Does Bitcoin Hedge Global Uncertainty? Evidence from Wavelet-Based Quantile-in-Quantile Regressions

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Abstract

In this study, we analyse whether Bitcoin can hedge uncertainty using daily data for the period of 17th March, 2011, to 7th October, 2016. Global uncertainty is measured by the first principal component of the VIXs of 14 developed and developing equity markets. We first use wavelets to decompose Bitcoin returns into various frequencies, i.e., investment horizons. Then, we apply standard OLS regressions and observe that uncertainty negatively affects raw Bitcoin return and its longer-term movements. However, given the heavy tails of the variables, we rely on quantile methods and reveal much more nuanced and interesting results. Quantile regressions indicate that Bitcoin does act as a hedge against uncertainty, that is, it reacts positively to uncertainty at both higher quantiles and shorter frequency movements of Bitcoin returns. Finally, when we use quantile-on-quantile regressions, we observe that hedging is observed at shorter investment horizons, and at both lower and upper ends of Bitcoin returns and global uncertainty.

JEL Codes: C22, G15.

Keywords: Bitcoin, global uncertainty, wavelet, quantile regressions.

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1. Introduction

Unlike a fiat currency that is backed by the government or central bank that issued it, Bitcoin is not a claim on anybody (Weber, 2014). Interestingly, this digital currency is fully decentralized thanks to its innovative ‘distributed ledger’ that allows it to be used in a decentralized payment system. More interestingly, Bitcoin was proposed by Satoshi Nakamoto in November 2008 during the global financial crisis, which led to ubiquitous financial and economic uncertainties that roiled the global financial systems and stock markets in both developed and emerging countries. At that time, confidence in the stability of the banking system and future economic security deteriorated rapidly, and market uncertainty—as measured by implied volatility—soared across the globe. Definitely, the launch of Bitcoin profited from such a highly uncertain environment that continued after the crisis (Weber, 2014), and Bitcoin’s controversial traits have fast gained the attention of practitioners, scholars and the financial press. The popularity and debate about Bitcoin have also been accentuated by later crises, namely the European sovereign debt crisis (ESDC) of 2010–2013 and the Cypriot banking crisis of 2012–2013. During these stress periods, Bitcoin spiked in value and gained more ground as many saw it as a shelter from uncertainty surrounding conventional economic and banking systems. Several press articles were released around that time pointing towards a flight from paper (fiat) currencies and bank deposits to Bitcoin, especially in geographic areas most affected by the ESDC and Cypriot crises such as Greece, Cyprus and Spain.1

More researchers have been interested in understanding Bitcoin price formation, showing that Bitcoin price is subject to factors that substantially differ from those affecting conventional assets such as stocks and bonds. Kristoufeck (2013) finds that internet (Google) searches determine Bitcoin price. Similar results are reported by Glaser et al. (2014) and Polasik et al. (2014). Garcia et al. (2014) extend these studies and show that word-of-mouth information on social media and information on both Google Trends and new Bitcoin users have a significant influence on Bitcoin price changes. The authors also argue that the cost of Bitcoin production via mining represents a lower bound for Bitcoin price. A similar finding has been shown by Hayes (2016). Further, Van Alstyne (2014) indicates that Bitcoin has technological value related to solving the so-called ‘double-spend problem’. Owens and Lavitch (2013) reveal that online gambling stimulates significant activity in the Bitcoin market. Yelowitz and

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Wilson (2015) argue that computer programming enthusiasts and illegal activities increase the interest in Bitcoin. Moreover, Ciaian et al. (2016) show that specific factors of supply/demand and digital currencies affect Bitcoin price, particularly the total number of unique Bitcoin transactions per day. The finance literature on Bitcoin has also been extended to cover the role of Bitcoin as an investment; although Bitcoin contains a considerable speculative component (Fry and Cheah, 2016), Bouoiyour et al. (2015) do not neglect its economic usefulness. Several studies point towards the valuable role of Bitcoin as an effective diversifier and hedge (Halaburda and Gandal, 2014; Baur et al., 2015; Eisl et al., 2015; Bouri, Azzi et al., 2016; Bouri, Molnár et al., 2016; Dyhrberg, 2016). Bitcoin has been found to be uncorrelated or negatively correlated with different equity indices, pointing towards its hedging ability (see, among others, Bouri, Molnár et al., 2016).

Interestingly, there is sufficient evidence to suggest that the implied volatility index, such as the US VIX, is negatively correlated with equities (e.g. Jubinski and Lipton, 2012). The VIX is a key market risk indicator that reflects market sentiment and investor expectation. It is widely used by market participants in their risk management strategy. Higher values of the VIX indicate more market uncertainty and vice-versa. Notably, an increase in the VIX generally results in ‘flights to safety’ (Thomas, 2015). For example, Jubinski and Lipton (2012) highlight the negative correlation between the VIX and treasury and investment grade bond yields. This means that yields fall in response to increases in implied volatility, suggesting that bond price increases as equity decreases. Accordingly, investors tactically move away from risky assets such as equities into safe haven assets such as gold and treasure bonds. Surprisingly, the link between market uncertainty and Bitcoin remains unexplored. Given that the information provided by the VIX serves as a valuable reference to investors, it is imperative to consider the VIX in any examination of Bitcoin’s ability to hedge or Bitcoin’s relation with other assets. Uncovering the nature and sign of such an association is important for market participants who are interested in revealing more about the hidden financial characteristics of Bitcoin. Notably, the use of a global uncertainty measure makes such an examination more vigorous.

Against this background, the purpose of this study is to analyse whether the relationship between global uncertainty and Bitcoin returns is positive at various frequencies, conditional on the state of the Bitcoin market (bear or bull), and also whether world uncertainty is high or low. For our purpose, we use daily data covering the period from 17th March, 2011, to 7th October, 2016, with global uncertainty being measured by the common component of the VIXs of 14 developed and developing equity markets. To the best of our knowledge, this is
the first study to formally analyse the ability of Bitcoin to hedge global uncertainty using standard OLS and two different quantile-based approaches (i.e. standard quantile and quantile-on-quantile [QQ] regressions) applied to wavelet-filtered data to capture movements of Bitcoin returns at various frequencies, that is investment horizons.

Our study makes three contributions to the finance literature. The first contribution stems from our unique methodological approach that combines the wavelet approach with QQ regressions. Wavelets decompose the time series in several frequencies, whereas the QQ approach not only models the heterogeneous relationship between Bitcoin returns and global uncertainty at various points of the conditional distribution of the former, similar to the typical quantile regression, but it also models the quantile of Bitcoin returns—and its various frequencies—as a function of the quantile of global uncertainty index. As such, the QQ approach allows the relationship between the two examined variables to vary at each point of their respective distributions. Our second contribution relates to the fact that although numerous studies have considered market uncertainty, as measured by the VIX, in studying the link between equities and uncertainty or financial assets and economic variables (e.g. Thomas, 2015; Basher and Sadorsky, 2016) no attention has been paid to the effect of (global) uncertainty in the Bitcoin market. Our third contribution arises from the use of a broad measure of market uncertainty that covers 14 developed and emerging equity markets, unlike most prior studies that have relied on the US VIX as a proxy of global uncertainty. In doing so, we provide a wide-ranging measure of uncertainty that is adequate for assessing the relation between global uncertainty and Bitcoin, given that the latter is used and traded extensively across the globe in both developed (i.e. US, Europe and Japan) and emerging economies (i.e. China), making the US VIX a restricted choice.

This paper is related to at least two strands of literature. The first is research on the diversification and hedging benefits of Bitcoin against equities (Halaburda and Gandal, 2014; Eisl et al., 2015; Bouri, Azzi et al., 2016; Bouri, Molnár et al., 2016; Dyhrberg, 2016) and against currencies or commodities (Bouri, Azzi et al., 2016; Bouri, Molnár et al., 2016; Dyhrberg, 2016). Related empirical work on this topic includes Rogojanu and Badea (2014) who explore the advantages and disadvantages of Bitcoin by comparing it with other alternative monetary systems. Similar to Popper (2015), they view Bitcoin as an alternative to conventional currencies in times of weak trust, referring to it as digital gold. The second strand is research focusing explicitly on uncertainty and its relation to conventional assets such as stocks and bonds and to commodities. For example, Jubinski and Lipton (2012) show that the negative association between the VIX and yields on treasury and investment-grade
bonds is in line with the flight-to-safety effect, confirming the view of Thomas (2015). Basher and Sadorsky (2016) concentrate on the hedging capability of the VIX against a falling stock market. Balcilar et al. (2016) find that uncertainty causes both gold returns and volatility. More interestingly, Bouri, Azzi et al. (2016) include the US VIX within a GARCH-based framework and find that Bitcoin volatility is negatively related with the US uncertainty. The remainder of the paper is organized as follows: Section 2 describes the data and the methodologies adopted, while Section 3 presents the results. Section 4 concludes.

2. Data and methodology

2.1 Data

Our study consists of two variables, namely the Bitcoin price and a measure of global uncertainty. The Bitcoin price is converted to returns using the first-differences of the natural logarithm of Bitcoin prices. The global uncertainty is based on the standardized (zero mean and unit variance) VIXs for the stock markets of Brazil, Canada, China, France, Germany, India, Japan, Mexico, Russia, South Africa, Sweden, Switzerland, the UK and the US. As can be seen, we include the equity markets of both developed and emerging economies, and thus we call this variable the World VIX (WVIX). Our data cover the daily period from 17th March, 2011, to 7th October, 2016, based on data availability, which, in turn, gives us a total of 1,452 observations. The VIX data are sourced from the DataStream of Thomson Reuters, while the Bitcoin price data in US dollars are collected from CoinDesk at www.coindesk.com/price. The CoinDesk Bitcoin Price Index represents an average of Bitcoin prices across leading Bitcoin exchanges and thus captures world Bitcoin prices better than other alternatives. Our proxy for the Bitcoin market is appropriate for analysing the hedging capabilities of Bitcoin relative to global uncertainty.

2.2 Wavelet multiscale decomposition

Wavelet analysis combines both the time and frequency domains. One strength of the wavelets over other existing econometric methods is the ability to decompose the time series in several wavelet scales or frequencies. Wavelets provide both an orthogonal timescale decomposition of the data and a nonparametric representation of each individual time series (Ramsey, 1999). They offer the possibility to perform frequency decomposition of the series, and, at the same time, preserve the time location. The wavelet transform is able to capture all
the information in the time series and associate it with specific time horizons and locations in
time (Gençay et al., 2002).

Following Ramsey (2002), it is possible to represent any function of time by the father ($\phi$) and mother ($\psi$) wavelets. Father wavelets integrate to one and are used to represent very long-scale smooth components of the signal. Mother wavelets, on the other hand, integrate to zero and represent deviations from the smooth components. Father wavelets generate scaling coefficients, while mother wavelets generate differencing coefficients.

The father wavelet is defined as follows:

$$
\phi_{j,k} = 2^{-j/2} \phi \left( \frac{t - 2^j k}{2^j} \right) \text{ with } \int \phi(t) dt = 1. \tag{1}
$$

The mother wavelet is defined as follows:

$$
\psi_{j,k} = 2^{-j/2} \psi \left( \frac{t - 2^j k}{2^j} \right) \text{ with } \int \psi(t) dt = 0. \tag{2}
$$

From the mother and father wavelets, one constitutes the basic functions from which a sequence of coefficients is defined.

The coefficients (smooth coefficients) of the father wavelets are as follows:

$$
s_{j,k} = \int f(t) \phi_{j,k}. \tag{3}
$$

The coefficients (detail coefficients) obtained from the mother wavelet are as follows:

$$
d_{j,k} = \int f(t) \psi_{j,k} \quad j = 1,...,J
$$

The maximal scale of the former is $2^j$, while the detailed coefficients are computed from the mother wavelets at all scales from 1 to J.

From the above coefficients, the function $f(.)$ is defined as follows:

$$
f(t) = \sum_k s_{j,k} \phi_{j,k}(t) + \sum_k d_{j,k} \psi_{j,k}(t) + \sum_k d_{j,k} \psi_{j,k}(t) + \sum_k d_{j,k} \psi_{j,k}(t), \tag{4}
$$

Though not relevant in our case as we work with stationary data, that is Bitcoin returns, an important feature of wavelets is the ability to capture events that are local in time, making it possible to handle nonstationary time series, unlike the Fourier transform that is suited for series with time-invariant spectral content.
which is simplified to

\[ f(t) = S_j + D_j + D_{j-1} + \ldots D_J + \ldots + D_1 \]  

(5)

with the following orthogonal components:

\[ S_j = \sum_k S_{j,k} \phi_{j,k}(t), \]
\[ D_j = \sum_k d_{j,k} \psi_{j,k}(t), \quad j=1,\ldots,J \]

The resulting multiresolution (multihorizon) decomposition of \( f(t) \) is \( \{S_j, D_{j-1}, \ldots, D_1\} \).

\( D_j \) defines the \( j^{th} \) level wavelet detail associated with changes in the series at scale \( \lambda_j \). \( S_j \) is a cumulative sum of variations at each detail scale and becomes smoother and smoother as \( j \) increases (Gençay et al., 2002).

In order to calculate the scaling and wavelet coefficients, the maximal overlap discrete wavelet transform (MODWT) is employed. The MODWT is preferred over discrete wavelet transform (DWT). While the DWT of level \( J_0 \) restricts the sample size to an integer multiple of \( 2^{J_0} \), there is no such limitation for the MODWT, which is defined for any sample size (Percival and Walden, 2000). The detail and smooth coefficients of a MODWT are associated with zero phase filters, which makes it possible to align the features of the original time series with features in the multiresolution analysis (MRA), and the MODWT variance estimator is asymptotically more efficient than the DWT-based estimator (Percival, 1995; Percival and Mofjeld, 1997; Gençay et al., 2002). Moreover, while the DWT uses weighted differences and averages contiguous pairs of observations, the MODWT uses a moving difference and average operator, and thus it keeps the exact amount of observations at each scale of the wavelet decomposition.

The series are decomposed using the Daubechies (a family of compactly supported wavelets) least asymmetric (LA) filter of length eight [hereafter LA(8)]. The LA(8) wavelet is relatively smooth when compared with Haar wavelet filters (Gençay et al., 2002) that have been used widely in previous studies. Moreover, the LA(8) filter yields coefficients that exhibit better uncorrelatedness across scales than the Haar\(^3\) filter (Cornish et al., 2006).

\(^3\)The Haar filter is equivalent to the Daubechies orthogonal wavelet D2. Haar filter is based on two non-zero coefficients, whereas LA(8) is based on eight non-zero coefficients. As the number of vanishing moments (decay towards low frequencies) is given by half of the length of the wavelet filter, Haar filter has one vanishing
The series are decomposed into wavelet coefficients $D_1$ to $D_3$. The detail coefficient $D_j$ provides a resolution of the data at scale $2^j$ to $2^{j+1}$. The wavelet scales $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ and $\lambda_6$ are associated with oscillations of periods of 2–4, 4–8, 8–16, 16–32, 32–64 and 64–128 days, respectively. The wavelet smooth $S6$ represents the long-term movements.

2.3 Quantile-on-Quantile approach

To study the relationship between global uncertainty with Bitcoin returns and its various frequencies, we start with linear regressions but then move into a quantile regression framework. The quantile regression analysis (QRA), since its introduction by Koenker and Bassett (1978), has become a common tool in modelling the time-varying degree and structure of dependence, as it involves a set of regression curves that differ across different quantiles of the conditional distribution of the dependent variable, with the quantiles capturing various (time-varying) phases of the dependent variable. Compared with a classical linear correlation or regression or even non-linear regression methods, the quantile functions provide a more precise and accurate result of the impact of covariates on the dependent variable (see Koenker, 2005). Further, the advantage of using QRA lies in its ability to provide information on tail dependence (i.e. upper and lower tails) in addition to the median, which can be considered to capture the normal phase of the dependent variable.\(^4\)

One shortcoming of the QRA approach is its inability to capture dependence in its entirety. Specifically, even though the QRA approach can estimate the heterogeneous relationship between Bitcoin returns and global uncertainty at various points of the conditional distribution of the former, it overlooks the possibility that the nature (i.e. big or small) of uncertainty could also influence the way Bitcoin is related to global uncertainty. Unfortunately, this cannot be captured by quantile regressions. Hence, rather than using the QRA approach, we follow Sim and Zhou (2015) who propose a QQ approach. Using the QQ approach, we are able to model the quantile of Bitcoin returns (and its various frequencies) as a function of the quantile of global uncertainty index, so that the relationship between these

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\(^4\) Quantile regression was introduced in the seminal paper by Koenker and Bassett (1978). It is a generalization of median regression analysis to other quantiles. The coefficients of the $\tau^{th}$ conditional quantile distribution are estimated as $\hat{\beta}(\tau) = \arg\min \sum \left( t - 1_{y_t < x_t^\tau \beta_t} \right) | y_t - x_t^\tau \beta_t |$, where the quantile regression coefficient $\beta(\tau)$ determines the connection between the vector of independent variables and the $\tau^{th}$ conditional quantile of the dependent variable, with $1_{y_t < x_t^\tau \beta_t}$ being the usual indicator function.
variables could vary at each point of their respective distributions. The QQ approach, thus, provides a more complete picture of dependence. It is implemented by selecting a number of quantiles of uncertainty and estimating the local effect these particular quantiles of uncertainty might have on the various quantiles of Bitcoin returns.

There are currently two approaches to model the QQ method: (1) a triangular system of equations based on Ma and Koenker (2006) and (2) a single equation regression approach based on Sim and Zhou (2015), which is also based on Ma and Koenker (2006). We use the latter approach in this study, which can be explained as follows:

Let $\theta$ superscript denote the quantile of Bitcoin returns, which are indicated by BR. We first postulate a model for the $\theta$-quantile of Bitcoin returns as a function of its past lag $BR_{t-1}$ and the global uncertainty, which is measured by the WVIX. Formally, this can be expressed as:

$$BR_{t} = \beta^{\theta}WVIX_{t} + \varepsilon_{t}^{\theta},$$

(6)

where $\varepsilon_{t}^{\theta}$ is an error term that has a zero $\theta$-quantile. We allow the relationship function $\beta^{\theta}(\cdot)$ to be unknown, since we do not have a prior on how the Bitcoin returns and WVIX changes are interlinked. To examine the linkage between the $\theta$-quantile of Bitcoin returns and $\theta$-quantile of WVIX, denoted by $WVIX^{\tau}$, we linearize the function $\beta^{\theta}(\cdot)$ by taking a first-order Taylor expansion of $\beta^{\theta}(\cdot)$ around $WVIX^{\tau}$, which yields the following:

$$\beta^{\theta}(WVIX) \approx \beta^{\theta}(WVIX^{\tau}) + \beta^{\theta}(WVIX^{\tau})(WVIX - WVIX^{\tau}).$$

(7)

Based on Sim and Zheng (2015)'s study, we can redefine $\beta^{\theta}(WVIX^{\tau})$ and $\beta^{\theta}(WVIX^{\tau})$, respectively, as $\beta_{0}(\theta, \tau)$ and $\beta_{1}(\theta, \tau)$.

Then, equation (2) can be re-written as follows:

$$\beta^{\theta}(WVIX) \approx \beta_{0}(\theta, \tau) + \beta_{1}(\theta, \tau)(WVIX - WVIX^{\tau}).$$

(8)

Ultimately, we substitute equation (8) into equation (7) to obtain the following:

$$BR_{t} = \beta_{0}(\theta, \tau) + \beta_{1}(\theta, \tau)(WVIX_{t} - WVIX^{\tau}) + \varepsilon_{t}^{\theta}.$$
corresponding to 2–4, 4–8, 8–16, 16–32, 32–64, 64–128 days and the long-term trend, respectively. Because we are interested in the effect exerted locally by the $\tau$-quantile of uncertainty, we employ a Gaussian kernel to weight the observations in the neighbourhood of the empirical quantile of uncertainty, based on a specific bandwidth. Refer to Sim and Zhou (2015) for complete details on the estimation.

3. Results

As discussed above, the raw data on Bitcoin returns is decomposed into seven frequency components using wavelets. The raw returns and various decomposed components as well as the WVIX are plotted in Figure A1 in the Appendix. Table A1 in the Appendix reports the summary statistics, which clearly show non-normal distributions for all the variables, and hence provide a good motivation for relying primarily on a quantile-based approach to accommodate for the heavy tails.

We start with simple OLS-estimation-based results to detect the relationship between Bitcoin returns and the global uncertainty, as reported in Table 1. The impact of uncertainty is negative and significant at the 1 percent level of significance. When we look at the various decomposed frequencies of Bitcoin returns, uncertainty is shown to affect the oscillations of the period of 64–128 days and the long-term movement. In other words, the negative impact of the WVIX on aggregate Bitcoin returns emerges from long-term oscillations in the data rather than from short-term frequencies. More importantly, Bitcoin does not seem to act as a hedge against uncertainty and behaves just like other equities (see Chuliá et al., 2016, for detailed literature regarding the impact of uncertainty on stock markets).

In Table 1, we also report the impact of uncertainty on Bitcoin returns and its various frequencies based on quantile regressions. Just like the OLS results for aggregate Bitcoin returns, uncertainty has a negative and significant impact on the entire conditional distribution of returns, and this behaviour is also observed at the longer frequency of 64–128 days and in the long-term movement of Bitcoin returns. However, if we look at oscillations of shorter frequencies (2–4, 4–8, 8–16, 16–32 and 32–64 days), we observe that while uncertainty does have a negative and significant impact on Bitcoin at lower quantiles, at higher quantiles, the relationship is positive and significant. Thus, our results imply that for the short-term frequencies, Bitcoin does hedge against risk when the market is in the bull
regime (i.e. upper quantiles) but not in the bear regime, where Bitcoin returns are negatively impacted by uncertainty. Our results highlight the importance of not only studying the entire conditional distribution of Bitcoin returns (based on quantile regressions), rather than just the conditional mean as in OLS estimations, but also looking at the various frequencies, that is investment horizons of Bitcoin returns. Clearly, for short-term investment horizons and when markets are performing well, Bitcoin does serve as a hedge against global uncertainty, as captured by the WVIX.

[INSERT TABLE 1 HERE]

To gain more insight into the results, we look at QQ regressions to analyse whether there is also a role for various levels of uncertainty in the conditional distribution behaviour of Bitcoin returns and its various frequencies. The results have been plotted in Figure 1 using three-dimensional graphs. We observe that although the results for the quantile regression generally carry over to the QQ case, for shorter frequencies of Bitcoin returns, the relationship with WVIX is also positive at lower quantiles of Bitcoin returns when we look at lower quantiles of WVIX. The positive relationship is also shown to hold at higher quantiles of the dependent and independent variables, again primarily for shorter frequency movements of Bitcoin returns. Thus, clearly, the ability of Bitcoin to act as a hedge against uncertainty is conditional on not only whether the market is in bear or bull regime but also whether global uncertainty is high or low. Specifically speaking, at shorter investment horizons, Bitcoin returns seem to hedge against the global uncertainty at extreme ends of both Bitcoin returns and uncertainty.5

[INSERT FIGURE 1 HERE]

4. Conclusion

The main objective of this study was to examine whether Bitcoin can act as a hedge against global uncertainty—an under-researched topic in the field of economic and financial analyses

5 In principle, what the QQ approach does is ‘decompose’ the QRA estimates so that they are specific for different quantiles of the explanatory variable. If the QQ approach does in fact ‘decompose’ the QRA estimates as claimed, it will be possible to use the QQ estimates to recover the key features of QRA estimates; otherwise, the QQ approach is methodologically flawed. To recover the QRA estimates from the QQ estimates, first, notice that the QRA parameters are indexed by $\theta$ only. Therefore, to construct parameters from the QQ model that are indexed by $\theta$ only, we summarize the estimated QQ parameters by averaging along $\tau$. As can be seen from Figure A2 in the Appendix, the response of the raw Bitcoin returns and its various frequencies for the QRA and QQ estimates are virtually inseparable, thus indicating that the QQ approach is methodologically correct.
of Bitcoin. Specifically, we used a global measure of market uncertainty based on the common component, that is the first principal component of the VIXs of 14 developed and developing stock markets. Besides using raw returns, we used wavelets to decompose Bitcoin returns into its various frequencies, that is, investment horizons. The results from standard OLS regressions showed that uncertainty negatively affects raw Bitcoin returns, with the effect on raw returns emanating at longer investment horizons. Given the heavy tails of the variables, we next applied quantile methods that are more appropriate and useful to detect left and right tail dependence. Quantile regressions indicated that Bitcoin does act as a hedge against uncertainty, that is, it reacted positively to it at higher quantiles, especially at shorter investment horizons. Finally, when we used QQ regressions, we observed that hedging is observed at both lower and upper ends of Bitcoin returns and global uncertainty, but again primarily at shorter investment horizons. Our findings highlight the importance of not only decomposing Bitcoin returns into its various investment horizons but also, more importantly, the role of accommodating for estimation methods that incorporate information from quantiles for both Bitcoin returns and global uncertainty. Thus, unlike conditional-mean-based results, Bitcoin is shown to serve as a hedge against uncertainty at the extreme ends of the Bitcoin market and global uncertainty, but at shorter investment horizons. Therefore, short-horizon investment in Bitcoin can help investors hedge global equity market uncertainty, especially when the market is functioning in bear and bull regimes and also when uncertainty is either low or high. Our interesting results add more detail to prior studies that show some hedging ability of Bitcoin against equities and commodities (e.g., Bouri, Azzi et al., 2016; Bouri, Molnár et al., 2016; Dyhrberg, 2016). Further research should examine whether our reported results are sensitive to the use of Bitcoin data denominated in a currency other than the USD.

References


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Note: Entries in **Bold Italic** indicate significance at the 1 percent (5 percent) level.
Figure 1. Quantile-on-Quantile Results
**Note:** BR: Raw Bitcoin returns; Bitcoin.di: 2–4, 4–8, 8–16, 16–32, 32–64 and 64–128 days; i: 1,2,...,6; Bitcoin.s6: long-term movement; Global uncertainty: World VIX (WVIX).
APPENDIX:

Figure A1. Data Plots

Note: See Notes to Figure 1.
**Table A1. Summary Statistics**

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*Note: Std.Dev. stands for standard deviation; Probability corresponds to the Jarque-Bera test with null of normality. See Notes to Figure 1.*
Figure A2. QRA versus QQ Estimates
Note: See Notes to Figure 1.