is a bandwidth, and h is the forecast horizon. Moreover, $y_{t+h}(\tau)$ represents the yield for maturity τ at time $t+h$. Y_s is an observed curve where one might want to evaluate the conditional expectation. For instance, $\hat{r}_h^{\tau}(Y_T)$ gives the h-steps ahead forecast from T. Therefore, we are working in the state domain. In particular, we are considering a time series $\{y_t(\tau)\}_{t=1}^{T}$ and each $y_{t+h}(\tau)$ is predicted based on curves $\{Y_j\}_{j=1}^{t}$, for a given horizon h. The bandwidth b_{opt} is selected by a typical leave-one-out cross-validation procedure.

Semi-metric Spaces. It is important to note that a curve is generally described as having domain in an interval of real numbers. Nevertheless the functional methodology for data analysis allows one to work with a finite set of observed points for each curve. In this regard, the statistical analysis of the discretized curve requires a careful choice of a measure of distance. It turns out that there is no equivalence between norms when considering infinite dimensional spaces. In fact, semi-metric spaces are more suitable to problems as those we are dealing with here[2]. There are many semi-metrics available in the literature, with each one appropriate for a particular problem, depending on the characteristics presented by the data used. A suitable semi-metric for this case is the Pricipal Component Analysis (PCA) semi-metric, which lies on the idea of dimensional reduction. More precisely, the empirical version of the PCA semi-metric used in the above estimator is defined for all $(Y_1, Y_2) \in E$ as

$$d_q(Y_1, Y_2) = \sqrt{\sum_{k=1}^{q} \left(\int (Y_1(s) - Y_2(s)) v_{k,n}(s) ds \right)^2}. \tag{2}$$

In practice we need to estimate q. One possibility is choosing q via a cross-validation type procedure.

Estimation details. It is well known that yield curves are non stationary in the mean, which is an explanation why a random walk model is so hard to outperform in forecasting exercises. In order to overcome this difficulty, we calculate the mean of each curve, i.e., we construct a time series $m_t = \frac{1}{n_\tau} \sum_{j=1}^{n_\tau} Y_t^j$, $t = 1, \cdots, T$, where n_τ is the number of maturities and Y_t^j is the yield for maturity j at time t. Then we calculate the demeaned curves $\ddot{Y}_t = Y_t - m_t 1'$, $t = 1, \cdots, T$, where $1'$ is a $1 \times n_\tau$ vector of ones. The proposed

[2]For details, see section 3.1 of Ferraty & Vieu (2006).

forecast of Y is then the sum of the NP-FDA forecast of $\ddot{Y}$ plus a random walk forecast of m.

The PCA semi-metric parameter, q, was set equal to one in all estimations. Caldeira & Torrent (2017) argues that for yield curve forecasts, q should be a small number like 1, 2 or 3. We considered these three values for q and the results (available upon request) were qualitatively similar. Hence, we present here the results for $q = 1$ only.

3. Competing Models

3.1. Random walk model

The main benchmark model adopted in the paper is the random walk (RW), and for which the $t+h$-step-ahead forecasts for an yield of maturity τ, are given by:

$$y_{t+h}(\tau) = y_t(\tau) + \varepsilon_t(\tau), \qquad \varepsilon_t(\tau) \sim \mathcal{N}\left(0, \sigma^2(\tau)\right). \tag{3}$$

In the RW, a h-step-ahead forecast, denoted $\widehat{y}_{t+h}(\tau)$, is simply equal to the most recently observed value $y_t(\tau)$. In practice, it is difficult to beat the RW in terms of out-of-sample forecasting accuracy, since yields are usually nonstationary or nearly nonstationary, and hence is a good benchmark for judging the relative prediction power of other models. Many other studies that consider interest rate forecasting have shown that consistently outperforming the random walk is difficult (see, for example, Duffee, 2002; Moench, 2008).

3.2. Univariate autoregressive model

It is possible to generalize the RW model and, forecast the maturity-τ yield Based on a first-order univariate autoregressive model (AR) estimated on the available data for that maturity:

$$y_t(\tau) = \alpha + \beta y_{t-1}(\tau) + \varepsilon_t. \tag{4}$$

The 1-step ahead forecast is produced as $\hat{y}_{t+1}(\tau) = \hat{\alpha} + \hat{\beta} y_{t-1}(\tau)$. The forecasts for h-step ahead horizon are obtained as:

$$\hat{y}_{t+h|t}(\tau) = \left(1 + \hat{\beta} + \hat{\beta}^2 + \ldots + \hat{\beta}^{h-1}\right)\hat{\alpha} + \hat{\beta}^h y_t(\tau).$$

3.3. Vector autoregressive model

The fact that the yield curve can be considered a vector process composed of yields of different maturities, implies that the cross-section information might be important in understanding yield curve movements. However, neither the RW nor the AR models exploit this information to produce the forecasts. Thus, a first-order unrestricted vector autoregressive model (VAR) for yields is a natural extension of the univariate AR model. The estimated model is:

$$y_t = A + B y_{t-1} + \varepsilon_t, \tag{5}$$

where $y_t = (y_t(\tau_1), y_t(\tau_2), \ldots, y_t(\tau_N))'$. The 1-step ahead forecast is produced as $\hat{y}_t = \hat{A} + \hat{B}y_{t-1}$, while the h-step ahead forecasts are obtained as:

$$\hat{y}_{t+h|t} = \left(I + \hat{B} + \hat{B}^2 + \ldots + \hat{B}^{h-1}\right)\hat{A} + \hat{B}^h y_t. \tag{6}$$

3.4. Dynamic Nelson-Siegel model

Diebold & Li (2006) have introduced dynamics into the original Nelson & Siegel (1987) model, and showed that the resulting model has good forecasting power. The Dynamic Nelson-Siegel model (DNS) is given by:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda_t\tau}\right) + \beta_{3t}\left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + \epsilon_t(\tau), \tag{7}$$

where β_1 can be interpreted as the level of the yield curve, β_2 as the slope, and β_3 as the curvature. The parameter λ determines the exponential decay of β_2 and of β_3. The vector of time-varying coefficients β_t follows a VAR process. Once forecasts of the factors are available, the corresponding forecasts of the yields can be retrieved simply by exploiting again the cross-sectional dimension of the system. The DNS can be interpreted as a dynamic factor model, and the Kalman filter can be used to obtain the likelihood function via the decomposition of the prediction error (Jungbacker & Koopman, 2015).

4. Data and Results

This paper focuses on the government bonds for the BRICS: Brazil, China, India, Russia, and South Africa. The daily yields of government bonds are sourced from the Datstream database of Thomson Reuters. The sample period covers January 1, 2010 to December 31, 2016, yielding a total of $T = 1488$ daily observations. The number of maturities varies between the datasets.[3]

The forecasting analysis is performed based on a pseudo real-time exercise, i.e. we never use information which is not available at the time the forecast is made. For computing our results we use a rolling estimation window of 500 daily observations (i.e., 2 years).[4] We produce forecasts for 1-week, 1-month, 3-month, 6-month, and 12-month-ahead. We use iterated forecasts instead of direct forecasts for the multi-period ahead predictions.

To compare the performance of out-of-sample forecasts, we compute the root mean square forecast error (RMSFE). Moreover, the Diebold & Mariano (1995) test is used assess whether each of the modeloutperforms the RW. Tables 1 and 2 report statistical

[3]More details on the datasets can be provided upon request from the authors.

[4]We have also estimated the models using an expanding window. However, the results obtained were qualitatively similar to those presented here, and are available upon request from thee authors.

measures of the out-of-sample forecasting performance at various horizons derived from the alternative models for the BRICS. The first row of entiries in each panel of the tables report the value of RMSFE (expressed in basis points) for the random walk model (RW), while all other rows report statistics relative to the RW. Asterisks (**10%, *5%) on the right indicate the level of significance for the forecast comparison test (i.e., a model has a lower RMSFE than the RW).

We start the model evaluation by investigating the performances of the interest rates forecasts ate 1-week and 1-month horizon (across all models and datasets). First, the RW is found to be a very competitive benchmark in forecasting the term-structure of bond yields, with it outperforming all other competing models, barring the NP-FDA. In sum, the RW forecasts are generally more accurate than those of most of the competing models, especially at short horizons.

The FDA is the only method that is able to systematically outperform the RW at all maturities and all datasets for long forecast horizons. The gains are over 10% at some maturities at the 3-month ahead horizon, and reaches to 8% for the 6-month- and 12-month-ahead horizons. The VAR and DNS-VAR models beat FDA for a few selected datasets and long forecast horizons but they perform much worse than the FDA in the remaining cases. Specifically, DNS-VAR model does a good job for the Brazilian yield curve at long-horizon forecast (i.e., 3-, 6-, and 12-month-ahead) and for Indian yields at 12-month-ahead forecasts.

5. Final remarks

Given the importance of bond markets of emerging economies for the purpose of portfolio diversification, in this paper, we develop a non-parametric functional data model to forecast the term-structure of BRICS countries. Our results show that while it is in general difficult to beat the random-walk model in the short-run (one-week- and one-month-ahead), at longer-horizons (3-month-, 6-month-, and 12-month-ahead) our proposed approach outperforms not only the random-walk model, but also other popular competitors used in this literature. Having said this, even in the short-run, there are instances when our proposed model outperforms the random-walk. As term-spread is known to predict recessions and inflation, our results also have important implications for the policymakers. Specifically, policy authorities would need to rely on a nonlinear functional data-based approach to produce forecasts of the yield curve to devise appropriate policies to ensure stable growth and inflation in the long-run.

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Table 1: **Relative Root Mean Square Forecast Errors, Brazil, China, and India Yields**

Note: In these tables we present the forecasting performance of the various models for selected maturities. The Table reports the Root Mean Squared Forecast Errors relative to the Random Walk (RW) model obtained by using individual yield models for the horizons 5-, 21-, 63-, 126, and 252-step-ahead. The evaluation sample is 2011:1 to 2016:12 ($\approx$ 1000 forecasts). The first line in each panel of the table reports the value of RMSFE (expressed in basis points) for the RW, while all other lines reports statistics relative to the RW. The following model abbreviations are used in the table: RW stands for the Random Walk, (V)AR for the first-order (Vector) Autoregressive Model, DNS for dynamic Nelson-Siegel model with a VAR specification for the factors, and NP stand for the non-parametric functional data analysis, respectively. Numbers smaller than one indicate that models outperform the random walk, whereas numbers larger than one indicate underperformance. The stars on the right of the cell entries signal the level at which the Diebold and Mariano (1995)'s test rejects the null of equal forecasting accuracy (*, and ** mean respectively rejection at 5%, and 10% level).

Models	***Brazil***					***China***					***India***				
	1-Year	2-Years	3-Years	4-Years	5-Years	1-Year	2-Years	3-Years	4-Years	5-Years	1-Year	2-Years	3-Years	4-Years	5-Years
Horizon = 1-week ahead															
RW	0.210	0.293	0.323	0.328	0.338	0.114	0.076	0.077	0.078	0.078	0.123	0.125	0.125	0.124	0.122
AR	1.101	1.091	1.090	1.091	1.092	1.233	1.571	1.487	1.463	1.436	1.064	1.063	1.063	1.065	1.069
VAR	1.115	1.085	1.099	1.099	1.167	1.142	1.227	1.209	1.223	1.258	1.080	1.075	1.073	1.074	1.075
DNS-VAR	1.131	1.039	1.040	1.049	1.094	2.124	1.125	1.150	1.189	1.234	1.114	1.086	1.074	1.066	1.062
NP-FDA	1.278	1.083	0.968**	0.980	1.095	1.220	1.078	1.058	1.042	1.031	1.097	1.020	0.987	0.985	0.999
Horizon = 1-month ahead															
RW	0.465	0.599	0.652	0.663	0.669	0.265	0.161	0.161	0.162	0.162	0.245	0.246	0.245	0.244	0.243
AR	1.068	1.055	1.049	1.047	1.044	1.320	1.918	1.815	1.778	1.735	1.072	1.084	1.095	1.103	1.108
VAR	1.072	0.988	0.986	1.007	1.064	1.121	1.231	1.278	1.326	1.374	1.131	1.116	1.104	1.093	1.082
DNS-VAR	1.010	0.938**	0.925**	0.916*	0.922*	1.443	1.185	1.241	1.311	1.380	1.312	1.263	1.235	1.218	1.207
NP-FDA	1.110	1.021	0.973	0.980	0.993	1.024	1.003	1.007	1.011	1.012	1.048	1.003	0.970**	0.950**	0.941**
Horizon = 3-months ahead															
RW	0.909	1.174	1.293	1.316	1.311	0.464	0.311	0.313	0.315	0.315	0.445	0.453	0.459	0.465	0.471
AR	1.126	1.099	1.086	1.079	1.077	1.310	1.617	1.554	1.522	1.493	1.051	1.061	1.067	1.067	1.064
VAR	1.159	0.983	0.924**	0.920**	0.946**	1.085	1.186	1.203	1.222	1.244	1.180	1.156	1.132	1.109	1.088
DNS-VAR	0.903**	0.831	0.792	0.784	0.781	1.210	1.206	1.197	1.202	1.215	1.221	1.177	1.153	1.138	1.127
NP-FDA	0.921**	0.886	0.879	0.884	0.894	1.017	0.982	0.963**	0.950**	0.941**	1.005	0.987	0.970**	0.955**	0.942**
Horizon = 6-months ahead															
RW	1.523	1.871	2.06	2.091	2.088	0.588	0.486	0.487	0.487	0.485	0.584	0.611	0.632	0.646	0.656
AR	1.215	1.168	1.143	1.126	1.117	1.250	1.192	1.168	1.156	1.148	1.005	0.993	0.982	0.972	0.963**
VAR	1.294	1.074	0.970**	0.947	0.944	1.052	1.043	1.047	1.054	1.065	1.116	1.079	1.053	1.035	1.022
DNS-VAR	0.797*	0.701*	0.664*	0.660*	0.653*	1.081	1.002	0.993	0.994	1.001	1.076	1.040	1.018	1.004	0.994
NP-FDA	1.081	1.005	0.971	0.975	0.980	0.919**	0.971**	0.952**	0.940**	0.933	0.975	0.961**	0.953**	0.951**	0.951**
Horizon = 12-months ahead															
RW	2.569	2.636	2.658	2.611	2.572	0.813	0.718	0.712	0.705	0.698	0.757	0.786	0.808	0.822	0.831
AR	1.467	1.380	1.332	1.278	1.224	1.016	0.855*	0.851*	0.850*	0.852*	0.897*	0.864*	0.846*	0.836*	0.831*
VAR	1.391	1.273	1.193	1.170	1.1380	0.980	0.866*	0.867*	0.871*	0.877*	1.066	1.004	0.964**	0.938**	0.920**
DNS-VAR	0.785*	0.743*	0.716*	0.706*	0.690*	1.050	0.834*	0.821*	0.816*	0.812*	0.937**	0.918**	0.905**	0.893*	0.882*
NP-FDA	1.074	0.992	0.934**	0.920**	0.899*	1.007	0.974	0.964**	0.951**	0.959**	0.991	0.974	0.963**	0.956**	0.953**

Table 2: **Relative Root Mean Square Forecast Errors, Russia and South Africa Yields**

Note: In these tables we present the forecasting performance of the various models for selected maturities. The Table reports the Root Mean Squared Forecast Errors relative to the Random Walk (RW) model obtained by using individual yield models for the horizons 5-, 21-, 63-, 126, and 252-step-ahead. The evaluation sample is 2011:1 to 2016:12 ($\approx$ 1000 forecasts). The first line in each panel of the table reports the value of RMSFE (expressed in basis points) for the RW, while all other lines reports statistics relative to the RW. The following model abbreviations are used in the table: RW stands for the Random Walk, (V)AR for the first-order (Vector) Autoregressive Model, DNS for dynamic Nelson-Siegel model with a VAR specification for the factors, and NP stand for the non-parametric functional data analysis, respectively. Numbers smaller than one indicate that models outperform the random walk, whereas numbers larger than one indicate underperformance. The stars on the right of the cell entries signal the level at which the Diebold and Mariano (1995)'s test rejects the null of equal forecasting accuracy (*, and ** mean respectively rejection at 5%, and 10% level).

Models	*Russia*					*South Africa*				
	1-Year	2-Years	3-Years	4-Years	5-Years	1-Year	2-Years	3-Years	4-Years	5-Years
Horizon = 1-week ahead										
RW	0.383	0.349	0.356	0.357	0.356	0.175	0.188	0.197	0.201	0.204
AR	1.203	1.344	1.278	1.197	1.153	1.096	1.098	1.098	1.096	1.093
VAR	1.180	1.233	1.228	1.194	1.165	1.129	1.127	1.124	1.116	1.108
DNS-VAR	1.431	1.389	1.288	1.236	1.206	1.041	1.005	1.005	1.006	1.007
NP-FDA	0.903**	0.831*	0.893*	0.903**	0.891*	1.800	1.605	1.582	1.542	1.520
Horizon = 1-month ahead										
RW	0.857	0.876	0.884	0.871	0.852	0.341	0.367	0.383	0.391	0.393
AR	1.284	1.401	1.331	1.233	1.151	1.067	1.055	1.047	1.041	1.035
VAR	1.281	1.255	1.230	1.197	1.167	1.114	1.099	1.083	1.064	1.049
DNS-VAR	1.223	1.123	1.085	1.090	1.133	1.023	1.016	1.018	1.018	1.018
NP-FDA	0.947**	0.977	0.991	0.984	0.971**	0.784*	0.754*	0.729*	0.706*	0.695*
Horizon = 3-months ahead										
RW	1.498	1.514	1.502	1.464	1.419	0.556	0.587	0.603	0.607	0.603
AR	1.410	1.444	1.374	1.302	1.227	1.118	1.110	1.103	1.093	1.085
VAR	1.372	1.298	1.253	1.216	1.189	1.116	1.109	1.087	1.066	1.047
DNS-VAR	1.326	1.265	1.249	1.266	1.307	1.031	1.041	1.050	1.053	1.052
NP-FDA	0.893*	0.946**	0.972	0.977	0.975	0.907**	0.917**	0.907**	0.896*	0.892*
Horizon = 6-months ahead										
RW	1.897	1.956	1.951	1.905	1.846	0.754	0.808	0.843	0.858	0.859
AR	1.471	1.351	1.261	1.202	1.156	1.200	1.178	1.152	1.128	1.108
VAR	1.458	1.318	1.25	1.209	1.184	1.151	1.116	1.063	1.016	0.980
DNS-VAR	1.376	1.253	1.213	1.221	1.268	1.093	1.119	1.128	1.128	1.126
NP-FDA	1.036	1.055	1.067	1.071	1.071	1.023	1.016	0.976	0.937**	0.907**
Horizon = 12-months ahead										
RW	2.718	2.812	2.804	2.730	2.633	1.096	1.166	1.209	1.225	1.219
AR	1.273	1.112	1.027	0.982	0.963**	1.185	1.164	1.124	1.088	1.055
VAR	1.282	1.154	1.094	1.065	1.052	1.135	1.076	0.996	0.927**	0.874*
DNS-VAR	1.060	0.971**	0.938**	0.940**	0.944**	1.267	1.284	1.282	1.277	1.276
NP-FDA	1.025	0.988	0.999	1.035	1.073	1.074	1.033	0.979	0.939**	0.914**