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Does Inequality Really Matter in Forecasting Real Housing Returns of the United Kingdom?

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Abstract

In this paper, we analyze the potential role of growth in inequality for forecasting real housing returns of the United Kingdom (UK). In our forecasting exercise, we use linear and nonlinear models, as well as, measures of absolute and relative consumption and income inequalities at quarterly frequency over the period of 1975 to 2016. Our results indicate that, while nonlinearity in the data generating process of real housing returns is important, growth in inequality does not necessarily carry important information in forecasting the future path of housing prices in the UK.

Keywords: Income and Consumption Inequalities; Real Housing Returns; Forecasting; Linear and Nonlinear Models; United Kingdom.

1. Introduction

The importance of the housing market, and in particular housing prices, in driving fluctuations in the real economy (as well as inflation) globally, especially in the wake of the recent financial crisis, is well-accepted now (see, [1, 2], [3], [4], [5] and [6] for detailed reviews of this literature). Naturally, accurate prediction of house prices is of tremendous importance to policymakers, to gauge the future path of the economy. Hence, not surprisingly, a large international literature exists (see for example, [7], [8], [9], [10], [11],

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[12], [13], [14], [15], [16], [17], [18] and references cited there in) that looks into the ability of various macroeconomic and financial variables based on alternative econometric approaches, in forecasting real estate prices.

In this regard, more recently, [19] points out that income inequality and house prices have risen sharply in developed countries during the last three decades. The authors argue that this co-movement is not a coincidence, but follows theoretically from two channels: First, an increase in income inequality raises the amount of people that are willing to pay high prices in order to access certain areas, when houses are considered as consumption goods; and second, inequality is expected to increase the absolute amount of savings (assuming that the propensity to consume is negatively related with higher incomes) when houses are considered as rent generating assets, which in turn raises the total demand for houses. In other words, inequality drives up house prices on the grounds that it raises the total demand for houses, which inflates housing prices given supply restrictions (see for example, [20], [21], [22], [23] [24] for detailed discussion of these theoretical channels).

When this hypothesis is tested for a panel of 18 The Organisation for Economic Co-operation and Development (OECD) countries for the period 1975-2010, the results of [19], suggest that income inequality and house prices in most OECD countries are positively correlated and co-integrated. Further, in the majority of cases absolute inequality Granger-causes house prices when measured in absolute terms. In addition, [19] shows that relative inequality is not co-integrated with house prices a result the authors point out to be expected given that total house demand depends on the absolute amount of investible income.

Against this backdrop, given the fact that in-sample predictability does not guarantee out-of-sample forecasting gain, and the suggestion in this regard that the ultimate test of any predictive model is its out-of-sample performance [25], the objective of this paper is to investigate for the first time whether inequality forecasts real housing returns in the United Kingdom (UK). We examine an unique data set at the (highest possible) quarterly frequency, over 1975Q1 to 2016Q1 which includes both income- and

consumption-based relative and absolute measures of inequality. Note that the choice of the UK as our case study is purely driven by data availability at a quarterly frequency, which is important, given the observation that the housing market leads the business cycle in the UK [26], and hence, accurate forecasting at quarterly frequency based on the information of inequality should be more relevant to policymakers than at the lower annual frequency. Recall that [19], analysed in-sample predictability of housing returns at the annual frequency using inequality data that is generally also available at the same frequency. Besides data-based reasons, when compared to 1975, real house prices in 2016 had appreciated by 124%, while income (consumption) inequality growth between this period has ranged between 10% to 21% (10% to 28%).[1] In addition, realizing that at higher frequency asset price movements are nonlinearly related with its predictors (as highlighted for stock returns and the same inequality dataset for the UK by [27]),[2] we not only use linear models for forecasting, but also nonparametric models. It is important to point out that our models are bivariate in nature and includes real housing returns and various measures of the growth rates of inequality (considered in turn), since the inequality on its own can be considered to encompass information of various other macroeconomic and financial variables as well, given the general equilibrium effects of inequality [30]. In fact, when we analyzed the correlation between our various inequality measures with two important predictors of the housing market (as suggested by the literature discussed above): output (real Gross Domestic Product (GDP)) and real interest rate (3-months Treasury bill rate less consumer price index (CPI) inflation rate) of the UK, the correlation was significant at 1% level of significance and consistently over 55%.[3]

[1] In the UK, Homes in popular towns and London boroughs have risen to 10 and 20 times local incomes, while rents account for up to 78% of earnings [29].

[2] Widespread evidence of nonlinearity in house prices of both emerging and advanced countries have been recently provided by [28].

[3] Interestingly, [19], could not detect any causality running from output to inequality for the OECD countries considered in their sample, but real interest rate did carry information of predictability for house prices.

The remainder of the paper is organized as follows: Section 2 outlines the alternative econometric models used for our forecasting analysis, while, Section 3 discusses the data and results, with Section 4 concluding the paper.

2. Model Description

2.1. Functional-Coefficient Autoregressive with Exogenous variables:

The Functional-Coefficient Autoregressive with Exogenous variables ($FARX$) formulates the time series y_t as follows [31, 32]:

$$y_t = \sum_{i=1}^{p} f_i(y_{t-d})y_{t-i} + \sum_{i=1}^{q} g_i(y_{t-d})x_{t,i} + \varepsilon_t,$$

where ε_t is white noise and $x_i (i = 1, \ldots, q)$ are exogenous variables (and may contain the exogenous variables' lags). The nonlinear functions $f_i(y_{t-d})$ and $g_i(y_{t-d})$ are estimated using local linear regression [31].

2.2. Nonlinear Additive Autoregressive with Exogenous variables:

The Nonlinear Additive Autoregressive with Exogenous variables ($NAARX$) uses the following formulation for time series modeling [33]:

$$y_t = \sum_{i=1}^{p} f_i(y_{t-i}) + \sum_{i=1}^{q} g_i(x_{t,i}) + \varepsilon_t,$$

where ε_t is white noise and $x_i (i = 1, \ldots, q)$ are exogenous variables (and may contain the exogenous variables' lags). The nonlinear functions $f_i(y_{t-i})$ and $g_i(x_{t,i})$ can be estimated using local linear regression [34].

2.3. Liner State Space Model:

A Liner State Space Model (LSS) uses following formulation to represent a linear ARX model:

$$\begin{cases} \boldsymbol{s}_t = \boldsymbol{A}\boldsymbol{s}_{t-1} + \boldsymbol{b}u_t \\ y_t = \boldsymbol{c}'\boldsymbol{s}_t + \boldsymbol{\beta}'\boldsymbol{x}_t + \varepsilon_t \end{cases}$$

where $\boldsymbol{s}_t$ is the state vector, u_t and ε_t are mutually *iid* Gaussian random variables (with variances η^2 and σ^2) and $\boldsymbol{x}_t$ is a vector of exogenous variables.

The system's matrices $\boldsymbol{A}$, $\boldsymbol{b}$, $\boldsymbol{c}$ and $\boldsymbol{\beta}$ and the exogenous vector are defined as follows [35]:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ \phi_p & \phi_{p-1} & \phi_{p-2} & \cdots & \phi_1 \end{bmatrix}_{p\times p},$$

$$\boldsymbol{b} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b \end{bmatrix}_{p\times 1}, \ \boldsymbol{c} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ c \end{bmatrix}_{p\times 1}, \ \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_q \end{bmatrix}_{(q+1)\times 1}, \ \boldsymbol{x}_t = \begin{bmatrix} 1 \\ x_{t,1} \\ \vdots \\ x_{t,q} \end{bmatrix}_{(q+1)\times 1}.$$

One may use an EM algorithm based on Kalman recursions to estimate the system's matrices [36].

2.4. Forecasting Evaluation

Suppose $E(y_t|\mathcal{F}_{\ t-1})$ is the forecast of real housing returns (conditional on the information set $\mathcal{F}_{\ t-1}$) and ε_t is the residual of the conditional mean model at time t:

$$\varepsilon_t = y_t - E(y_t|\mathcal{F}_{\ t-1}),$$

Root Mean Square Error ($RMSE$):

$$RMSE = \left(\frac{1}{n}\sum_{t=1}^{n}(\varepsilon_t)^2\right)^{\frac{1}{2}}$$

Diebold-Mariano test: Suppose there is two forecasting models to forecast time series $y_t; (t = 1, \ldots, n)$. The Diebold Mariano test (DM test) compares the accuracy of two forecasts, regarding some accuracy measure $g(.)$ [37]. The null hypothesis and the alternative in two tailed DM test are as follows:

$$\begin{cases} H_0 : \text{The accuracy of two forecasts are the same} \\ H_1 : \text{The accuracy of two forecasts are not the same} \end{cases}$$

If $(e_{t,1}, e_{t,2}); (t = 1, \ldots, n)$ are h-steps ahead forecast errors generated by two forecasting models, the DM tests $H_0 : E(g(e_{t,1}) - g(e_{t,1})) = 0$, vs. $H_1 : E(g(e_{t,1}) - g(e_{t,1})) \neq 0$. Two popular accuracy measures in DM test, are Square Error, SE, (i.e. $g(x) = x^2$) and Absolute Error, AE, (i.e. $g(x) = |x|$).

3. Data and Results

3.1. Data Description

Data on real house price for the UK is obtained from the OECD,[4] which originally sources the data from the Department for Communities and Local Government, with the house price corresponding to the sales of all types of newly-built and existing residential dwellings across the whole country. Nominal house price is divided using the private consumption expenditure deflator from the national account statistics of the OECD.[5] The three measures of inequality used are the Gini coefficient, standard deviation (of the data in natural logarithms), and the difference between the 90th and 10th percentile (with the data in natural logarithms). In other words, we include both absolute and relative measures of inequality, the importance of which has been highlighted by [19]. The various inequality measures are calculated using survey data on income and consumption from the family expenditure survey.[6] Further details on the construction of the data and the survey are documented in [38].[7] Note that we work with the growth rates of both real housing prices and the inequality measures to ensure that our variables under consideration is stationary as required by the empirical models. We abbreviate the growth rates of the three income-based inequality measures as x_1, x_2, and x_3, while the growth rates of the three consumption-based inequality measures are denoted as x_4, x_5, and x_6, and y is used to depict real housing (log) returns.

[4]http://www.oecd.org/eco/outlook/focusonhouseprices.htm.

[5]http://www.oecd.org/sdd/oecdmaineconomicindicatorsmei.htm.

[6]The data is downloadable from: https://discover.ukdataservice.ac.uk/series/?sn=200016 and https://discover.ukdataservice.ac.uk/series/?sn=2000028.

[7]We would like to thank Professor Haroon Mumtaz for kindly sharing the inequality data with us.

3.2. Results

Tables 1 and 2 show the RMSE for out of sample y forecasting using different models and predictors. Note, given that we have 164 observations to work with, following [39], we use 50% of the observations as in-sample, while the remaining 50% is used as the out-of-sample period, over which all our models are recursively estimated to mimic a pseudo out-of-sample forecasting scenario. We conduct the forecasting exercise over horizons of one, two, and four-quarters-ahead, i.e., for $h = 1$, 2, and 4. As it can be seen, the best model and predictors (in the sense of minimum RMSE), for one step ahead forecasting ($h = 1$) is the linear $ARMAX$ model and x_5 respectively. In other forecasting horizons ($h = 2, 4$), the best out of sample forecast are given by FAR model without any predictors. Table 3 summarizes the best models for the three forecasting horizons considered. Note that the importance of an absolute measure of inequality in predicting real housing returns at $h = 1$, is in line with [19]. The relevance of consumption over income inequality is possibly an indication of housing serving as a consumption rather than an investment good, which has traditionally been the case in the UK ([40]). Given this, and the fact that wealth effects are important in defining consumption movements (see for example, [41]), inequality in consumption is possibly bringing in the information of the wealth channel, and hence, is more important than income-based measures of inequality. In addition, the role of nonlinearity in forecasting housing returns is in line with the overwhelming evidence that house prices do not evolve in a linear manner across the world by [28].

Table 1 appears here.

Table 2 appears here.

Table 3 appears here.

Although the RMSE metric suggests that the best model to forecast y, are linear $ARMAX$ (with x_5 as predictor) and FAR, concluding which models and predictors are the best, needs a statistical hypothesis testing. One may

use DM statistic to test null hypothesis under which a given model has the same forecasting accuracy as the best model (in the sense of minimum RMSE). Tables 4 and 5 show the p-values for DM test, comparing the models and predictors with the minimum RMSE model (as summarized in Table 3). Table 6 shows the models and predictors for which the DM test's null hypothesis is retained under $\alpha = 0.05$ significance level, (i.e. the models and predictors with same accuracy as the minimum RMSE model).

Table 4 appears here.

Table 5 appears here.

Table 6 appears here.

According to the DM results, for one-step-ahead forecasts, the linear models ARX and $ARMAX$ (with variety of predictors), the nonlinear model $NAARX$ (with variety of predictors) and the models without predictors (liner and nonlinear), as well as the Random Walk, RW, have the same out of sample forecasting accuracy as the minimum RMSE model (i.e. the $ARMAX$ with x_5 predictor), at 5% significance level. At two-step-ahead forecasting horizon, the $NAARX$ model with predictors x_1 and x_3 has the same performance as minimum RMSE model, FAR. However, none of the linear models has the same performance as the minimum RMSE model. Finally, the four-step-ahead forecasting results show that the linear models ARX, $ARMAX$, AR and $ARMA$ and nonlinear models $FARX$, $NAARX$ and $NAAR$ have the same performance as the FAR model. However, the FAR model produces better performance in comparison to the RW. As the results show, the FAR model can be used as the best forecasting model for the forecasting horizons considered over a year, since it has the minimum RMSE model for h=2, and 4, and has the same forecasting accuracy as the minimum RMSE model for h=1. But more importantly, now after conducting formal tests of forecast comparison, we can conclude that, across all forecasting horizons considered in this paper, the inequality variables do not statistically improve the forecasting accuracy of real housing returns, but

what is more important is incorporating nonlinearity instead.[8] In the process, from a general perspective, our results also highlight the importance of conducting out-of-sample evaluation to determine the importance of a predictor, as we show that in-sample evidence of predictability, as provided in [19], might not carry over to forecasting.

4. Conclusion

Recent theoretical models have related inequality with housing prices, and some empirical support to this line of research has also been provided based on in-sample tests of causality. However, there is widespread acceptance of the fact that in-sample predictability does not necessarily translate into out-of-sample forecasting gains, and hence, it is tests of forecasting accuracy that actually provides a more robust measure of predictability. Given this, we investigate whether income- and consumption-based relative and absolute measures of inequality can forecast real housing returns in the United Kingdom (UK), based on an unique high-frequency (quarterly) data set over 1975Q1 to 2016Q1. Using an array of univariate and bivariate linear and nonlinear models, we find that, while nonlinearity in the data generating process of real housing returns matter, growth in inequality does not necessarily additional information in forecasting housing prices in the UK. So, based on a more powerful empirical approach of forecasting relative to in-sample tests of causality, we show that theoretical predictions do not hold for high-frequency data from the UK.

As part of future research, given that inequality data is traditionally only available at annual frequency, it would be interesting to extend our analysis to multiple countries using panel data-based forecasting methods. This will, in the process, provide a more robust test (from the perspective of obtaining cross-country evidence) of the theoretical claims relating inequality with movements in housing prices.

[8] Using the Minimum Absolute Error (MAE) and the corresponding AE function in DM test produces qualitatively similar results. These results are available upon request from the authors.

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Tables

Table 1: Out-of-sample RMSE for real housing (log) returns forecasting

Predictor	Model	$h = 1$	$h = 2$	$h = 4$
x_1	$FARX$	0.13494	0.12450	0.10824
	$NAARX$	0.03982	1.51128	0.05360
	LSS	0.02261	0.02105	0.02578
	ARX	0.01825	0.02166	0.02395
	$ARMAX$	0.01816	0.02197	0.02387
x_2	$FARX$	0.13120	0.13786	0.13305
	$NAARX$	0.01769	0.02049	0.02801
	LSS	3.31657	3.10756	2.70853
	ARX	0.01951	0.02277	0.02565
	$ARMAX$	0.01976	0.02329	0.02545
x_3	$FARX$	0.17185	0.32063	0.33945
	$NAARX$	0.03915	0.44031	0.04747
	LSS	4.99911	4.07168	4.10167
	ARX	0.01671	0.02066	0.02428
	$ARMAX$	0.01658	0.02076	0.02417
x_4	$FARX$	0.24093	0.74025	58131.880
	$NAARX$	0.01762	0.02231	0.04581
	LSS	4.28459	3.57179	3.72588
	ARX	0.01554	0.01929	0.02350
	$ARMAX$	0.01547	0.01938	0.02356

Table 2: Out-of-sample RMSE for real housing (log) returns forecasting (continued)

Predictor	Model	$h = 1$	$h = 2$	$h = 4$
x_5	$FARX$	0.18429	0.31920	37.23637
	$NAARX$	0.01537	0.01914	0.02397
	LSS	4.20324	3.66080	3.86548
	ARX	0.01538	0.01941	0.02357
	$ARMAX$	0.01520	0.01948	0.02360
x_6	$FARX$	0.16372	0.21802	0.32259
	$NAARX$	0.01578	0.01933	0.02354
	LSS	4.60095	3.93811	3.89997
	ARX	0.01615	0.01994	0.02396
	$ARMAX$	0.01593	0.02000	0.02395
Without Predictors	$FARX$	0.01652	0.00149	0.02318
	$NAARX$	2.55958	0.02529	0.02874
	LSS	0.25128	0.37653	0.63382
	ARX	0.01558	0.01940	0.02360
	$ARMAX$	0.01547	0.01957	0.02380
	RW	0.01629	0.02145	0.02945

Table 3: Summary table (minimum out of sample RMSE models and predictors for real housing (log) returns forecasting)

	h = 1	h = 2	h = 4
Model	$ARMAX$	FAR	FAR
Predictor	x_5	.[a]	.[a]

[a]. Without Predictors .

Table 4: DM test P-values (two tailed) for comparing the out of sample forecasts to minimum RMSE real housing (log) returns forecast.[a]

	$h = 1$	$h = 2$	$h = 4$
Minimum RMSE model →	$ARMAX$ (x_5)	FAR	FAR
Comparint to ↓			
$FARX$ (x_1)	0.00000	0.00000	0.00000
$NAARX$ (x_1)	0.14263	0.30752	0.19281
LSS (x_1)	0.00150	0.00000	0.08747
ARX (x_1)	0.00135	0.00000	0.89039
$ARMAX$ (x_1)	0.00135	0.00000	0.89039
$FARX$ (x_2)	0.00000	0.00000	0.00000
$NAARX$ (x_2)	0.00575	0.00000	0.00877
LSS (x_2)	0.00000	0.00000	0.00000
ARX (x_2)	0.00000	0.00000	0.80025
$ARMAX$ (x_2)	0.00000	0.00000	0.80025
$FARX$ (x_3)	0.00000	0.00000	0.00000
$NAARX$ (x_3)	0.30176	0.30668	0.15273
LSS (x_3)	0.00000	0.00000	0.00000
ARX (x_3)	0.01723	0.00001	0.82139
$ARMAX$ (x_3)	0.01723	0.00001	0.82139
$FARX$ (x_4)	0.00000	0.00158	0.22123
$NAARX$ (x_4)	0.19141	0.00001	0.12051
LSS (x_4)	0.00000	0.00000	0.00000
ARX (x_4)	0.48888	0.00001	0.83342
$ARMAX$ (x_4)	0.48888	0.00001	0.83342

[a]. The test is based on SE .

Table 5: DM test P-values (two tailed) for comparing the out of sample forecasts to minimum RMSE real housing (log) returns forecast.[a] (continue)

	$h = 1$	$h = 2$	$h = 4$
Minimum RMSE model →	$ARMAX$ (x_5)	FAR	FAR
Comparint to ↓			
$FARX$ (x_5)	0.00000	0.00000	0.15692
$NAARX$ (x_5)	0.95882	0.00009	0.68433
LSS (x_5)	0.00000	0.00000	0.00000
ARX (x_5)	0.95892	0.00001	0.84244
$ARMAX$ (x_5)		0.00001	0.84244
$FARX$ (x_6)	0.00000	0.00000	0.00030
$NAARX$ (x_6)	0.51018	0.00008	0.85039
LSS (x_6)	0.00000	0.00000	0.00000
ARX (x_6)	0.00859	0.00002	0.85152
$ARMAX$ (x_6)	0.00859	0.00002	0.85152
FAR	0.47903		
$NAAR$	0.31408	0.00000	0.12092
LSS	0.00000	0.00000	0.00000
(Without Independents)			
AR	0.49541	0.00007	0.84953
$ARMA$	0.49541	0.00007	0.84953
RW	0.26137	0.00004	0.00000

[a]. The test is based on SE .

Table 6: Forecasts similar to the Minimum RMSE for real housing (log) returns forecasting.[a]

Minimum RMSE model →	$h = 1$	$h = 2$	$h = 4$
	$ARMAX$ (x_5)	FAR	FAR
	$NAARX$ (x_1)	$NAARX$ (x_1)	$NAARX$ (x_1)
	$NAARX$ (x_3)	$NAARX$ (x_3)	LSS (x_1)
	$NAARX$ (x_4)		ARX (x_1)
	ARX (x_4)		$ARMAX$ (x_1)
	$ARMAX$ (x_4)		ARX (x_2)
	$NAARX$ (x_5)		$ARMAX$ (x_2)
	ARX (x_5)		$NAARX$ (x_3)
	$NAARX$ (x_6)		ARX (x_3)
Similar forecasts	FAR		$ARMAX$ (x_3)
$(\alpha = 0.05)$	$NAAR$		$FARX$ (x_4)
	AR		$NAARX$ (x_4)
	$ARMA$		ARX (x_4)
	RW		$ARMAX$ (x_4)
			$FARX$ (x_5)
			$NAARX$ (x_5)
			ARX (x_5)
			$ARMAX$ (x_5)
			$NAARX$ (x_6)
			ARX (x_6)
			$ARMAX$ (x_6)
			$NAAR$
			AR
			$ARMA$

[a]. H_0 Retained at 0.05 significance level .