Forecasting Stock Market (Realized) Volatility in the United Kingdom: Is there a Role for Economic Inequality?

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Abstract

This paper explores the potential role of economic inequality for forecasting the stock market volatility of the United Kingdom (UK). Utilizing linear and nonlinear models as well as measures of consumption and income inequalities over the period of 1975 to 2016, we find that linear models incorporating the information of growth in inequality indeed produce lower forecast errors. These models, however, do not necessarily outperform the univariate linear and nonlinear models based on formal statistical forecast comparison tests, especially in short- to medium-runs. On the other hand, at a one-year-ahead horizon, absolute measure of consumption inequality results in significant statistical gains for stock market volatility predictions. We argue that the long-run predictive power of consumption inequality is driven by its informational content over both political and social uncertainty in the long-run.

Keywords: Income and Consumption Inequalities; Stock Markets; Realized Volatility; Forecasting; Linear and Nonlinear Models; United Kingdom.

\textit{JEL:} C22, G1.
1. Introduction

Stock market fluctuations reflect not only firm- and aggregate-level changes in economic fundamentals, but also changes in investors’ perception of risk and economic stability. Although the literature provides ample evidence linking stock market volatility to real economic activity (e.g. Hamilton and Lin (1996); Schwert (2011)) and the business cycle (e.g. Choudhry (2016)), the approach has largely been from a cashflow perspective, focusing on how economic fundamentals drive fluctuations in earnings and cashflow projections, which then contribute to volatility at the aggregate market level. From a non-cashflow perspective, however, one might argue that investors’ perception of economic stability (or lack thereof), which may be driven by social and political risk factors, also plays a role in driving fluctuations in financial markets as investors adjust their expectations of risk exposures with respect to economic instability worries. To that end, a growing strand of the literature presents an opening by relating volatility in financial markets to economic inequality although the evidence on the direction of the relationship is mixed (e.g. Blau (2015)). This study contributes to this debate by exploring the potential role of economic inequality for forecasting the stock market volatility of the United Kingdom (UK) via a battery of linear and nonlinear models that utilize a unique data set of alternative measures of economic inequality. By doing so, it enlarges our understanding of the channels in which political and social risks relate to financial market dynamics.

Clearly, accurate forecasting of the process of volatility has implications for portfolio selection, the pricing of derivative securities and risk management (Poon and Granger, 2003). In addition, financial market volatility, as witnessed during the recent global financial crisis, can have widespread repercussions on the economy as a whole, via its effect on real economic activity and public confidence. Hence, forecasts of market volatility, can serve as a measure for the vulnerability (uncertainty) of financial markets and the economy (Gupta et al., 2018a), and can help policymakers design appropriate policies to neutralize the negative impacts. Not surprisingly, given the importance of information on volatility for both investors and in policy-making, the literature on forecasting of volatility is huge (see Rapach et al. (2008), Babikir et al. (2012) and Ben Nasr et al. (2014, 2016) for details reviews). While prediction of volatility has historically relied on high-frequency uni-
variate (Generalized Autoregressive Conditional Heteroskedasticity (GARCH)-
type) models, more recently, Engle and Rangel (2008), Rangel and Engle
(2011) and Engle et al. (2013) have highlighted the importance of low-
frequency financial and macroeconomic variables in capturing future move-
ments in the volatility process of financial assets. In this regard, given an
upward trend in economic inequality globally (Piketty and Saez, 2014), which
in turn, can lead to both political and social uncertainty (Barro, 2000), one
could hypothesize that inequality might contribute to instability in financial
markets, with possible second-moment effects on stock prices (specifically,
increased volatility). From an opposing perspective, however, one can also
argue that income inequality would foster skilled decision making at the cor-
porate level as it represents a higher payoff for human capital (Becker and
Chiswick, 1966; Lucas, 1977; Becker and Murphy, 2007), such that highly
skilled corporate decision makers bring about stability in stock prices, which
in turn, results in lower stock market volatility, as observed in an in-sample

Against this backdrop, given the fact that in-sample predictability does
not guarantee out-of-sample forecasting gains, and the suggestion that the ul-
timate test of any predictive model is its out-of-sample performance (Camp-
bell, 2008), the objective of this paper is to investigate for the first time
whether inequality forecasts stock market volatility in the United Kingdom
(UK). In this regard, we use a unique data set at the (highest possible) quar-
terly frequency, over 1975Q1 to 2016Q1 which includes both income- and
consumption-based relative and absolute measures of inequality. Given that
stock market data over this period is available at daily frequency, we capture
the latent process of volatility using a model-free estimate, namely realized
volatility - sum of daily squared returns over a quarter.

Realizing that realized volatility is nonlinearly related with its predic-
tors (as highlighted by Gupta et al. (2018c)), we not only use linear models
for forecasting, but also nonparametric models to control against possible
misspecification. Our findings suggest that linear models incorporating the

\[ \text{Note that, a recent line of research has already related prediction of stock market returns with measures of inequality (see for example, Brogaard et al. (2015), Christou et al. (2017) and Gupta et al. (2018b) for detailed reviews of the theoretical and empirical literature in this regard).} \]
information of growth in inequality indeed produce lower forecast errors. These models, however, do not necessarily outperform the univariate linear and nonlinear models based on formal statistical forecast comparison tests, especially in short- to medium-runs. On the other hand, at a one-year-ahead horizon, absolute measure of consumption inequality results in significant statistical gains for stock market volatility predictions. We argue that the long-run predictive power of consumption inequality is driven by its informational content over both political and social uncertainty in the long-run.

The remainder of the paper is organized as follows: Section 2 outlines the alternative econometric models used for our forecasting analysis, Section 3 discusses the data and results and Section 4 concludes the paper.

2. Forecasting Models and Accuracy Measures

2.1. Functional-Coefficient Autoregressive with Exogenous variables:

The Functional-Coefficient Autoregressive with Exogenous variables (FARX) formulates the time series $y_t$ as follows (Cai et al., 2000; Chen and Tsay, 1993a):

\[ y_t = \sum_{i=1}^{p} f_i(y_{t-i})y_{t-i} + \sum_{i=1}^{q} g_i(y_{t-d})x_{t,i} + \epsilon_t, \]

where $\epsilon_t$ is white noise and $x_i(i = 1, \ldots, q)$ are exogenous variables (and may contain the exogenous variables’ lags). The nonlinear functions $f_i(y_{t-d})$ and $g_i(y_{t-d})$ are estimated using local linear regression (Cai et al., 2000).

2.2. Nonlinear Additive Autoregressive with Exogenous variables:

The Nonlinear Additive Autoregressive with Exogenous variables (NAARX) uses the following formulation for time series modeling (Chen and Tsay, 1993b):

\[ y_t = \sum_{i=1}^{p} f_i(y_{t-i}) + \sum_{i=1}^{q} g_i(x_{t,i}) + \epsilon_t, \]

where $\epsilon_t$ is white noise and $x_i(i = 1, \ldots, q)$ are exogenous variables (and may contain the exogenous variables’ lags). The nonlinear functions $f_i(y_{t-i})$ and $g_i(x_{t,i})$ can be estimated using local linear regression (Cai and Masry, 2000).
2.3. Linear State Space Model:

A Linear State Space Model (LSS) uses following formulation to represent a linear ARX model:

\[
\begin{align*}
    s_t &= A s_{t-1} + b u_t \\
    y_t &= c' s_t + \beta' x_t + \varepsilon_t
\end{align*}
\]

where \( s_t \) is the state vector, \( u_t \) and \( \varepsilon_t \) are mutually iid Gaussian random variables (with variances \( \eta^2 \) and \( \sigma^2 \)) and \( x_t \) is a vector of exogenous variables. The system’s matrices \( A, b, c \) and \( \beta \) and the exogenous vector are defined as follows (Pearlman, 1980):

\[
A = \begin{bmatrix}
    0 & 1 & 0 & \cdots & 0 \\
    0 & 0 & 1 & \cdots & 0 \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & 0 & \cdots & 1 \\
    \phi_p & \phi_{p-1} & \phi_{p-2} & \cdots & \phi_1
\end{bmatrix}_{p \times p},
\]

\[
b = \begin{bmatrix}
    0 \\
    \vdots \\
    0
\end{bmatrix}_{p \times 1},
\]

\[
c = \begin{bmatrix}
    0 \\
    \vdots \\
    c
\end{bmatrix}_{p \times 1},
\]

\[
\beta = \begin{bmatrix}
    \beta_0 \\
    \beta_1 \\
    \vdots \\
    \beta_q
\end{bmatrix}_{(q+1) \times 1},
\]

\[
x_t = \begin{bmatrix}
    1 \\
    x_{t,1} \\
    \vdots \\
    x_{t,q}
\end{bmatrix}_{(q+1) \times 1}.
\]

One may use an EM algorithm based on Kalman recursions to estimate the system’s matrices (Shumway and Stoffer, 2011).

2.4. Heterogeneous Autoregressive Model of Realized Volatility

Consider the classical estimator of realized volatility (RV) of a market or an asset (Andersen and Bollerslev, 1998):

\[
RV_t^\Omega = \sqrt{\frac{1}{M} \sum_{i=1}^M r_{t,i}^2} \quad (1)
\]

where \( \Omega \) is the frequency which \( RV \) is calculated in (i.e. daily, weekly, monthly, quarterly, etc.) and \( r_{t,i}, (i = 1, \ldots, M) \) are log-return (first-differences of the natural logarithmic values) of the market index or asset price in period \( t \) (in \( \Omega \) frequency). Note that \( RV \) is an approximation to the volatility of high frequency data (Andersen et al., 2001a; Barndorff-Nielsen and Shephard, 2003).
The Heterogeneous Autoregressive Model of Realized Volatility ($HAR - RV$) is a cascade model based on $RV$s in lower frequencies (Corsi, 2009).

\[
RV_{t+1}^\Omega = \beta_0 + \beta_1 RV_{t}^{\omega_1} + \cdots + \beta_k RV_{t}^{\omega_k} + \nu_{t+1},
\]

where $\omega_1 = 1$, $RV_{t}^{j\Omega} = \frac{1}{j} \left( RV_{t}^{\Omega} + \cdots + RV_{t-j+1}^{\Omega} \right)$, $(j > 1)$, are $RV$ in lower frequencies and $\nu_{t+1}$ is the innovation term. The sequence $\omega_1, \ldots, \omega_k$ shows the lag-structure of the $HAR - RV$ model.

2.5. Forecasting Evaluation

Suppose $E(RV_t|F_{t-1})$ is the realized volatility forecast and the $\varepsilon_t$ is the square residual of the conditional mean model at time $t$:

\[
\varepsilon_t = (RV_t - E(RV_t|F_{t-1}))^2.
\]

The Root Mean Square Error is formulated as follows:

\[
RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^{n} \varepsilon_t}.
\]

In order to compare two forecasting models, we use the Kolmogorov-Smirnov Prediction Accuracy test (KSPA test) of Hassani and Silva (2015).

The null hypothesis and the alternative for the two-tailed KSPA test are as follows:

\[
\begin{aligned}
H_0 : F_{\varepsilon_{t,1}}(z) &= F_{\varepsilon_{t,2}}(z) \\
H_1 : F_{\varepsilon_{t,1}}(z) &\neq F_{\varepsilon_{t,2}}(z),
\end{aligned}
\]

where $\varepsilon_{t,i}$ is the h-step ahead out-of-sample forecast square errors generated by $i$-th forecasting model and $F_{\varepsilon_{t,i}}(\cdot)$ is the cumulative distribution function.

Rejecting the null hypothesis implies that the two competing models have different forecasting accuracy.

\footnote{It should be noted that the original $HAR - RV$ model in Corsi (2009) is formulated based on daily, weekly and monthly frequencies. The formulation is generalized to match the structure of data in this research. Details on structure of the data is given in next section.}
3. Data and Results

Data on daily FTSE All Share Stock Index (ALSI) for the UK is obtained from Data stream of Thomson Reuters. Since the inequality data is available quarterly, we compute the quarterly realized volatility of the FTSE ALSI using daily data and obtain $RV$ in quarterly frequency (given by $\Omega = \text{Quarter}$). In the case of inequality, we use three alternative measures, i.e. (i) the Gini coefficient, (ii) standard deviation (of the data in natural logarithms), and (iii) the difference between the 90th and 10th percentile (with the data in natural logarithms). In other words, we include both absolute and relative measures of inequality. Various measures of economic inequality (taken into account one at a time) are calculated using survey data on income and consumption from the family expenditure survey. Further details on the construction of the data and the survey are documented in Mumtaz and Theophilopoulou (2017). Note that we work with the growth rates of the inequality measures to ensure that the predictors under consideration are stationary as required by the empirical models. The growth rates of the three income-based inequality measures are denoted as $x_1$, $x_2$, and $x_3$, while the growth rates of the three consumption-based inequality measures are denoted as $x_4$, $x_5$, and $x_6$.

Tables 1 and 2 show the RMSE values for out-of-sample forecasting of $RV$ using different models and predictors. Note, given that we have 164 observations to work with, following Rapach et al. (2005), we use 50% of the observations as in-sample, while the remaining 50% is used as the out-of-sample period, over which all our models are recursively estimated to mimic a pseudo out-of-sample forecasting scenario. As it can be seen, the best model with a specific-type of inequality (in the sense of minimum RMSE), is the linear $ARX$ model with $x_3$ (i.e., the income inequality measure as given by the difference between the 90th and 10th percentile) for $h = 1, 2$. For $h = 4$, the best model is $LSS$ with the $x_5$ (i.e., the consumption inequality measure as given by the standard deviation) as the predictor. Table 3 summarizes the

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3 The data is downloadable from: https://discover.ukdataservice.ac.uk/series/?sn=200016 and https://discover.ukdataservice.ac.uk/series/?sn=2000028.

4 We would like to thank Professor Haroon Mumtaz for kindly sharing the inequality data with us.
best models for the three horizons considered. Although the RMSE metric suggests that the models with the highest accuracy in forecasting $RV$ are the linear $ARX$ and the $LSS$ with predictor variables $x_3$ and $x_5$, respectively, one needs statistical hypothesis testing in order to identify the best models and predictors. For this purpose, we use the KSPA statistic to test the null hypothesis that a model has the same forecasting accuracy as the best performing model (in the sense of minimum RMSEs).

Tables 4 and 5 show the p-values for the KSPA test, comparing the models and predictors with the minimum RMSE model in terms of the out-of-sample forecasts of $RV$. Table 6 shows the models and predictors for which the null hypothesis of the KSPA test is retained at $\alpha = 0.05$ significance level, (i.e. the models and predictors with the same accuracy as the minimum RMSE model).

According to the KSPA test results, for one-step-ahead forecasts, the linear $ARX$, $HAR−RV$ and $NAARX$ models with predictors, have the same accuracy as the minimum RMSE model. Further, there is no significant difference between the accuracy of the minimum RMSE model and the $NAAR$, $AR$, $HAR−RV$ models without any predictors. In the case of the two-step-ahead forecasts, we observe similar results although the $NAARX$ model with $x_2$ as the predictor and the $NAAR$ model are not found to have the same accuracy as the minimum RMSE, at two-step-ahead forecasting. Accordingly, the effect of $x_3$ in short- and medium-term forecasting of $RV$ is not significant. Furthermore, using the $ARX$ model with $x_3$ as the predictor, does not improve the short-term forecasting accuracy of the $RW$ model. At the one-year-ahead forecasting horizon, however, there is a significant improvement to the forecasting ability of the RW model, using $LSS$. Furthermore, we observe that using $x_5$ (i.e., the consumption inequality measure as given by the standard deviation) as predictor, improves the accuracy of one-year-ahead forecasting.\footnote{Using the Minimum Absolute Error and $AE$ function in KSPA test tends to provide similar results, which are available upon request from the authors.}

Note that, as indicated in the introduction, the theoretical predictions suggest that inequality can either increase volatility by contributing to both political and social uncertainty, or reduce volatility if income inequality is a
signal about skilled decision making at the corporate level. To that end, the
lack of predictive evidence of inequality for stock market volatility, partic-
ularly at the short- to medium-runs could be an indication that these two
effects are possibly canceling out each other in our data set for the UK. How-
ever, the information content in the increased absolute consumption inequal-
ity (as given by the standard deviation), is likely to enhance stock market
volatility in the longer run via heightened political and social risks that is
generated.

4. Conclusion

Financial market volatility is used as an important input in investment
decisions, option pricing and financial market regulation, thus making fore-
casting of volatility an important area of research for academics, investors and
policymakers. Given this, we investigate whether income- and consumption-
based relative and absolute measures of inequality possess any predictive
power over stock market realized volatility of the UK, based on a unique
high-frequency (quarterly) data set over 1975Q1 to 2016Q1. Using an ar-
ray of univariate and bivariate linear and nonlinear models, we find that,
while linear models with inequality can produce lower forecast errors, their
performance is not statistically different from other univariate (and even bi-
variate) linear and nonlinear models in short- to medium-runs. At the same
time, we also observe that growth in inequality, and in particular absolute
consumption inequality, carries additional information in forecasting stock
market volatility in the UK in the long-horizon.

In short, our findings imply that the long-run predictive power of con-
sumption inequality over stock market volatility is possibly driven by its
informational content over both political and social uncertainty in the long-
run. This finding further supports the possible role of non-cashflow related
factors on the stability of financial markets, although their predictive power is
limited to longer horizons. As part of future research, given that inequality
data is traditionally only available at annual frequency, it would be inter-
esting to extend our analysis to multiple countries using panel data-based
forecasting methods. This will, in the process, provide a more robust test
(from the perspective of obtaining cross-country evidence) of the theoretical
claims relating inequality to instability in financial markets.
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casting, 21(1), 137-166.
Table 1: Out-of-sample RMSE for $RV$ forecasting (based on 82 out-of-sample forecasts)

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Model</th>
<th>$h = 1$</th>
<th>$h = 2$</th>
<th>$h = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$FARX$</td>
<td>1.5104</td>
<td>228.410</td>
<td>2.671E+03</td>
</tr>
<tr>
<td></td>
<td>$NAARX$</td>
<td>0.3472</td>
<td>0.3961</td>
<td>0.4301</td>
</tr>
<tr>
<td></td>
<td>$LSS$</td>
<td>5.1227</td>
<td>5.3190</td>
<td>5.7348</td>
</tr>
<tr>
<td></td>
<td>$ARX$</td>
<td>0.3413</td>
<td>0.3954</td>
<td>0.4278</td>
</tr>
<tr>
<td></td>
<td>$HAR - RV^a$</td>
<td>0.3484</td>
<td>0.4075</td>
<td>0.4293</td>
</tr>
<tr>
<td>$x_2$</td>
<td>$FARX$</td>
<td>2.2922</td>
<td>3.115E+04</td>
<td>7.633E+04</td>
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<tr>
<td></td>
<td>$NAARX$</td>
<td>0.6657</td>
<td>5.9579</td>
<td>0.9081</td>
</tr>
<tr>
<td></td>
<td>$LSS$</td>
<td>4.3694</td>
<td>4.5020</td>
<td>4.7727</td>
</tr>
<tr>
<td></td>
<td>$ARX$</td>
<td>0.3380</td>
<td>0.3935</td>
<td>0.4254</td>
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<tr>
<td></td>
<td>$HAR - RV^a$</td>
<td>0.3474</td>
<td>0.4073</td>
<td>0.4286</td>
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<tr>
<td>$x_3$</td>
<td>$FARX$</td>
<td>1.5233</td>
<td>449.42</td>
<td>396.288</td>
</tr>
<tr>
<td></td>
<td>$NAARX$</td>
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<td>0.3934</td>
<td>0.4980</td>
</tr>
<tr>
<td></td>
<td>$LSS$</td>
<td>4.7007</td>
<td>4.8085</td>
<td>5.1005</td>
</tr>
<tr>
<td></td>
<td>$ARX$</td>
<td>0.3358</td>
<td>0.3921</td>
<td>0.4236</td>
</tr>
<tr>
<td></td>
<td>$HAR - RV^a$</td>
<td>0.3449</td>
<td>0.4062</td>
<td>0.4277</td>
</tr>
<tr>
<td>$x_4$</td>
<td>$FARX$</td>
<td>1.3477</td>
<td>38.6539</td>
<td>1.520E+06</td>
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<tr>
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<td>4.8258</td>
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<tr>
<td></td>
<td>$HAR - RV^a$</td>
<td>0.3508</td>
<td>0.4067</td>
<td>0.4287</td>
</tr>
</tbody>
</table>

$a$. The lag-structure of the model is $\omega_1 = 1, \omega_2 = 4$. 
Table 2: Out-of-sample RMSE for RV forecasting (continued)

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Model</th>
<th>$h = 1$</th>
<th>$h = 2$</th>
<th>$h = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_5$</td>
<td>FARX</td>
<td>1.3637</td>
<td>51.6935</td>
<td>1.299E+07</td>
</tr>
<tr>
<td></td>
<td>NAARX</td>
<td>0.3414</td>
<td>0.3992</td>
<td>0.4394</td>
</tr>
<tr>
<td></td>
<td>LSS</td>
<td>1.2157</td>
<td>0.5571</td>
<td>0.1744</td>
</tr>
<tr>
<td></td>
<td>ARX</td>
<td>0.3414</td>
<td>0.3946</td>
<td>0.4274</td>
</tr>
<tr>
<td></td>
<td>HAR - RV$^a$</td>
<td>0.3514</td>
<td>0.4070</td>
<td>0.4282</td>
</tr>
<tr>
<td>$x_6$</td>
<td>FARX</td>
<td>1.4523</td>
<td>82.0759</td>
<td>9.746E+06</td>
</tr>
<tr>
<td></td>
<td>NAARX</td>
<td>0.3456</td>
<td>0.3953</td>
<td>0.4291</td>
</tr>
<tr>
<td></td>
<td>LSS</td>
<td>4.3086</td>
<td>4.4380</td>
<td>4.6928</td>
</tr>
<tr>
<td></td>
<td>ARX</td>
<td>0.3403</td>
<td>0.3951</td>
<td>0.4268</td>
</tr>
<tr>
<td></td>
<td>HAR - RV$^a$</td>
<td>0.3495</td>
<td>0.4087</td>
<td>0.4289</td>
</tr>
<tr>
<td>Without Predictors</td>
<td>FARX</td>
<td>1.3657</td>
<td>51.1659</td>
<td>2.801E+03</td>
</tr>
<tr>
<td></td>
<td>NAARX</td>
<td>0.3941</td>
<td>0.4053</td>
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<td></td>
<td>LSS</td>
<td>3.7939</td>
<td>3.8810</td>
<td>4.0633</td>
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<tr>
<td></td>
<td>ARX</td>
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</tr>
<tr>
<td></td>
<td>HAR - RV$^a$</td>
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<td>0.4063</td>
<td>0.4278</td>
</tr>
<tr>
<td></td>
<td>RW</td>
<td>0.3593</td>
<td>0.4272</td>
<td>0.4890</td>
</tr>
</tbody>
</table>

$^a$. The lag-structure of the model is $\omega_1 = 1, \omega_2 = 4$.

Table 3: Summary table (minimum out-of-sample RMSE models and predictors for RV forecasting)

<table>
<thead>
<tr>
<th>Model</th>
<th>Predictors</th>
<th>$h = 1$</th>
<th>$h = 2$</th>
<th>$h = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARX</td>
<td>$x_3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARX</td>
<td>$x_3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LSS</td>
<td>$x_5$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4: KSPA test p-values (two tailed) for comparing the forecasting models to minimum RMSE RV forecast. (based on 82 out-of-sample forecasts)

<table>
<thead>
<tr>
<th>Minimum RMSE model → Comparing to ↓</th>
<th>$h = 1$</th>
<th>$h = 2$</th>
<th>$h = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$FARX (x_1)$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$NAARX (x_1)$</td>
<td>0.7027</td>
<td>0.9794</td>
<td>0.0000</td>
</tr>
<tr>
<td>$LSS (x_1)$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$ARX (x_1)$</td>
<td>0.9806</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$HAR – RV^a (x_1)$</td>
<td>0.9806</td>
<td>0.8219</td>
<td>0.0000</td>
</tr>
<tr>
<td>$FARX (x_2)$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$NAARX (x_2)$</td>
<td>0.0562</td>
<td>0.0216</td>
<td>0.0000</td>
</tr>
<tr>
<td>$LSS (x_2)$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0003</td>
</tr>
<tr>
<td>$ARX (x_2)$</td>
<td>0.9981</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$HAR – RV^a (x_2)$</td>
<td>0.8277</td>
<td>0.9220</td>
<td>0.0000</td>
</tr>
<tr>
<td>$FARX (x_3)$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$NAARX (x_3)$</td>
<td>0.7027</td>
<td>0.9794</td>
<td>0.0000</td>
</tr>
<tr>
<td>$LSS (x_3)$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0006</td>
</tr>
<tr>
<td>$ARX (x_3)$</td>
<td></td>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>$HAR – RV^a (x_3)$</td>
<td>0.7027</td>
<td>0.8219</td>
<td>0.0000</td>
</tr>
<tr>
<td>$FARX (x_4)$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$NAARX (x_4)$</td>
<td>0.7027</td>
<td>0.9220</td>
<td>0.0000</td>
</tr>
<tr>
<td>$LSS (x_4)$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0311</td>
</tr>
<tr>
<td>$ARX (x_4)$</td>
<td>0.9806</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$HAR – RV^a (x_4)$</td>
<td>0.9254</td>
<td>0.9220</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

$^a$. The lag-structure of the model is $\omega_1 = 1, \omega_2 = 4$.
Table 5: KSPA test p-values (two tailed) for comparing the forecasting models to minimum RMSE $RV$ forecast. (continue)

<table>
<thead>
<tr>
<th></th>
<th>$h = 1$</th>
<th>$h = 2$</th>
<th>$h = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum RMSE model →</td>
<td>$ARX (x_3)$</td>
<td>$ARX (x_3)$</td>
<td>$LSS (x_5)$</td>
</tr>
<tr>
<td>Comparing to ↓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$FARX (x_5)$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$NAARX (x_5)$</td>
<td>0.8277</td>
<td>0.9220</td>
<td>0.0000</td>
</tr>
<tr>
<td>$LSS (x_5)$</td>
<td>0.0000</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>$ARX (x_5)$</td>
<td>0.9981</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$HAR - RV^a (x_5)$</td>
<td>0.4462</td>
<td>0.9794</td>
<td>0.0000</td>
</tr>
<tr>
<td>$FARX (x_6)$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$NAARX (x_6)$</td>
<td>0.5705</td>
<td>0.9794</td>
<td>0.0000</td>
</tr>
<tr>
<td>$LSS (x_6)$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$ARX (x_6)$</td>
<td>0.9806</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$HAR - RV^a (x_6)$</td>
<td>0.5705</td>
<td>0.9220</td>
<td>0.0000</td>
</tr>
<tr>
<td>$FAR$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$NAAR$</td>
<td>0.8277</td>
<td>0.9794</td>
<td>0.0000</td>
</tr>
<tr>
<td>$LSS$ (Without Predictors)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$AR$</td>
<td>0.9981</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$HAR - RV^a$ (Without Predictors)</td>
<td>0.5705</td>
<td>0.8219</td>
<td>0.0000</td>
</tr>
<tr>
<td>$RW$</td>
<td>0.1245</td>
<td>0.6953</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

$^a$. The lag-structure of the model is $\omega_1 = 1, \omega_2 = 4$. 

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Table 6: Forecasts similar to the Minimum RMSE for RV forecasting.\textsuperscript{a}

<table>
<thead>
<tr>
<th>Minimum RMSE model</th>
<th>$h = 1$</th>
<th>$h = 2$</th>
<th>$h = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$ARX (x_3)$</td>
<td>$ARX (x_3)$</td>
<td>$LSS (x_5)$</td>
</tr>
<tr>
<td>Similar forecasts</td>
<td>$NAARX (x_1)$</td>
<td>$NAARX (x_1)$</td>
<td></td>
</tr>
<tr>
<td>$(\alpha = 0.05)$</td>
<td>$HAR - RV^b (x_1)$</td>
<td>$HAR - RV^b (x_1)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$NAARX (x_2)$</td>
<td>$ARX (x_2)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$ARX (x_2)$</td>
<td>$HAR - RV^b (x_2)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$HAR - RV^b (x_2)$</td>
<td>$NAARX (x_3)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$NAARX (x_3)$</td>
<td>$HAR - RV^b (x_3)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$ARX (x_4)$</td>
<td>$AR (x_4)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$HAR - RV^b (x_4)$</td>
<td>$NAARX (x_5)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$NAARX (x_5)$</td>
<td>$AR (x_5)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$ARX (x_5)$</td>
<td>$HAR - RV^b (x_5)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$HAR - RV^b (x_5)$</td>
<td>$NAARX (x_6)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$NAARX (x_6)$</td>
<td>$AR (x_6)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$ARX (x_6)$</td>
<td>$HAR - RV^b (x_6)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$HAR - RV^b (x_6)$</td>
<td>$AR$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$NAAR$</td>
<td>$HAR - RV^b$</td>
<td>(Without Predictors)</td>
</tr>
<tr>
<td></td>
<td>$AR$</td>
<td>$RW$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$HAR - RV$</td>
<td>$RW$</td>
<td>(Without Predictors)</td>
</tr>
</tbody>
</table>

\textsuperscript{a} $H_0$ Retained at 0.05 significance level

\textsuperscript{b} The lag-structure of the model is $\omega_1 = 1, \omega_2 = 4$