The Predictability of \( cay \) and \( cay^{MS} \) for Stock and Housing Returns: A Nonparametric Causality in Quantile Test

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The predictability of $cay$ and $cay^{MS}$ for stock and housing returns:
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We use a nonparametric causality-in-quantiles test to compare the predictive ability of $cay$ and $cay^{MS}$ for excess and real stock and housing returns and their volatility using quarterly data for the US over the periods of 1952:Q1-2014:Q3 and 1953:Q2-2014:Q3 respectively. Our results reveal strong evidence of nonlinearity and regime changes in the relationship between asset returns and $cay$ or $cay^{MS}$, which corroborates the relevance of this econometric framework. Moreover, we confirm the outperformance of $cay^{MS}$ vis-à-vis $cay$ and their relevance for excess stock returns. Furthermore, we show that $cay^{MS}$ is particularly useful at forecasting certain quantiles of the conditional distribution. As for housing returns, the empirical evidence suggests that the predictive ability of $cay$ and $cay^{MS}$ is relatively low. Yet, $cay$ outperforms $cay^{MS}$ over the majority of the quantiles of the conditional distribution of the variance of real housing returns.
1. **INTRODUCTION**

The seminal contribution of Lettau and Ludvigson (2001) opened an important line of investigation that has been looking at the consumption-wealth ratio \(cay\) and the extent to which it captures the dynamics of the equity risk premium and the investors’ expectations about future asset returns. Ever since, a large number of studies have confirmed this finding (Sousa, 2010; Rapach and Zhou, 2013; Caporale and Sousa, forthcoming). More recently, Caporale and Sousa (forthcoming) and Caporale *et al.* (forthcoming) have analysed the importance of \(cay\) at predicting housing returns for emerging and developed market economies, respectively.

More recently, Bianchi *et al.* (2015) provide evidence of infrequent shifts, or breaks, in the mean of \(cay\). One may interpret this as a troubling feature of stock returns (for example, the presence of asset price bubbles) or as reflecting irregular changes in the moments of the distribution. As a result, the authors introduce a Markov-switching version of the consumption-wealth ratio i.e., \(cay^{MS}\), and show that it has superior forecasting power for quarterly excess stock market returns compared to the conventional \(cay\).

It should be also noted that, as is standard practice in the literature of asset returns predictability (Rapach and Zhou, 2013), the existing studies by Lettau and Ludvigson (2001), Bianchi *et al.*, (2015), Caporale and Sousa (forthcoming) and Caporale *et al.* (forthcoming) rely on linear predictive regression frameworks.

Against this backdrop, the objective of our paper is to compare the predictive ability of \(cay\) and \(cay^{MS}\) not only for excess and real stock and housing returns of the US, but also their volatility. We accomplish this goal by using a nonparametric causality-in-quantiles test that has been recently developed by Balcilar *et al.* (2015).

This test studies higher order causality over the entire conditional distribution and is inherently based on a nonlinear dependence structure between the variables. It essentially combines the causality-in-quantile test of Jeong *et al.* (2012) and the higher-moment \(k^{th}\)-order nonparametric causality of Nishiyama *et al.* (2011).

Its main novelties are as follows. First, it is robust to mis-specification errors, as it detects the underlying dependence structure between the examined dependent variables (i.e. excess and real stock and housing returns) vis-à-vis the regressors (i.e. \(cay\) and \(cay^{MS}\)). This could prove to be particularly important, as it is well-known that financial markets data tend to display nonlinear dynamics. Second, it tests for causality that may exist at the tails of the joint distribution of the variables. Therefore, it assesses causality not only in the mean asset return (i.e. the first moment), but also in the volatility of the asset return (i.e. higher...
moments). Consequently, we are able to investigate causality-in-variance (thereby, volatility spillovers), as sometimes one does not uncover causality in the conditional mean, but higher order interdependencies emerge.

Our analysis relies on quarterly data for the US over the period of 1952:Q1-2014:Q3 for stock returns, and 1953:Q2-2014:Q3 for housing returns. We find evidence of nonlinearity and regime changes between asset returns and $cay$ or $cay^{MS}$, which supports the use of the nonparametric causality-in-quantiles test. Moreover, while the linear Granger causality tests provide overwhelming evidence of the predictability for both excess and real stock returns, with $cay^{MS}$ outperforming $cay$, the causality-in-quantiles approach shows that these two predictors are only relevant for excess stock returns. Furthermore, while the entire conditional distribution of excess stock returns can be forecasted by both $cay$ and $cay^{MS}$, the latter is only a strong predictor at certain quantiles. In what concerns the predictability for excess housing returns and their variance, as well as real housing returns, neither $cay$ nor $cay^{MS}$ appear to display a large predictive ability. However, $cay$ outperforms $cay^{MS}$ in forecasting the variance of real housing returns over the majority of the quantiles of the conditional distribution.

To the best of our knowledge, this is the first paper that uses a nonparametric causality-in-quantiles framework to investigate the forecasting power of $cay$ and $cay^{MS}$ for excess and real stock and housing returns, as well as their volatility. Yet, our study is related with the works of Ludvigson and Ng (2007) and Bekiros and Gupta (2015). While the former analyses the predictive ability of $cay$ for both excess returns and their volatility using a linear predictive regression framework, the latter investigates the predictability of real stock returns and its volatility emanating from $cay$ and $cay^{MS}$ using the $k^{th}$-order nonparametric causality test of Nishiyama et al. (2011). Note, the causality-in-quantiles test that we employ in this paper is more general than the Nishiyama et al. (2011) test used by Bekiros and Gupta (2015), since our approach allows us to study the entire conditional distribution of returns and volatility. In addition, unlike Ludvigson and Ng (2007) and Bekiros and Gupta (2015), we also analyse housing returns and volatility over and above stock returns and volatility.

The rest of the paper is organized as follows. Section 2 presents a brief literature review. Section 3 describes the econometric framework of quantile and higher-moment nonparametric causality. Section 4 presents the data and discusses the empirical results. Finally, Section 5 concludes.
2. **A Brief Literature Review**

A relevant strand of the empirical literature investigates the joint dynamics of consumption, wealth and a series of macroeconomic aggregates, and their relevance in terms of capturing time-variation in expectations about future stock returns (Lettau and Ludvigson, 2001; Sousa, 2010, 2015).

As for housing returns, the literature primarily focuses on determining the macroeconomic drivers, such as business cycle fluctuations, income growth, industrial production or employment rate (Leung, 2004; Hwang and Quigley, 2006; Kallberg et al., 2014), and the wealth effects that it generates (Ludvigson and Steindel, 1999; Lettau and Ludvigson, 2004; Case et al., 2005, 2011). Other studies in the empirical finance literature include features of the housing market dynamics into asset pricing models of equity risk premium (Kallberg et al., 2002; Lustig and van Nieuwerburgh, 2005; Yogo, 2006; Piazzesi et al., 2007; Leung et al., 2006; Leung, 2007; Sousa, 2010; Pakos, 2011; Quijano, 2012; Ren et al., 2014).

Despite this, there is a lack of empirical work dealing with the specific question of predictability of housing risk premium. This is somewhat surprising in the light of: (i) the strong the linkages between the housing sector, the financial system and the real economic activity, as exposed by the financial turmoil of 2007-2009, (ii) the transmission of asset market volatility during periods of financial stress (Blenman, 2004); and (iii) the fact that housing is the most important asset in households’ portfolios, providing both utility and collateral services (Banks et al. 2004).

In this context, some recent works try to pave the way for further analysis on the issue of forecasting housing returns. For instance, Caporale et al. (forthcoming) show that the predictability of housing risk premium depends on whether investors perceive financial and housing assets as being substitutes or complements. Caporale and Sousa (forthcoming) also validate empirically the predictive power of $cay$ for both equity and housing risk premia in a set of emerging countries.

3. **Nonparametric Quantile Causality Testing**

In this section, we present a novel methodology for the detection on nonlinear causality via a hybrid approach developed by Balcilar et al. (2015) and based on the frameworks of Nishiyama et al. (2011) and Jeong et al. (2012). This approach is robust to extreme values in the data and captures general nonlinear dynamic dependencies.
We start by denoting asset returns (i.e. excess or real stock and housing returns) by \( y_t \) and the predictor variable (in our case, \( cay \) or \( cay^{MS} \)) as \( x_t \). Following Jeong et al. (2012), the quantile-based causality is defined as:\(^1\)

\( x_t \) does not cause \( y_t \) in the \( \theta \)-quantile with respect to the lag-vector of \( \{y_{t-1}, \ldots, y_{t-p}, x_{t-1}, \ldots, x_{t-p}\} \) if

\[
Q_\theta \{ y_t \mid y_{t-1}, \ldots, y_{t-p}, x_{t-1}, \ldots, x_{t-p} \} = Q_\theta \{ y_t \mid y_{t-1}, \ldots, y_{t-p} \}, \quad \text{and} \quad (1)
\]

\( x_t \) is a prima facie cause of \( y_t \) in the \( \theta \)th quantile with respect to \( \{y_{t-1}, \ldots, y_{t-p}, x_{t-1}, \ldots, x_{t-p}\} \) if

\[
Q_\theta \{ y_t \mid y_{t-1}, \ldots, y_{t-p}, x_{t-1}, \ldots, x_{t-p} \} \neq Q_\theta \{ y_t \mid y_{t-1}, \ldots, y_{t-p} \}. \quad (2)
\]

where \( Q_\theta \{ y_t \mid \cdot \} \) is the \( \theta \)th quantile of \( y_t \) depending on \( t \) and \( 0 < \theta < 1 \).

Let \( Y_{t-1} \equiv (y_{t-1}, \ldots, y_{t-p}) \), \( X_{t-1} \equiv (x_{t-1}, \ldots, x_{t-p}) \), \( Z_t = (X_t, Y_t) \) and \( F_{y_t|Z_{t-1}}(y_t, Z_{t-1}) \) and \( F_{y_t|X_{t-1}}(y_t, Y_{t-1}) \) denote the conditional distribution functions of \( y_t \) given \( Z_{t-1} \) and \( Y_{t-1} \), respectively. The conditional distribution \( F_{y_t|Z_{t-1}}(y_t, Z_{t-1}) \) is assumed to be absolutely continuous in \( y_t \) for almost all \( Z_{t-1} \).

If we denote \( Q_\theta(Z_{t-1}) \equiv Q_\theta(y_t \mid Z_{t-1}) \) and \( Q_\theta(Y_{t-1}) \equiv Q_\theta(y_t \mid Y_{t-1}) \), we have \( F_{y_t|Z_{t-1}}(Q_\theta(Z_{t-1}) \mid Z_{t-1}) = \theta \) with probability one. Consequently, the hypotheses to be tested based on definitions (1) and (2) are:

\[
H_0 = P\{ F_{y_t|Z_{t-1}}(Q_\theta(Y_{t-1}) \mid Z_{t-1}) = \theta \} = 1, \quad (3)
\]

\[
H_1 = P\{ F_{y_t|Z_{t-1}}(Q_\theta(Y_{t-1}) \mid Z_{t-1}) \neq \theta \} < 1. \quad (4)
\]

Jeong et al. (2012) employ the distance measure \( J = \{ \varepsilon_t E(\varepsilon_t \mid Z_{t-1}) f_z(Z_{t-1}) \} \), where \( \varepsilon_t \) is the regression error term and \( f_z(Z_{t-1}) \) is the marginal density function of \( Z_{t-1} \). The regression error \( \varepsilon_t \) emerges based on the null hypothesis in (3), which can only be true if and only if \( E[1\{ y_t \leq Q_\theta(Y_{t-1}) \mid Z_{t-1} \}] = \theta \) or, equivalently, \( 1\{ y_t \leq Q_\theta(Y_{t-1}) \} = \theta + \varepsilon_t \), where \( 1\{ \cdot \} \) is an indicator function. The authors specify the distance function as follows:

\[
J = E[\{ F_{y_t|Z_{t-1}}(Q_\theta(Y_{t-1}) \mid Z_{t-1}) - \theta \}^2 f_z(Z_{t-1})]. \quad (5)
\]

\(^1\)The discussion in this section closely follows Nishiyama et al. (2011) and Jeong et al. (2012).
It is important to note that \( J \geq 0 \) i.e., the equality holds if and only if \( H_0 \) in Eq. (3) is true, while \( J > 0 \) holds under the alternative hypothesis, \( H_1 \), in Eq. (4). Jeong et al. (2012) show that the feasible kernel-based test statistic for \( J \) has the following form:

\[
\hat{J}_T = \frac{1}{T(T-1)h^2} \sum_{t=p+1}^{T} \sum_{s=p+1, s \neq t} \sum_{t=p+1}^{T} K \left( \frac{Z_{t-1} - Z_{s-1}}{h} \right) \hat{\varepsilon}_t \hat{\varepsilon}_s.
\]

where \( K(\cdot) \) is the kernel function with bandwidth \( h \), \( T \) is the sample size, \( p \) is the lag order, and \( \hat{\varepsilon}_t \) is the estimate of the unknown regression error, which is estimated as follows:

\[
\hat{\varepsilon}_t = 1 \{ y_t \leq Q_{\theta}(Y_{t-1}) - \theta \}.
\]

\( \hat{Q}_{\theta}(Y_{t-1}) \) is an estimate of the \( \theta \)th conditional quantile of \( y_t \) given \( Y_{t-1} \), and we estimate \( \hat{Q}_{\theta}(Y_{t-1}) \) using the nonparametric kernel method as

\[
\hat{Q}_{\theta}(Y_{t-1}) = \hat{F}_{Y \mid Y_{t-1}}^{-1}(\theta \mid Y_{t-1}),
\]

where \( \hat{F}_{Y \mid Y_{t-1}}(y_t \mid Y_{t-1}) \) is the Nadarya-Watson kernel estimator given by

\[
\hat{F}_{Y \mid Y_{t-1}}(y_t \mid Y_{t-1}) = \frac{\sum_{s=p+1, s \neq t}^{T} L \left( \frac{Y_{t-1} - Y_{s-1}}{h} \right) 1 \{ y_s \leq y_t \}}{\sum_{s=p+1, s \neq t}^{T} L \left( \frac{Y_{t-1} - Y_{s-1}}{h} \right)},
\]

with \( L(\cdot) \) denoting the kernel function and \( h \) the bandwidth.

In an extension of Jeong et al. (2012)'s framework, we develop a test for the second moment. In particular, we want to test the volatility causality between \( cay \) (or \( cay^{MS} \)) and asset returns. Causality in the \( k \)th moment generally implies causality in the \( m \)th moment for \( k < m \).

We employ the nonparametric Granger quantile causality approach by Nishiyama et al. (2011). For the \( y_t \) process, the authors assume that

\[
y_t = g(Y_{t-1}) + \sigma(X_{t-1}) \varepsilon_t,
\]

where \( \varepsilon_t \) is a white noise process, and \( g(\cdot) \) and \( \sigma(\cdot) \) are unknown functions that satisfy certain conditions for stationarity. However, this specification does not allow for Granger-type causality testing from \( x_t \) to \( y_t \), but could possibly detect the “predictive power” from \( x_t \) to \( y_t^2 \) when \( \sigma(\cdot) \) is a general nonlinear function. Hence, the Granger causality-in-variance definition does not require an explicit specification of the squares of \( X_{t-1} \).
We re-formulate Eq. (10) into a null and an alternative hypotheses for causality in variance as follows:

\[ H_0 = P\{F_{Y_t|Z_{t-1}} \mid Q_\theta(Y_{t-1}) \mid Z_{t-1} \} = \theta\} = 1 \]  
(11)

\[ H_1 = P\{F_{Y_t|Z_{t-1}} \mid Q_\theta(Y_{t-1}) \mid Z_{t-1} \} = \theta\} < 1. \]  
(12)

To obtain a feasible statistics for testing the null hypothesis in Eq. (10), we replace \( y_t \) in Eq. (6) - (9) with \( y_t^2 \). By incorporating Jeong et al. (2012)'s approach, we overcome the problem that causality in the conditional first moment (i.e. the mean) imply causality in the second moment (i.e. the variance). In order to overcome this problem, we specify the causality in higher order moments using the following model:

\[ y_t = g(X_{t-1}, Y_{t-1}) + \varepsilon_t. \]  
(13)

Thus, higher order quantile causality can be specified as:

\[ H_0 = P\{F_{Y_t|Z_{t-1}} \mid Q_\theta(Y_{t-1}) \mid Z_{t-1} \} = \theta\} = 1 \quad \text{for } k = 1,2,\ldots, K \]  
(14)

\[ H_1 = P\{F_{Y_t|Z_{t-1}} \mid Q_\theta(Y_{t-1}) \mid Z_{t-1} \} = \theta\} < 1 \quad \text{for } k = 1,2,\ldots, K \]  
(15)

Integrating the entire framework, we define that \( x_t \) \textit{Granger causes } \( y_t \) \textit{in quantile } \( \theta \) \textit{up to the } \( k^{th} \) \textit{moment} using Eq. (11) to construct the test statistic of Eq. (6) for each \( k \). However, it can be shown that it is not easy to combine the different statistics for each \( k = 1,2,\ldots, K \) into one statistic for the joint null in Eq. (11), because the statistics are mutually correlated (Nishiyama et al., 2011).

To efficiently address this issue, we include a sequential-testing method as described Nishiyama et al. (2011) with some modifications. First, we test for the nonparametric Granger causality in the first moment (i.e. \( k = 1 \)). Rejecting the null of non-causality means that we can stop and interpret this result as a strong indication of possible Granger quantile causality-in-variance. Nevertheless, failure to reject the null for \( k = 1 \) does not automatically leads to no-causality in the second moment. Thus, we can still construct the tests for \( k = 2 \). Finally, we can test the existence of causality-in-variance or the causality-in-mean and variance, successively.

The empirical implementation of causality testing via quantiles entails specifying three important choices: the bandwidth \( h \), the lag order \( p \), and the kernel type for \( K(\cdot) \) and \( L(\cdot) \) in Eq. (6) and (9) respectively. In our study, the lag order of one is determined using the Schwarz Information Criterion (SIC) under a VAR comprising of excess or real returns on stock and housing prices and \( cay \) or \( cay^{MS} \) respectively. The bandwidth value is selected
using the least squares cross-validation method. Lastly, for $K(\cdot)$ and $L(\cdot)$ we employ Gaussian-type kernels.

4. **DATA ANALYSIS AND EMPIRICAL RESULTS**

4.1. **DATA**

Our quarterly dataset comprises excess and real stock and housing returns, $cay$ and $cay^{MS}$. The data on $cay$ and $cay^{MS}$ span over the period 1952:Q1-2014:Q3 and are obtained from Sydney C. Ludvigson’s website: [http://www.econ.nyu.edu/user/ludvigsons/](http://www.econ.nyu.edu/user/ludvigsons/). As we want to compare the predictive ability of both measures, we standardize them by dividing the actual series by the corresponding standard deviations.

Excess stock market returns are computed as the excess returns of a market index ($exsr$) over the risk-free asset return, which is common in the relevant literature. Specifically we calculate the continuously compounded log return of the S&P 500 index (including dividends) minus the 3-month Treasury bill rate. We also compute the volatility of excess stock market returns ($exsv$) using the standard deviation of the series. Data are sourced from the Center for Research in Security Prices (CRSP).

Real stock returns ($rsr$) are computed as the difference between the nominal stock returns and consumer price index (CPI – All Urban Consumers, with base year 1982-1984) inflation. The volatility of real stock returns ($rsv$) is then computed as the squared values of $rsr$. Data on the value-adjusted CSRP for the S&P500 index, the risk free rate and CPI inflation are obtained from Amit Goyal's website: [http://www.hec.unil.ch/agoyal/](http://www.hec.unil.ch/agoyal/).

Nominal and real house prices (obtained by deflating the nominal house price with the Consumer Price Index (CPI)) come from Shiller (2015), which is available at Robert J. Shiller's website: [http://www.econ.yale.edu/~shiller/data.htm](http://www.econ.yale.edu/~shiller/data.htm). Data are available at the monthly frequency since January 1953, which we convert into quarterly frequency by taking three-month averages. We calculate the difference between continuously compounded log nominal housing returns and the risk-free rate to derive excess housing returns ($exhr$) since 1953:Q2. The volatility of excess housing returns ($exhv$) is measured as the squared values of $exhr$. Real housing returns ($rhr$) and their volatility ($rhv$) are computed in the same way as their stock market counterparts.

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2 To the best of our knowledge, this is the longest available (monthly) house price data for the US economy. Other house price data at monthly or quarterly frequencies can be obtained from Freddie Mac and the Federal Housing Finance Agency (FHFA) since 1975, and from the Lincoln Institute of Land Policy since 1960.
4.2. **Empirical Results**

In Table 1, we start by presenting the summary statistics of excess and real stock and housing returns, $cay$ and $cay^{MS}$. As can be seen, all variables display excess kurtosis and, barring $exhr$ and $cay^{MS}$, are skewed to the left. Normality is strongly rejected for all the returns, but is accepted for $cay^{MS}$ and rejected only at the 10% level for $cay$. This non-normality of asset returns provides a preliminary motivation to look into causality based on the entire conditional distribution, rather than just on the conditional mean. Not surprisingly, stock market returns are more volatile than housing market returns.

[ INSERT TABLE 1 HERE. ]

Though our objective is to analyse the causality-in-quantiles running from $cay$ and $cay^{MS}$ to asset returns and their volatilities, for the sake of completeness and comparability, we also conduct the standard linear Granger causality test based on VAR(1) models.

The results are reported in Table 2. The null hypothesis that $cay$ and $cay^{MS}$ do not Granger-cause stock returns ($exsr$ and $rsr$) are overwhelmingly rejected at the 1% significance level, with $cay^{MS}$ being a stronger predictor than $cay$ - a result that is consistent with Bianchi et al. (2015). However, there is no evidence of predictability originating from $cay$ or $cay^{MS}$ for housing returns ($exhr$ and $rhr$). The lack of predictability of $cay$ is in line with Caporale et al. (forthcoming). Moreover, it is relevant to highlight that, as we show below based on the tests of nonlinearity and structural breaks, the linear models for the predictability analysis are mis-specified and, hence, the results from the standard Granger causality test cannot be deemed robust.

[ INSERT TABLE 2 HERE. ]

To further motivate the use of the nonparametric quantile-in-causality approach, we investigate two features of the relationship between asset returns and the two predictors, namely, nonlinearity and structural breaks. To assess the existence of nonlinearity, we apply the Brock et al. (1996) BDS test on the residuals of an AR(1) model for excess and real returns, and the excess or real returns equation in the VAR(1) model involving $cay$ or $cay^{MS}$. The $p$-values of the BDS test are reported in Table 3 and, in general, they reject the null hypothesis of no serial dependence. These results provide strong evidence of nonlinearity in not only excess and real stock and housing returns, but also in their relationship with $cay$ or
Consequently, the evidence of predictability for the stock market and the lack of it in the case of the housing market emanating from the linear Granger causality test cannot be relied upon.

[ INSERT TABLE 3 HERE. ]

Next, we turn to the Bai and Perron (2003)'s tests of multiple structural breaks, applied again to the AR(1) model for asset returns, and the asset return equations from a VAR(1) model involving \( cay \) or \( cay^{MS} \). The results are summarized in Table 4 and corroborate the existence of structural breaks. Therefore, the Granger causality tests based on a linear framework are, again, likely to suffer from mis-specification.

[ INSERT TABLE 4 HERE. ]

In this context, we now turn our attention to the nonparametric causality-in-quantiles test, i.e. a framework that, by design, is robust to the above mentioned econometric problems. Figures 1 to 4 display the results from the causality-in-quantiles test for excess (\( exsr \)) and real stock returns (\( rsr \)) and their volatilities (\( exsv \) and \( rsv \)), while Figures 5 to 8 report the same evidence for excess (\( exhr \)) and real housing returns and associated volatilities. We find that \( cay \) and \( cay^{MS} \) fail to predict \( rsr \) and \( rsv \) over their conditional distributions, a result that is in line with the work of Bekiros and Gupta (2015). Moreover, \( cay \) and \( cay^{MS} \) predict \( exsr \), but not \( exsv \) over the entire conditional distribution. However, it is important to note that \( cay^{MS} \) performs better than \( cay \) at certain quantile of the distribution of \( exsr \), such as 0.10, 0.30, 0.40, 0.80 and 0.90.

In what concerns excess housing returns (\( exhr \)) and its volatility (\( exhv \)), there is no evidence of predictability emanating from \( cay \) or \( cay^{MS} \). Additionally, while \( cay \) and \( cay^{MS} \) still fail to predict \( rhr \) (real housing returns), these two variables tend to forecast the volatility of real housing returns (\( rhv \)) over the entire conditional distribution.\(^3\)

\(^3\) Since the house price data of Shiller (2015) does not include housing rents, we recomputed returns on housing including rents, with the data on house price as well as rents obtained from the Lincoln Institute of Land Policy and the Organisation for Economic Co-operation and Development (OECD). The data based on the Lincoln Institute of Land Policy started in 1960:Q1 till 2014:Q3, while the data set from the OECD covered the period of 1970:Q1-2013:Q4. The results based on the first data set from the Lincoln Institute of Land Policy showed no evidence of predictability for \( exhr \), \( exhv \), \( rhr \) and \( rhv \) originating from \( cay \) or \( cay^{MS} \) using the causality-in-quantiles test. For the OECD database, again there was no evidence of predictability from either \( cay \) or \( cay^{MS} \) for \( exhr \), \( exhv \) and \( rhr \), but we observed that \( cay \) and \( cay^{MS} \) caused \( rhv \) over the quantiles 0.45 to 0.60, and 0.50 to 0.60 respectively, i.e., around the median of the conditional distribution. In other words, evidence of
Summing up, while the linear Granger causality tests provide evidence of predictability for both \( exsr \) and \( rsr \), with \( cay^{MS} \) outperforming \( cay \), the causality-in-quantiles approach shows that the two predictors are only relevant for \( exsr \). In addition, while the entire conditional distribution of \( exsr \) can be predicted by both \( cay \) and \( cay^{MS} \), the latter is only a strong predictor at certain quantiles. As for housing returns, there is evidence of predictability over the entire conditional distribution of \( rhv \), with \( cay \) performing better than \( cay^{MS} \) in the majority of the quantiles of the distribution.

5. Conclusion

This paper compares the predictive ability of \( cay \) and the Markov-switching \( cay \) (\( cay^{MS} \) introduced by Bianchi et al. (2015)) for stock and housing returns in the US over the period 1953Q2-2014Q3, as well as their volatility, using a nonparametric causality-in-quantiles test developed by Balcilar et al. (2015).

We find strong evidence of nonlinearity and regime changes in the relationship between stock and housing returns and \( cay \) or \( cay^{MS} \), which gives support to the use of nonparametric causality-in-quantiles test.

Our results also indicate that the two predictors are mainly relevant for excess stock returns but not for real stock returns, with \( cay^{MS} \) outperforming \( cay \). Furthermore, the entire conditional distribution of excess stock returns can be predicted by both \( cay \) and \( cay^{MS} \), with the latter being a strong predictor at certain quantiles.

With regard to housing returns, we only find evidence of predictability emanating from \( cay \) or \( cay^{MS} \) in the case of the conditional distribution of the variance of real housing returns. In this case, \( cay \) beats \( cay^{MS} \) in the majority of the quantiles of the distribution.

As part of future research, it would be interesting to extend our study in order to examine if these results continue to hold in an out-of-sample exercise (Rapach and Zhou, 2013; Bonaccolto et al., 2015).

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 predictability for \( rhv \) based on the OECD housing returns that include rent, was found to be weaker than that obtained using the house price of Shiller (2015). Complete details of these results are available upon request from the authors.
References


**List of Tables**

**Table 1. Summary statistics.**

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<thead>
<tr>
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<th>exsr</th>
<th>rsr</th>
<th>exhr</th>
<th>rhr</th>
<th>cay</th>
<th>cay&lt;sub&gt;MS&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.0181</td>
<td>0.0206</td>
<td>-0.0015</td>
<td>0.0011</td>
<td>1.59E-11</td>
<td>-0.0021</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>0.0277</td>
<td>0.0282</td>
<td>-0.0021</td>
<td>0.0013</td>
<td>0.000125</td>
<td>-0.0025</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>0.2145</td>
<td>0.2109</td>
<td>0.0531</td>
<td>0.0478</td>
<td>0.043397</td>
<td>0.0291</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>-0.2723</td>
<td>-0.2848</td>
<td>-0.0538</td>
<td>-0.0482</td>
<td>-0.047730</td>
<td>-0.0401</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td>0.0797</td>
<td>0.0802</td>
<td>0.0148</td>
<td>0.0135</td>
<td>0.019354</td>
<td>0.0121</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>-0.5816</td>
<td>-0.5586</td>
<td>0.1964</td>
<td>-0.3651</td>
<td>-0.205892</td>
<td>0.0424</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>3.9044</td>
<td>3.8772</td>
<td>4.5536</td>
<td>4.7524</td>
<td>2.437870</td>
<td>2.8493</td>
</tr>
<tr>
<td><strong>Jarque-Bera test</strong></td>
<td>22.7032</td>
<td>21.1029</td>
<td>26.3227</td>
<td>36.9440</td>
<td>5.078114</td>
<td>0.3125</td>
</tr>
<tr>
<td><strong>Probability</strong></td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.078941</td>
<td>0.8553</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>251</td>
<td>251</td>
<td>246</td>
<td>246</td>
<td>251</td>
<td>251</td>
</tr>
</tbody>
</table>

**Table 2. Linear Granger causality test.**

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>$\chi^2(1)$ test statistic</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>cay does not Granger cause exsr</td>
<td>9.9107***</td>
<td>0.0016</td>
</tr>
<tr>
<td>cay&lt;sub&gt;MS&lt;/sub&gt; does not Granger cause exsr</td>
<td>14.9947***</td>
<td>0.0001</td>
</tr>
<tr>
<td>cay does not Granger cause rsr</td>
<td>12.0734***</td>
<td>0.0005</td>
</tr>
<tr>
<td>cay&lt;sub&gt;MS&lt;/sub&gt; does not Granger cause rsr</td>
<td>15.8214***</td>
<td>0.0001</td>
</tr>
<tr>
<td>cay does not Granger cause exhr</td>
<td>0.2259</td>
<td>0.6346</td>
</tr>
<tr>
<td>cay&lt;sub&gt;MS&lt;/sub&gt; does not Granger cause exhr</td>
<td>0.8740</td>
<td>0.3498</td>
</tr>
<tr>
<td>cay does not Granger cause rhr</td>
<td>0.4094</td>
<td>0.5223</td>
</tr>
<tr>
<td>cay&lt;sub&gt;MS&lt;/sub&gt; does not Granger cause rhr</td>
<td>0.3117</td>
<td>0.5767</td>
</tr>
</tbody>
</table>

*Note: exsr, rsr, exhr and rhr stand for excess stock returns, real stock returns, excess housing returns and real housing returns, respectively. *** indicates rejection of the null hypothesis at the 1% significance level.*
### Table 3. Brock et al. (1996) BDS test.

<table>
<thead>
<tr>
<th>Dimension</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1): exsr</td>
<td>0.0103</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR(1): rsr</td>
<td>0.0138</td>
<td>0.0004</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR(1): exhr</td>
<td>0.0039</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR(1): rhr</td>
<td>0.0021</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>VAR(1): [exsr, cay]</td>
<td>0.0099</td>
<td>0.0004</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>VAR(1): [exsr, cay&lt;sup&gt;MS&lt;/sup&gt;]</td>
<td>0.0693</td>
<td>0.0156</td>
<td>0.0014</td>
<td>0.0008</td>
<td>0.0002</td>
</tr>
<tr>
<td>VAR(1): [rsr, cay]</td>
<td>0.0142</td>
<td>0.0005</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>VAR(1): [rsr, cay&lt;sup&gt;MS&lt;/sup&gt;]</td>
<td>0.0606</td>
<td>0.0093</td>
<td>0.0012</td>
<td>0.0007</td>
<td>0.0002</td>
</tr>
<tr>
<td>VAR(1): [exhr, cay]</td>
<td>0.0058</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>VAR(1): [exhr, cay&lt;sup&gt;MS&lt;/sup&gt;]</td>
<td>0.0045</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>VAR(1): [rhr, cay]</td>
<td>0.0014</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>VAR(1): [rhr, cay&lt;sup&gt;MS&lt;/sup&gt;]</td>
<td>0.0014</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**Note:** See notes to Table 2. $p$-value of the BDS test statistic, with the test applied to the residuals recovered from the AR(1) models of exsr, rsr, exhr and rhr, and the residuals from the exsr, rsr, exhr and rhr equations of the VAR(1) model comprising these returns and cay or cay<sup>MS</sup>.

### Table 4. Bai and Perron (2003)'s test of multiple structural breaks.

<table>
<thead>
<tr>
<th>Models</th>
<th>Break Dates</th>
</tr>
</thead>
</table>

**Note:** See notes to Table 2. Break dates are based on the Bai and Perron (2003) test of multiple structural breaks applied to the AR(1) models of exsr, rsr, exhr and rhr, and the the exsr, rsr, exhr and rhr equations of the VAR(1) model comprising of these returns and cay or cay<sup>MS</sup>.
List of Figures

Figure 1. Causality-in-quantiles: Excess stock returns \((exsr)\), \(cay\) and \(cay^{MS}\).

![Figure 1](image1)

*Note:* \(cay^{\{MS\}}\) stands for \(cay^{MS}\).

Figure 2. Causality-in-quantiles: Volatility of excess stock returns \((exsv)\), \(cay\) and \(cay^{MS}\).

![Figure 2](image2)

*Note:* See note to Figure 1.
Figure 3. Causality-in-quantiles: Real stock returns ($rsr$), $cay$ and $cay^{MS}$.

Figure 4. Causality-in-quantiles: Volatility of real stock returns ($rsv$), $cay$ and $cay^{MS}$.

Note: See note to Figure 1.
Figure 5. Causality-in-quantiles: Excess housing returns \((exhr)\), \(cay\) and \(cay^{MS}\).

**Note:** See Note to Figure 1.

Figure 6. Causality-in-quantiles: Volatility of excess housing returns \((exhv)\), \(cay\) and \(cay^{MS}\).

**Note:** See note to Figure 1.
Figure 7. Causality-in-quantiles: Real housing returns ($rhr$), $cay$ and $cay^{MS}$.

Note: See note to Figure 1.

Figure 8. Causality-in-quantiles: Volatility of real housing returns ($rhv$), $cay$ and $cay^{MS}$.

Note: See note to Figure 1.