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Abstract

This paper contributes to the literature on stock market predictability by exploring the causal relationships between equity return dispersion, stock market volatility and excess returns via multivariate nonlinear causality tests. Both bivariate and multivariate nonlinear causality tests yield significant evidence of causality from return dispersion to both stock market volatility and equity premium, even after controlling for the state of the economy. While we find significant causality from business conditions to return dispersion, we see that expansionary (contractionary) market states are associated with low (high) level of equity return dispersion, indicating asymmetries in the relationship between equity return dispersion and economic conditions. Overall, our findings suggest that both return dispersion and business conditions are valid joint forecasters of both the stock market volatility and excess market returns and that return dispersion possesses incremental information regarding future stock return dynamics beyond which can be explained by the state of the economy.

Keywords: Equity return dispersion, Stock market volatility, Business cycle, Multivariate causality

JEL Codes: C32, E32, G10

1. Introduction

Recent literature has documented that equity return dispersion, measured by the cross-sectional standard deviation of stock returns, either at the individual stock or disaggregate portfolio level, carries reliable information regarding the state of the economy and future stock market volatility. While earlier studies including Christie and Huang (1994) and Duffee (2001) associate dispersion of stock returns with business cycles, later studies including Stivers (2003) and Connolly and Stivers (2006) show that return dispersion possesses incremental information regarding idiosyncratic as well as aggregate stock market volatility. In a more recent application to G7 countries, Angelidis et al. (2015) further support the role of return dispersion as an economic state variable and show that return dispersion reliably predicts the time-variation in stock market returns, volatility as well as the value and momentum premia observed in the cross-section of stock returns. Similarly, Maio (2016) shows that return dispersion consistently forecasts a decline in the excess market returns, with superior out-of-sample performance in predicting the equity premium, compared to alternative predictors including the dividend yield, term spread, etc.

Meanwhile, another strand of the literature provides ample evidence linking stock market volatility to real economic activity (e.g., Hamilton and Lin, 1996; Schwert, 2011) and stock market volatility to future aggregate stock returns (e.g. Goyal and Santa-Clara, 2003; Bali et al., 2005; Pollet and Wilson, 2010; Garcia et al.,2014). In a recent study, applying linear and nonlinear causality tests, Choudry et al. (2016) show that a bidirectional causal relationship exists between stock market volatility and the business cycle in a sample of four major economies without utilizing return dispersion in their multivariate tests. Given the ample evidence on the predictive power of equity return dispersion on stock market volatility and the evidence of bidirectional causality between stock market volatility and the business cycle, a natural research question is whether the predictive power of return dispersion is driven by a common fundamental factor that drives both stock market volatility and the dispersion of stock returns. To that end, multivariate causality tests provide a valuable avenue for empirical analysis as we are able to test for causality between return dispersion and stock market premium and volatility after controlling for the state of the economy.

This paper contributes to the literature on stock market predictability by exploring the causal relationship between return dispersion and stock market volatility and excess returns via multivariate nonlinear causality tests recently developed by Bai et al. (2010, 2011, 2018) and Chow et al. (2018). The advantage of multivariate causality tests, as opposed to bivariate alternatives that are often employed in the literature, is that it allows us to control for business cycles via the business conditions index that we utilize in our tests. Given the recent evidence by Choudhry et al. (2016) of a bidirectional causal relationship between stock market volatility and the business cycle, the multivariate causality tests that control for business cycles in the causal relationship between return dispersion and stock market volatility allows us to explore whether return dispersion possesses any incremental information regarding stock market return dynamics even after controlling for business cycles, and thus enlarges our understanding of the role of return dispersion as an economic state variable. The issue is of interest for not only from the perspective of stock market predictability, but also has significant applicability to the pricing of derivatives, hedging and portfolio diversification as volatility forecasts are integral part of these exercises.

Performing a combination of linear vs. nonlinear and bivariate vs. multivariate causality tests, we find that linear causality tests generally fail to detect causal effects from return dispersion to excess market returns and volatility. While we observe some evidence of causality from return dispersion to both stock market volatility and excess returns, we see that causality disappears when we control for the business conditions via the Aruoba-Diebold-Scotti business conditions index. Furthermore, we find that the predictive power of business conditions on return dispersion is concentrated on contractionary periods only, suggesting the presence of asymmetric causal interactions between business conditions, equity return dispersion and stock market volatility.

Both bivariate and multivariate nonlinear causality tests, however, yield significant evidence of causality from return dispersion to both stock market volatility and equity premium. While we find significant causality from business conditions to return dispersion, we see that expansionary (contractionary) market states are associated with low (high) level of equity return dispersion, in line with the findings in Angelidis et al. (2015) that high return dispersion is associated with a deterioration of business conditions. Overall, our findings suggest that both return dispersion and business conditions are valid joint forecasters of both the stock market volatility and excess market return and that return dispersion indeed possesses incremental information regarding future stock return dynamics beyond which can be explained by the state of the economy.

The results have significant implications for stock market forecasting models as well as for policy makers to take into account the cross-sectional variation in stock returns and nonlinearities when assessing the predictors of stock market dynamics. The rest of the paper is organized as follows. Section 2 provides a brief review of the literature on equity return dispersion in asset pricing and investments. Section 3 presents the data and the methodology for linear and nonlinear multivariate causality tests. Section 4 presents the empirical findings and Section 5 concludes the paper.

2. Literature Review

The literature provides ample evidence that associates equity return dispersion with different aspects of risk. In earlier studies focusing on U.S. stock returns, Christie and Huang (1994) and Duffee (2001) associate return dispersion with economic expansions and recessions, documenting asymmetries in the cross-sectional dispersion of stock returns with respect to stock market movements and business cycles. Similarly, Loungani et al. (1990) find that an index that measures the dispersion among stock prices from different industries has predictive power over unemployment. To that end, early research establishes evidence of an association between equity return dispersion and macroeconomic indicators.

In another strand of the literature that is related to portfolio management, studies including Lillo and Mantegna (2000), Solnik and Roulet (2000), Baur (2006), Statman and Scheid (2005, 2008) and Demirer (2013) relate return dispersion to the association of asset returns and examine this statistic in the context of portfolio diversification. While Baur (2006) notes that return dispersion can be used to obtain additional information about market linkages that is not provided by correlation, Eiling and Gerard (2011) utilize a variant of the dispersion measure in order to examine the time variation in linkages among global stock markets.

In studies that are more related to the focus of our empirical analysis, studies including Stivers (2003), Connolly and Stivers (2006) establish a link between return dispersion, aggregate market volatility and idiosyncratic volatility and suggest that return dispersion provides signals about future aggregate stock market volatility. Further extending the role of return dispersion to asset pricing models, Stivers and Sun (2010) and Bhootra (2011) associate the time variation in the value and momentum premia with the variation in the market's cross-sectional return dispersion. Similarly, studies including Jiang (2010), Demirer and Jategaonkar (2013) and Demirer et al. (2015) show that return dispersion serves as a systematic risk factor, carrying a positive price of risk in the cross-section of stock returns, while Demirer and Jategaonkar (2013) show that return dispersion risk is asymmetrically priced, conditional on the market return.

Building on the recent evidence from asset pricing tests, Chichernea et al. (2015) further support the role of return dispersion as a systematic risk factor and document that return dispersion has explanatory power for accrual and investment anomalies, associating high level of return dispersion exposure with conditions that are not conducive to growth and investment. A natural research question, therefore, is what drives the predictive value of return dispersion for future returns and volatility and whether this predictive ability is indeed driven by the information return dispersion possesses regarding the state of the economy. To that end, multivariate causality tests provide an interesting opening as they allow us not only to account for possible nonlinearities in the time series, but also examine the causal associations between return dispersion and stock market return and volatility after controlling for business conditions.

3. Data and Methodology

3.1. Data

The primary variables of interest in our causality tests are equity return dispersion and stock market volatility, with the Aruoba-Diebold-Scotti business conditions index used as a control variable in our multivariate tests. The sample period covers July 1963 to February 2017. Equity return dispersion (RD_t) for day t is expressed as the cross-sectional standard deviation of daily stock returns calculated as

$$RD_t = \sqrt{w_{i,t} \sum_{i=1}^{N} \left(r_{i,t} - r_{m,t}\right)^2} \qquad \textbf{(1)}$$

where $r_{i,t}$ and $r_{m,t}$ are the return for stock *i* and the market for day *t*, respectively; $w_{i,t} = 1/N$ for the equally-weighted cross-sectional dispersion of equity returns and N is the number of stocks. Following Stivers and Sun (2010), Angelidis et al. (2015) and Maio (2016), we compute the cross-sectional standard deviation of daily returns on 100 portfolios sorted on size and book-to-market ratios, obtained from Ken French's website as an estimate for equity return dispersion. Maio (2016) argues that the use of portfolios in the computation of return dispersion mitigates estimation errors due to the presence of illiquid and small stocks in the cross-section of individual stocks. Likewise, we obtain data for daily excess returns on the market, defined as the CRSP value-weighted index return minus the one-month Treasury bill rate, from Ken French's website.

Solnik and Roulet (2000) use the market model benchmark to show that return dispersion relates to the cross-sectional correlation of asset returns. However, unlike traditional measures of correlation and volatility, return dispersion provides an aggregate measure of co-movement in a portfolio for a given time period. To that end, the equity return dispersion measure in Equation (1) can be regarded as a measure of directional similarity in stock returns for a given day. In the case of stock market volatility, we follow Choudry et al. (2016) and estimate stock market volatility (SV) by means of the univariate GARCH(1,1) model of CRSP market index returns.

Figure 1 presents the time series plots for daily equity return dispersion (RD) and stock market volatility (SV) during the sample period. Not surprisingly, we observe several notable spikes in both series particularly during the Asian crisis period in the late 1990s and the global financial crisis periods, in line with the previous studies associating high stock market volatility with recessionary periods and periods of market stress (e.g., Schwert, 2011; Hamilton and Lin, 1996). It is interesting that return dispersion values also exhibit similar spikes during these periods. Demirer et al. (2018) note that these periods are also associated with spikes observed in the level of global risk aversion, driving equity market correlations higher globally. To that end, the high level of equity return dispersion observed in Figure 1 during periods when stock market volatility also rises suggests that these two series are possibly driven by a common fundamental factor related to the economy.

Motivated by studies including Stivers and Sun (2010) and Angelidis et al. (2015) suggesting that equity return dispersion can predict the time-variation in economic activity, we supplement our multivariate causality tests with the Aruoba-Diebold-Scotti Business Conditions Index *(ADS)* in order to account for economic conditions in the causal effect of return dispersion on stock market return and volatility. The ADS index developed in Aruoba et al. (2009) measures economic activity at high frequency using a dynamic factor model that includes a number of economic variables. We obtain the data for the ADS index from the Philadelphia Fed's website and utilize this index in our multivariate causality tests in order to track the predictive ability of business conditions along with return dispersion over stock market volatility and premium.

3.2. Methodology

3.2.1 Bivariate linear causality tests

In order to examine the bivariate linear causal relationship between any pair of equity return dispersion (RD_t), stock market volatility (SV_t), equity market premium (MP_t), business conditions index (ADS_t), and the positive [negative] ADS business conditions index values $(ADS1_t)$ [$(ADS2_t)$], we let x_t and y_t present any pair of RD_t, SV_t, MP_t, ADS_t, $ADS1_t$, and $ADS2_t$ that we are interested in studying and utilize the widely accepted vector autoregression (VAR) specification and the corresponding Granger causality test (Granger, 1969). Consider the following two-equation model:

$$x_t = a_1 + \sum_{i=1}^{p} \alpha_i x_{t-i} + \sum_{i=1}^{p} \beta_i y_{t-i} + \varepsilon_{1t} \quad , \tag{2a}$$

$$y_t = a_2 + \sum_{i=1}^{p} \gamma_i x_{t-i} + \sum_{i=1}^{p} \delta_i y_{t-i} + \varepsilon_{2t} \quad , \tag{2b}$$

where x_t and y_t are stationary variables, p is the optimal lag in the system based on the well-known information criteria such as the Akaike information criterion (AIC), and ε_{1t} and ε_{2t} are the disturbances satisfying the regularity assumptions of the classical linear normal regression model. The variable $\{y_t\}$ is said not to Granger cause $\{x_t\}$ if $\beta_i = 0$ in Equation (2a), for any $i = 1, \ldots, p$. In other words, the past values of $\{y_t\}$ do not provide any additional information on $\{x_t\}$. Similarly, $\{x_t\}$ dose not Granger cause $\{y_t\}$ if $\gamma_i = 0$ in Equation (2b), for any $i = 1, \ldots, p$. In order to test for Granger causality, we use the following null hypotheses separately:

$$H_0^1 : \beta_1 = \beta_2 = \cdots = \beta_p = 0 \, , \tag{3}$$

$$H_0^2 : \gamma_1 = \gamma_2 = \cdots = \gamma_P = 0 \, ,$$

and use the standard F-test to empirically test them.

There are four different situations for the causality relationships between RD_t and SV_t in (2a) and (2b): (a) rejecting H_0^1 but not rejecting H_0^2 implies a unidirectional causality from SV_t to RD_t, (b) rejecting H_0^2 but not rejecting H_0^1 implies a unidirectional causality from RD_t to SV_t, (c) rejecting both H_0^1 and H_0^2 implies the existence of feedback relations, and (d) not rejecting both H_0^1 and H_0^2 implies that RD_t and SV_t are not rejected to be independent. Readers may refer to Bai et al. (2010, 2011, 2018), Chow et al. (2018), and the references therein for the details of testing H_0^1 and/or H_0^2.

3.2.2 Nonlinearity tests

In this paper, we first perform linear causality test, and thereafter, conduct nonlinear causality tests to test whether there is any linear and nonlinear causality among RD_t, SV_t, MP_t, ADS_t, $ADS1_t$, and $ADS2_t$. If it is necessary to conduct nonlinear causality tests on the variables, we believe that the residuals obtained from performing the linear causality should contain nonlinearity. In addition, RD_t, SV_t, MP_t, ADS_t , $ADS1_t$, and $ADS2_t$ should contain some nonlinear elements so that linear causality cannot eliminate nonlinearity. Thus, in this paper, we conduct a nonlinear test on RD_t, SV_t, MP_t, ADS_t, $ADS1_t$, and $ADS2_t$. We will let Y_t represent RD_t, SV_t, MP_t, ADS_t , $ADS1_t$, and $ADS2_t$. In order to test for nonlinearity in the variable Y_t, we first remove the linear components in the series $\{Y_t\}$ by using an AR specification and compute the residuals series of $\{Y_t\}$ without loss of generality, we also let $\{Y_t\}$ to be the residuals series of $\{Y_t\}$ if there is no confusion. The series $\{Y_t\}$ does not possess any nonlinearity if and only if, for any t, the law of corresponding residuals $\{Y_t\}$ satisfies $L(Y_t|Y_{t-1}) = L(Y_t)$ and we define $C_1(\tau) \equiv \Pr(Y_{t-1} < \tau, Y_t < \tau)$, $C_2(\tau) \equiv \Pr(Y_{t-1} < \tau)$, and $C_3(\tau) \equiv \Pr(Y_t < \tau)$. Since

$\Pr(Y_t < \tau|Y_{t-1} < \tau) = \frac{C_1(\tau)}{C_2(\tau)}$, we can test the following hypothesis when testing the existence of the nonlinear of a sequence $\{Y_t\}$:

$$H_0: \frac{C_1(\tau)}{C_2(\tau)} - C_3(\tau) = 0 . \tag{4}$$

For a residual sequence $\{Y_t\}$, the dependence test statistic is given by

$$T_n = \sqrt{n}\left(\frac{C_1(\tau,n)}{C_2(\tau,n)} - C_3(\tau,n)\right), \tag{5}$$

where $C_1(\tau,n) \equiv \frac{1}{n}\sum_{t=2}^{T} I_{(y_{t-1}<\tau)} \cdot I_{(y_t<\tau)}$, $C_2(\tau,n) \equiv \frac{1}{n}\sum_{t=2}^{T} I_{(y_{t-1}<\tau)}$, $C_1(\tau,n) \equiv \frac{1}{n}\sum_{t=2}^{T} I_{(y_t<\tau)}$, $n = T-1$, and T is the length of residual $\{Y_t\}$. Under this condition, if the residual $\{Y_t\}$ is iid, then the test statistic $T_n \rightarrow N\big(0,\sigma^2(\tau)\big)$, as n is large enough and the hypothesis:$H_0: \frac{C_1(\tau)}{C_2(\tau)} - C_3(\tau) = 0$ is rejected at level α if $|T_n|/\widehat{\sigma}^2(\tau) > z_{\alpha/2}$. In this situation, the series $\{Y_t\}$

possesses any nonlinearity. The reader is referred to Hui et al. (2017) and others for more information.

3.2.3 Multivariate Granger Causality tests

In this section, we will review the theory of both linear and nonlinear causality and discuss how to apply the linear and nonlinear Granger causality tests to identify the causality relationships among RD_t, ADS_t , $ADS1_t$, and $ADS2_t$ to SV_t and MP_t . To test the linear and nonlinear causality relationship between a vector of stationary variables from RD_t, ADS_t , $ADS1_t$, and $ADS2_t$ and another vector of stationary variable of either SV_t and MP_t , we let $x_t = (x_{1,t}, x_{2,t}, \ldots x_{n_1,t})'$ and $y_t = (y_{1,t}, y_{2,t}, \ldots y_{n_2,t})'$ with $n_1 = 2$ and $n_2 = 1$, $x_{1,t} = RD_t$, $x_{2,t} = ADS_t$, $ADS1_t$, or $ADS2_t$, $x_{1,t} = SV_t$ or MP_t, and $n_1 + n_2 = n$ series in total.

3.2.3.1 *Multivariate linear causality*

To test the linear causality relationship between a vector of stationary variables from $x_t = (x_{1,t}, x_{2,t}, \ldots x_{n_1,t})'$ and $y_t = (y_{1,t}, y_{2,t}, \ldots y_{n_2,t})'$, one could construct the following $n-$VAR equations:

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} A_{x[n_1\times 1]} \\ A_{y[n_2\times 1]} \end{pmatrix} + \begin{pmatrix} A_{xx[n_1\times n_1]}(L) & A_{xy[n_1\times n_2]}(L) \\ A_{yx[n_2\times n_1]}(L) & A_{yy[n_2\times n_2]}(L) \end{pmatrix} \begin{pmatrix} x_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} e_x \\ e_y \end{pmatrix} , \qquad \textbf{(6)}$$

where $A_{x[n_1\times 1]}$ and $A_{y[n_2\times 1]}$ are two vectors of intercept terms, and $A_{xx[n_1\times n_1]}(L)$, $A_{yx[n_2\times n_1]}(L)$, $A_{xy[n_1\times n_2]}(L)$, and $A_{yy[n_2\times n_2]}(L)$ are matrices of lag polynomials.

In order to test the following null hypotheses separately:

(1) $H_0^1 : A_{xy}(L) = 0$,

(2) $H_0^2 : A_{yx}(L) = 0$, and

(3) both $H_0^1: A_{xy}(L)=0$ and $H_0^2: A_{yx}(L)=0$,

we should obtain the residual covariance matrix Σ from the full model in by using ordinary least squares estimation (OLSE) for each equation without imposing any restriction on the parameters, compute the residual covariance matrix Σ_0 from the restricted model in (6) by using OLSE for each equation with the restriction on the parameters imposed by the null hypothesis H_0^1, H_0^2, or both H_0^1 and H_0^2, and obtain the following statistic:

$$(T-c)\left(\log|\Sigma_0| - \log|\Sigma|\right) \quad , \tag{7}$$

where T is the number of usable observations, c is the number of parameters estimated in each equation of the unrestricted system, and $\log|\Sigma_0|$ and $\log|\Sigma|$ are the natural logarithms of the determinants of restricted and unrestricted residual covariance matrices, respectively. When the null hypothesis is true, this test statistic has an asymptotic χ^2 distribution with the degree of freedom equal to the number of restrictions on the coefficients in the system.

3.2.3.2 *Multivariate nonlinear causality*

After applying the VAR model to identify the linear causality relationships from RD_t, ADS_t, $ADS1_t$, and $ADS2_t$ to SV_t and MP_t, we obtain their corresponding residuals $\{\hat{\varepsilon}_{1t}\}$ and $\{\hat{\varepsilon}_{2t}\}$ to test the nonlinear causality with the residual series. For simplicity, in this section we denote $\mathrm{X_t} = (\mathrm{X_{1,t}}, \dots, \mathrm{X}_{n_1,\mathrm{t}})'$ and $y_t = \left(y_{1,t}, \dots, y_{n_2,t}\right)'$ to be the corresponding residuals of any two vectors of variables to be examined. We define the lead vector and lag vector of a time series, say $\mathrm{X_{i,t}}$, as follows: for $\mathrm{X_{i,t}}, \mathrm{i} = 1, \dots, \mathrm{n}$, the $\mathrm{m_{x_i}}$-length lead vector, and the $\mathrm{L_{x_i}}$-length lag vector of $\mathrm{X_{i,t}}$ to be:

$$\mathrm{X_{i,t}^{m_{x_i}}} \equiv \left(\mathrm{X_{i,t}}, \mathrm{X_{i,t+1}}, \dots, \mathrm{X_{i,t+m_{x_i}-1}}\right), \mathrm{m_{x_i}} = 1,2, \dots, \mathrm{t} = 1, 2, \dots,$$

$X_{i,t-L_{x_i}}^{L_{x_i}} \equiv \left(X_{i,t-L_{x_i}}, X_{i,t-L_{x_i}+1}, \dots, X_{i,t-1}\right), L_{x_i} = 1, 2, \dots, t = L_{x_i} + 1, L_{x_i} + 2, \dots,$ respectively. We denote $M_x = \left(m_{x1}, \dots, m_{x_{n_1}}\right)$, $L_x = \left(L_{x1}, \dots, L_{x_{n_1}}\right), m_x = \max\left(m_{x1}, \dots, m_{n_1}\right)$, and $l_x = \max\left(L_{x1}, \dots, L_{x_{n_1}}\right)$. The m_{y_i}-length lead vector, $Y_{i,t}^{m_{y_i}}$, the L_{y_i}-length lag vector, $Y_{i,t-L_{y_i}}^{L_{y_i}}$, of $Y_{i,t}$, and M_y, L_y, m_y, and l_y can be defined similarly.

To test the null hypothesis, H_0, that Y_t does not strictly Granger cause $X_t = (X_{1,t}, \dots, X_{n_1,t})'$ under the assumptions that the time series vector variables $X_t = (X_{1,t}, \dots, X_{n_1,t})'$ and $y_t = \left(y_{1,t}, \dots, y_{n_2,t}\right)'$ are strictly stationary, weakly dependent, and satisfy the mixing conditions stated in Denker and Keller (1983), we first define the following four events given that m_x, m_y, L_x, L_y, and $e > 0$:

$\{\|X_t^{M_x} - X_s^{M_x}\| < e\} \equiv \left\{\left\|X_{i,t}^{M_{x_i}} - X_{i,s}^{m_{x_i}}\right\| < e, \text{for any } i = 1, \dots, n_1\right\};$

$\{\|X_{t-L_x}^{L_x} - X_{s-L_x}^{L_x}\| < e\} \equiv \left\{\left\|X_{i,t-L_{x_i}}^{L_{x_i}} - X_{i,s-L_{x_i}}^{L_{x_i}}\right\| < e, \text{for any } i = 1, \dots, n_1\right\};$

$\left\{\left\|Y_t^{M_y} - Y_s^{M_y}\right\| < e\right\} \equiv \left\{\left\|Y_{i,t}^{m_{y_i}} - Y_{i,s}^{m_{y_i}}\right\| < e, \text{for any } i = 1, \dots, n_2\right\};$ and

$\left\{\left\|Y_{t-L_y}^{L_y} - Y_{s-L_y}^{L_y}\right\| < e\right\} \equiv \left\{\left\|Y_{i,t-L_{y_i}}^{L_{y_i}} - Y_{i,s-L_{y_i}}^{L_{y_i}}\right\| < e, \text{for any } i = 1, \dots, n_2\right\};$

where $\|\cdot\|$ denotes the maximum norm which is defined as $\|X - Y\| = \max(|x_1 - y_1|, |x_2 - y_2|, \dots, |x_n - y_n|)$ for any two vectors $X = (x_1, \dots, x_n)$ and $Y = (y_1, \dots, y_n)$. The vector series $\{Y_t\}$ is said not to strictly Granger cause another vector series $\{X_t\}$ if

$$Pr\left(\|X_t^{M_x} - X_s^{M_x}\| < e \middle| \|X_{t-L_x}^{L_x} - X_{s-L_x}^{L_x}\| < e, \left\|Y_{t-L_y}^{L_y} - Y_{s-L_y}^{L_y}\right\| < e,\right)$$

$$= Pr\left(\|X_t^{M_x} - X_s^{M_x}\| < e \middle| \|X_{t-L_x}^{L_x} - X_{s-L_x}^{L_x}\| < e\right) \quad (8)$$

where $\Pr(\cdot \mid \cdot)$ denotes conditional probability.

If the null hypothesis, H_0, is true, the test statistic

$$\sqrt{n}\left(\frac{C_1(M_x+L_x,L_y,e,n)}{C_2(L_x,L_y,e,n)}-\frac{C_3(M_x+L_x,e,n)}{C_4(L_x,e,n)}\right), \tag{9}$$

is distributed as $N\left(0,\sigma^2\left(M_x,L_x,L_y,e\right)\right)$. When the test statistic is too far away from zero, we reject the null hypothesis. Readers may refer to Bai, et al. (2010, 2011, 2018) and Chow, et al. (2018) for the definitions of C_1, C_2, C_3, and C_4, and more information on the estimates of Equation (9).

4. Empirical results

Although not reported due to space considerations, the summary statistics reveal evidence of non-normality, indicated by highly significant Jarque-Bera statistics, with all four time series (i.e. RD, SV, ADS and equity market premium, MP) exhibiting significant kurtosis. We also observe significant skewness for both RD and SV, suggesting greater likelihood of experiencing large values for these variables. These preliminary observations provide support for the subsequent causality tests based a nonlinear specification as our nonlinearity will formally confirm later. Finally, in unreported findings, unit root tests based on the augmented Dickey and Fuller (1979) shows that the series are stationary.

4.1 Bivariate causality tests

We begin our discussion by presenting the findings from bivariate causality tests. Table 1 presents the findings for the bivariate linear Granger causality tests. The optimal lag length for each case based on the well-known information criteria, such as BIC and AIC are also presented along with the test statistics. Examining the findings in Panel A, we observe significant causality from equity return dispersion to both the stock market volatility and equity market premium, consistent with the evidence in Angelidis et al. (2015). Interestingly, however, we see that the causality from return dispersion becomes insignificant after controlling for business conditions measured by the ADS index. Following the suggestion by Angelidis et al. (2015) that a relatively high return dispersion predicts a deterioration in business conditions, we distinguish between good and bad business conditions and create two additional variables ADS1 (ADS2) representing the positive (negative) ADS business conditions index values, respectively. However, we see in Panel A that differentiating between good and bad business conditions still yield insignificant causal effects from return dispersion, suggesting that business conditions serves as the primary driver of stock market volatility, rendering the predictive power of return dispersion insignificant.

The findings in Panel B further support these observations, suggesting that business conditions have significant predictive power over both stock market volatility and equity return dispersion. However, interestingly, the predictive power of business conditions is concentrated on contractionary periods only, suggesting asymmetric causal interactions between business conditions, equity return dispersion and stock market volatility. Overall, the findings in Table 1 show that the level of economic activity plays a significant role in studying linear causality from return dispersion to both stock market volatility and equity market premium.

Table 2 presents the results from the nonlinearity tests based on Hui et al. (2016) presented in Section 3.2.2. The tests indicate significant evidence of nonlinearity in all time series at the highest significance level, justifying the use of subsequent nonlinear causality tests. Table 3 presents the results from bivariate nonlinear causality tests. Examining the results in Panels A and B, we observe a significant linear causal relationship from return dispersion to both stock market volatility and equity market premium even after including the ADS business conditions index. Furthermore, we observe in Panel C that there exists significant causality from business conditions to return dispersion, however, with some degree of asymmetry such that expansionary (contractionary) market states are associated with low (high) level of equity return dispersion, indicating higher (lower) degree of directional similarity in stock returns, respectively. To that end, the findings from bivariate tests clearly indicate that the predictive power of equity return dispersion over stock market volatility and equity premium is largely asymmetric with regime specific patterns. This finding is indeed significant for not only stock market forecasting models, but also in the pricing of stock options as volatility forecasts are crucial in pricing derivatives as well as the estimation of optimal hedge ratios.

4.2 Multivariate Granger Causality tests

Having established evidence suggesting that the level of economic activity plays a significant role in studying causality from return dispersion to both stock market volatility and equity market premium, we now proceed with the multivariate causality tests. Table 4 presents the findings for the multivariate linear Granger causality tests explained in Section 3.2.4. We observe in Panel A that multivariate linear Granger causality exists from the return dispersion and business conditions to stock market volatility at the highest significance level, while the same does not hold for equity market premium, regardless of the distinction between expansionary or contractionary business conditions.

On the other hand, similar to the findings observed for the bivariate case, when we examine the findings from the multivariate nonlinear tests, presented in Table 5, we observe that equity return dispersion and business conditions together have significant predictive power over both the stock market volatility and equity market premium at the highest statistical significance level. The predictive power of RD and ADS together is robust regardless of the state of economic activity, implied by significant findings for both ADS1 and ADS2. To that end, our findings underline the significance of nonlinearity in the causal relationship between return dispersion and stock market premium and volatility, but also suggest that equity return dispersion along with a measure of economic conditions can be used to improve forecasting models for both return and volatility of stock market returns.

5. Conclusion

This paper contributes to the literature on stock market predictability by exploring the causal relationships between equity return dispersion, stock market volatility and excess returns via multivariate nonlinear causality tests recently developed by Bai et al. (2010, 2011, 2018). Performing a combination of linear vs. nonlinear and bivariate vs. multivariate causality tests, we find that linear causality tests generally fail to detect causal effects from return dispersion to excess market returns and volatility. Both bivariate and multivariate nonlinear causality tests, however, yield significant evidence of causality from return dispersion to both stock market volatility and equity premium, even after controlling for the state of the economy. Overall, our findings suggest that both return dispersion and business conditions are valid joint forecasters of both the stock market volatility and excess market return and that return dispersion indeed possesses incremental information regarding future stock return dynamics beyond which can be explained by the state of the economy.

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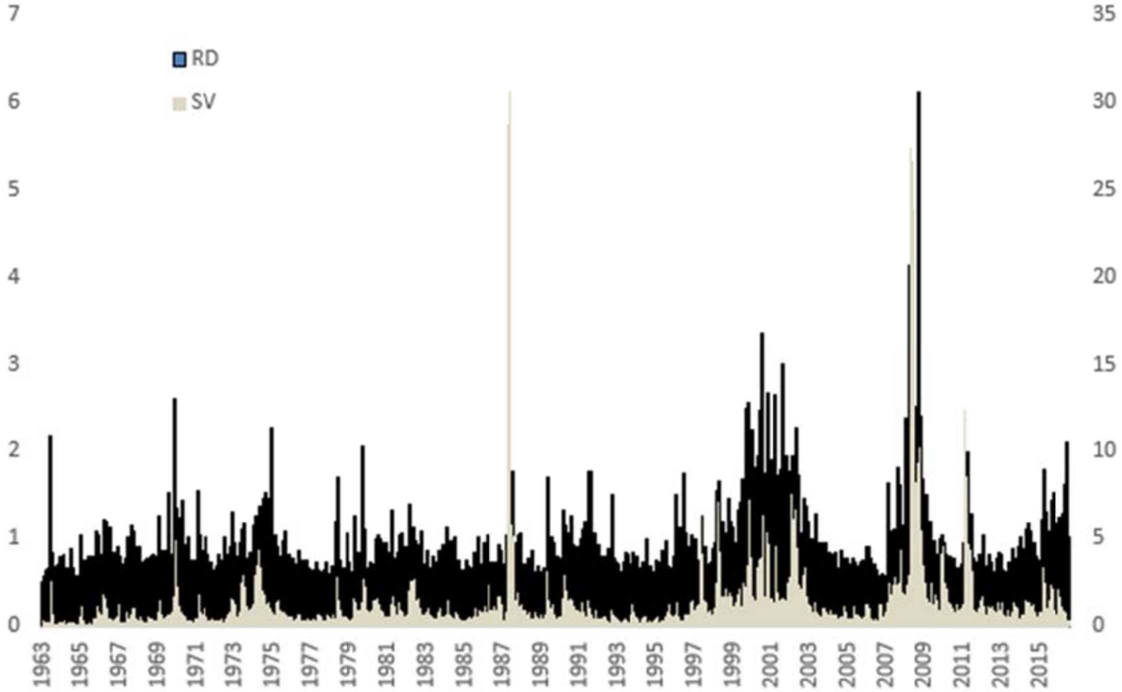
Figure 1. Daily equity return dispersion (RD) and stock market volatility (SV).



Note: RD and SV are equity return dispersion and stock market volatility, respectively.

Table 1. Bivariate linear causality tests

	Panel A: The Predictive Power of Equity Return Dispersion			
	RD → SV	RD → MP	RD → SV |ADS	RD → MP|ADS
Lags	15	9	16	16
F-Stat	188.760***	3.196***	9.716×10^{-7}	1.136×10^{-8}
	RD → SV|ADS1	RD → SV|ADS2	RD →MP |ADS1	RD →MP |ADS2
Lags	9	9	9	9
F-Stat	1.729×10^{-6}	1.714×10^{-6}	1.744×10^{-8}	1.749×10^{-8}
	Panel B: The Predictive Power of Business Conditions			
	ADS1 → SV	ADS2 → SV	ADS1 →MP	ADS2 →MP
Lags	16	16	9	9
F-Stat	1.146	3.579***	0.738	1.768
	ADS → RD	ADS1 → RD	ADS2 → RD	
Lags	9	9	9	
F-Stat	4.068***	0.513	5.967***	

Note: RD, SV, MP and ADS refer to equity return dispersion, stock market volatility, equity market premium, and business conditions index, respectively. ADS1 (ADS2) represents the positive (negative) business conditions index values, respectively. The notation "→" indicates causality and "RD → SV|ADS" indicates causality from RD to SV after controlling for ADS. *, **, *** indicate significance at 5, 1, and 0.1 percent level, respectively.

Table 2. Nonlinearity Tests

	ADS	ADS1	ADS2	RD	SV	MP
Lags	11	10	16	10	15	2
T-Stat	7.734***	7.845***	7.893***	8.970***	3.574***	8.547***

Note: RD, SV, MP and ADS refer to equity return dispersion, stock market volatility, equity market premium, and business conditions index, respectively. ADS1 (ADS2) represents the positive (negative) business conditions index, respectively.

Table 3. Bivariate nonlinear causality tests

Lags				
Panel A: The predictability of stock market volatility				
Lags	RD→SV	RD→SV|ADS	RD→SV|ADS1	RD→SV|ADS2
1	7.879***	7.8190***	7.758***	7.824***
2	7.718***	7.665***	7.533***	7.525***
3	7.637***	7.659***	7.533***	7.621***
4	7.908***	7.871***	7.745***	7.772***
5	7.461***	7.484***	7.309***	7.449***
6	7.155***	7.207***	7.141***	7.279***
7	6.770***	6.813***	6.611***	6.662***
8	6.617***	6.721***	6.461***	6.535***
9	5.984***	6.169***	5.741***	5.884***
10	5.918***	6.067***	5.646***	5.742***
Panel B: The predictability of equity market premium				
Lags	RD→MP	RD→MP |ADS	RD→MP|ADS1	RD→MP |ADS2
1	11.365***	11.379***	11.363***	11.302***
2	12.910***	13.079***	12.904***	12.877***
3	12.878***	13.053***	12.867***	12.928***
4	13.357***	13.643***	13.364***	13.428***
5	13.275***	13.693***	13.272***	13.420***
6	12.519***	12.931***	12.527***	12.694***
7	11.823***	12.206***	11.844***	12.038***
8	11.805***	12.155***	11.807***	12.048***
9	11.716***	11.996***	11.695***	11.950***
10	11.104***	11.405***	11.068***	11.321***
Panel C: The Predictive Power of Business Conditions				
Lags	ADS→RD	ADS1→RD	ADS2→RD	
1	-1.122	-5.676***	1.755*	
2	-1.366	-6.626***	1.808*	
3	-1.352	-6.930***	2.627**	
4	-2.015*	-6.917***	2.317*	
5	-0.820	-4.650***	2.711**	
6	-1.718*	-5.231***	2.311*	
7	-2.147*	-5.412***	2.425**	
8	-2.148*	-4.913***	1.708*	
9	-1.919*	-4.669***	1.928*	
10	-1.987*	-4.427***	1.053	

Note: RD, SV, MP and ADS refer to equity return dispersion, stock market volatility, equity market premium, and business conditions index, respectively. The notation "→" indicates causality and "RD → SV|ADS" indicates causality from RD to SV after controlling for ADS. *, **, *** indicate significance at 5, 1, and 0.1 percent level, respectively.

Table 4. Multivariate linear causality tests

	Panel A: The predictability of stock market volatility		
	RD+ADS→SV	RD+ADS1→SV	RD+ADS2→SV
Lags	10	9	9
LR	535.909***	560.136***	573.599***
	Panel B: The predictability of equity market premium		
	RD+ADS→MP	RD+ADS1→MP	RD+ADS2→MP
Lags	10	9	9
LR	37.812	37.456	39.096

Note: RD, SV, MP and ADS refer to equity return dispersion, stock market volatility, equity market premium, and business conditions index, respectively. ADS1 (ADS2) represents the positive (negative) business conditions index values, respectively. The notation "RD+ADS→X" indicates RD and ADS together predict variable X. *, **, *** indicate significance at 5, 1, and 0.1 percent level, respectively.

Table 5. Multivariate nonlinear causality tests

Panel A: The predictability of stock market volatility			
Lags	RD+ADS→SV	RD+ADS1→SV	RD+ADS2→SV
1	7.706***	7.661***	7.614***
2	7.529***	7.454***	7.217***
3	7.140***	7.286***	7.037***
4	6.736***	7.496***	6.565***
5	6.321***	6.954***	5.967***
6	5.818***	6.610***	5.694***
7	5.380***	6.107***	4.963***
8	5.447***	6.016***	4.969***
9	4.731***	5.387***	4.095***
10	4.665***	5.168***	4.108***
Panel B: The predictability of equity market premium			
Lags	RD+ADS→MP	RD+ADS1→MP	RD+ADS2→MP
1	11.271***	11.296***	11.260***
2	12.523***	12.655***	12.280***
3	12.557***	12.594***	12.370***
4	13.092***	12.988***	12.846***
5	12.590***	12.727***	11.980***
6	11.753***	11.797***	11.288***
7	10.6764***	10.733***	10.478***
8	10.749***	10.493***	10.584***
9	10.662***	10.577***	10.141***
10	10.075***	9.923***	9.601***

Note: RD, SV, MP and ADS refer to equity return dispersion, stock market volatility, equity market premium, and business conditions index, respectively. ADS1 (ADS2) represents the positive (negative) business conditions index values, respectively. The notation "RD+ADS→X" indicates RD and ADS together predict variable X. *, **, *** indicate significance at 5, 1, and 0.1 percent level, respectively.