University of Pretoria
Department of Economics Working Paper Series

Forecasting Stock Market (Realized) Volatility in the United Kingdom: Is there a Role for Economic Inequality?

Hossein Hassani
Bournemouth University
Mohammad Reza Yeganegi
Islamic Azad University Central Tehran Branch
Rangan Gupta
University of Pretoria
Riza Demirer
Southern Illinois University Edwardsville
Working Paper: 2018-80
November 2018

Department of Economics
University of Pretoria
0002, Pretoria
South Africa
Tel: +27 12 420 2413

Forecasting Stock Market (Realized) Volatility in the United Kingdom: Is There a Role for Economic Inequality?

Hossein Hassani[a,*], Mohammad Reza Yeganegi[b], Rangan Gupta[c], Riza Demirer[d]

[a] *The Statistical Research Centre, Bournemouth University, Bournemouth, UK.*
[b] *Department of Accounting, Islamic Azad University Central Tehran Branch, Iran.*
[c] *Department of Economics, University of Pretoria, Pretoria, 0002, South Africa.*
[d] *Department of Economics and Finance, Southern Illinois University Edwardsville, Edwardsville, IL 62026-1102, USA.*

Abstract

This paper explores the potential role of economic inequality for forecasting the stock market volatility of the United Kingdom (UK). Utilizing linear and nonlinear models as well as measures of consumption and income inequalities over the period of 1975 to 2016, we find that linear models incorporating the information of growth in inequality indeed produce lower forecast errors. These models, however, do not necessarily outperform the univariate linear and nonlinear models based on formal statistical forecast comparison tests, especially in short- to medium-runs. On the other hand, at a one-year-ahead horizon, absolute measure of consumption inequality results in significant statistical gains for stock market volatility predictions. We argue that the long-run predictive power of consumption inequality is driven by its informational content over both political and social uncertainty in the long-run.

Keywords: Income and Consumption Inequalities; Stock Markets; Realized Volatility; Forecasting; Linear and Nonlinear Models; United Kingdom.
JEL: C22, G1.

---

*Corresponding author

Email addresses: hhassani@bournemouth.ac.uk (Hossein Hassani), m.yeganegi@iauctb.ac.ir (Mohammad Reza Yeganegi), rangan.gupta@up.ac.za (Rangan Gupta), rdemire@siue.edu (Riza Demirer)

1. Introduction

Stock market fluctuations reflect not only firm- and aggregate-level changes in economic fundamentals, but also changes in investors' perception of risk and economic stability. Although the literature provides ample evidence linking stock market volatility to real economic activity (e.g. Hamilton and Lin (1996); Schwert (2011)) and the business cycle (e.g. Choudhry (2016)), the approach has largely been from a cashflow perspective, focusing on how economic fundamentals drive fluctuations in earnings and cashflow projections, which then contribute to volatility at the aggregate market level. From a non-cashflow perspective, however, one might argue that investors' perception of economic stability (or lack thereof), which may be driven by social and political risk factors, also plays a role in driving fluctuations in financial markets as investors adjust their expectations of risk exposures with respect to economic instability worries. To that end, a growing strand of the literature presents an opening by relating volatility in financial markets to economic inequality although the evidence on the direction of the relationship is mixed (e.g. Blau (2015)). This study contributes to this debate by exploring the potential role of economic inequality for forecasting the stock market volatility of the United Kingdom (UK) via a battery of linear and nonlinear models that utilize a unique data set of alternative measures of economic inequality. By doing so, it enlarges our understanding of the channels in which political and social risks relate to financial market dynamics.

Clearly, accurate forecasting of the process of volatility has implications for portfolio selection, the pricing of derivative securities and risk management (Poon and Granger, 2003). In addition, financial market volatility, as witnessed during the recent global financial crisis, can have widespread repercussions on the economy as a whole, via its effect on real economic activity and public confidence. Hence, forecasts of market volatility, can serve as a measure for the vulnerability (uncertainty) of financial markets and the economy (Gupta et al., 2018a), and can help policymakers design appropriate policies to neutralize the negative impacts. Not surprisingly, given the importance of information on volatility for both investors and in policy-making, the literature on forecasting of volatility is huge (see Rapach et al. (2008), Babikir et al. (2012) and Ben Nasr et al. (2014, 2016) for details reviews).

While prediction of volatility has historically relied on high-frequency univariate (Generalized Autoregressive Conditional Heteroskedasticity (GARCH)-type) models, more recently, Engle and Rangel (2008), Rangel and Engle (2011) and Engle et al. (2013) have highlighted the importance of low-frequency financial and macroeconomic variables in capturing future movements in the volatility process of financial assets. In this regard, given an upward trend in economic inequality globally (Piketty and Saez, 2014), which in turn, can lead to both political and social uncertainty (Barro, 2000), one could hypothesize that inequality might contribute to instability in financial markets, with possible second-moment effects on stock prices (specifically, increased volatility). From an opposing perspective, however, one can also argue that income inequality would foster skilled decision making at the corporate level as it represents a higher payoff for human capital (Becker and Chiswick, 1966; Lucas, 1977; Becker and Murphy, 2007), such that highly skilled corporate decision makers bring about stability in stock prices, which in turn, results in lower stock market volatility, as observed in an in-sample analysis by Blau (2015)[1].

Against this backdrop, given the fact that in-sample predictability does not guarantee out-of-sample forecasting gains, and the suggestion that the ultimate test of any predictive model is its out-of-sample performance (Campbell, 2008), the objective of this paper is to investigate for the first time whether inequality forecasts stock market volatility in the United Kingdom (UK). In this regard, we use a unique data set at the (highest possible) quarterly frequency, over 1975Q1 to 2016Q1 which includes both income- and consumption-based relative and absolute measures of inequality. Given that stock market data over this period is available at daily frequency, we capture the latent process of volatility using a model-free estimate, namely realized volatility - sum of daily squared returns over a quarter.

Realizing that realized volatility is nonlinearly related with its predictors (as highlighted by Gupta et al., 2018c), we not only use linear models for forecasting, but also nonparametric models to control against possible misspecification. Our findings suggest that linear models incorporating the

[1] Note that, a recent line of research has already related prediction of stock market returns with measures of inequality (see for example, Brogaard et al. (2015), Christou et al. (2017) and Gupta et al. (2018b) for detailed reviews of the theoretical and empirical literature in this regard).

information of growth in inequality indeed produce lower forecast errors. These models, however, do not necessarily outperform the univariate linear and nonlinear models based on formal statistical forecast comparison tests, especially in short- to medium-runs. On the other hand, at a one-year-ahead horizon, absolute measure of consumption inequality results in significant statistical gains for stock market volatility predictions. We argue that the long-run predictive power of consumption inequality is driven by its informational content over both political and social uncertainty in the long-run.

The remainder of the paper is organized as follows: Section 2 outlines the alternative econometric models used for our forecasting analysis, Section 3 discusses the data and results and Section 4 concludes the paper.

2. Forecasting Models and Accuracy Measures

2.1. Functional-Coefficient Autoregressive with Exogenous variables:

The Functional-Coefficient Autoregressive with Exogenous variables ($FARX$) formulates the time series y_t as follows (Cai et al., 2000; Chen and Tsay, 1993a):

$$y_t = \sum_{i=1}^{p} f_i(y_{t-d})y_{t-i} + \sum_{i=1}^{q} g_i(y_{t-d})x_{t,i} + \varepsilon_t,$$

where ε_t is white noise and $x_i(i = 1, \dots, q)$ are exogenous variables (and may contain the exogenous variables' lags). The nonlinear functions $f_i(y_{t-d})$ and $g_i(y_{t-d})$ are estimated using local linear regression (Cai et al., 2000).

2.2. Nonlinear Additive Autoregressive with Exogenous variables:

The Nonlinear Additive Autoregressive with Exogenous variables ($NAARX$) uses the following formulation for time series modeling (Chen and Tsay, 1993b):

$$y_t = \sum_{i=1}^{p} f_i(y_{t-i}) + \sum_{i=1}^{q} g_i(x_{t,i}) + \varepsilon_t,$$

where ε_t is white noise and $x_i(i = 1, \dots, q)$ are exogenous variables (and may contain the exogenous variables' lags). The nonlinear functions $f_i(y_{t-i})$ and $g_i(x_{t,i})$ can be estimated using local linear regression (Cai and Masry, 2000).

2.3. Linear State Space Model:

A Liner State Space Model (LSS) uses following formulation to represent a linear ARX model:

$$\begin{cases} \boldsymbol{s}_t = \boldsymbol{A}\boldsymbol{s}_{t-1} + \boldsymbol{b}u_t \\ y_t = \boldsymbol{c}'\boldsymbol{s}_t + \boldsymbol{\beta}'\boldsymbol{x}_t + \varepsilon_t \end{cases}$$

where $\boldsymbol{s}_t$ is the state vector, u_t and ε_t are mutually iid Gaussian random variables (with variances η^2 and σ^2) and $\boldsymbol{x}_t$ is a vector of exogenouse variables. The system's matrices $\boldsymbol{A}$, $\boldsymbol{b}$, $\boldsymbol{c}$ and $\boldsymbol{\beta}$ and the exogenouse vector are defined as follows (Pearlman, 1980):

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ \phi_p & \phi_{p-1} & \phi_{p-2} & \cdots & \phi_1 \end{bmatrix}_{p\times p},$$

$$\boldsymbol{b} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b \end{bmatrix}_{p\times 1}, \boldsymbol{c} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ c \end{bmatrix}_{p\times 1}, \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_q \end{bmatrix}_{(q+1)\times 1}, \boldsymbol{x}_t = \begin{bmatrix} 1 \\ x_{t,1} \\ \vdots \\ x_{t,q} \end{bmatrix}_{(q+1)\times 1}.$$

One may use an EM algorithm based on Kalman recursions to estimate the system's matrices (Shumway and Stoffer, 2011).

2.4. Heterogeneous Autoregressive Model of Realized Volatility

Consider the classical estimator of realized volatility (RV) of a market or an asset(Andersen and Bollerslev, 1998):

$$RV_t^{\Omega} = \sqrt{\sum_{i=1}^{M} r_{t,i}^2} \tag{1}$$

where Ω is the frequency which RV is calculated in (i.e. daily, weekly, monthly, quarterly, etc.) and $r_{t,i}$, $(i = 1, \ldots, M)$ are log-return (first-differences of the natural logarithmic values) of the market index or asset price in period t (in Ω frequency). Note that RV is an approximation to the volatility of high frequency data(Andersen et al., 2001a,b; Barndorff-Nielsen and Shephard, 2002a,b). The Heterogeneous Autoregressive Model of Realized Volatility ($HAR - RV$) is a cascade model based on RVs in lower frequencies(Corsi, 2009)[2]:

$$RV_{t+1}^{\Omega} = \beta_0 + \beta_1 RV_t^{\omega_1 \Omega} + \cdots + \beta_k RV_t^{\omega_k \Omega} + \nu_{t+1},$$

where $\omega_1 = 1$, $RV_t^{j\Omega} = \frac{1}{j}\left(RV_t^{\Omega} + \cdots + RV_{t-j+1}^{\Omega}\right)$, $(j > 1)$, are RV in lower frequencies and ν_{t+1} is the innovation term. The sequence $\omega_1, \ldots, \omega_k$ shows the lag-structure of the $HAR - RV$ model.

2.5. Forecasting Evaluation

Suppose $E(RV_t|\mathcal{F}_{\ t-1})$ is the realized volatility forecast and the ε_t is the square residual of the conditional mean model at time t:

$$\varepsilon_t = \left(RV_t - E(RV_t|\mathcal{F}_{\ t-1})\right)^2.$$

The Root Mean Square Error is formulated as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n} \varepsilon_t}.$$

In order to compare two forecasting models, we use the Kolmogorov-Smirnov Prediction Accuracy test (KSPA test) of Hassani and Silva (2015). The null hypothesis and the alternative for the two-tailed KSPA test are as follows:

$$\begin{cases} H_0 : F_{\varepsilon_{t,1}}(z) = F_{\varepsilon_{t,2}}(z) \\ H_1 : F_{\varepsilon_{t,1}}(z) \neq F_{\varepsilon_{t,2}}(z) \end{cases},$$

where $\varepsilon_{t,i}$ is the h-step ahead out-of-sample forecast square errors generated by i-th forecasting model and $F_{\varepsilon_{t,i}}(.)$ is the cumulative distribution function. Rejecting the null hypothesis implies that the two competing models have different forecasting accuracy.

[2]It should be noted that the original $HAR - RV$ model in Corsi (2009) is formulated based on daily, weekly and monthly frequencies. The formulation is generalized to match the structure of data in this research. Details on structure of the data is given in next section.

3. Data and Results

Data on daily FTSE All Share Stock Index (ALSI) for the UK is obtained from Data stream of Thomson Reuters. Since the inequality data is available quarterly, we compute the quarterly realized volatility of the FTSE ALSI using daily data and obtain RV in quarterly frequency (given by (1) with $\Omega = Quarter$). In the case of inequality, we use three alternative measures, i.e. (i) the Gini coefficient, (ii) standard deviation (of the data in natural logarithms), and (iii) the difference between the 90th and 10th percentile (with the data in natural logarithms). In other words, we include both absolute and relative measures of inequality. Various measures of economic inequality (taken into account one at a time) are calculated using survey data on income and consumption from the family expenditure survey[3]. Further details on the construction of the data and the survey are documented in Mumtaz and Theophilopoulou (2017)[4]. Note that we work with the growth rates of the inequality measures to ensure that the predictors under consideration are stationary as required by the empirical models. The growth rates of the three income-based inequality measures are denoted as x_1, x_2, and x_3, while the growth rates of the three consumption-based inequality measures are denoted as x_4, x_5, and x_6.

Tables 1 and 2 show the RMSE values for out-of-sample forecasting of RV using different models and predictors. Note, given that we have 164 observations to work with, following Rapach et al. (2005), we use 50% of the observations as in-sample, while the remaining 50% is used as the out-of-sample period, over which all our models are recursively estimated to mimic a pseudo out-of-sample forecasting scenario. As it can be seen, the best model with a specific-type of inequality (in the sense of minimum RMSE), is the linear ARX model with x_3 (i.e., the income inequality measure as given by the difference between the 90th and 10th percentile) for $h = 1, 2$. For $h = 4$, the best model is LSS with the x_5 (i.e., the consumption inequality measure as given by the standard deviation) as the predictor. Table 3 summarizes the

[3]The data is downloadable from: `https://discover.ukdataservice.ac.uk/series/?sn=200016` and `https://discover.ukdataservice.ac.uk/series/?sn=2000028`.

[4]We would like to thank Professor Haroon Mumtaz for kindly sharing the inequality data with us.

best models for the three horizons considered. Although the RMSE metric suggests that the models with the highest accuracy in forecasting RV are the linear ARX and the LSS with predictor variables x_3 and x_5, respectively, one needs statistical hypothesis testing in order to identify the best models and predictors. For this purpose, we use the KSPA statistic to test the null hypothesis that a model has the same forecasting accuracy as the best performing model (in the sense of minimum RMSEs).

Tables 4 and 5 show the p-values for the KSPA test, comparing the models and predictors with the minimum RMSE model in terms of the out-of-sample forecasts of RV. Table 6 shows the models and predictors for which the null hypothesis of the KSPA test is retained at $\alpha = 0.05$ significance level, (i.e. the models and predictors with the same accuracy as the minimum RMSE model).

According to the KSPA test results, for one-step-ahead forecasts, the linear ARX, $HAR-RV$ and $NAARX$ models with predictors, have the same accuracy as the minimum RMSE model. Further, there is no significant difference between the accuracy of the minimum RMSE model and the $NAAR$, AR, $HAR-RV$ models without any predictors. In the case of the two-step-ahead forecasts, we observe similar results although the $NAARX$ model with x_2 as the predictor and the $NAAR$ model are not found to have the same accuracy as the minimum RMSE, at two-step-ahead forecasting. Accordingly, the effect of x_3 in short- and medium-term forecasting of RV is not significant. Furthermore, using the ARX model with x_3 as the predictor, does not improve the short-term forecasting accuracy of the RW model. At the one-year-ahead forecasting horizon, however, there is a significant improvement to the forecasting ability of the RW model, using LSS. Furthermore, we observe that using x_5 (i.e., the consumption inequality measure as given by the standard deviation) as predictor, improves the accuracy of one-year-ahead forecasting.[5]

Note that, as indicated in the introduction, the theoretical predictions suggest that inequality can either increase volatility by contributing to both political and social uncertainty, or reduce volatility if income inequality is a

[5]Using the Minimum Absolute Error and AE function in KSPA test tends to provide similar results, which are available upon request from the authors.

signal about skilled decision making at the corporate level. To that end, the lack of predictive evidence of inequality for stock market volatility, particularly at the short- to medium-runs could be an indication that these two effects are possibly canceling out each other in our data set for the UK. However, the information content in the increased absolute consumption inequality (as given by the standard deviation), is likely to enhance stock market volatility in the longer run via heightened political and social risks that is generated.

4. Conclusion

Financial market volatility is used as an important input in investment decisions, option pricing and financial market regulation, thus making forecasting of volatility an important area of research for academics, investors and policymakers. Given this, we investigate whether income- and consumption-based relative and absolute measures of inequality possess any predictive power over stock market realized volatility of the UK, based on a unique high-frequency (quarterly) data set over 1975Q1 to 2016Q1. Using an array of univariate and bivariate linear and nonlinear models, we find that, while linear models with inequality can produce lower forecast errors, their performance is not statistically different from other univariate (and even bivariate) linear and nonlinear models in short- to medium-runs. At the same time, we also observe that growth in inequality, and in particular absolute consumption inequality, carries additional information in forecasting stock market volatility in the UK in the long-horizon.

In short, our findings imply that the long-run predictive power of consumption inequality over stock market volatility is possibly driven by its informational content over both political and social uncertainty in the long-run. This finding further supports the possible role of non-cashflow related factors on the stability of financial markets, although their predictive power is limited to longer horizons. As part of future research, given that inequality data is traditionally only available at annual frequency, it would be interesting to extend our analysis to multiple countries using panel data-based forecasting methods. This will, in the process, provide a more robust test (from the perspective of obtaining cross-country evidence) of the theoretical claims relating inequality to instability in financial markets.

References

Hamilton, J.D., Lin, G., 1996. Stock market volatility and the business cycle. Journal of Applied Econometrics 11 (5), 573593.

Schwert, G. W., 2011. Stock Volatility during the Recent Financial Crisis. European Financial Management 17 (5), 789-805.

Choudhry, T., Papadimitriou, F. I., Shabi, S. 2016. Stock market volatility and business cycle: Evidence from linear and nonlinear causality tests. Journal of Banking & Finance 66, 89101.

Blau, B.M. (2015). Income Inequality and Stock Market Volatility. Available at SSRN: http://dx.doi.org/10.2139/ssrn.2708496.

Poon, S-H, and Granger, C. W. J. (2003).Forecasting Volatility in Financial Markets: A Review. Journal of Economic Literature, 41(2), 478-539.

Gupta, R., Ma, J., Risse, M., and Wohar, M.E. (2018a). Common business cycles and volatilities in US states and MSAs: The role of economic uncertainty. Journal of Macroeconomics, **57(C)**, 317-337.

Rapach, D.E., Strauss, J.K., and Wohar, M.E. (2008). Forecasting stock return volatility in the presence of structural breaks, in Forecasting in the Presence of Structural Breaks and Model Uncertainty, in David E. Rapach and Mark E. Wohar (Eds.), Vol. 3 of Frontiers of Economics and Globalization, Bingley, United Kingdom: Emerald (May 2008), pp. 381-416.

Babikir, A., Gupta, R., Mwabutwa, C., and Owusu-Sekyere , E. (2012). Structural Breaks and GARCH Models of Stock Return Volatility: The Case of South Africa. Economic Modelling, 29(6), 2435-2443.

Ben Nasr, A. Ajmi, A.N., and Gupta, R. (2014).Modeling the Volatility of the Dow Jones Islamic Market World Index Using a Fractionally Integrated Time Varying GARCH (FITVGARCH) Model. Applied Financial Economics, 24(14), 993-1004.

Ben Nasr, A. Lux, T., Ajmi, A.N., and Gupta, R. (2016).Forecasting the volatility of the Dow Jones Islamic stock market index: Long memory vs.

regime switching. International Review of Economics and Finance, 45(1), 559-571.

Engle, R.F., and Rangel, J.G. (2008).The Spline-GARCH Model for Low-Frequency Volatility and Its Global Macroeconomic Causes. Review of Financial Studies 21(3), 1187-1222.

Rangel, J.G., and Engle, R.F. (2011).The Factor-Spline-GARCH Model for High and Low Frequency Correlations. Journal of Business & Economic Statistics 30(1), 109-124.

Engle, R.F., Ghysels, E., and Sohn, B. (2013).Stock Market Volatility and Macroeconomic Fundamentals. The Review of Economics and Statistics 95(3), 776-797.

Piketty, T. and Saez, E. (2014).Inequality in the long run, Science, 344, 383843.

Barro, R.J., 2000. Inequality and Growth in a Panel of Countries. Journal of Economic Growth, 5, 5-32.

Becker, G. and B. Chiswick, 1966.Education and the Distribution of Earnings. American Economic Review, 56, 350-369.

Lucas, R.E.B., 1977. Is There a Human Capital Approach to Income Inequality? Journal of Human Resources, 12, 387-395.

Becker, G. and K.M. Murphy, 2007.The Upside of Income Inequality. The American, 1: 20.

Brogaard, J., Detzel, A., and Ngo, P.T.H. (2015).Inequality and risk premia. Available at SSRN: http://dx.doi.org/10.2139/ssrn.2649558.

Christou, C., Gupta, R., and Jawadi, F. (2017). Does inequality help in forecasting equity premium in a panel of G7 countries? Department of Economics, University of Pretoria, Working Paper No. 201720.

Gupta, R., Pierdzioch, C., Vivian, A.J., and Wohar, M.E. (2018b). The predictive value of inequality measures for stock returns: An analysis of long-span UK Data using quantile random forests. Finance Research Letters. DOI: https://doi.org/10.1016/j.frl.2018.08.013.

Campbell, J.Y. (2008). Viewpoint: estimating the equity premium, Canadian Journal of Economics, 41, 121.

Gupta, R., Pierdzioch, C., Selmi, R., and Wohar, M.E. (2018a). Does Partisan Conflict Predict a Reduction in US Stock Market (Realized) Volatility? Evidence from a Quantile-on-Quantile Regression Model. North American Journal of Economics and Finance, **43**, 87-96.

Cai, Z., Fan, J. and Yao, Q. (2000), Functional-coefficient regression models for nonlinear time series, *Journal of the American Statistical Association*, **95**, 941-956.

Chen, R. and Tsay, R.S. (1993), Functional-coefficient autoregressive models, *Journal of the American Statistical Association*, **88**, 298-308.

Chen, R. and Tsay, R.S. (1993), Nonlinear additive ARX models, *Journal of the American Statistical Association*, **88**, 955-967.

Cai, Z. and Masry, E. (2000), Nonparametric estimation of additive nonlinear ARX time series: local linear fitting and projections, *Econometric Theory*, **16**,465-501.

Pearlman, J.G. (1980). An algorithm for the exact likelihood of a high-order autoregressive-moving average process, *Biometrika*, **67**, 232-233.

Shumway, R.H. and Stoffer, D.S. (2011), *Time Series Analysis and Its Applications With R Examples*, Springer, New York.

Andersen T.G., and Bollerslev T. (1998). Answering the skeptics: yes, standard volatility models do provide accurate forecasts. International Economic Review, 39 (4), 885-905.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Labys, P. (2001), The distribution of realized exchange rate volatility, *Journal of the American Statistical Association*, **96**, 42-55.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Ebens, H. (2001) The distribution of stock returns volatilities, *Journal of Financial Economics*, **61**, 43-76.

Barndorff-Nielsen, O., and Shephard, N. (2002a), Econometric analysis of realized volatility and its use in estimating stochastic volatility models, *Journal of the Royal Statistical Society, B*, **64**, 253-280.

Barndorff-Nielsen, O., and Shephard, N. (2002b), Estimating quadratic variation using realized variance, *Journal of Applied Econometrics*, **17**, 457-477.

Corsi, F. (2009), A simple approximate long-memory model of realized volatility, *Journal of Financial Econometrics*, **7**, 174-196.

Hassani, H. and Silva, E.S., 2015. A Kolmogorov-Smirnov based test for comparing the predictive accuracy of two sets of forecasts, *Econometrics*, **3**, 590-609.

Mumtaz, H., and Theophilopoulou, A. (2017).The impact of monetary policy on inequality in the UK.An empirical analysis. European Economic Review, 98, 410-423.

Rapach, D.E., Wohar, M.E., and Rangvid, J. (2005). Macro Variables and International Stock Return Predictability. International Journal of Forecasting, **21(1)**, 137-166.

Table 1: Out-of-sample RMSE for RV forecasting (based on 82 out-of-sample forecasts)

Predictor	Model	$h=1$	$h=2$	$h=4$
x_1	$FARX$	1.5104	228.410	2.671E+03
	$NAARX$	0.3472	0.3961	0.4301
	LSS	5.1227	5.3190	5.7348
	ARX	0.3413	0.3954	0.4278
	$HAR-RV^a$	0.3484	0.4075	0.4293
x_2	$FARX$	2.2922	3.115E+04	7.633E+04
	$NAARX$	0.6657	5.9579	0.9081
	LSS	4.3694	4.5020	4.7727
	ARX	0.3380	0.3935	0.4254
	$HAR-RV^a$	0.3474	0.4073	0.4286
x_3	$FARX$	1.5233	449.42	396.288
	$NAARX$	0.3649	0.3934	0.4980
	LSS	4.7007	4.8085	5.1005
	ARX	0.3358	0.3921	0.4236
	$HAR-RV^a$	0.3449	0.4062	0.4277
x_4	$FARX$	1.3477	38.6539	1.520E+06
	$NAARX$	4.7721	1.7204	0.5971
	LSS	4.6861	4.8258	5.1411
	ARX	0.3422	0.3949	0.4283
	$HAR-RV^a$	0.3508	0.4067	0.4287

[a]. The lag-structure of the model is $\omega_1 = 1, \omega_2 = 4$.

Table 2: Out-of-sample RMSE for RV forecasting (continued)

Predictor	Model	$h=1$	$h=2$	$h=4$
x_5	$FARX$	1.3637	51.6935	1.299E+07
	$NAARX$	0.3414	0.3992	0.4394
	LSS	1.2157	0.5571	0.1744
	ARX	0.3414	0.3946	0.4274
	$HAR-RV^a$	0.3514	0.4070	0.4282
x_6	$FARX$	1.4523	82.0759	9.746E+06
	$NAARX$	0.3456	0.3953	0.4291
	LSS	4.3086	4.4380	4.6928
	ARX	0.3403	0.3951	0.4268
	$HAR-RV^a$	0.3495	0.4087	0.4289
Without Predictors	$FARX$	1.3657	51.1659	2.801E+03
	$NAARX$	0.3941	0.4053	0.4198
	LSS	3.7939	3.8810	4.0633
	ARX	0.3384	0.3938	0.4257
	$HAR-RV^a$	0.3452	0.4063	0.4278
	RW	0.3593	0.4272	0.4890

[a]. The lag-structure of the model is $\omega_1 = 1, \omega_2 = 4$.

Table 3: Summary table (minimum out-of-sample RMSE models and predictors for RV forecasting)

	$h=1$	$h=2$	$h=4$
Model	ARX	ARX	LSS
Predictor	x_3	x_3	x_5

Table 4: KSPA test p-values (two tailed) for comparing the forecasting models to minimum RMSE RV forecast. (based on 82 out-of-sample forecasts)

	$h=1$	$h=2$	$h=4$
Minimum RMSE model → Comparing to ↓	ARX (x_3)	ARX (x_3)	LSS (x_5)
$FARX$ (x_1)	0.0000	0.0000	0.0000
$NAARX$ (x_1)	0.7027	0.9794	0.0000
LSS (x_1)	0.0000	0.0000	0.0000
ARX (x_1)	0.9806	1.0000	0.0000
$HAR-RV^a$ (x_1)	0.9806	0.8219	0.0000
$FARX$ (x_2)	0.0000	0.0000	0.0000
$NAARX$ (x_2)	0.0562	0.0216	0.0000
LSS (x_2)	0.0000	0.0000	0.0003
ARX (x_2)	0.9981	1.0000	0.0000
$HAR-RV^a$ (x_2)	0.8277	0.9220	0.0000
$FARX$ (x_3)	0.0000	0.0000	0.0000
$NAARX$ (x_3)	0.7027	0.9794	0.0000
LSS (x_3)	0.0000	0.0000	0.0006
ARX (x_3)			0.0000
$HAR-RV^a$ (x_3)	0.7027	0.8219	0.0000
$FARX$ (x_4)	0.0000	0.0000	0.0000
$NAARX$ (x_4)	0.7027	0.9220	0.0000
LSS (x_4)	0.0000	0.0000	0.0311
ARX (x_4)	0.9806	1.0000	0.0000
$HAR-RV^a$ (x_4)	0.9254	0.9220	0.0000

[a]. The lag-structure of the model is $\omega_1 = 1, \omega_2 = 4$.

Table 5: KSPA test p-values (two tailed) for comparing the forecasting models to minimum RMSE RV forecast. (continue)

	$h=1$	$h=2$	$h=4$
Minimum RMSE model $\rightarrow$	ARX (x_3)	ARX (x_3)	LSS (x_5)
Comparing to $\downarrow$			
$FARX$ (x_5)	0.0000	0.0000	0.0000
$NAARX$ (x_5)	0.8277	0.9220	0.0000
LSS (x_5)	0.0000	0.0000	
ARX (x_5)	0.9981	1.0000	0.0000
$HAR-RV^a$ (x_5)	0.4462	0.9794	0.0000
$FARX$ (x_6)	0.0000	0.0000	0.0000
$NAARX$ (x_6)	0.5705	0.9794	0.0000
LSS (x_6)	0.0000	0.0000	0.0000
ARX (x_6)	0.9806	1.0000	0.0000
$HAR-RV^a$ (x_6)	0.5705	0.9220	0.0000
FAR	0.0000	0.0000	0.0000
$NAAR$	0.8277	0.9794	0.0000
LSS (Without Predictors)	0.0000	0.0000	0.0000
AR	0.9981	1.0000	0.0000
$HAR-RV^a$ (Without Predictors)	0.5705	0.8219	0.0000
RW	0.1245	0.6953	0.0000

[a]. The lag-structure of the model is $\omega_1 = 1, \omega_2 = 4$.

Table 6: Forecasts similar to the Minimum RMSE for RV forecasting.[a]

Minimum RMSE model →	$h = 1$ ARX (x_3)	$h = 2$ ARX (x_3)	$h = 4$ LSS (x_5)
	$NAARX$ (x_1)	$NAARX$ (x_1)	
	ARX (x_1)	ARX (x_1)	
	$HAR-RV^b$ (x_1)	$HAR-RV^b$ (x_1)	
	$NAARX$ (x_2)	ARX (x_2)	
	ARX (x_2)	$HAR-RV^b$ (x_2)	
	$HAR-RV^b$ (x_2)	$NAARX$ (x_3)	
	$NAARX$ (x_3)	$HAR-RV^b$ (x_3)	
Similar forecasts	$HAR-RV^b$ (x_3)	$NAARX$ (x_4)	
($\alpha = 0.05$)	$NAARX$ (x_4)	ARX (x_4)	
	ARX (x_4)	$HAR-RV^b$ (x_4)	
	$HAR-RV^b$ (x_4)	$NAARX$ (x_5)	
	$NAARX$ (x_5)	ARX (x_5)	
	ARX (x_5)	$HAR-RV^b$ (x_5)	
	$HAR-RV^b$ (x_5)	$NAARX$ (x_6)	
	$NAARX$ (x_6)	ARX (x_6)	
	ARX (x_6)	$HAR-RV^b$ (x_6)	
	$HAR-RV^b$ (x_6)	AR	
	$NAAR$	$HAR-RV^b$ (Without Predictors)	
	AR	RW	
	$HAR-RV$ (Without Predictors)		
	RW		

[a]. H_0 Retained at 0.05 significance level .

[b]. The lag-structure of the model is $\omega_1 = 1, \omega_2 = 4$.