University of Pretoria
Department of Economics Working Paper Series

Tuition Grant and Equity-Efficiency Tradeoff in Stages of Higher Education Development

Yoseph Getachew
University of Pretoria
Working Paper: 2018-82
December 2018

Department of Economics
University of Pretoria
0002, Pretoria
South Africa
Tel: +27 12 420 2413

Tuition Grant and Equity–Efficiency Tradeoff in Stages of Higher Education Development

Yoseph Getachew[a]

[a] *Department of Economics, University of Pretoria, 0028, Pretoria, South Africa, e-mail: yoseph.getachew@up.ac.za, tel.: +27760722472*

Abstract

We study how taxes and alternative higher education financing such as universal, scholarship, and means tested tuition grant schemes affect different groups of individuals, and the implications to equity–efficiency trade off at *different phases* of higher education development. We based on a simple overlapping generations model where agents are heterogenous in terms of their initial human capital and their ability to learn. Only individuals who afford to pay the minimum tuition fee up front join college while government engages in different types of education subsidy and financing programs. Despite the economy starts from an early stage where only few elites have access to higher education, through a positive externality effects of education that increases individuals' productivity, it ends up to a highly advanced economy where the majority invest in higher education. Among our findings, a scholarship program is the most efficient higher-education-subsidy program at all stages of higher education development due to its highly regressive nature. Wealth distribution under means-tested Lorenz dominates the one in scholarship in all stages. In the early stages, laissez faire is second best followed by universal grant, when it comes to mitigating inequality. In the late stage, universal subsidy Lorenz dominates laissez faire in general and it Lorenz dominates the rest of the schemes for the bottom poor, followed by the laissez fair. However, if the purpose is to narrow the gap between the top earners and the rest of the society, scholarship is the second best. We also find at this stage enrollment rate increases in universal subsidy but decreases in other policies, implying the recent shift away from universal grant scheme in the UK could lead to a decline in enrollment rate.

Key words:
Education subsidy and financing, public policy, inequality, Growth
JEL Classification:

*Corresponding author

"*For me, education has never been simply a policy issue – it's personal. Neither of my parents and hardly anyone in the neighborhood where I grew up went to college. But thanks to a lot of hard work and plenty of financial aid, I had the opportunity to attend some of the finest universities ... .*"
Michelle Obama

1. Introduction

College funding is personal. In September 19, 2016, the South African higher education minister Blade Nzimande announced that higher institutions in the country could hike next year's fees by a maximum of 8%. The fee hikes, he goes on, may not affect poor students and those individuals who earn less than $43, 000 a year (the so called the "missing middle"). Anyway this has led to a national student protest movement in the country, students calling for free and quality higher education. The protests have led to at least seventeen (out of twenty six) major universities closure in the country for weeks, leading to major disruptions in academic activities. In 2010, students across Britain protest that turned violent tuition hikes in after the government's plan to lower the government subsidy to higher education significantly. Subsidy to higher education is lowered by about eighty percent in 2012, leading sophomore students to pay triple of the tuition fee that their seniors paid. The government consider alternative funding such as increasing low interest loans, and subsidizing students from poor backgrounds although the impacts of the policy shift on efficiency and inequality continue to be debatable.

For many economists, however, the problem of higher education financing is another question of equity-efficiency trade-off. The debate often revolves around its regressive and/or externality impacts on the economy. On one hand, higher education subsidies and grants are belittled for transferring resources away from unskilled workers towards the skilled ones (e.g., Hanson and Weisbrod, 1969; Fernandez and Rogerson, 1995; Garcia-Penalosa and Walde, 2000; De Fraja, 2002). On the other, they are justified on the basis of externality effects of human capital[1] and the perva-

[1] There are some support from the empirical literature with respect to human capital externality

siveness of borrowing constraints that prevent individuals from investing optimally by borrowing against future human capital (e.g., Barham et al. 1995; Fender and Wang, 2003). There is also a third case for education subsidies, for alleviating the distortions in human capital caused by redistributive policies such as progressive taxation (see, for e.g., Benabou, 2002, Bovenbreg and Jacobs, 2005, Krueger and Ludwig, 2016).

A common feature of this literature is its failure to draw the problem along with a country's system of higher education. Ironically, the latter largely determines the equity and efficiency impact of any higher education financing policy, and, it even becomes more important when comparing the impacts of alternative policies. For instance, a universal tuition grant where the enrollment rate is close to 90 percent (such as in the US) will not have the same regressive effect on places where less than 5 percent of the age group (such as in Uganda) have access to tertiary education.[2] In fact, in many developing countries higher education is at a stage where enrolling in higher education is considered a luxury enjoyed by few elites (see Table 1); in contrast, in many advanced economies, the "massification" of higher education is at an advanced stage where the majority of the population have access to it (See Table 2). In closing the gap, the current work examines the efficiency and equity impacts of alternative higher education financing schemes that accounts for the transition and different forms and phases of higher education.

but not without dispute. Moretti (2004) estimates of human capital externality (in terms of the effects of one more year of average education on income) up to a 25% for the US. In contrast, Krueger and Lindahl (2001) and Acemoglu and Angrist (2000) argue the difference between the social and private return of education is not significantly different from zero for the US. According to Benhabib and Spiegel (1994), the relationship between human capital and growth could be zero but they found a positive relation between human capital and total factor productivity.

[2] South Africa and the UK may seem to share similar problem in how to pay for higher education but there is a stark difference between their higher education enrollment structure. In South Africa, only 14% of the black population (that accounts more than 80% of the population) enrolls in higher education whereas more than 50% of the white (that accounts only 10% of the population) enroll in higher education.

Table 1: Higher education enrollment of age group, sample of low income countries

Country	**2013**
Benin	15.3628
Burkina Faso	4.77591
Burundi	4.40817
Congo, Dem. Rep.	6.64076
Guinea	10.3789
Madagascar	4.24579
Mozambique	5.04323
Rwanda	7.52925
Tanzania	3.64732
Togo	10.0422
Zimbabwe	5.87175

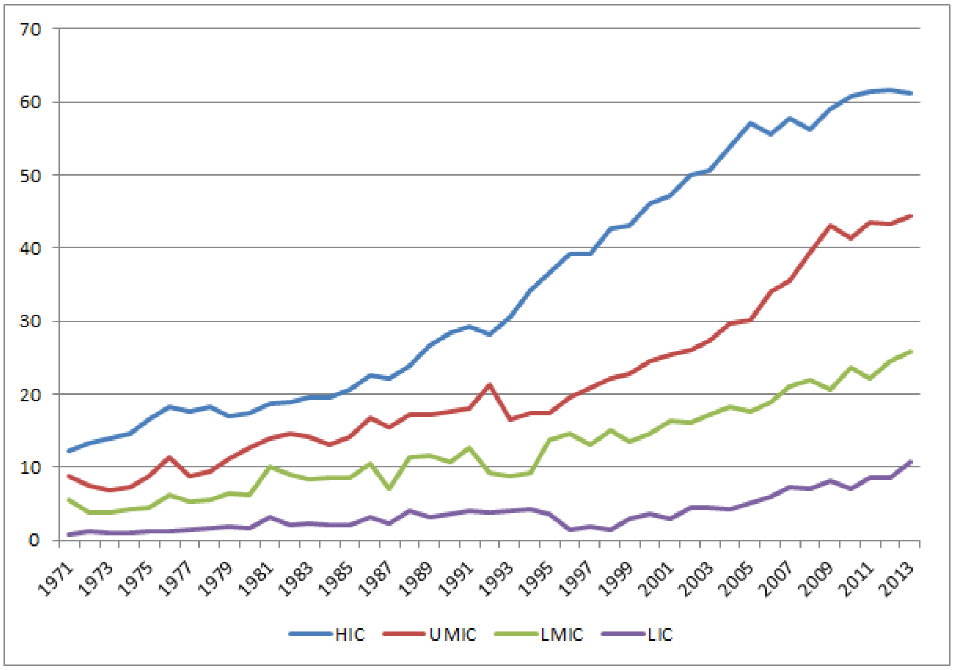
More than four decades ago, in a seminal work, Trow (1973) predicted the transformation of a higher education system from *elite*, to *mass* and to *universal* system. An *elite* phase is where less than 15% of the high school cohorts moves beyond the secondary level and higher education is mainly about the shaping of the ruling class and preparing them for an elite role. The *mass* phase is where 16-50% of high school graduates continue their educations and the mass prepare for broader range of technical roles. The *universal* phase is where over 50% percent of graduates continue their higher education and the majority of the population prepare itself to embrace technological progress.[3] It is now believed that the "massification" of higher education is real and many of today's industrialized economies has, more or less, passed through Trow's phases of development since the Second World War (see Table 2).

[3] In a latter work, Trow (2007) argued that the industrialized society is currently moving beyond that towards a situation he described as "a learning society," with very large parts of the population engaged in formal education of one form or another.

Table 2: Higher education enrollment of age group, high income countries

Country	**1971**	**2013**
Argentina	15.3701	79.9867
Australia	17.0328	86.5546
Austria	12.2113	80.3868
Belgium	16.8641	72.3096
Chile	11.1577	83.8164
Czech Republic	8.92373	65.3774
Denmark	18.8583	81.237
Finland	13.1341	91.0658
France	18.5413	62.1469
Hong Kong SAR, China	6.83597	67.2759
Hungary	10.0217	57.0167
Ireland	10.5903	73.1685
Italy	16.8803	63.4551
Japan	17.6406	62.4116
Korea, Rep.	7.24645	95.3454
Malta	6.51885	45.6805
New Zealand	16.9108	79.7143
Norway	15.7949	76.1179
Panama	10.3144	38.7393
Poland	13.3588	71.1587
Portugal	7.27266	66.2216
Spain	8.66966	87.0658
Sweden	21.7328	63.3929
Switzerland	10.0385	56.2682
United Kingdom	14.5679	56.8701
United States	47.3235	88.8086

Figure 1: Evolution and stages of higher education of countries at different stages of development



Note: UIC – Upper income countries; HMIC – Upper middle income countries; LMIC – Lower middle income countries; LIC – Lower income countries.

The current work thus provides a more comprehensive analysis of the impact that alternative higher education policies have on inequality and aggregate efficiency at different phases of higher education development. We develop a simple overlapping generations model that captures the endogenous transition of higher education through different phases in line with Trow's (1973) work. The economy start from an early stage where only few elites have access to higher education and ends up to a highly advanced economy where all individuals invest in higher education. The phases of higher education are proportional to the stages of economic development that countries exist, as reflected in the data (Figure 1).

In the model, agents are heterogenous in terms of their initial human capital and their ability to learn. We differentiate individuals rich and poor, based on their family background (wealth); high and regular ability, based on their ability to accumulate human capital. Only individuals whose parents afford to pay the minimum tuition fee up front join college while government engages in different types of education subsidy and financing programs. For those families who do not afford the tuition fee, their children are destined to join the unskilled labor force when becoming adult and earn a lower wage income. Individuals with college education receive an additional skill premium. Skilled (human capital) and raw labor are the only factors of production at the aggregate level. Individual productivity and hence their labor income depend on the level of aggregate human capital. Since households' capacity to pay for education depend on their income, aggregate human capital (among individuals' ability to learn and parental background) is a critical component of education investment threshold that determines what type of individuals invest in higher education. At initial, when the level of aggregate human capital is relatively too low, only high ability individuals from affluent family background invest in higher education (*Stage I* – an *elite* phase). As the economy continues to grow, there would be a positive externality effects of education that increases individuals' productivity and hence their income leading more families to join higher education investment: first, the rest of affluent families with regular ability children send their children to higher education (*Stage II* – a *mass* phase); then, poor families with high ability children follow the suit (*Stage III* – a *universal* phase); finally, poor families with regular ability children would be able to afford the tuition fee and invest in higher education (*Stage IV* – an *advanced* phase).

In all these stages, government could involve in one of commonly practiced tuition subsidy programs, which are financed by a flat rate tax. It could choose *a universal subsidy scheme* that targets any individual that joins college; *a scholarship scheme* that targets high ability individuals regardless of their family background; and, *a means-tested* program that target high ability individual with poor family background. We examine how each of these policies affect individuals' ability and decision to invest in higher education and their implications to equity–efficiency trade off at the different phases of higher education development compared to a laissez faire system. Our analysis do not include student loan which seem extensively studied in recent literature in higher education financing.[4] While the loan system is widely practiced in many advanced countries but it is not necessarily the right framework for the rest of the world due to the lack of institutions enforcing the loan system.[5]

Among our findings: a scholarship program is the most efficient higher-education-subsidy program at all phases of higher education development due to its highly regressive nature. Means-tested is the least efficient policy in the early *Stages I & II*, as few are eligible to this program during these stages. However, it is the most efficient one (along scholarship scheme) in *Stages III and IV* through mobilizing resources to the most able individuals in the economy. Laissez faire is preferable at the initial or last stage but not in the middle *Stages II & III*; particularly, it is the least efficient one in *Stage III* when high ability individuals of poor family background have also access to higher education. At this stage, government intervention in any form is preferable to ensure resource poor but high ability individuals would not be left behind. A universal subsidy scheme performs the second best in most of developmental phases.

However, the effects of higher education policies on inequality are rather ambiguous. In general, the distribution under means-tested Lorenz dominates the one in scholarship in all stages. In *Stages I & II*, laissez faire is second best followed by universal grant. In the early stages, means tested, leaves every one worse off, as non of the groups who invest in education at these stages qualify for the program. Even though all pay tax and none are qualified to the grant scheme; still taxation seems to hurt the high ability individuals more. In the latter stages (particularly in *Stage IV*), we have established that universal subsidy Lorenz dominates laissez faire. More interestingly, the former Lorenz dominates the rest of the schemes when it comes to the poorest of the poor section of the society followed by the laissez fair. However, if the purpose is to narrow the gap between the top earners and the rest of the society,

[4] See for instance, Garcoa-Penalosa (2000), De Fraja, 2002), De Rey and Racionero (2010), Abott et al. (2013), Gary-Bobo and Trannoy (2015), Heijdra eta . (2017) among others.

[5] For instanc, in South Africa, a country with a much developed institution and economy in the continent, only the percentage of loan recovered between 2017 and is about 6% of the total students loan granted.

scholarship is the second best. It even ties with means tested in narrowing the gap between individuals below and above the middle class.

We have also made analysis how these policies affect enrollment rate. In the elite stage, compared to laissez faire, universal grant and scholarship have similar positive effects; means tested has a negative effect. In Stage II, enrollment increases in universal grant but decreases in other policies. Means tested is the first best in increasing enrollment, in *Stage III*, whereas scholarship and universal are the second and the third best, respectively. We find enrollment rate increases in universal subsidy but decreases in other policies in *Stage IV*. This result particularly confirms with other studies that find the policy shift in 2012 has led a decline in enrollment rate in the UK (Geven, 2015).

The paper is related to strands of literature. Particularly, it is closely related to the literature that compares the efficiency and equity effects of different financing system: a non-comprehensive list includes Garcia-Penalosa and Walde (2000), Caucutt and Kumar (2003), Cigno and Luporini (2009), Del Rey and Racionero (2010) and Abott et al. (2013) among others. Garcia-Penalosa and Walde (2000), for instance, examine the equity and efficiency effects of a general tax-subsidy, pure and income contingent loan schemes and graduate tax.[6] They argue efficiency targets could be achieved with the general tax-subsidies scheme but not equity and efficiency targets at the same time, as the scheme is regressive. Loan schemes and graduate tax fare better than the traditional tax-subsidy system in achieving efficiency-equity where the latter is preferable when education outcome is uncertain as it could provide a partial insurance.[7] However, there is no externality effects, in Garcia-Penalosa and Walde (2000), from human capital investment to the general population which is the deriving force of societal transformation in the current work. A more comprehensive

[6]Cigno and Luporini (2009) argue that all students loans basically are income-contingent loans because anyway if unsuccessful it would be difficult to enforce a repayment.

[7]Del Rey and Racionero (2010) build on this and rather divide the income-contingent loans into two types: those with risk sharing and risk pooling, the difference being unpaid costs from unsuccessful students to be covered by the general population and successful cohorts, respectively. They also do not model externality and only analyze the efficiency and participation effects of alternative fianancing schemes with a focus on the role of insurance.

and unifying work has been done by Abott et al. (2013) who consider individuals' decision through different stages of their life cycle – from high school to retirement: whether to attend and high school and college and whether to complete or dropout high school and college. They also consider uncertain return to investment in education, endogenous life span and parental transfer of resources; furthermore, there are different types of human capital that correspond to different levels of education such as high school and college. They then calibrate their model for the US economy and conclude that the current financial system in the US is welfare improving. In particular, they find the partial and general equilibrium effects of different financing programs such as means-tested, ability-tested and general expansion of future grant to be welfare improving.[8] They don't address equity issues, though. Beside, their focus on highly advanced country is in sharp contrast to ours that examines different stages of higher education development *analytically.*

The paper is also related to the literature that focuses on altruistic parents that face a warm glow utility and human capital investment threshold (e.g., Galor and Zier, 1993, Moav, 2002, Galor and Moav, 2004, Galor and Mountford, 2008), which defines individual investment and consumption decision. However, this literature fully abstracts from education policies but inequality and growth issues.

The paper is structured as follows: the next Section provides the model with a laissez faire condition. Section 3 characterizes the transitional phases of higher education under laissez fair. In section 4, we introduce government; and, in Section 5 and 6, we assess the enrollment and efficiency effects of different policies at different stages of the economy, respectively. In Section 7, we construct a Lorenz curve associated to each policy and examine the equity impact of each policy. Section 8 concludes. The proofs for all propositions are provided in the Appendix.

[8] This finding is supported by Akyol and Athreya (2005) who argue that not only existing higher education subsidies in the US are welfare improving but even more higher subsidies could be benefitial since it encourages students to invest in higher education, which is risky and lumpy, by reducing college failure risk.

2. The model

Suppose heterogenous households in OGM. The size of these households is one where there is no population growth. At the beginning, λ of the households are college educated, the rest, $1-\lambda$, are non-college educated. Individuals are also different in their ability to learn. In general there are two types of individuals: those who are highly talented (those that we assume are very few in numbers) and the rest of the population.[9] The probability to be born as talented is p. Individuals live two periods as a young and as an adult. Children born with a unit of time. Conditioned on parental investment on them (covering a fixed college tuition fee plus other variable cost such as books, laptop, etc.) they could built on it by joining a college. College education is a possibility only if the minimum tuition fee is paid up-front. Therefore only households that can afford the tuition fee (and, finds optimal to do so) will send their children to college. Otherwise the child joins the unskilled labor force when she becomes an adult.

2.1. Human capital and Preferences

The human capital of individual i who is born at date t is given as follows:

$$h^j_{it+1} = \epsilon^j e_{it} + 1 \tag{1}$$

e_{it} represents additional parental investment (other than a fixed tuition cost) in education. Implicit in condition (1) is human capital will be fully depreciated at the end of each period. Not only such specification enables us to obtain closed form solutions but also it might also be quite appropriate given that human capital is embedded on individuals that have a finite life. With a slight abuse of notation, we denote the fixed tuition cost which depends on whether an individual invests in education as $s|e_{it}$. If the parent chooses $e_{it}=0$, then she doesn't need to pay the tuition fee, thus $s|e_{it}=0$. But her child grows as unskilled worker while her human

[9] Throughout the paper we use "college educated", "rich" and "skilled", interchangeably, in their loosely meaning, as we will "non-college educated", "poor", or "unskilled". We also loosely use "gifted" "top-ten" and "talented" interchangeably, as we will "regular" and "ordinary".

capital is given by $h_{it+1} = 1$. Therefore, even without a formal college education, individuals will have a basic knowledge when joining the labor force.[10] Otherwise, she pays a constant tuition fee cost, $s|e_{it} = s$. ϵ^j represents the learning ability of a child, $j \equiv \{g, r\}$. If j takes g that implies the child i is among top-ten students; otherwise she is a regular student. $i \equiv \{c, n\}$, where c and n stand for college-educated and non-college-educated individuals.

Suppose the following "warm-glow" utility function with logarithmic preferences:[11]

$$u_{it} \equiv \ln c_{it} + \beta \ln h^j_{it+1} \tag{2}$$

The utility of the ith agent is subject to the budget constraints:

$$c_{it} + s|e_{it} + e_{it} = I_{it} \tag{3a}$$

where

$$I_{it} = \begin{cases} \omega_t \text{ if } e_{it-1} = 0 \\ \omega_t + \phi_t h_{it} \text{ if } e_{it-1} \neq 0 \end{cases} \tag{3b}$$

and

$$c_{it} \geq 0,\ e_{it} \geq 0 \tag{3c}$$

$$s > 1,\ \epsilon^j > 1 \tag{3d}$$

where c_{it} is the household consumption. I_{it} is the gross labor income of the adult; its value will be determined based on the individual's education background, i.e.,

[10] An alternative interpretation of this is that since all children go through a compulsory primary and secondary school education, they have at least a minimum level of skill when joining the labor force.

[11] The use of such utility function is ubiquitous in the literature (see for instance, Glomm and Ravikumar, 1992, Galor and Zier, 1993, Banerjee and Newman, 1993, Galor and Weil, 2000, Benabou, 2000, Galor and Maov, 2004 among many other). Its main advantage (*vis á vis* other dynastic altruistic models that assume parents derive utility from the utility of their children) is its greater analytical tractability while the qualitative results of the model remains unaffected.

whether or not she has received a college education when child, at $t-1$. ω_t and $\phi_t h_{it}$ are the wage rate per unit of labor and the skill premium per se, respectively. We see later, $\phi_t = \phi$. Eq. (3c) represents a no-borrowing condition, as individuals are restricted to have a non-negative consumption and they are not allowed to carryover a negative asset in the future.

Note that the specification in (2) acknowledges the parent's good knowledge of her child's ability. According to Caucutt (2003), such assumption is reasonable given that the parent lives together with her child for an extended period of time. Our setting is in contrast from the literature that emphasizes that parents care only about a bequest they leave to their children (see, for e.g., Galor and Moav, 2004). In the current setting, the parent rather cares for the human capital of her child, but not only just for the bequest she leaves. This does not necessarily make parents more altruistic but make them to consider additional factors in their investment in the current model, which, we believe, is a better reflection of the reality. In particular, parents are aware of their children's possession of a unit of human capital (in addition to their knowledge of their ability) regardless of their investment, which affects their marginal benefit of investing in their children education.

2.2. The Firm

There is a representative firm that operates in a perfectly competitive market. The firm uses both skilled and unskilled labors to produce the final product where the later is augmented by the aggregate capital stock in the economy (in the spirit of Romer, 1986). Prices per unit of unskilled and skilled labor are thus given by, respectively:

$$\omega_t = (1-\alpha) A h_t \tag{4a}$$

$$\phi = \alpha A \tag{4b}$$

where A is a constant TFP; α is a factor share and h_t is the aggregate human capital at time t. Implicit in condition (4b) is perfect substitutability (or homogeneity) among skilled workers. Both gifted and regular individuals receive similar rate per unit of human capital holdings. The only difference between these individuals is thus on the quantity but quality of human capital that they possess.

2.3. Optimal Education Investment

The solution for the ith household education investment is given by

$$e_{it}^{j*} = b\left(I_{it} - s\right) - b/\beta\epsilon^{j} \tag{5}$$

where $b \equiv \beta/\left(1+\beta\right)$.

Therefore, if the parent is an unskilled worker ($h_{it}^{j} = 1$), and she chooses to invest on her child education, then $e_{t}^{j*} = b\left(\omega_t - s\right) - b/\beta\epsilon^{j}$. However, if the parent is college educated ($h_{it}^{j} > 1$) and chooses to invest in her child, $e_{it}^{j*} = b\left(\omega_t + \phi_t h_{it}^{j} - s\right) - b/\beta\epsilon^{j}$.[12] Three observations immediately follow, from eq. (5). First, individuals with total income below the tuition fee, s, *cannot afford* to send their children to college, given that they face borrowing constraint. Second, even those who could afford the fixed college tuition fee may not necessarily invest in higher education, as they may not find it optimal. Third, parents with high ability children are more likely to send their kids to college than their counterparts. Therefore, all income, tuition and ability are important factors in determining whether a child will have a college education or not.

Thus, effective college investment is given by:

$$e_{it} = \max\left(0, e_{it}^{j*}\right) \tag{6}$$

The economy thus features two types of households. The first are those households whose consumption decision entails consuming the full amount of their income, and do not invest in education, either because their income falls short of the tuition fee, it is not optimal to invest in education, or both. The second are those who send their kids to college.

[12]Note that i and j in h_{it}^{j} represent the grandparent's class and the parent's ability, respectively.

From (1), (4) and (5), the human capital of a young individual who is born at time t and joins college is given by

$$h_{it+1}^{j*} = \begin{cases} b\left(\epsilon^{j}\left(\omega_t - s\right) + 1\right) \text{ if } h_{it} = 1 \\ b\left(\epsilon^{j}\left(\omega_t + \phi h_{it}^{j} - s\right) + 1\right) \text{ if } h_{it} \neq 1 \end{cases} \quad j \in \{g, r\} \tag{7}$$

If the individual does not join college, simply $h_{it+1}^{j} = 1-$ the human capital of any individual i and ability level j. Therefore, the optimal human capital associated to the jth child is given by

$$h_{it+1}^{j} = \max\left(1, h_{it+1}^{j*}\right) \tag{8}$$

The first and the second lines in eq. (7) show that the human capital of a young individual with unskilled and skilled parents, respectively. The terms in the right hand side represent the respective incomes of the parents; a fraction of the incomes will be invested in the children education that form their human capital whereas the rest are consumed by the household. In addition to their family background, children also differ in their ability to learn (as shown by the superscript j) in this economy. Condition (8) includes the corner solution for education investment and follows from (6). We can rewrite (7), using (9), as

$$h_{it+1}^{j*} = \begin{cases} b\left(\epsilon^{j}\left(\left(1-\alpha\right) A h_t - s\right) + 1\right) \text{ if } h_{it} = 1 \\ b\left(\epsilon^{j}\left(B h_t - s\right) + 1\right) \text{ if } h_{it} \neq 1 \end{cases} \tag{9}$$

where[13]

$$B \equiv (1-\alpha) A + \alpha A/\lambda$$

is the average income of college educated parents at time t.

[13] In deriving the second equation in (9), substitute (9) in to (7) to get

$$h_{it+1}^{j*} = b\left(\epsilon^{j}\left(\left(1-\alpha\right) A + \alpha A h_{it}^{j} - s\right) + 1\right)$$

But, given λ number of individuals have attended higher education at time t, $h_t = \lambda h_{it}^{j}$. Substituting that into the above leads to (9).

2.4. Education Investment Threshold

Since household education investment is a function their labor income(s), which depends on aggregate productivity, the level of aggregate human capital is what in essence determines individuals' education investment. Considering (9) and (9), the threshold level of human capital in the economy below which individuals do not invest in education is given by

$$\overline{h}_c^j = \left(\frac{1}{\beta\epsilon^j} + s\right) B^{-1} \tag{10a}$$

$$\overline{h}_n^j = \left(\frac{1}{\beta\epsilon^j} + s\right) \left(\left(1-\alpha\right) A\right)^{-1} \tag{10b}$$

We see from (10a) and (10b), an individual's investment in education on her child depends on her patience, college tuition, the capacity of the economy and her child's ability. $\overline{h}_c^j$ and $\overline{h}_n^j$ represent the threshold levels of aggregate human capital beyond which college-educated and non college educated parents invest in their children's education, respectively. The superscript j shows the thresholds are different for people with different ability children. If there are no background differences among parents, those with high ability children are more likely to invest in their children than those with lower ability ones. If there are background differences, however, both the parents' background (whether or not they are college educated) and the children's ability are important in determining who more likely attend college. It can be easily shown from (10a) and (10b) that $\overline{h}_c^g < \overline{h}_c^r$ and $\overline{h}_n^g < \overline{h}_n^r$ hold given $\epsilon^g > \epsilon^r$ but comparison between $\overline{h}_n^g$ and $\overline{h}_c^r$ and is rather less clear cut. But, in general, we consider the case $\overline{h}_n^j > \overline{h}_c^j$, which implies that regardless of their ability, poor individuals are less likely to afford college education by themselves. However, although it is a possibility that $\overline{h}_n^g < \overline{h}_c^r$, which implies poor but highly talented individuals are more likely to go to college than regular rich kids, it might be in contrary to intuition and empirical evidences. Moreover, to allow such a scenario, the ability gap between gifted and regular ones would be unrealistically high.[14] Therefore considering that the following relation holds:

$$\overline{h}_c^g < \overline{h}_c^r < \overline{h}_n^g < \overline{h}_n^r \tag{11}$$

Parents with college education and high ability children are most likely to invest in child education whereas skilled parents with regular kids are better suited to invest in education compared with parents without a college education. Individuals with no college education and regular kids are least likely to invest in education.

The following proposition follows directly from (10a) and (10b):

Proposition 1. 1. *In any of the group, individuals invest in education more likely if they have gifted children, if* ***they are more patient****, there is a lower tuition fee or/and a higher TFP.*

2. *The higher the labor factor share the more likely unskilled individuals invest in education.*

3. Aggregate Capital Dynamics

The aggregate human capital is the total human capital in the economy with regular and high-ability individuals in the population, with skilled and unskilled parents. Thus, if there are λ number of individuals (parents) who have a college education at time t and the probability of being born with high ability is p, then the aggregate human capital (h_{t+1}) in the economy at time $t+1$ is given by:

$$h_{t+1} = \lambda \left[p h_{ct+1}^g + (1-p) h_{ct+1}^r \right] + (1-\lambda) \left[p h_{nt+1}^g + (1-p) h_{nt+1}^r \right] \tag{12}$$

where h_{ct+1}^g and h_{ct+1}^r represent the total human capital of talented and regular individuals with skilled parents, respectively; h_{nt+1}^g and h_{nt+1}^r represent the total human capital of high and regular ability individuals with unskilled parents, respectively. The subscripts c and n show the background of the specific individual whether its

[14] Even with the unlikely condition of zero tuition fee ($s = 0$), the ability gap between the two should be more than three times, when calibrated with reasonable values of $\alpha = 0.33$ and $\lambda = 0.25$.

from a college or non-college educated parents, respectively. The first term (in square bracket) is the total number of skilled individuals with college educated parents while the second is of those with parents of no college education. In each group, there are high-ability individuals with probability p and regular ability individuals with probability $1-p$. Condition (12) implicitly assumes that *all* individuals in the economy invest in education. If only part of the population invest in education, then the aggregate human capital becomes smaller, accordingly.[15] **Note also that given that individuals are homogenous within each group their descendants are also homogenous.** This also implies that it will not be possible for some individuals from one group to invest in education while others from the same group to not. A specific group is identified based on parental background and ability of the child: (λ, p).

3.1. Stage of Development and Aggregate Human Capital Dynamics

Using eqs. (9), (10) to (12), the dynamic system that characterizes the economy's developmental stages could be derived:[16]

$$h_{t+1} = \begin{cases} b\lambda p\left[\epsilon^g\left(Bh_t - s\right) + 1\right] \text{ if } \overline{h}_c^g < h < \overline{h}_c^r \\ b\lambda\left\{\left(p\epsilon^g + \epsilon^r\left(1-p\right)\right)\left[Bh_t - s\right] + 1\right\} \text{ if } \overline{h}_c^r < h < \overline{h}_n^g \\ bp\left(\epsilon^g\left(Ah_t - s\right) + 1\right) + b\lambda\left(1-p\right)\left(\epsilon^r\left[Bh_t - s\right] + 1\right) \text{ if } \overline{h}_n^g < h < \overline{h}_n^r \\ b\left(\left(p\epsilon^g + \left(1-p\right)\epsilon^r\right)\left(Ah_t - s\right) + 1\right) \text{ if } h > \overline{h}_n^r \end{cases} \quad (13)$$

The developmental stage are associated to the evolution of higher education enrollment: the economy may start from an early stage where only few elites have access to higher education and end up to a highly advanced economy where all individuals invest in higher education.

The next period aggregate human capital investment is unity if the initial aggregate capital is too small, below the threshold level $\overline{h}_c^g$ (i.e., if $h_{t+1} = 1$ if $h < h_c^g$).

[15] For instance, if only parents with college education invest in education, then the second term in the square brackets will immediately disappears and the total human capital in the economy becomes: $h_{t+1} = \lambda\left[ph_{ct+1}^g + \left(1-p\right)h_{ct+1}^r\right]$.

[16] See Appendix A for details.

Even the most rich and highly talented ones do not find it optimal to invest in education, as it yields too low return. We see in the first line in eq. (13) some individuals, in particular, college educated parents with high ability children to begin investing in college education. In this case, the current aggregate human capital stock should be greater than $\overline{h}_c^g$ but less than $\overline{h}_c^r$ (the threshold capital required for *all rich* parents to send their kids to education). The total aggregate education investment is $\lambda p \epsilon^g b\,(Bh_t - s)$: $\lambda p \epsilon^g$ is to imply only the rich, with high-ability, with probability p, send their kids to college. $Bh_t - s$ is the *average* income of college educated parents, net of college tuition fee. However if the current capital is greater than $\overline{h}_c^r$, all rich parents regardless of child ability invest in college education (second line). If it is greater than $\overline{h}_n^g$, then all rich parents and some poor parents with high ability children invest in higher education (third line). $Ah_t - s$ is the average income of *all* parents, net of college tuition fee. The first term shows education investment by all types of parents with high ability children while the second captures investment by the rich parents with regular children. Only then when the aggregate human capital stock passes $\overline{h}_n^r$, non-college educated parents with regular children start to invest in education (fourth line). At this stage, education investment in the economy is simply a fraction of aggregate income net of tuition fee.

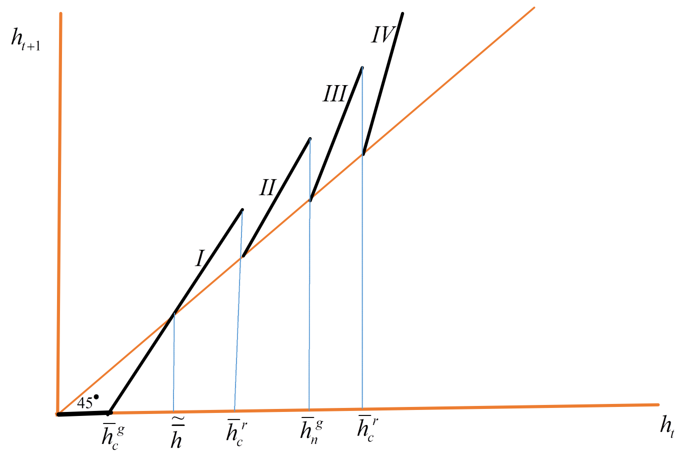
As a requirement for a growing economy, the following restriction is imposed:

$$b\lambda p \epsilon^g B - 1 > 0 \tag{A1}$$

It implies that the slope of the curve in the initial stage of the economy shall be greater than unity.

Figure 2 shows the different developmental stages that the economy experiences based on eq. (13). As shown in the horizontal line, $h_{t+1} = 1$ for any initial capital $h < \overline{h}_c^g$. But if $\overline{h}_c^g < h < \overline{h}_c^r$, the economy will be in Stage I where $h_{t+1} \neq 1$ because some individuals, in particular, parents with college education background and high ability children begin to invest in human capital. But in this stage, if the initial capital stock is not sufficiently high, the dynamic could go back to the stable equilibrium, $h_{t+1} = 1$. The economy escapes the low equilibrium only if $h_{t+1} \geq h_t$.

Figure 2: Stages of Development: The economy kicks off only if the initial capital stock exceeds $\tilde{h}$



The associated threshold is then defined, eq. (13) second line, by $h^T \equiv h_{t+1} = h_t$:

$$h^T = \frac{b\lambda p}{b\lambda p\epsilon^g B - 1}\left(s\epsilon^g - 1\right) \tag{A2}$$

The economy continues to grow as long as the initial aggregate human capital is greater than this threshold level: $h_0 > h^T$; in the next steps it eventually passes the thresholds required for other individuals to begin investing in human capital, through productivity spillover that boosts individual labor and capital incomes. Stage II of development begins when $\overline{h}_c^r < h < \overline{h}_n^g$. At this stage, all individuals with college educated parents invest in college education. This is followed by Stage III and Stage IV, when $\overline{h}_n^g < h < \overline{h}_n^r$ and $\overline{h}_n^r > h$, respectively. The latter represents the long-run path of the economy where all individuals (rich and poor) invest in college education whereas the former represents a middle stage where all rich households and those poor households with talented children invest in college education.

4. Higher Education Policy

The previous model is based on a laissez-faire condition where there is no government intervention at any of the phases of higher education development. In this section, we introduce a government that engages with a provision of different types of higher education grants. We let the government taxes labor and capital incomes (skill premium) to finance education subsidy. We consider three higher education policies that are commonly applied: (i) a universal grant, (ii) a scholarship or (iii) a means-tested scheme. The policies differ in terms of eligibility criteria that they associate to. In the first, grant is available for any individual who enrolls to higher education. This happens if the government has *no* knowledge of individuals' ability and background and thus provides grant for any member of the society that joins a college or a university. In the second, tuition grant is available for individuals based on their *merit*. This happens if the government has knowledge of individuals' ability but their family background. In the third scheme, the government provide tuition grant for high ability individuals from poor family background. This could happen if the government has knowledge of both individuals' ability and family background.

4.1. Government Budget

We assume the government has a balanced budget. Given that there are λ college-educated and $1 - \lambda$ non-college educated individuals at time t, the total number of tax-payer individuals is unity.

$$z_t \equiv \tau_w \omega_t + \tau_y \phi h_t \tag{14}$$

This implies the total government revenue, which is the sum of taxes collected from labor income of skilled ($\lambda \omega_t$) and unskilled individuals ($(1 - \lambda)\, \omega_t$) and capital incomes ($\lambda \phi h_{it}$), is equal to the total education expenditure (z_t). Using (9) this can be rewritten as:

$$z_t = [(1-\alpha)\,\tau_w + \alpha\tau_y]\,Ah_t = \theta Ah_t \tag{15a}$$

$$\theta \equiv \tau_w\,(1-\alpha) + \tau_y\alpha \tag{15b}$$

where θ represents the *grant ratio* – the fraction of aggregate income that are used for public subsidy.

Note that z_t is the aggregate tuition grant available at time t. The amount of tuition subsidy available per person at time t (call it x_t) depends on *eligibility*, which in turn, depends on the *type of the scheme* (whether it is a universal, a scholarship or a means-tested) and the *enrollment rate*. The latter depends on the stage of higher education development. The per capita tuition subsidy is the total tuition subsidy (z) divided by the number of *eligible individuals who are enrolled* to college at the time. The amount of x could thus be different at different phases and/or for different scheme due to variation in the number of eligible individuals enrolled to higher education. For instance, if higher education is at elite stage (where *only* the rich could afford to invest in education) and if government involves in a universal scheme – subsidy is available for *every* individual who invests in higher education – then the numbers of eligible individuals with access to college are λ and the amount of tuition subsidy available to an individual is $x_t = z_t/\lambda$. However, if it is a scholarship scheme, then the number of individuals with college access who are eligible is λp and hence the per capita tuition grant is $x_t = z_t/\lambda p$. If the program is means-tested then non of the individuals who enroll to college receives grant as no one is eligible: $x_t = 0$.

In determining the values of x, for sake of comparison, we adopt the same enrollment trend that we have in the laissez faire case (11). That is, grant or no grant, the super rich (λp) would most likely to invest in college, followed by the rich, $\lambda\,(1-p)$, and then the middle class, $p\,(1-\lambda)$ while the poor, $(1-\lambda)\,(1-p)$, are the least likely ones to invest in higher education. Table 3 below summarizes the per capita allocation of tuition grants that are available at different phases of higher education development and for different public programs:

Table 3: Tuition subsidy provision at different stages and programs

Stages	**Grant available per person (x_t)**		
	Universal	*Scholarship*	*Means tested*
Stage I	$z_t/\lambda p$	$z_t/\lambda p$	0
Stage II	z_t/λ	$z_t/\lambda p$	0
Stage III	$z_t/\left(\lambda+(1-\lambda)\,p\right)$	z_t/p	$z_t/\left(1-\lambda\right)p$
Stage IV	z_t	z_t/p	$z_t/\left(1-\lambda\right)p$

4.2. Household Budget, Education Investment and Human Capital

With government intervention, the ith household budget constraints are given as follows:

$$I_{it} \equiv c_{it} + s'|e_{it} + e_{it} \tag{16a}$$

$$= \begin{cases} (1-\tau_w)\,\omega_t \text{ if } e_{it-1}=0 \\ (1-\tau_w)\,\omega_t + (1-\tau_y)\,\phi h_{it}^{j} \text{ if } e_{it-1}\neq 0 \end{cases} \tag{16b}$$

where

$$s'|e_{it} \equiv \begin{cases} s - x_t \text{ if eligible for subsidy} \\ s \text{ otherwise} \end{cases} ;\; e_{it}\neq 0 \tag{16c}$$

τ_w and τ_h denote the fixed tax rates imposed in wage and capital incomes respectively. I_{it} is the ith household disposable income. For a skilled individual, her disposable income now constitutes labor income and skill premium, minus the respective labor and capital taxes; for an unskilled person, it is after-tax labor income. The fixed tuition cost that an eligible household has to pay up-front, if it chooses to send the child to college ($e_{it}\neq 0$), is now $s' \equiv s - x_t$. Ineligible households, however, incur the full tuition cost $s' \equiv s$ and still pay their taxes accordingly. Of course, for families who do not participate in higher education ($e_{it} = 0$), $s' = 0$. The solution for the household problem under government intervention becomes:

$$e_{it}^{j*} = b\,(I_{it} - s') - b/\beta\epsilon^{j} \tag{17}$$

Eq. (17) is basically similar to eq. (5), except that now the tuition fee for those eligible individuals who invest in education is reduced by x_t. However, individual incomes that are available for investment are also reduced due to tax duties.

From (1), (9), (16) and (17), it follows that the optimal human capital of a young individual who is born at time t and receives a college education during the same period is given by

$$h_{it+1}^{j} = \begin{cases} \epsilon^{j} b\,(A'h_t - s') + b \text{ if } h_{it} = 1 \\ \epsilon^{j} b\,(B'h_t - s') + b \text{ if } h_{it} \neq 1 \end{cases} \tag{18}$$

and

$$\begin{aligned} A' &\equiv (1-\alpha)(1-\tau_w)\,A \\ B' &\equiv B - ((1-\alpha)\,\tau_w + \tau_y\alpha/\lambda)\,A \end{aligned}$$

$A'h_t - s'$ and $B'h_t - s'$ are the *average after tax* income of non-college and college educated parents, respectively, net of college tuition fee and subsidy. Note that eq. (18) is in the spirit of eq. (9) whereas the two converge if $\tau_y = \tau_w = 0$. Apparently, whether or not an individual is better off or worse from government intervention depends on the net effects of taxes that she pays and the subsidy that she receives (if any).

By substituting for x_t from Table 3 into (18), one obtains individuals' optimal human capital associated to a given stage of development and type of tuition subsidy. Individuals' optimal human capital may differ at different grant scheme and developmental phase, due to differences in per capita grant provision (x_t). For instance, substituting x_t, from

Table 3 column 2, in (18) gives the individuals' optimal rules under the universal grant program, from Stage I-IV; substituting x_t from column 3 and column 4 give the optimal rules under the scholarship and means tested programs, respectively.

5. Higher Education Policies and Enrollment Rates

By comparing the investment thresholds associated to the different grant schemes with the laissez faire one, we can study how different higher education policies affect college enrollment rate. **Because access to college is categorized based on class in each stages of development, we make the comparison of the threshold associated to each type of group of individuals within the same stage.** In Stage I, for instance, only the elite have access to higher education; thus, we can examine how a given policy (vis-a-vis laissez faire) affect their likelihood to enroll in higher education. Similarly, in Stage II, individuals with regular ability but from affluent families have access to higher education. In Stage III and IV, high and regular ability individuals from poor families, respectively, will have access to college. The question is that: how does a given policy impact the threshold investment associated to *each type* of group of individuals?

The investment threshold related to the different grant schemes at different stages of development are derived in Appendix B, by combining Table 3 and eq. (18). The following proposition follows from that:

Proposition 2.

1. *Stage I: universal and scholarship programs have similar positive effect in enrollment; means tested has a negative effect.*
2. *Stage II: enrollment increases in universal grant but decreases in other policies.*
3. *Stage III: Means tested is the first best in increasing enrollment; scholarship and universal are the second and third best, respectively.*
4. *Stage IV: enrollment increases in universal grant but decreases in other policies.*

In Stage I, individuals who are likely to enroll in college do not qualify in the means-tested scheme despite they pay taxes. In the universal and scholarship schemes, they are better off compared to the laissez faire because the tuition grant that they receive is higher than the taxes they pay. Note that in Stage II and IV, the investment threshold only holds for regular ability individuals and these individuals are not qualified for the scholarship and means-tested schemes despite they pay taxes. They are thus better off with the universal grant, as they receive more grant than the amount of taxes they pay. In Stage II, the additional fund comes from those who do not enroll in college; in Stage IV, it comes from individuals with rich background (capital tax revenue). In Stage III, the investment threshold holds for high ability individual but poor family background; means-test has the most effect as the whole fund is available for them. But in the scholarship (or universal) scheme the fund is distributed among a larger section of the society.

6. Phases of Higher Education Development with Government Intervention

In this section, we characterize the different phases of higher education development, under government intervention, in a similar fashion to one in the laissez faire. The difference from laissez faire is that this time the dynamics reflect the taxes that individuals pay and the tuition grant they receives under alternative grant schemes such as universal grant, scholarship and means-tested. Different grant schemes may have different implication to efficiency and equity due to differences in their exclusiveness and differences in their ability to mobilize resources from individuals who do not invest in college to those who do and in ability to mobilize resources from low ability to high ability individuals.

6.1. Universal Grant

From (12), (16c), (18) and Table 3 (*column 2*), the dynamics of aggregate human capital under the universal grant program at time $t+1$ are given by (see Appendix A):

$$h_{t+1} = \begin{cases} \lambda pb\left\{\epsilon^{g}\left(B'h_t - s\right)+1\right\} + b\epsilon^{g} z_t \text{ if } h_c^g < h_t < h_c^r \\ b\lambda\left\{\left(p\epsilon^{g} + (1-p)\,\epsilon^{r}\right)\left(B'h_t - s\right)+1\right\} + b\left(p\epsilon^{g} + (1-p)\,\epsilon^{r}\right) z_t \text{ if } h_c^r < h_t < h_n^g \\ b\left\{p\left[\epsilon^{g}\left((1-\theta)\,Ah_t - s\right)+1\right] + \lambda(1-p)\left[\epsilon^{r}\left(B'h_t - s\right)+1\right]\right\} + \vartheta b z_t/\omega \text{ if } h_n^g < h_t < h_n^r \\ b\left(p\epsilon^{g} + (1-p)\,\epsilon^{r}\right)\left(Ah_t - s\right) + b \text{ if } h_t > h_n^r \end{cases} \tag{19}$$

where

$$\omega \equiv \lambda + (1-\lambda)\,p$$
$$\vartheta \equiv p\epsilon^{g} + \lambda\epsilon^{r}\,(1-p)$$

Eq. (19), which is comparable to eq. (13), characterizes the dynamics of an economy that passes through four different phases of higher education development, under a universal tuition grant government scheme. First terms, from Stage I to III, in curly brackets, show fractions of *after tax* average income invested in education; second terms show the amount of tuition grant provided.[17] The last term is similar to that of the last term in eq. (13).

If the initial capital at the economy level is smaller than the minimum investment threshold (h_c^g), then no one in the economy will enroll in higher education (i.e., if $h_{t+1} = 1$ if $h < h_c^g$). However, the economy will be in Stage I if the current aggregate capital is greater than the minimum investment threshold ($h_t > h_c^g$). The threshold levels associated to different stages of development are derived in eq. (28). Using similar logic as in the laissez faire, we identify the *threshold for take off* ($h_t > h^{T\prime}$):

$$h^{T\prime} = \frac{b\lambda p}{b\lambda p\epsilon^{g}\left(B' + \theta A\right) - 1}\left(s\epsilon^{g} - 1\right) \tag{20}$$

[17] In Stage I and II, λp and λ individuals invest in education while each receives $z_t/\left(p\lambda\right)$ and z_t/λ tuition grant. In Stage III and IV, as more and more individuals invest in education, per capita tuition grant reduces to z_t/ω and z_t, respectively. ω is the number of eligible individuals for the grant in Stage III: ϑ shows that the grant is distributed to p poor and rich high ability individuals and $(1-p)\,\lambda$ rich and regular ability individuals.

In Stage I, only families with college education adults and highly talented children invest in education. The economy continues growing and other families will start to join in education investment (through productivity spillovers) ones economy's capital stock is sufficiently higher than the kickoff threshold ($h_t > h^{T\prime}$). If not, the dynamics could go back to the stable equilibrium, $h_{t+1} = 1$.

Since $h^{T\prime} < h^T$, take off starts earlier under the universal tuition grant than the laissez faire case. During the transition periods of the economy (Stage I to III), growth is relatively higher than the ones in laissez faire. The laissez faire conditions are inferior in every stages of higher education development process, except at the last stage where all individuals attend college, as seen from comparing the terms in the brackets in eq. (13) and (19). In the latter, additional resources are mobilized for the public program from individuals who are not attending college and consume the full amount of their income. Note only individuals who send their children to college bear the cost of tuition subsidy, those who do not invest in college education also share the burden.

The gain in efficiency by moving from the laissez faire to a universal grant program at different stages are given by the difference between aggregate capital investment in each stage:

$$
\begin{aligned}
&\textit{Stage I}\text{: } b\epsilon^g \left(1 - \chi p \lambda\right) z_t \\
&\textit{Stage II}\text{: } b\left(p\epsilon^g + (1-p)\,\epsilon^r\right)\left(1 - \lambda\chi\right) z_t \\
&\textit{Stage III}\text{:} bp\epsilon^g \left(1/\omega - 1\right) z_t + b\lambda\left(1-p\right)\epsilon^r \left(1/\omega - \chi\right) z_t \\
&\textit{Stage IV}\text{:} 0
\end{aligned}
\tag{21}
$$

where

$$
\chi \equiv \frac{\tau_w\left(1-\alpha\right) + \alpha\tau_y/\lambda}{\tau_w\left(1-\alpha\right) + \alpha\tau_y}
$$

where χz_t is the tax contribution by wealthy individuals.[18]

The gain in efficiency mainly comes from resource mobilization and redistribution

[18] Since before minus after tax income is: $Bh_t - B'h_t = \chi z_t$.

from those who do not invest in college to those who do.[19] In Stage I, for instance, the tax contribution by the $p\lambda$ elite is $p\lambda\chi z_t$ but the same individuals receive $z_t/p\lambda$ each or z_t in total. Similarly, in Stage II, the tax contributions by the wealthy λ individuals is $\lambda\chi z_t$ whereas the same individuals receive z_t/λ each or z_t in total. Therefore, first and second lines in (21) show the resources that are redistributed regressively to these sections of the society in the form of tuition grant. There are in general $1-p\lambda$ rich & poor individuals in Stages I[20] and $1-\lambda$ poor individuals in Stage II who pay taxes but do not invest in higher education. In Stage III, there are $(1-\lambda)(1-p)$ poor individuals who pay taxes but do not enroll in college. First and second terms capture *net* grant received by p high ability and $(1-p)\lambda$ regular ability rich individuals, respectively. **In Stage IV, all individuals who pay taxes also enroll in higher education. In general, the gain in efficiency would reduce when moving up of stages, which disappears eventually, as the number of college participants increases.**

6.2. Scholarship

With the scholarship program, the dynamics of aggregate human capital at time $t+1$ are then given by, from (12), (16c), (18), and Table 3, *column 3*, (see Appendix A):

$$h_{t+1} = \begin{cases} \lambda pb\left\{\epsilon^g\left(B'h_t - s\right)+1\right\} + b\epsilon^g z_t \text{ if } h_c^g < h_t < h_c^r \\ \lambda b\left\{\left[p\epsilon^g + (1-p)\,\epsilon^r\right]\left(B'h_t - s\right)+1\right\} + b\epsilon^g z_t \text{ if } h_c^r < h_t < h_n^g \\ b\left\{p\left[\epsilon^g\left((1-\theta)\,Ah_t - s\right)+1\right] + \lambda(1-p)\left[\epsilon^r\left(B'h_t - s\right)+1\right]\right\} + b\epsilon^g z_t \text{ if } h_n^g < h_t < h_n^r \\ b\left\{\left[p\epsilon^g + (1-p)\,\epsilon^r\right]\left(Ah_t - s\right)+1\right\} + b(1-p)\left(\epsilon^g - \epsilon^r\right) z_t \text{ if } h_t > h_n^r \end{cases} \tag{22}$$

[19] It is straightforward to see the first and the second equations are positive, in (21) since $\lambda\chi < 1$. To see the third equation is also positive, first rewerite it as as

$$\text{Stage III: } b(1-p)\,\epsilon^r\left(\lambda\Theta - \lambda\chi\right) z_t$$

where $\Theta \equiv \left(1 + p\epsilon^g(1-\lambda)/\left(\epsilon^r\lambda\right)\right)/\omega$. It can then be confirmed that $\lambda\Theta - \lambda\chi > 0$ if $\epsilon^g > \epsilon^r$.

[20] These are $(1-p)\lambda$ rich and $1-\lambda$ poor individuals.

Eq. (22) represents the different phases of higher education development when the government provides scholarship – a tuition grant that targets high ability individuals. First terms in curly brackets, in Stages I to III, first to third lines, show the after tax average income invested in education by λp, λ and ω individuals , respectively. The second term $b\epsilon^g z_t$ captures the total tuition grant that are provided to high ability individuals at each stage. Unlike the previous cases, in the last stage of development (Stage IV), there is a redistribution of resources from regular ability to high ability individuals.[21] Note also that in Stage I, aggregate human capital is similar to the universal grant case, due to similarity in the amount of per capita grant available during this time $(z_t/\lambda p)$. This also implies that the two economies face *similar take off condition*, defined in (20).[22]

The scholarship program is the most efficient one compared to a laissez faire and universal tuition subsidy due to its regressive nature. Comparing and contrasting (19) and (22), we see the latter is greater at every stage of development, except in the first stage where they are tied. There is a constant $b(1-p)(\epsilon^g - \epsilon^r) z_t$ gain in efficiency by moving from a universal education grant to a scholarship program, from Stage II to Stage IV:

$$\begin{aligned} &\textit{Stage I}: 0 \\ &\textit{Stage II} \text{ to } \textit{IV}: b(1-p)(\epsilon^g - \epsilon^r) z_t \end{aligned} \tag{23}$$

Such a gain comes from mobilizing resources to highly productive individuals, in the human capital production sector of the economy. As the skill gap $(\epsilon^g - \epsilon^r)$ widens it becomes more efficient to shift to the scholarship program; $1-p$ in (23) indicates that the extra resource comes from the regular ability sect of the population, as the economy is moving to the scholarship program.

To see the difference between the scholarship program and the laissez faire, add eq. (23), which is the difference between the universal subsidy and the laissez faire ones, to eq. (21), which is the difference between the universal subsidy and the laissez

[21] If $\epsilon^g = \epsilon^r$, aggregate investment in education becomes similar to the previous cases.

[22] The thresholds associated to the rest of Stages I to IV are given in (29).

faire conditions:

$$\begin{array}{l} \textit{Stage I}\colon b\epsilon^{g}\left(1-\chi p\lambda\right)z_{t} \\ \textit{Stage II}\colon b\left(p\epsilon^{g}+\left(1-p\right)\epsilon^{r}\right)\left(1-\lambda\chi\right)z_{t}+b\left(1-p\right)\left(\epsilon^{g}-\epsilon^{r}\right)z_{t} \\ \textit{Stage III}\colon\ z_{t}b\left(1-p\right)\left[\epsilon^{r}\left(\lambda\vartheta-1\right)+\left(\epsilon^{g}-\epsilon^{r}\lambda\chi\right)\right] \\ \textit{Stage IV}\colon\ b\left(1-p\right)\left(\epsilon^{g}-\epsilon^{r}\right)z_{t} \end{array} \tag{24}$$

We see immediately that the equations associated to Stage I and IV in (24) are all positive.[23]

6.3. Means-tested

Similarly from (12), (16c), (18), and Table 3 (*column 4*), the stages of higher education development for the case where government provides tuition subsidy based on both merit and need basis are given by (see Appendix A):

$$h_{t+1}=\begin{cases} \lambda pb\left\{\epsilon^{g}\left(B'h_{t}-s\right)+1\right\}\ \text{if } h_{c}^{g}<h_{t}<h_{c}^{r} \\ \lambda b\left\{\left(p\epsilon^{g}+\left(1-p\right)\epsilon^{r}\right)\left(B'h_{t}-s\right)+1\right\}\ \text{if } h_{c}^{r}<h_{t}<h_{n}^{g} \\ b\left\{p\left[\epsilon^{g}\left(\left(1-\theta\right)Ah_{t}-s\right)+1\right]+\lambda\left(1-p\right)\left[\epsilon^{r}\left(B'h_{t}-s\right)+1\right]\right\}+b\epsilon^{g}z_{t}\ \text{if } h_{n}^{g}<h_{t}<h_{n}^{r} \\ b\left\{\left(p\epsilon^{g}+\left(1-p\right)\epsilon^{r}\right)\left(Ah_{t}-s\right)+1\right\}+\left(1-p\right)b\left(\epsilon^{g}-\epsilon^{r}\right)z_{t}\ \text{if } h_{t}>h_{n}^{r} \end{cases} \tag{25}$$

As in the precious two cases, the first terms in the curly brackets show after tax average income while the second terms (if any) show total tuition grant. One may have noticed, in Stages I and II, because only rich individuals are investing in education, there are no tuition grants provided by government. These are the cases where the government collects taxes and the revenues are being "thrown to the ocean".[24] Of

[23]The equation in Stage III is greater than zero, given that $\lambda\vartheta>1$ and $\lambda\chi<1$, and hence:

$$\text{sign}\left(\lambda\vartheta-1\right)=\text{sign}\left(\epsilon^{g}-\epsilon^{r}\lambda\chi\right)=+$$

[24]It may be argued though, it would be counterintuitive for the government to conduct such distortionary policy. An alternative is then to consider rather the case where there is no any government involvment in Stage I and II but latter stages. In this case, in the first two Stages, aggregate capital dynamics is identical to the laizes faire conditions.

course, this would have the immediate effects of lowering aggregate efficiency in every aspect during these stages. As a result, the economy may take off much later than any of the earlier cases. The respective threshold to take off then easily computed as above:[25]

$$h^{T\prime\prime} = \frac{b\lambda p}{b\lambda p\epsilon^g B' - 1}\left(s\epsilon^g - 1\right) \tag{26}$$

Only in Stage III, individuals who are eligible to the grant begin to invest in education. At this stage and the next, government revenue will then be available as tuition grant for these households. It is interesting to note that, in Stage III and IV, aggregate capital under means tested is similar to that of the scholarship program. Therefore, basically there is no difference in the effects on aggregate efficiency between government provision of subsidy on a scholarship or a need & ability basis during these relatively advanced stages of higher education development.

Comparing and contrasting the programs with respect to their aggregate efficiency, we see the scholarship program is the most efficient education subsidy regardless of the higher education developmental stage. Table 2 below ranks the public programs based on their efficiency impact on each of the developmental phase.

Table 4: The ranking of different higher education programs based on their impacts on aggregate efficiency

	Laissez faire	**Universal grant**	**Scholarship**	**Means-tested**
Stage I	2nd	1st	1st	3nd
Stage II	3rd	2nd	1st	4th
Stage III	3rd	2nd	1st	1st
Stage IV	2nd	2nd	1st	1st

The Proposition summarizes Table 4 and the above discussion:

[25] The threshold levels associated to Stage I to IV are given in (30).

Proposition 3. 1. *Universal grant and scholarship are the most efficient ones in Stage I followed by laissez faire and means-tested.*

2. *In Stage II, means-tested are the least efficient whereas scholarship is the most efficient followed by universal grant as the second best efficient.*
3. *In Stage III and IV, the scholarship and means-tested programs are the first best.*
4. *In Stage III, universal grant is the second and laissez faire is the last whereas they are tied in Stage IV.*

7. Inequality

7.1. The Lorenz Curve

In analyzing the effects of different higher education policies in inequality, we construct the Lorenz curves associated to each phases of higher education development that capture the cumulative aggregate wealth (human capital) ratio of the respective cumulative population. Recall first that at time t, the size of the population is unity and then individuals can be categorized into four classes. These are the bottom $(1-\lambda)(1-p)$ *poor* individuals, the $(1-\lambda)\,p$ *lower middle class* individuals, the $(1-p)\,\lambda$ *middle class* individuals and the top $p\lambda$ *upper class* individuals.[26]

[26] Alternative naming of classes may be adopted.

Table 5: Lorenz curve ratios

	Cumulative aggregate human capital ratios			
Cumulative population ratios	*Stage I*	*Stage II*	*Stage III*	*Stage IV*
0	0	0	0	0
$(1-\lambda)(1-p)$	0	0	0	$\frac{(1-\lambda)(1-p)h^r_{nt+1}}{\lambda\left[ph^g_{ct+1}+(1-p)h^r_{ct+1}\right]+(1-\lambda)\left[ph^g_{nt+1}+(1-p)h^r_{nt+1}\right]}$
$1-\lambda$	0	0	$\frac{(1-\lambda)ph^g_{nt+1}}{\lambda\left[ph^g_{ct+1}+(1-p)h^r_{ct+1}\right]+(1-\lambda)ph^g_{nt+1}}$	$\frac{(1-\lambda)\left(ph^g_{nt+1}+(1-p)h^r_{nt+1}\right)}{\lambda\left[ph^g_{ct+1}+(1-p)h^r_{ct+1}\right]+(1-\lambda)\left[ph^g_{nt+1}+(1-p)h^r_{nt+1}\right]}$
$1-\lambda p$	0	$\frac{\lambda(1-p)h^r_{ct+1}}{\lambda ph^g_{ct+1}+\lambda(1-p)h^r_{ct+1}}$	$\frac{(1-\lambda)ph^g_{nt+1}+\lambda(1-p)h^r_{ct+1}}{\lambda\left[ph^g_{ct+1}+(1-p)h^r_{ct+1}\right]+(1-\lambda)ph^g_{nt+1}}$	$\frac{\lambda(1-p)h^r_{ct+1}+(1-\lambda)\left(ph^g_{nt+1}+(1-p)h^r_{nt+1}\right)}{\lambda\left[ph^g_{ct+1}+(1-p)h^r_{ct+1}\right]+(1-\lambda)\left[ph^g_{nt+1}+(1-p)h^r_{nt+1}\right]}$
1	1	1	1	1

The first column in Table 5 shows the cumulative ratios of the population, which is ranked in increasing size whereas the rest of the columns show the corresponding cumulative ratios of aggregate human capital in different stages. In Stage I, investment is made only by the top λp individuals while investment by the rest of the population is zero; therefore, the cumulative aggregate human capital ratio is either zero or unity.

In Stage II, $\lambda p h^g_{ct+1} + \lambda (1-p) h^r_{ct+1}$ human capital investment is made by the λ rich (middle and upper class) individuals while the rest of the population below them does not invest in education. Thus, the share of the $\lambda (1-p)$ middle class individuals who invest in education is given by the second raw from the last. This is also the cumulative ratio for the $1-\lambda p$ size of the population, since the $1-\lambda$ (poor & lower middle class) individuals below them do not invest in education.

In Stage III, the total investment is the sum of investments by the λ (middle & upper class) individuals and $(1-\lambda) p$ lower middle class individuals (as shown in the denominators, in Stage III). The cumulative investment of the poor & lower middle class individuals, $1-\lambda$, is simply the investment made by the $(1-\lambda) p$ lower middle class individuals which is $(1-\lambda) p h^g_{nt+1}$ as shown in the nominator, third from the last raw. $1-\lambda p$ is the cumulative population ratio of poor & lower middle class $(1-\lambda)$ plus middle class $(\lambda (1-p))$ individuals. Their cumulative aggregate investment ratio is shown, second from the last raw.

In Stage IV, every individual in the economy invests in education, as shown in the denominators of the last column. The fourth raw from the last, shows the total investment by the poor, $(1-\lambda)(1-p)$; the second from the last, the cumulative investment ratio by the poor & lower middle class individuals, $1-\lambda$; the second from the last, the cumulative ratio by the poor, middle & upper class individuals, $1-\lambda p$.

7.2. Tuition Grant and Inequality

Definition 1. *Let v and w represent certain distributions and $L(v)$ and $L(w)$ are the associated Lorenz curves, respectively. If*

$$L(v) \geq L(w)$$

then we say $L(v)$ is Lorenz dominance of $L(w)$.[27]

Since we have closed form solutions for all the values in Table 5, computing the Lorenz curves for each stage is straightforward and presented in Appendix C. In particularly, Table 7 to 9 provide the respective Lorenz curves for Stage II to IV, associated to the laissez faire and different tuition grant.

Stage I. In the elite stage, all human capital investment are made by the top λp percentile of the population;[28] therefore, the economy will remain perfectly unequal as, when only the upper class invest in education. Regardless of the tuition subsidy program implemented at this stage at time t, Lorenz inequality will not change in the next period. But of course, compared to laissez faire or means tested, in the scholarship and universal grant programs, the rich become more richer but in all cases they represent the entire investment in higher education.

Stage II. In this stage, education investment is made by the λ percentile of the population. Therefore, *basically* comparison is made between the middle class $\lambda(1-p)$ and upper class λp percentiles. Because, the rest $1-\lambda$ percentile does not invest in education.

Proposition 4. *Given $\epsilon^g > \epsilon^r$, in Stage II, the effects of higher policies in inequality can be ranked as:*

	Laissez faire	***Universal***	***Scholarship***	***means tested***
Rank	*2nd*	*3rd*	*4th*	*1st*

Means tested tops in terms of reducing inequality followed by laissez faire and then universal grant. Means tested, on the contrary, leaves every one worse off, as non of the groups who invest in education at this stage qualify for the program. Even though all pay tax and none are qualified to the grant scheme, taxation seems to hurt the high ability individuals more. At this early phases of higher education development, even if all groups are equally benefitted from a program (like in the

[27] See Davies and Hoy (1995) for more in the subject.

[28] Of course, λp is a fraction and it should be multiplied by 100 to call it (strictly) a 'percentile'.

universal grant scheme), then laissez faire would be better when easing inequality is the policy target. Although universal grant benefits the middle class ($100\lambda(1-p)$ percentile), it benefits more than proportional the top class ($100\lambda p$ percentile). Apparently a scholarship program is quite regressive and benefits only those individuals with high ability (the top $100\lambda p$ percentile). The higher the difference in ability, the higher the regressivity of the policy becomes.

Stage III. In this stage, both poor and rich individuals invest in higher education; analytical comparison of the effect of all programs on inequality may not thus possible. But, comparison can be made analytically between means-tested and scholarship where the former is found the better policy in term of reducing inequality. This can be seen by comparing the respective columns in Table 8; in all cases the nominators are relatively greater for means tested while the denominators are smaller.

Stage IV . At this advanced stage all individuals invest in the economy. Table 6 below summarizes the ranks of the different public programs based on their Lorenz dominance at this stage:

Table 6: Ranking of the public programs based on Lorenze dominance

	Laissez faire	**Universal**	**Scholarship**	**Means-tested**
$(1-\lambda)(1-p)$	2nd	1st	3rd	3rd
$1-\lambda$	3rd	2nd	1st	1st
$1-\lambda p$	4th	3rd	2nd	1st

Note: 1st implies a greatest lorenze dominance while 4th is the least.

We can make easy comparison between universal grant and laissez faire, and, between means-tested and scholarship as one Lorenz dominates for every quantile of the population:

Proposition 5. 1. *The distribution in the universal grant scheme Lorenz dominates the distribution in laissez faire.*

2. *The distribution under mean-tested Lorenz dominates the distribution in the scholarship program.*

These results are quite intuitive. In the case where all individuals are eligible for tuition subsidy (universal grant) and have access to college education, basically labor tax has *no any effect* on individual human capital investment; but, capital tax has a direct redistributive effect. This is because each individual pays a labor tax and receives it back in the form of tuition grant. We can see that immediately by rewriting (18), using Table 3, as follows:

$$h_{it+1}^{j} = \begin{cases} \epsilon^{j} b\left(\omega_t + \tau_y \phi h_t - s\right) + b \text{ if } h_{it} = 1 \\ \epsilon^{j} b\left(\omega_t + \left[1 - (1-\lambda)\tau_y\right]\phi h_t/\lambda - s\right) + b \text{ if } h_{it} \neq 1 \end{cases} \tag{27}$$

where

$$\psi \equiv 1 - (1-\lambda)\tau_y$$

As we see from (27), the labor tax has disappeared from the equation. We also see each of the $1-\lambda$ non-educated parents are now subsidized by an amount of $\tau_y \phi h_t$ (the first equation). This is paid by λ number of college-educated household heads, in the amount of $\tau_y \phi h_t (1-\lambda)/\lambda$ per head. Thus, the term in the square bracket in the second equation is the left over of a dollar of a skill premium after tax deduction. Therefore, subsidizing tuition fee under universal grant at this stage is nothing but redistribution of income from skilled to unskilled households:

$$\underbrace{1-\lambda}_{\substack{\text{Number of} \\ \text{unskilled parents}}} \times \underbrace{\tau_y \phi h_t}_{\substack{\text{Subsidy received} \\ \text{per head}}} = \underbrace{\tau_y \phi h_t (1-\lambda)/\lambda}_{\substack{\text{Skill premium} \\ \text{paid per head}}} \times \underbrace{\lambda}_{\substack{\text{Number of} \\ \text{skilled parents}}}$$

In comparing the rest of the programs, as shown in Table 6, we have the following Propositions:

Proposition 6. 1. *Universal grant Lorenz dominates scholarship for the bottom* $100(1-\lambda)(1-p)$ *percent of the population.*

2. *But scholarship Lorenz dominates universal grant for the poor & middle-class,* $100(1-\lambda)$ *percent of the population.*
3. *For the* $100(1-p\lambda)$ *percent of the population, scholarship Lorenz dominates universal grant if* $\epsilon^g(1-\lambda) > \epsilon^r$.

The follows Corollary follows from Proposition 5 and 6:

Corollary 1. 1. *The same relationship holds as in the last Proposition 6 when comparing universal grant with means-tested programs except that a more weaker condition than* $\epsilon^g(1-\lambda) > \epsilon^r$ *may be required.*

2. *Both scholarship and means tested programs Lorenz dominate laissez faire for* $100(1-\lambda)$ *and* $100(1-p\lambda)$ *percent of the population while they are both Lorenz dominated for the poorest section of the society,* $100(1-\lambda)(1-p)$.

Therefore, at the more advanced stage, the poorest section of the society are relatively better off from a universal grant; both means-tested and scholarship seem to benefit disproportionately the poor & lower middle sections of the society. In terms of income redistribution from the top class to the rest, means tested seems the most effective policy.

8. Conclusion

References

Abbott, B., G. Gallipoli, C. Meghir, and G. L. Violante (2013): "Education policy and intergenerational transfers in equilibrium," Discussion paper, National Bureau of Economic Research.

Acemoglu, D., and J. Angrist (2000): "How large are human-capital externalities? Evidence from compulsory schooling laws," *NBER macroeconomics annual*, 15, 9–59.

Banerjee, A. V., and A. F. Newman (1993): "Occupational Choice and the Process of Development," *The Journal of Political Economy*, 101(2), 274–298.

Barham, V., R. Boadway, M. Marchand, and P. Pestieau (1995): "Education and the poverty trap," *European Economic Review*, 39(7), 1257–1275.

Benabou, R. (2000): "Unequal Societies: Income Distribution and the Social Contract," *The American Economic Review*, 90(1), 96–129.

——— (2002): "Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?," *Econometrica*, 70(2), 481–517.

Benhabib, J., and M. M. Spiegel (1994): "The role of human capital in economic development evidence from aggregate cross-country data," *Journal of Monetary economics*, 34(2), 143–173.

Bovenberg, A. L., and B. Jacobs (2005): "Redistribution and education subsidies are Siamese twins," *Journal of Public Economics*, 89(11-12).

Davies, J., and M. Hoy (1995): "Making inequality comparisons when Lorenz curves intersect," *The American Economic Review*, pp. 980–986.

De Fraja, G. (2002): "The design of optimal education policies," *The Review of Economic Studies*, 69(2), 437–466.

Del Rey, E., and M. Racionero (2010): "Financing schemes for higher education," *European Journal of Political Economy*, 26(1), 104–113.

Fender, J., and P. Wang (2003): "Educational policy in a credit constrained economy with skill heterogeneity," *International Economic Review*, 44(3), 939–964.

Fernandez, R., and R. Rogerson (1995): "On the political economy of education subsidies," *The Review of Economic Studies*, 62(2), 249–262.

Galor, O., and O. Moav (2004): "From Physical to Human Capital Accumulation: Inequality and the Process of Development," *Review of Economic Studies*, 71(4), 1001–1026.

Galor, O., and J. Zeira (1993): "Income Distribution and Macroeconomics," *The Review of Economic Studies*, 60(1), 35–52.

Garcia-Penalosa, C., and K. Wälde (2000): "Efficiency and equity effects of subsidies to higher education," *Oxford Economic Papers*, 52(4), 702–722.

Gary-Bobo, R. J., and A. Trannoy (2015): "Optimal student loans and graduate tax under moral hazard and adverse selection," *The RAND Journal of Economics*, 46(3), 546–576.

Geven, K. (2015): "How Did the Latest Increase in Fees in England Affect Student Enrolment and Inequality?," in *The European Higher Education Area*, pp. 479–500. Springer.

Glomm, G., and B. Ravikumar (1992): "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," *The Journal of Political Economy*, 100(4), 818–834.

Hansen, W. L., and B. A. Weisbrod (1969): "The distribution of costs and direct benefits of public higher education: the case of California," *Journal of human resources*, pp. 176–191.

Heijdra, B. J., F. Kindermann, and L. S. Reijnders (2017): "Life in shackles? The quantitative implications of reforming the educational financing system," *Review of Economic Dynamics*, 25, 37–57.

Krueger, A. B., and M. Lindahl (2001): "Education for growth: Why and for whom?," *Journal of economic literature*, 39(4), 1101–1136.

Krueger, D., and A. Ludwig (2016): "On the optimal provision of social insurance: Progressive taxation versus education subsidies in general equilibrium," *Journal of Monetary Economics*, 77, 72 – 98.

Moav, O. (2002): "Income distribution and macroeconomics: the persistence of inequality in a convex technology framework," *Economics Letters*, 75(2), 187–192.

MORETTI, E. (2004): "Human capital externalities in cities," in *Handbook of regional and urban economics*, vol. 4, pp. 2243–2291. Elsevier.

ODED GALOR, D. N. W. (2000): "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and beyond," *The American Economic Review*, 90(4), 806–828.

ROMER, P. M. (1986): "Increasing Returns and Long-run Growth," *Journal of Political Economy*, 94(5), 1002–1037.

TROW, M. (1973): "Problems in the transition from elite to mass higher education.," .

——— (2007): "Reflections on the transition from elite to mass to universal access: Forms and phases of higher education in modern societies since WWII," in *International handbook of higher education*, pp. 243–280. Springer.

Appendix

A. Aggregation

In deriving (13), we take into consideration that any child could have a gifted or a regular parent, as any parent could have a gifted or regular child. To derive the first line of the equation, first, substitute (7) into the first term of (12) to obtain:

$$\begin{aligned} h_{t+1} &= \lambda p h^g_{ct+1} \\ &= \lambda p \left[\epsilon^g b \left(\omega_t + \phi h^j_{it} - s\right) + b\right] \end{aligned}$$

But considering that the parent itself could be gifted (or not) with probability p $(1-p)$, this becomes:

$$h_{t+1} = \lambda p \left[\epsilon^g b \left(\omega_t + \phi\lambda \left[p h^g_{ct} + (1-p)\, h^r_{ct}\right]/\lambda - s\right) + b\right]$$

Since, considering (12), $h_t = \lambda \left[p h^g_{ct} + (1-p)\, h^r_{ct}\right]$, thus

$$h_{t+1} = \lambda p \left[\epsilon^g b \left(\omega_t + \phi h_t/\lambda - s\right) + b\right]$$

using (9), we get the second equation in (13). Similar procedures can be used to derive the rest of the equations.

We derive eq.(18), second line as follows from (1) (9), (16b), (17) and (14),

$$\begin{aligned} h^j_{t+1} &= \epsilon^j \left(b \left((1-\tau_w)\, \omega_t + (1-\tau_y)\, \phi h^j_{it} \; - (1-T_t)\, s\right) - b/\beta\epsilon^j\right) + 1 \text{ if } h_{it} \neq 1 \\ &= \epsilon^j b \left(\omega_t + (1-\tau_y)\, \phi h^j_{it} \; - s + \tau_y \phi h_t\right) + b \text{ if } h_{it} \neq 1 \end{aligned}$$

Given that we have λ educated individuals, $h_t = \lambda h^j_{it}$

$$h^j_{t+1} = \epsilon^j b \left(\omega_t + (1 \; - \tau_y\,(1-\lambda))\, \phi h^j_{it} - s\right) + b \text{ if } h_{it} \neq 1$$

In deriving the first line of (19), note that in Stage I, only λp number of high ability individuals from college educated parents have access to higher education.

Therefore, from (12), the aggregate human capital, in Stage I, is given by

$$h_{t+1} = \lambda p h^g_{ct+1}$$

Then, substitute the second equation from (18) into the above to obtain

$$h_{t+1} = \lambda p \left[\left(\epsilon^g b \left(B' h_t - s' \right) + b \right) \right]$$

Under the universal grant anyone who enrolls to college is eligible for tuition grant; therefore, given (16c), $s' = s - x_t$ where x_t is given by (from Table 1, *column & row 2*) $x_t = z_t / \lambda p$. Substituting that into the above gives the first equation in (18):

$$\begin{aligned} h_{t+1} &= \lambda p \left[\left(\epsilon^g b \left(B' h_t - s + z_t / \lambda p \right) + b \right) \right] \\ &= \lambda p b \left\{ \epsilon^g \left(B' h_t - s \right) + 1 \right\} + b \epsilon^g z_t \end{aligned}$$

Similar procedures can be used to derive the rest of the equations, Eqs. (22) and (25).

B. Human Capital Investment Thresholds

B.1. Investment Thresholds under Education Policy

We compute here the threshold levels of aggregate human capital beyond which parents begin to invest in their children's education for different higher education policy scheme. The per capita tuition grant is different not only at different phases of higher education but also for different grant schemes. This is because the number of eligible individuals depends on both the type of the scheme and the enrollment rate where the latter depends at the economy's stage of development. By construction, the enrollment rate at each stage, from Stage I to IV, are λp, λ, $\lambda + (1 - \lambda) p$ and 1, respectively.

The three columns in Table3 associate to the per capita tuition provided in each of the three grant schemes. By substituting column 1 to 3 into (18), one then could compute the threshold levels associated to the universal, scholarship and means tested grant schemes respectively, in a similar fashion as in (10).

Universal Grant:. Given (9), (18) and Table3, column 2:

$$\overline{\overline{h}}_c^j \equiv \left(\frac{1}{\epsilon^j \beta} + s\right) \left(B' + A\theta/\kappa^u\right)^{-1} \tag{28a}$$

$$\overline{\overline{h}}_n^j \equiv \left(\frac{1}{\beta \epsilon^j} + s\right) \left[A' + A\theta/\kappa^u\right]^{-1} \tag{28b}$$

where $\overline{\overline{h}}_i^j$ is the threshold associated to the ith person of j ability if the grant scheme is universal. $\kappa^u \in \{\lambda p,\ \lambda,\ \lambda + (1-\lambda)\, p, 1\}$ is the number of eligible individuals at each stage, from Stage I to IV, respectively. The enrollment rate at each stage is similar to the number of eligible individuals for this scheme because anyone who enrolls to college is automatically eligible for a tuition grant according to this scheme.

Scholarship:. Given (9), (18) and Table3, column 3:

$$\widehat{h}_c^g \equiv \left(\frac{1}{\epsilon^g \beta} + s\right) \left(B' + A\theta/\kappa^s\right)^{-1} \tag{29a}$$

$$\widehat{h}_c^r \equiv \left(\frac{1}{\epsilon^r \beta} + s\right) B'^{-1} \tag{29b}$$

$$\widehat{h}_n^g \equiv \left(\frac{1}{\beta \epsilon^g} + s\right) \left[A' + A\theta/\kappa^s\right]^{-1} \tag{29c}$$

$$\widehat{h}_n^r \equiv \left(\frac{1}{\beta \epsilon^r} + s\right) A'^{-1} \tag{29d}$$

where $\widehat{h}_i^j$ is the threshold associated to the ith person of j ability if the grant scheme is scholarship. where $\kappa^s \in \{\lambda p,\ \lambda p,\ p,\ p\}$ is the number of eligible individuals at each stage, from Stage I to IV, respectively. Note that the number of eligible individuals are different from the enrollment rate here as the scheme naturally excludes some individuals. In the first Stage, λp individuals enroll where all are eligible for the grant. In Stage II, λ rich individuals enroll but only the λp rich and high ability individuals are eligible. In Stage III and IV, $\lambda + (1-\lambda)\, p$ and 1 individuals enroll, respectively, but only the p high ability individuals are eligible.

Means-tested:. Given (9), (18) and Table3, column 3:

$$\widetilde{h}_c^j = \left(\frac{1}{\epsilon^j \beta} + s\right) B'^{-1} \tag{30a}$$

$$\widetilde{h}_n^g = \left(\frac{1}{\beta \epsilon^g} + s\right) \left[A' + A\theta / \left((1-\lambda)\, p\right)\right]^{-1} \tag{30b}$$

$$\widetilde{h}_n^r = \left(\frac{1}{\beta \epsilon^r} + s\right) A'^{-1} \tag{30c}$$

where $\widetilde{h}_i^j$ is the threshold associated to the ith person of j ability if the grant scheme is means-tested. Note that no one is eligible in this scheme in Stage I and II, ; but, in Stage III and IV, $(1-\lambda)\, p$ high ability poor individuals are eligible.

C. Inequality

C.1. The Lorenz curves

We construct here the Lorenz curves associated to different stages of higher education development.

Stage I. Apparently in Stage I, there is a *perfect inequality* between the classes, with a unity of Gini a coefficient, as all investments are made by the top λp percentile of the population.

Stage II. We build the Lorenz curves for Stage II, from Table 5, column 3, eqs. (9), (18) and Table 3.

Stage III. From Table 5, column 4, eqs. (9), (18) and Table 3, we construct the Lorenz curves associated to Stage III.

Stage IV. From Table 5, column 5, eqs. (9), (18) and Table 3, we construct the Lorenz curves associated to Stage IV:

Table 7: Lorenz curves associated to faizzes faire and different grant schemes in Stage II

	Cumulative aggregate human capital ratios			
Cumulative population ratios	**Laissez faire**	**Universal**	**Scholarship**	**Means tested**
0	0	0	0	0
$(1-\lambda)(1-p)$	0	0	0	0
$1-\lambda$	0	0	0	0
$1-\lambda p$	$\frac{(1-p)[\epsilon^r(Bh_t-s)+1]}{[p\epsilon^g+(1-p)\epsilon^r](Bh_t-s)+1}$	$\frac{(1-p)\left[\epsilon^r\left(B'h_t-s+\frac{z_t}{\lambda}\right)+1\right]}{[p\epsilon^g+(1-p)\epsilon^r]\left(B'h_t-s+\frac{z_t}{\lambda}\right)+1}$	$\frac{(1-p)[\epsilon^r(B'h_t-s)+1]}{[p\epsilon^g+(1-p)\epsilon^r](B'h_t-s)+\epsilon^g\frac{z_t}{\lambda}+1}$	$\frac{(1-p)[\epsilon^r(B'h_t-s)+1]}{[p\epsilon^g+(1-p)\epsilon^r](B'h_t-s)+1}$
1	1	1	1	1

Table 8: Lorenz curves associated to faizzes faire and different grant schemes in Stage III

	Cumulative aggregate human capital ratios	
Cumulative population ratios	**Laissez faire**	**Universal**
0	0	0
$(1-\lambda)(1-p)$	0	0
$1-\lambda$	$\frac{(1-\lambda)p[\epsilon^g((1-\alpha)Ah_t-s)+1]}{p(\epsilon^g(Ah_t-s)+1)+\lambda(1-p)(\epsilon^r(Bh_t-s)+1)}$	$\frac{(1-\lambda)p[\epsilon^g(A'h_t-s)+1]+(1-\lambda)p\epsilon^g z_t/\omega}{p[\epsilon^g(Ah_t-s)+1]+\lambda(1-p)[\epsilon^r(B'h_t-s)+1]+\varrho z_t/\omega}$
$1-\lambda p$	$\frac{(1-\lambda)p(\epsilon^g((1-\alpha)Ah_t-s)+1)+\lambda(1-p)(\epsilon^r(Bh_t-s)+1)}{p(\epsilon^g(Ah_t-s)+1)+\lambda(1-p)(\epsilon^r(Bh_t-s)+1)}$	$\frac{(1-\lambda)p(\epsilon^g(A'h_t-s)+1)+\lambda(1-p)(\epsilon^r(B'h_t-s)+1)+\psi z_t/\omega}{p[\epsilon^g(Ah_t-s)+1]+\lambda(1-p)[\epsilon^r(B'h_t-s)+1]+\varrho z_t/\omega}$
1	1	1

	Cumulative aggregate human capital ratios	
Cumulative population ratios	**Scholarship**	**Means tested**
0	0	0
$(1-\lambda)(1-p)$	0	0
$1-\lambda$	$\frac{(1-\lambda)p[(A'h_t-s)+1]+(1-\lambda)\epsilon^g z_t}{p[\epsilon^g(Ah_t-s)+1]+\lambda(1-p)[\epsilon^r(B'h_t-s)+1]+(1-p)\epsilon^g z_t}$	$\frac{(1-\lambda)p[\epsilon^g(A'h_t-s)+1\ ]+\epsilon^g z_t}{p[\epsilon^g(Ah_t-s)+1]+\lambda(1-p)[\epsilon^r(B'h_t-s)+1]+(1-p)\epsilon^g z_t}$
$1-\lambda p$	$\frac{(1-\lambda)p(\epsilon^g(A'h_t-s)+1)+\lambda(1-p)(\epsilon^r(B'h_t-s)+1)+(1-\lambda)\epsilon^g z_t}{p[\epsilon^g(Ah_t-s)+1]+\lambda(1-p)[\epsilon^r(B'h_t-s)+1]+(1-p)\epsilon^g z_t}$	$\frac{(1-\lambda)p(\epsilon^g b(A'h_t-s)+1)+\lambda(1-p)(\epsilon^r b(B'h_t-s)+1)+\epsilon^g z_t}{p[\epsilon^g(Ah_t-s)+1]+\lambda(1-p)[\epsilon^r(B'h_t-s)+1]+(1-p)\epsilon^g z_t}$
1	1	1

Table 9: Lorenz curves associated to faizzes faire and different grant schemes in Stage IV

	Cumulative aggregate human capital ratios	
Cumulative pop ratios	**Laissez faire**	**Universal**
0	0	0
$(1-\lambda)(1-p)$	$\frac{(1-\lambda)(1-p)\epsilon^r[((1-\alpha)Ah_t-s)+1]}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+1}$	$\frac{(1-\lambda)(1-p)\{[\epsilon^r(A'h_t-s)+1]+\epsilon^r z_t\}}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+1}$
$1-\lambda$	$\frac{(1-\lambda)\{[p\epsilon^g+(1-p)\epsilon^r]((1-\alpha)Ah_t-s)+1\}}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+1}$	$\frac{(1-\lambda)\{[p\epsilon^g+(1-p)\epsilon^r](A'h_t+z_t-s)+1+[p\epsilon^g+(1-p)\epsilon^r]z_t\}}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+1}$
$1-\lambda p$	$\frac{(1-p)\epsilon^r(Ah_t-s)+(1-\lambda)p\epsilon^g((1-\alpha)Ah_t-s)+1-\lambda p}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+1}$	$\frac{(1-p)\epsilon^r[Ah_t-s]+(1-\lambda)p\epsilon^g(A'h_t-s)+1-\lambda p+(1-\lambda)p\epsilon^g z_t}{(p\epsilon^g+(1-p)\epsilon^r)(Ah_t-s)+1}$
1	1	1

	Cumulative aggregate human capital ratios	
Cumulative pop ratios	**Scholarship**	**Means tested**
0	0	0
$(1-\lambda)(1-p)$	$\frac{(1-\lambda)(1-p)[\epsilon^r(A'h_t-s)+1]}{[p\epsilon^g+(1-p)\epsilon^r](Ah_t-s)+(1-p)(\epsilon^g-\epsilon^r)z_t+1}$	$\frac{(1-\lambda)(1-p)(\epsilon^r(A'h_t-s)+1)}{[p\epsilon^g+(1-p)\epsilon^r](Ah_t-s)+(1-p)(\epsilon^g-\epsilon^r)z_t+1}$
$1-\lambda$	$\frac{(1-\lambda)\{([p\epsilon^g+(1-p)\epsilon^r](A'h_t-s)+1)+\epsilon^g z_t\}}{[p\epsilon^g+(1-p)\epsilon^r](Ah_t-s)+(1-p)(\epsilon^g-\epsilon^r)z_t+1}$	$\frac{(1-\lambda)([p\epsilon^g+(1-p)\epsilon^r](A'h_t-s)+1)+\epsilon^g z_t}{[p\epsilon^g+(1-p)\epsilon^r](Ah_t-s)+(1-p)(\epsilon^g-\epsilon^r)z_t+1}$
$1-\lambda p$	$\frac{(1-p)\epsilon^r(Ah_t-s)+(1-\lambda)p\epsilon^g(A'h_t-s)+1-\lambda p+\Omega z_t}{[p\epsilon^g+(1-p)\epsilon^r](Ah_t-s)+(1-p)(\epsilon^g-\epsilon^r)z_t+1}$	$\frac{(1-p)\epsilon^r[Ah_t-s]+(1-\lambda)p\epsilon^g(A'h_t-s)+1-\lambda p+\Omega' z_t}{[p\epsilon^g+(1-p)\epsilon^r](Ah_t-s)+(1-p)(\epsilon^g-\epsilon^r)z_t+1}$
1	1	1

where

$$
\begin{aligned}
\varrho &\equiv p\epsilon^g (1-\omega) + \lambda\epsilon^r (1-p) \\
&= (1-p)\left[(1-\lambda)\, p\epsilon^g + \lambda\epsilon^r\right] \\
\psi &\equiv p\epsilon^g (1-\lambda) + \lambda\epsilon^r (1-p) \\
\Omega &\equiv (1-\lambda)\,\epsilon^g - (1-p)\,\epsilon^r = \Omega' - \lambda\epsilon^g \\
\Omega' &\equiv \epsilon^g - (1-p)\,\epsilon^r
\end{aligned}
$$

Note that, in Stage IV, from the last equations in (22) and (25), the denominators in scholarship and means tested schemes, and, from the last equations in (13) and (19), the denominators for laissez faire and universal grants are the same.

D. Proofs for Propositions

D.1. Proposition 1

Proof.

$\overline{h}_i^j$ decreases in β, A and ϵ^j and increases in s. Also, $\overline{h}_n^j$ decreases in $1-\alpha$.

■

D.2. Proposition 2

Proof.

Comparing the thresholds associated to laissez faire ($\overline{h}_i^j$), universal ($\overline{\overline{h}}_i^j$), scholarship ($\widehat{h}_i^j$) and means-tested ($\widetilde{h}_i^j$) schemes in (10), (28), (29) and (30), respectively: (1) $\widetilde{h}_c^g > \overline{h}_c^g > \overline{\overline{h}}_c^g = \widehat{h}_c^g$; i.e., the threshold for means tested at this stage is the largest followed by laissez faire (2) $\widehat{h}_c^g = \widetilde{h}_c^g > \overline{h}_c^r > \overline{\overline{h}}_c^r$; i.e., the thresholds for the universal grant at this stage is the least followed by laissez faire (3) $\overline{h}_n^g > \overline{\overline{h}}_n^g > \widehat{h}_n^g > \widetilde{h}_n^g$; the threshold for means tested is the smallest followed by the threshold for scholarship; the one for laissez faire is the largest (4) $\widehat{h}_n^r = \widetilde{h}_n^r > \overline{h}_n^r > \overline{\overline{h}}_n^r$; the threshold for universal grant is the smallest followed by the one for laissez faire.

■

D.3. Proposition 3

Proof.

It is straightforward that follows from Table 4.

■

D.4. Proposition 4

Proof.

Let the Lorenz curves associated to the laissez faire, universal grant, scholarship and means tested are defined as $L(l)$, $L(u)$, $L(s)$ and $L(m)$, respectively. Then from Table 7, given, $\epsilon^g > \epsilon^r$, we want to prove that for the $100\,(1-\lambda p)$ percent (since the rest are zero):

$$L^{II}(m) > L^{II}(l) > L^{II}(u) > L^{II}(s) \tag{31}$$

where the superscript in $L^{II}(.)$ implies the Lorenz curve is associated to Stage II.

$$\begin{aligned}
L^{II}(l) &= \frac{(1-p)\left[\epsilon^r\left(Bh_t - s\right)+1\right]}{\left[p\epsilon^g + (1-p)\,\epsilon^r\right]\left(Bh_t - s\right)+1}\\
L^{II}(u) &= \frac{(1-p)\left[\epsilon^r\left(B'h_t - s + \frac{z_t}{\lambda}\right)+1\right]}{\left[p\epsilon^g + (1-p)\,\epsilon^r\right]\left(B'h_t - s + \frac{z_t}{\lambda}\right)+1}\\
L^{II}(s) &= \frac{(1-p)\left[\epsilon^r\left(B'h_t - s\right)+1\right]}{\left[p\epsilon^g + (1-p)\,\epsilon^r\right]\left(B'h_t - s\right)+\epsilon^g\frac{z_t}{\lambda}+1}\\
L^{II}(m) &= \frac{(1-p)\left[\epsilon^r\left(B'h_t - s\right)+1\right]}{\left[p\epsilon^g + (1-p)\,\epsilon^r\right]\left(B'h_t - s\right)+1}
\end{aligned}$$

Now let's define, for convenience:

$$a_1 \equiv Bh_t - s; a_2 \equiv B'h_t - s;\; b_1 \equiv p\epsilon^g + (1-p)\,\epsilon^r \tag{32a}$$

$$c_1 \equiv Ah_t - s; c_2 \equiv A'h_t - s \tag{32b}$$

Substituting these into the above we get:

$$\begin{aligned}
L^{II}(l) &= \frac{(1-p)\left[\epsilon^r a_1 + 1\right]}{b_1 a_1 + 1} \\
L^{II}(u) &= \frac{(1-p)\left[\epsilon^r \left(a_2 + \frac{z_t}{\lambda}\right) + 1\right]}{b_1 \left(a_2 + \frac{z_t}{\lambda}\right) + 1} \\
L^{II}(s) &= \frac{(1-p)\left[\epsilon^r a_2 + 1\right]}{b_1 a_2 + \epsilon^g \frac{z_t}{\lambda} + 1} \\
L^{II}(m) &= \frac{(1-p)\left[\epsilon^r a_2 + 1\right]}{b_1 a_2 + 1}
\end{aligned}$$

First, note that

$$\begin{aligned}
&: L^{II}(m) > L^{II}(l) \\
&\Rightarrow \frac{(1-p)\left[\epsilon^r a_2 + 1\right]}{b_1 a_2 + 1} > \frac{(1-p)\left[\epsilon^r a_1 + 1\right]}{b_1 a_1 + 1} \\
&\Rightarrow \left[\epsilon^r a_2 + 1\right](b_1 a_1 + 1) > \left[\epsilon^r a_1 + 1\right](b_1 a_2 + 1) \\
&\Rightarrow \epsilon^r a_2 b_1 a_1 + \epsilon^r a_2 + b_1 a_1 + 1 > \epsilon^r a_1 b_1 a_2 + \epsilon^r a_1 + b_1 a_2 + 1 \\
&\Rightarrow \epsilon^r a_2 + b_1 a_1 > \epsilon^r a_1 + b_1 a_2 \\
&\Rightarrow b_1 > \epsilon^r
\end{aligned} \tag{33a}$$

since $\epsilon^g > \epsilon^r \Rightarrow \epsilon^r < b_1$.

Second, we can easily see:

$$\begin{aligned}
&: L^{II}(u) > L^{II}(s) \\
&\Rightarrow \frac{\epsilon^r a_2 + 1 + \frac{z_t}{\lambda}\epsilon^r}{b_1 a_2 + 1 + b_1 \frac{z_t}{\lambda}} > \frac{\epsilon^r a_2 + 1}{b_1 a_2 + 1 + \epsilon^g \frac{z_t}{\lambda}}
\end{aligned} \tag{33b}$$

The numerator in the left handside is higher (since $\frac{z_t}{\lambda}\epsilon^r > 0$) while the denominator is smaller (since $b_1 < \epsilon^g$) implying () holds.

Third, we can show

$$
\begin{aligned}
&: L^{II}(l) > L^{II}(u)\\
&\Rightarrow \frac{(1-p)\left[\epsilon^r a_1 + 1\right]}{b_1 a_1 + 1} > \frac{(1-p)\left[\epsilon^r \left(a_2 + \frac{z_t}{\lambda}\right) + 1\right]}{b_1 \left(a_2 + \frac{z_t}{\lambda}\right) + 1}\\
&\Rightarrow (\epsilon^r a_1 + 1)\left(b_1\left(a_2 + \frac{z_t}{\lambda}\right) + 1\right) > \left(\epsilon^r\left(a_2 + \frac{z_t}{\lambda}\right) + 1\right)(b_1 a_1 + 1)\\
&\Rightarrow \epsilon^r a_1 b_1 \left(a_2 + \frac{z_t}{\lambda}\right) + \epsilon^r a_1 + b_1\left(a_2 + \frac{z_t}{\lambda}\right) + 1 > \epsilon^r\left(a_2 + \frac{z_t}{\lambda}\right) b_1 a_1 + b_1 a_1 + \epsilon^r\left(a_2 + \frac{z_t}{\lambda}\right) + 1\\
&\Rightarrow \epsilon^r a_1 + b_1\left(a_2 + \frac{z_t}{\lambda}\right) > b_1 a_1 + \epsilon^r\left(a_2 + \frac{z_t}{\lambda}\right)\\
&\Rightarrow \epsilon^r a_1 - b_1 a_1 > \epsilon^r\left(a_2 + \frac{z_t}{\lambda}\right) - b_1\left(a_2 + \frac{z_t}{\lambda}\right)\\
&\Rightarrow a_1 < a_2 + \frac{z_t}{\lambda} \qquad (33c)
\end{aligned}
$$

As long as $0 < \lambda < 1$, the last relation holds.

Fourth, (33a), (33b) and (33c) together imply (31) holds.

■

D.5. Proposition 5

Proof.

We see immediately from Table 9:

1. $L^{IV}(u) > L^{IV}(l)$, because the nominators associated to the former are greater than that of the latter for each cumulative population ratio whereas the denominators are the same.
2. $L^{IV}(m) \geq L^{IV}(s)$, because the nominator for means-tested for the $100\,(1-\lambda p)$ percent of the population are higher than that of the latter; for any other percent of the population, they remain equal.

■

D.6. Proposition 6

Proof.

From Table 9:

1. For the bottom $100(1-\lambda)(1-p)$ percent of population, the nominator of the Lorenz curve associated to the universal grant is greater than that of the scholarship scheme; but the dominator is relatively smaller for the former.
2. For the $100(1-\lambda)$ percent of population, the Lorenz curves associated to the universal ($L^{IV}(u)$) and scholarship schemes ($L^{IV}(s)$) are given by

$$L^{IV}(u) = (1-\lambda)\frac{\{[p\epsilon^g + (1-p)\epsilon^r](A'h_t - s) + [p\epsilon^g + (1-p)\epsilon^r]z_t + 1\}}{(p\epsilon^g + (1-p)\epsilon^r)(Ah_t - s) + 1} \tag{34}$$

$$L^{IV}(s) = (1-\lambda)\frac{\{[p\epsilon^g + (1-p)\epsilon^r](A'h_t - s) + 1 + \epsilon^g z_t\}}{[p\epsilon^g + (1-p)\epsilon^r](Ah_t - s) + (1-p)(\epsilon^g - \epsilon^r)z_t + 1} \tag{35}$$

   Let's define

$$\begin{aligned} d &\equiv [p\epsilon^g + (1-p)\epsilon^r](A'h_t - s) + [p\epsilon^g + (1-p)\epsilon^r]z_t + 1 \\ f &\equiv (p\epsilon^g + (1-p)\epsilon^r)(Ah_t - s) + 1 \\ g &\equiv (1-p)(\epsilon^g - \epsilon^r)z_t \\ &= \epsilon^g z_t - [p\epsilon^g + (1-p)\epsilon^r]z_t \end{aligned}$$

   Then, we can rewrite (34) and (35) as

$$\begin{aligned} L^{IV}(u) &= (1-\lambda)\frac{d}{f} \\ L^{IV}(s) &= (1-\lambda)\frac{d+g}{f+g} \end{aligned}$$

   Note that $(1-\lambda)d/f$ and $(1-\lambda)(d+g)/(f+g)$ are simply the cumulative ratio associated to the distribution in the universal and scholarship schemes, respectively, for the $100(1-\lambda)$ percent of population. Thus, we see immediately since $f > d$, then $L^{IV}(s) > L^{IV}(u)$ holds.
3. For the large $100(1-\lambda p)$ percent of the population, we want to show the

distribution under scholarship scheme Lorenz dominates universal grant:

$$L^{IV}(s) > L^{IV}(u)$$

where

$$L^{IV}(u) = \frac{(1-p)\,\epsilon^r\,[Ah_t - s] + (1-\lambda)\,p\epsilon^g\,(A'h_t - s) + 1 - \lambda p + (1-\lambda)\,p\epsilon^g z_t}{(p\epsilon^g + (1-p)\,\epsilon^r)\,(Ah_t - s) + 1} \tag{36}$$

$$L^{IV}(s) = \frac{(1-p)\,\epsilon^r\,(Ah_t - s) + (1-\lambda)\,p\epsilon^g\,(A'h_t - s) + 1 - \lambda p + \Omega z_t}{[p\epsilon^g + (1-p)\,\epsilon^r]\,(Ah_t - s) + (1-p)\,(\epsilon^g - \epsilon^r)\,z_t + 1} \tag{37}$$

Again let's make the following definitions for simplicity

$$\begin{aligned}
q &\equiv (1-p)\,\epsilon^r\,[Ah_t - s] + (1-\lambda)\,p\epsilon^g\,(A'h_t - s) + 1 - \lambda p + (1-\lambda)\,p\epsilon^g z_t \\
f &\equiv (p\epsilon^g + (1-p)\,\epsilon^r)\,(Ah_t - s) + 1 \\
m &\equiv (1-p)\,(\epsilon^g - \epsilon^r)\,z_t \\
r &\equiv \lambda\epsilon^g\,(1-p)\,z_t \\
m - r &= \Omega z_t - (1-\lambda)\,p\epsilon^g z_t
\end{aligned}$$

Then we can rewrite (36) and (37) using our definitions:

$$\begin{aligned}
L^{IV}(u) &= \frac{q}{f} \\
L^{IV}(s) &= \frac{q + m - r}{f + m}
\end{aligned}$$

Thus, q/f and $(q+m-r)\,/\,(q+m)$ are the cumulative ratio associated to universal grant and scholarship, respectively, for the $100\,(1-\lambda p)$ percent of the population. And, $q/f < (q+m-r)\,/\,(f+m)$ if $m - r > 0$ or

$$\epsilon^g\,(1-\lambda) > \epsilon^r$$

since

$$\begin{aligned} m - r &= \Omega z_t - (1-\lambda)\, p\epsilon^g z_t \\ &= \left((1-\lambda)\,\epsilon^g - (1-p)\,\epsilon^r\right) z_t - (1-\lambda)\, p\epsilon^g z_t \\ &= \left[(1-\lambda)\,\epsilon^g - \epsilon^r\right](1-p)\, z_t \end{aligned}$$

■