Reducing illegal immigration to South Africa: A dynamic CGE analysis

Heinrich R. Bohlmann

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Abstract

South African authorities are attempting to limit inflows of illegal immigrants. Evidence for the United States presented in Dixon et al (2011) suggests that a policy-induced reduction in labour supply from illegal immigrants generates a welfare loss for legal residents. I use a similar labour market mechanism within a dynamic CGE model for South Africa, but take into consideration a number of well-known facts about the local economy. With high unemployment rates among low skilled workers and a legal minimum wage in place, I find a net gain in employment and welfare for legal residents in South Africa when reducing the inflow of illegal immigrants.

JEL codes: J61, C68

Keywords: Illegal immigration, dynamic CGE modelling

1 Introduction

The economic landscape in South Africa is as diverse as its people. The Rainbow Nation is the largest economy on the African continent. It produces a wide variety of goods and services for both the domestic and export market. Since the first democratic elections in 1994 economic growth in South Africa has been strong. This is largely attributable to improvements in total factor productivity and increased openness to trade and capital flows (Du Plessis & Smit, 2007). However, all is not well. Many socio-economic problems still plague the country. Foremost is overall unemployment which has remained above 30 percent, contributing to widespread poverty and inequality (OECD, 2008; StatsSA, 2009).¹

Unfortunately, many other African countries face even more trying circumstances. Relatively poor economic and political conditions persist in many of South Africa’s closest neighbours. Motivated by the gap in potential earnings

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¹ Overall unemployment here refers to the expanded definition of unemployment which includes both the officially unemployed and discouraged work-seekers.
and relative employment conditions, many desperate workers who fail to meet immigration requirements continue to look for jobs in South Africa. Large flows of illegal immigrants (and refugees) into South Africa have been a direct consequence.

This paper aims to inform policy discussions by analysing the impact of reducing employment of illegal immigrants in South Africa. This is achieved via a reduction in the preferences of foreign-born workers with illegal status for moving to and earning money in South Africa. A change in such supply-side preferences may be brought on in a number of ways. What is envisioned here is the introduction of policies that increase border security around South Africa or improve economic and political stability in neighbouring countries.

I evaluate the economic consequences of this policy-induced cut to illegal immigration using ZAR-M, a computable general equilibrium (CGE) model of South Africa, under two different scenarios or sets of assumptions. The first uses labour market and wage adjustment assumptions equivalent to that in the Dixon et al. (2008) study of illegal immigration in the United States. This includes the modelling of upward-sloping labour supply curves and equilibrium rates of unemployment. The second set of assumptions attempts to simulate a more realistic picture of the South African labour market at its lower end. Two important factors are accounted for here: high unemployment rates among low skilled workers and a legal minimum wage. The model is solved using GEMPACK\(^2\) (Harrison & Pearson, 1996) and implemented on a database representing the South African economy for the base year 2006 (Bohlmann, 2010).

## 2 Overview of the ZAR-M Model

ZAR-M is a large scale recursive-dynamic CGE model of the South African economy. Its theoretical structure closely follows that of the MONASH model (Dixon & Rimmer, 2002). To facilitate the analysis of migrants it incorporates a detailed labour market specification similar to that introduced in the USAGE-M model (Dixon et al., 2011).

The linearised system of equations that make up ZAR-M describes the theory underlying the behaviour of participants in the economy. Demand and supply equations for industries and households are derived from the solutions to the optimisation problems which are assumed to underlie the behaviour of private sector agents in conventional neo-classical microeconomics. Each industry minimises cost subject to given input prices and a constant returns to scale production function. Households maximise a Klein-Rubin utility function subject to their budget constraint. Units of new industry-specific capital are determined as cost-minimising combinations of domestic and imported commodities. Imperfect substitutability between foreign and domestic sources of commodities is

\(^2\)GEMPACK is a suite of software applications designed specifically for general equilibrium modelling. I used the Euler 32-step solution method in this paper. Multi-step methods are used to largely eliminate the linearisation error arising from the derivative approach to solving economy-wide models.
modelled using the Armington CES assumptions. The export demand for any local commodity is inversely related to its foreign-currency price. The price of imports is exogenously determined, consistent with the assumption of South Africa being a small open economy. Government consumption and the details of direct and indirect taxation are also recognised in the model. Markets are assumed to be competitive which implies that zero pure profits are captured in any sector or activity.

The dynamic elements of ZAR-M allow for inter-temporal links describing i) physical capital accumulation, ii) financial asset/liability accumulation, and iii) lagged adjustment processes for labour. Capital accumulation is specified separately for each industry, and linked to industry specific net investment. Investment in each industry is positively related to its expected rate of return on capital. Adjustments to the national net foreign liability position are related to the annual investment/savings imbalance. In policy simulations, the labour market follows a lagged adjustment path where wage rates are allowed to respond over time to gaps between demand and supply for labour. This behaviour is partly modified though under the second set of assumptions for which the simulation is run.

The 2006 database features 27 multi-product industries and 27 multi-industry commodities. Final users of commodities include investors, households, government and exporters. For the labour market I specify 11 local occupations, two types of unemployment and two foreign occupations. The first of these foreign occupations, \textit{zmn} capture employment in source countries for potential low skilled illegal migrants. The second foreign occupation, \textit{auk} facilitates the modelling of skilled migration, but is of little importance for this paper which is mainly concerned with low skilled migration.

No reliable data on the current stock or gross flows of illegal migrants in South Africa exist. Based on earlier studies, the number of deportations, applications for asylum, and evidence that has emerged after the recent spate of xenophobic attacks, the number of illegal workers in South Africa has been conservatively estimated at upward of 2 million, or 10 per cent of the total labour force (Solomon, 2000; StatsSA, 2005, 2010a; Walmsley \textit{et al}, 2007; UNDESA, 2009; UNDP, 2009; UNHCR, 2010; M&G Online, 2010). With no firm data available, initial estimates of illegal migrant numbers and relevant substitution elasticities in ZAR-M necessarily represent own judgments, incorporating as much available information as possible.

3 ZAR-M Labour Market Specification

The detailed modelling of migrant flows within a recursive-dynamic environment is made possible by the labour market specification in ZAR-M. It includes i) the division of the labour force into categories at the start of each year reflecting labour force functions of people in the previous year; ii) the identifi-

\textsuperscript{3}ZAR-M is modelled with 1 representative household and 1 central government in this paper.
cation of labour force activities, that is, what people do during the year; iii) the determination of labour supply from each category to each activity; iv) the determination of labour demand in employment activities; v) the specification of wage adjustment processes reflecting labour demand and supply; and vi) the determination of everyone’s activity, that is, who finds employment and what happens to those who don’t.

The labour force in ZAR-M takes on an expanded definition. It includes workers who are employed in any local occupation, the recently unemployed and discouraged work-seekers. In addition, it also includes potential legal and illegal migrants working outside South Africa. Categories (cat) and activities (act) are defined over dimensions that describe the birthplace (b), legal status (s) and labour force function (f) of workers. Each set within the (b, s, f) dimension contain elements used to detail the characteristics of workers. Birthplace is domestic (dom) or foreign (fgn) and status is legal (leg) or illegal (ill). Labour force functions include employment in three skilled and eight low skilled local occupations (o), short-term (S) and long-term (L) unemployment, one foreign low skilled occupation (zmn) and one foreign skilled occupation (auk). An exogenously specified number of new entrants (new) to the labour market are introduced at the start of each year. At the end of each year I assume that one per cent of people in every activity drop out of the labour force through either death or retirement. The link between people in different activities in year t − 1 and the number of people in each category at the start of year t is specified by the following equations:

\[ CAT_t(b, s, f) = ACT_{t-1}(b, s, f) \times 0.99 \quad \text{for all } b, s \text{ and } f \neq \text{new} \quad (1) \]

\[ CAT_t(b, s, 'new') = \text{exogenous} \quad \text{for all } b \quad (2) \]

As with the standard USAGE-M labour market specification, employment activities that people in a given category undertake each year are determined mainly by their willingness to offer their services to that activity, relative to offers from people in other categories, and by employers’ demand for the services of that activity (Dixon et al, 2008). In policy simulations, after-tax real wages for workers in local occupations adjust according to the following equation:

\[ \frac{ATRW_t(b, s, o)}{ATRW_{base}^t(b, s, o)} - \frac{ATRW_{t-1}(b, s, o)}{ATRW_{base}^{t-1}(b, s, o)} = \beta(s, o) \]

\[ \frac{D_t(b, s, o)}{D_{base}^t(b, s, o)} - \frac{L_t(b, s, o)}{L_{base}^t(b, s, o)} \quad \text{for all } b, s \text{ and local occupations } o \quad (3) \]

In equation (E3) \( \beta(s, o) \) is a positive parameter controlling the adjustment or sensitivity of after-tax real wages (ATRW) over time to gaps between labour demand (D) and labour supply (L).
With ATRW endogenous in the model’s policy closure, (E3) shows that any policy-induced increase in labour demand relative to supply will elevate after-tax real wages relative to their baseline values. As pointed out in Dixon et al. (2008), this process is also compatible with popular labour market theories such as search models and efficiency-wage theory. These modelling assumptions regarding the wage adjustment process of workers will hereafter be referred to as Scenario 1.

Under my alternative set of assumptions, I model this process differently for low skilled legal workers. High unemployment rates combined with a legal minimum wage renders the standard optimising behaviour for these workers almost meaningless. To address this, labour supply curves for lower skilled legal workers are simulated as being close to perfectly elastic under this alternative set of assumptions. That is, the wage adjustment behaviour in (E3) is altered for lower skilled legal workers in the policy run. Any policy-induced change to labour demand for low skilled legal workers would then be met by a corresponding shift in labour supply without inducing any real wage pressure. These modelling assumptions will hereafter be referred to as Scenario 2.

The final task in the labour market specification of ZAR-M involves the determination of everyone’s activity during the year, that is, who finds employment and what happens to those who don’t. This is required because under (E3) markets for local occupations do not clear. That is, labour demand and labour supply are not equal in the short-run. Figure 1 illustrates this task by showing the general labour flows accounted for in ZAR-M, aggregated over all birthplaces and legal status workers.

Each type of flow from a start-of-year category to activity is described by an equation. Here I will focus only on equations (E4–E6) shown below that describe flows from categories to local employment activities, that is, the shaded area 1 in Figure 1. The equations describing in areas 2–5 can be found in the Appendix. Areas in which a zero appears indicate flows which are not permitted in ZAR-M.

\[ V_t(\text{act}) = E_t(\text{act}) - H_t[\text{act}; \text{act}] \quad \text{for all local employment activities act} \quad (4) \]

\[ H_t(\text{cat}; \text{act}) = V_t(\text{act}) \times \left[ \frac{L_t(\text{cat}; \text{act})}{\sum_{v \neq \text{act}} L_t(v; \text{act})} \right] \quad \text{for all categories cat ≠ activities act} \quad \text{for all local employment activities act} \quad (5) \]

\[ H_t(\text{cat}; \text{cat}) = \text{CAT}_t(\text{cat}) - \sum_{\text{act} \neq \text{cat}} H_t(\text{cat}; \text{act}) \quad \text{for all employment categories cat} \quad (6) \]

4 With a positive value for \( \beta(s,o) \) and ATRW endogenous in the policy run, equation (E3) implies the existence of equilibrium rates of unemployment in local occupations.

5 This is achieved by either altering the model closure to reflect fixed ATRW in the policy run or setting \( \beta(s,o) \) to a suitably small number for legal workers in low skilled occupations.
Equation (E4) defines vacancies in local employment activity \( \text{act} \) in year \( t \) as employment in the particular activity during year \( t \) minus the number of jobs filled by incumbents in the activity.

Equation (E5) models the flow of non-incumbents to local employment activity \( \text{act} \) as being proportional to the vacancies in that activity and the share of category \( \text{cat} \) in the supply of labour to activity \( \text{act} \) from workers outside category \( \text{act} \). In modelling (E5) I assume there is always competition for jobs, that is, I assume the number of people from outside category \( \text{act} \) who plan to work in employment activity \( \text{act} \) is greater or equal to the number of vacancies in \( \text{act} \). This ensures that \( \sum_{\text{cat}} H_t (\text{cat}; \text{act}) \) will be less than or equal to \( \sum_{\text{cat}} L_t (\text{cat}; \text{act}) \) for all categories \( \text{cat} \neq \text{act} \) and that \( V_t (\text{act}) \) will not become negative.

Incumbents from employment category \( \text{cat} \) who remain in activity \( \text{cat} \) are defined in equation (E6) as the number of workers in category \( \text{cat} \) minus the number who move out of activity \( \text{cat} \) during the year. Workers in employment-category \( \text{cat} \) who planned to work in a different activity \( \text{cat} \neq \text{act} \) but who are unable to move to \( \text{act} \) due to insufficient vacancies simply remain in \( \text{act} \).

4 Simulations

One of the main purposes of CGE models is to provide projections of the impact of economic policy changes on a wide variety of economic variables. To accomplish this and generate results with ZAR-M, a baseline scenario incorporating available forecast data is first simulated. The baseline forecast simulation aims to produce a believable business-as-usual picture of the future evolution of the economy, excluding the impact of the particular policy under consideration. A perturbed scenario incorporating the relevant policy shocks is then run and compared to the outcome of the baseline scenario, with deviations usually reported as percentage changes. This allows the model to produce a more realistic estimate of the policy’s impact. That is, if we wish to know what the difference in migrant flows will be in 2020 as a result of imposing certain policies, we must first establish what the flow of migrants would have looked like in 2020 without the imposition of such policies.

ZAR-M recognises that the results of any simulation depend on the economic environment and assumptions under which it is run. This is reflected by the choice of model closure in which users must select an appropriate combination of exogenous variables for each type of simulation.\(^6\) The model closures used in this study largely follow those for the baseline forecast and policy simulations described in Dixon & Rimmer (2002). Simulation results are projected for a 14-year period up to 2020.\(^7\)

\(^6\)Large scale CGE models such as ZAR-M contain many more variables \( n \) than equations \( m \). To close the model and compute a solution, \( n - m \) variables must therefore be treated as exogenous.

\(^7\)The initial solution for ZAR-M is based on 2006 data. At the time of writing, historical data up to 2009 were available for most macroeconomic variables. This information was incorporated into the baseline forecast to give a more accurate description of the economy’s
Policy Shocks – Reducing Illegal Immigration

The policy simulations conducted here take a counterfactual approach. That is, it estimates the impact on the South African economy, relative to the baseline, had the proposed policy interventions aimed at reducing illegal immigration been successfully implemented. The policy shock is introduced in ZAR-M as a 44 per cent reduction in the marginal utility to potential illegal immigrants from earning money in South Africa. Equation (E7) describes the labour supply functions in ZAR-M used to carry this shock. This equation is derived by solving an optimisation problem whereby it is assumed that at the start of year t, people in category cat decide their offers to activity act during year t by maximising their expected utility from earning wages in those activities.\(^8\)

\[
L_t(\text{cat}; \text{act}) = \text{CAT}_t(\text{cat}) \cdot \left[ \frac{[\text{PREF}_t(\text{cat}; \text{act}) \cdot \text{ATRW}_t(\text{act})]^n}{\sum_q [\text{PREF}_t(\text{cat}; q) \cdot \text{ATRW}_t(q)]^n} \right] \quad (7)
\]

for all cat & act

In equation (E7) \( \eta \) is a positive parameter reflecting the ease with which people feel they can shift between activities, \( \text{PREF}_t(\text{cat}; \text{act}) \) is a variable reflecting the preference of people in category cat for earning money in activity act in year t, and \( \text{ATRW}_t(\text{act}) \) is the after-tax real wage rate of labour in activity act.

In terms of (E7), the shocks in this policy simulation are a 44 per cent reduction in \( \text{PREF}_t(\text{cat}; \text{act}) \) for cat = fgn, ill, zim or new, and act = (fgn, ill, o) where o is any local South African occupation. The categories zim and new capture foreign-born workers plus new entrants in neighbouring countries that may be viewed as potential illegal immigrants to South Africa. The shocks are introduced as a 30 per cent reduction in 2008 and a 20 per cent reduction in 2009. The policy shocks are identical in both scenarios.

Understanding and interpreting the policy shock implemented via equation (E8) is made easier when written in its linearised percentage-change form.

\[
ls_t(\text{cat}; \text{act}) = \text{cat}_t(\text{cat}) + \eta \cdot [\text{atrw}_t(\text{act}) - \text{atrw}_{\text{ave}}^t(\text{act})] + \eta \cdot [\text{pref}_t(\text{cat}; \text{act}) - \text{pref}_{\text{ave}}^t(\text{cat})] \quad \text{for all cat & act} \quad (E8)
\]

In the linearised equation (E2.41), also previously described in Chapter 2, the lowercase symbols \( ls_t(\text{cat}; \text{act}), \text{cat}_t(\text{cat}), \text{atrw}_t(\text{act}) \) and \( \text{pref}_t(\text{cat}; \text{act}) \) are percentage changes in the variables denoted by their corresponding uppercase symbols in (E7). The symbols \( \text{atrw}_{\text{ave}}^t(\text{cat}) \) and \( \text{pref}_{\text{ave}}^t(\text{cat}) \) are weighted averages, with the weights reflecting the share of activity q in the offers from people in category cat. From here, interpretation of the policy shocks imposed on the \( \text{pref}_t(\text{cat}; \text{act}) \) variable and the role of the parameter \( \eta \) in the labour evolution following the Global Financial Crisis.

\(^8\)Wages earned in different activities are treated as imperfect substitutes via (E7), with \( \eta \) indicating the relevant substitution elasticity. In this application of ZAR-M \( \eta \) is set at 1.5.

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supply function becomes clear. In the next section we present and interpret the policy simulation results under Scenario 1, followed by an analysis and comparison of results under Scenario 2. The policy shocks are identical in both scenarios.

As noted earlier, I run this simulation under two different scenarios. The assumptions contained in Scenario 1 reflect labour market conditions similar to that depicted in Dixon et al (2008) for the U.S. economy. This includes the modelling of upward-sloping labour supply curves and equilibrium rates of unemployment. Scenario 2 reflects alternative labour market conditions which I believe are more appropriate for the South African economy. In this scenario I account for high unemployment rates among low skilled workers and a legal minimum wage.

**Policy Results – Scenario 1**

Figure 2 shows the employment paths for illegal workers in the baseline forecast and policy simulation under Scenario 1 assumptions. In the baseline, employment of illegal migrants grows from 1.760 million in 2007 to 2.343 million in 2020. This represents an increase of 33.1 per cent between 2007 and 2020. The total number of illegal migrants in South Africa, including those in unemployment, grows from 2.169 million in 2007 to 2.861 million in 2020. Growth in employment of legal residents is projected to grow at a similar rate in the baseline, increasing from 11.877 million in 2007 to 15.820 million in 2020. The share of illegals in aggregate employment is projected to remain constant at around 12.9 per cent up to 2020. This result is mainly due to our forecast assumptions which allow little change in relative wage rates between labour groups, and exogenously projects only the change in aggregate employment.

Because illegal migrants are assumed to have low paid jobs, their share in the total wage bill is considerably less than their share in total employment. Over the forecast period, the wage bill share of illegals is projected to decline slightly from 3.94 per cent in 2007, to 3.84 per cent in 2020. In the policy simulation, employment of illegal migrants falls to 1.609 million in 2020. This represents a drop of 734,000 illegal workers in South Africa compared to the baseline in 2020. The total number of illegals in South Africa, including those in unemployment, falls to 2.034 million in 2020. The policy therefore has the effect of reducing illegal employment in South Africa by 31.3 per cent, and the total number of illegal migrants in South Africa by 28.9 per cent, over the forecast period.

Figure 3 shows that the policy shocks affect flows of illegal migrants in both directions. The shocks have a direct effect on inflows by reducing the number of low skilled people outside South Africa who want to move illegally into local employment activities. The shocks also have an indirect effect on outflows by lowering the number of illegal migrants in South Africa and thereby lowering

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9 The parameter \( \eta \) has the value 1.5. This suggests that the number of people who wish to change jobs is quite sensitive to changes in relative wage rates between activities or exogenous changes to their preferences for earning money in a particular activity.

10 Readers who view these estimates as conservative should bear in mind that I do not attempt to model those migrants who operate outside the given economic framework of ZAR-M.
the number who seek to go home.

Two features of Figure 3 warrant further comment. First, it implies that the net inflow of 60,000 illegals to the local labour force in 2007 was generated by a gross inflow of about 152,000 and a gross outflow of about 92,000. Despite a fair degree of uncertainty, these estimates are consistent with strong inflows of illegal migrants reported over the past decade as well as illegal migrants making frequent trips home.

The second notable feature of Figure 3 is the sharp decline in the early years of the policy run in the net and gross inflows of illegals to local employment activities, followed by recovery in later years. It appears that the policy would have a much greater effect on flows of illegal migrants in the short run than in the long run. The policy-induced decline in gross inflow shown in Figure 3 for 2009 is 94.7 per cent and the net inflow in the policy run is negative. Indeed, the model projects a negative net inflow for each year from 2008–2011. The decline in labour supply from \( z_{im} \) causes an increase in wage rates for illegal workers as dictated by (E3), and a decrease in local demand for illegal labour via the nested labour demand equations in ZAR-M. The growth rate in demand for illegal labour for the period 2008–2011 turns from positive in the baseline to negative in the policy run. Because the level of net inflow of illegals depends on local growth in demand for their services, negative growth in this demand translates into negative net inflow requiring a dramatic reduction in gross inflow. Eventually, wages for illegal workers rise sufficiently to reconcile demand with the reduced supply at which point demand in the policy run recommences growth at approximately the same rate as in the baseline. This allows net and gross inflow of foreign illegals to partially recover. In 2020 net inflow in the policy run is 40.6 per cent less (58,600 versus 98,800) than in the baseline and gross inflow is 36.5 per cent less (127,600 versus 201,100).

Figures 4 and 5 show the policy impacts on macroeconomic variables in South Africa. In each case, impacts are expressed as percentage deviations from the baseline forecast. Figure 4 is concerned with the supply side of the economy. It shows that the policy causes a relatively large reduction in the number of employed workers. In 2020, the total number of jobs is 775,000 or 4.3 per cent lower in the policy run than in the baseline. This mainly reflects the reduction of 734,000 in the number of illegal jobs. Since the lost jobs are mainly for low paid illegal workers, the reduction in wage bill weighted labour input in 2020 is less than 4.3 per cent. Using wage bill weights, we might expect the percentage loss in labour input to be about 31.3 per cent of 3.84 per cent (that is, the reduction in illegal employment times the illegal share in the baseline wage bill for 2020). However, this gives only a 1.2 per cent loss in labour input compared to the actual projected loss of 2.9 per cent. The additional loss in labour input in Scenario 1 is mainly due to the restriction of illegal employment causing the occupational mix or composition of future employment to shift towards lower paid occupations. A similar result was also found in Dixon et al (2011). Reasons for this projected shift in the occupational mix of employment and its welfare implications for legal residents are explained later.

The reduction in the capital stock of 2.2 per cent up to 2020 carries the
same trend as the reduction in labour input. Indeed, the longer the simulation period is extended for, the closer these two results tend towards each other. We assume no change in either long-run rates of return or technical change in the policy run, implying little change to the K/L ratio. With capital and labour inputs down by their respective amounts and no change in technology, GDP is projected to be 2.4 per cent lower over the simulation period in Scenario 1. This is equivalent to a reduction in the average annual growth of GDP from 4.5 per cent in the baseline to around 4.3 per cent in the policy run. As a result of the cut in illegal immigration, the total size of the labour force in South Africa falls by 3.1 per cent, indicating a slight increase in overall GDP per capita. However, with the number of legal residents seeking job opportunities largely unaffected in the long run, this does not translate into a welfare gain for the legal population.

Figure 5 is mainly concerned with the demand side of the economy. The long-run impacts of the policy on all aggregate expenditure variables are shown to be negative and ranged around that for GDP. I assume that the policy would have no effect on the private to public consumption ratio. Both these macro variables fall by 2.2 per cent at the end of the simulation period. The drop is slightly less severe than that in GDP since the policy improves the South African terms of trade. This boosts GNP relative to GDP. To explain this result we start by considering the negative impact of the policy on the overall size of the economy, or GDP. With a smaller economy, the long-run deviation for exports is negative as shown in Figure 5. With no shock to foreign-demand curves for South African exports in the policy run, the cut in export volumes is accompanied by an increase in their foreign-currency prices. On the import side we assume that changes in local demand have no effect on foreign-currency prices. An improvement in the terms of trade, that is, the price of exports relative to the price of imports, allows the local economy to increase its consumption (both public and private) relative to its GDP.

As witnessed by the trends in Figure 5, the eventual increase in consumption relative to GDP causes a deteriorating real trade balance (X–M), supported by long-run real appreciation. Investment falls relative to GDP mainly because the capital stock is not fully adjusted after the policy shock and still falling relative to the baseline at the end of the simulation period in 2020. If we extend the simulation period we find that perturbed capital growth reaches a steady-state level relative to the baseline slightly below its 2020 level and that results for investment relative to GDP tend even closer over time.

The short-run results in Figure 5 are driven by the economy’s adjustment towards a lower capital stock in the policy run relative to the baseline. In the short run, the policy causes a relatively sharp reduction in investment along with a real devaluation. This temporarily stimulates exports whilst limiting imports. As the adjustment in capital stock nears completion in 2020, investment recovers, causing the local currency to appreciate, exports to fall and imports to rise.

Figure 6 shows the change in legal employment by occupation type. The reduction in skilled employment of legislators and managers (lsm), professionals
and technical and associate professionals (tch) may at first seem surprising given the policy under consideration has no direct impact on legal or skilled employment. This result is due to the occupational-mix effect alluded to earlier. The policy shock directly reduces the labour supply of potential illegal immigrants to local South African occupations. The first round effect of the policy shock therefore is to reduce labour supply and consequently illegal employment in lesser skilled occupation groups. Real wages in these occupations rise in the short run as labour becomes scarcer. The second round effect of the policy shock relates to the increase in job vacancies at the lower end of the market as a result of the policy shock. Combined with the real wage increase, lower skilled occupation groups are now expected to attract relatively more legal labour offers in future years.

The greater the share of illegals employed in any occupation, the larger these first and second round impacts will be in ZAR-M. Local occupations such as agriculture and fishery (sag), craft and related trades (crf), elementary (elt) and domestic (dwk) employ a relatively large share of illegal labour in the baseline. As a result, we find that new legal entrants to the labour market, and those in unemployment, who previously may have considered paying large sums in further training and education to find a skilled job, are now more likely to offer their services to one of these lower skilled jobs.

As shown in Figure 6, there is a positive deviation in legal jobs in some lower skilled occupations. As suggested, the largest positive deviations occur in those jobs that previously employed the highest share of illegal labour, that is, the sag, crf, elt and dwk occupations. These jobs therefore yielded the greatest number of vacancies and percentage increase in real wage to workers as a result of the policy. The policy-induced diversion of legal job offers towards lesser skilled occupations reduces the number of workers employed in the three well-paid skilled occupations (lsm, prf, tch) by an average of around 2.6 per cent over the simulation period. This explains the greater than expected loss in wage bill weighted labour input referred to earlier.

Figure 7 summarises the impact of the policy on aggregate employment and real wages. The long-run deviation in illegal employment of -31.3 per cent is equivalent to the loss of 734,000 illegal jobs reported in Figure 2. Real wages for illegals rise quickly to 13 per cent above the baseline before steadying. The total number of legal jobs falls by close to 0.3 per cent, or 41,000 workers, with the average real wage increasing by 2.2 per cent. The combined loss of legal and illegal jobs therefore adds up to the 775,000 shown earlier in Figure 4.

The overall loss of legal jobs in Scenario 1, most notably in skilled occupations, is mainly due to the occupational-mix effect. This is traced to a shift in the occupational composition of legal employment towards lesser skilled oc-

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11 It is assumed that there are no illegal migrants employed in skilled jobs. That is, all workers in the three local skilled occupation groups (lsm, prf, tch) are considered to have legal status.

12 The theoretical specification of ZAR-M prevents the occupational-mix effect from impacting directly on incumbent skilled workers as they are not allowed to offer to any of the lower skilled occupations.
occupations in which there are higher equilibrium rates of unemployment. That is, the occupation-mix effect described before pushes legal workers towards occupations where relatively high rates of unemployment can be sustained with little wage pressure. This allows the deviation in aggregate employment of legal workers to be negative without producing an employment-increasing reduction in their average real wage rate.

As a corollary to the policy, it also has a negative impact on the overall welfare of legal residents measured via a combination of their private and public consumption expenditure. This measure is closely linked to the household disposable income of legal residents which falls by 2.0 per cent over the simulation period. Similar to many of the other results reported in this section, we find that this outcome closely mirrors that of the Dixon et al (2011) study on illegal immigration in the United States. The welfare loss using our ZAR-M model for South Africa can also be traced to a number of factors. These include i) the direct illegal labour effect, ii) legal employment effect, and iii) occupation-mix effect. The direct illegal labour effect occurs as a result of moving towards a smaller economy in the perturbed scenario. Assuming the wage rate of illegal migrants equal the value of their marginal product to employers, the direct loss to South African GDP can easily be approximated using a simple demand and supply diagram that reflects the shock to illegal labour supply in the policy run. The loss of welfare to legal residents can then be tied to the deadweight loss of producer surplus and the increased transfers from local employers to illegal migrants because of higher wage rates. Additional welfare loss linked to the direct illegal labour effect can be expected from reduced tax revenue on wages earned by illegals. The legal employment effect is tied to the loss of legal jobs as a result of the policy which imposes a direct welfare loss on legal residents.

Other factors detailed in Dixon et al (2011) play a small role in determining the net impact on welfare. These include iv) the capital effect, v) public expenditure effect, and vi) terms of trade price effect. The capital effect is tied to the reduction in capital stock as a result of the policy, and the share of local versus foreign-owned capital in South Africa. Taking into consideration the trade deficit and data on foreign-owned capital in South Africa (SARB, 2010), we expect the combined loss of capital income for legals and taxes collected from capital income to negatively affect the welfare of legal residents. The public expenditure effect is tied to the reduced amount of spending required by government on the number of illegals in the country. In ZAR-M we assume that public expenditure per illegal migrant is half the public expenditure per legal resident. With the 28.9 per cent reduction in the number of illegal migrants, government spending reaching illegals is also reduced. This amounts to a small welfare gain for legal residents. The terms of trade price effect also generates a minor welfare gain for legal residents. This effect is derived via the increase in the GDP price deflator relative to the consumer price index (CPI) and also the GNE price deflator as a result of the terms of trade improvement of 0.7 per cent. This effectively increases the consuming power of income earned by legal residents.

Despite small welfare gains attributable to the public expenditure and terms
of trade effect, the net impact on overall welfare of legal residents as a result of the policy remains negative. Analysis shows that the largest negative contributions to consumption by legal residents are the occupation-mix effect and the direct illegal labour effect.

It is worth pausing at this stage to review the modelling evidence presented to date under Scenario 1. Most results have ready explanations. However, the reduction in overall employment of legal workers may be interpreted as a counterintuitive outcome. In explaining this result, it is worth noting that this does not imply that large numbers of existing skilled workers would give up their jobs in high paid occupations and shift towards lower paid occupations. This effect mainly influences employment outcomes for future new entrants and those in unemployment. For each occupation, two different factors influencing legal employment must be considered. The first is an increase in job opportunities or vacancies for legal workers in those lower skilled jobs previously held by illegal workers. The second is a general reduction in opportunities for all legal workers as a result of moving towards a smaller economy.

As seen in Figure 6, the positive replacement effect for legals dominate in the low paid occupations that employ large numbers of illegal migrants in the baseline. The negative effect of having a smaller economy dominates in the higher paid occupations that employ very few illegal migrants. Higher equilibrium rates of unemployment in lower skilled occupations a direct consequence of my assumptions employed in Scenario 1, play an important role in this shift of occupational composition. The increase in vacancies in low paid, low skilled occupations relative to the high paid occupations allow these occupations to absorb an increased proportion of both new legal entrants and unemployed workers. In terms of equation (E5) described earlier, the policy has the second round impact of increasing $V_t(\text{act})$ in the lower skilled occupations relative to high skilled occupations. This results in more non-incumbents finding employment in these occupations.

It is often asserted that a reduction in illegal immigration would reduce unemployment rates of lower skilled local workers. This idea is counteracted by long-run evidence of the occupation-mix effect under Scenario 1. As pointed out in Dixon et al (2008), under such labour market conditions, it may even be true that with cuts in illegal immigration, lower skilled workers find themselves under increased pressure from high skilled workers who can no longer find vacancies in high skilled occupations.

Thus, the legal employment outcome is interpretable too – under Scenario 1. The question therefore becomes whether the assumptions implied by Scenario 1 is believable. A study of the local labour market quickly reveals the concept of equilibrium rates of unemployment for lower skilled South Africans to be inappropriate. My alternative set of assumptions in Scenario 2 attempts to address this issue.

Policy Results – Scenario 2

Equation (E3) shows that an increase in labour demand relative to supply would place upward pressure on wages over time. With steady-state conditions in the base year and unemployment present in the model, (E3) implies the
existence of equilibrium rates of unemployment. Workers who are unemployed in the base under Scenario 1 are therefore seen as unwilling to lower their asking wage in order to obtain employment. That is, they continue to present upward-sloping supply curves despite not being able to find a job at the prevailing wage rate.

The reality in South Africa is different though. Millions of workers are actively looking for jobs every week. Most are unsuccessful. With a minimum wage in place, unemployed workers are not legally allowed to reduce their wage to match their marginal productivities to employers. My alternative labour market assumptions underlying Scenario 2 directly account for these factors. By effectively ‘switching off’ the optimising behaviour of low skilled legals in (E3), increases in labour demand for these workers can be met with matching shifts of the labour supply curve without inducing any wage pressure. This is consistent with a situation in which an excess supply of labour exists at a given legal minimum wage level.

Figures 8 to 13 show the impacts of the policy-induced cut to illegal immigration under Scenario 2 assumptions. Since the policy shock has remained the same, it is not surprising that simulation results show a similar pattern to those generated under Scenario 1. It is immediately evident though that the local economy is better off under the conditions implied in Scenario 2. The divergence between the two sets of results is naturally traced to the change in assumptions between the two scenarios.

The policy shock reduces the supply of low skilled illegal labour via a direct change to their preferences in (E7). This reduces demand for these workers as a result of increased wage rates over time. During the adjustment period, a large number of vacancies in the jobs previously done by illegal workers become available. With illegal labour now relatively more expensive, employers shift some of their demand for low skilled labour to legal workers. It is at this stage where the two scenarios produce meaningfully different results. Under Scenario 1, increased demand for low skilled legals generated an increase in their real wage via (E3). The increase in demand for low skilled workers in later years was therefore choked off somewhat as a result. Along with increased competition from workers who previously might have pursued a skilled job, employment for legal workers only increased in four lower-skilled occupations. As shown in Figure 13, the increase in demand for legal workers produces almost no increase in their real wage under Scenario 2. This is because Scenario 2 allows the excess supply of low skilled legals to accommodate the increase in their demand as a result of the policy shock. With virtually no wage pressure, Figure 12 shows that demand for low skilled legal workers now increases in

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13 Not surprisingly, most evidence suggests that a legal minimum wage contribute to increased levels of unemployment, especially among low skilled workers (Neumark & Wascher, 2007).

14 As discussed in the policy results for Scenario 1, the occupations that employed the largest share of illegal workers, i.e. sag, crf, elt and dwk also opened up the largest amount of vacancies for legals leading to increased employment in these low skilled jobs for legals in the policy run relative to the baseline.
virtually all low skilled occupations relative to the baseline. This result at the lower end of the market, also contributes to an improved outcome for skilled workers in Scenario 2 via a reduction in the occupation-mix effect. Significantly, the policy now generates a healthy boost to overall legal employment. Whereas the number of legal jobs previously fell by around 0.3 per cent, the same policy shock now generates an increase of 2.0 per cent, or 313,000 legal jobs overall. This result drives the divergence in macro outcomes between Scenario 1 and Scenario 2.

Close inspection of Figure 11 shows that industry output results can largely be explained by deviations in closely linked macro variables.\textsuperscript{15} In the short run, investment-related industries such as construction are most severely affected. This reflects the adjustment of the economy to a lower capital stock which causes a sharp negative deviation in investment relative to GDP as shown in Figure 10. Trade-exposed industries do comparatively better in the short run as a result of the real devaluation associated with the weakening of the investment to GDP ratio. This includes tourism-related industries such as hotels and transport services, and import-competing industries such as agriculture and textiles.

In the long run, the simulation shows a real appreciation of the currency. Consequently, trade-exposed industries, including the manufacturing sector, start to deteriorate more rapidly in later years relative to the baseline. In the mining sector, coal and other mining do better than gold since all production by the gold industry is exported. Demand elasticities for exports are typically higher than for other final uses. Virtually all of the commodity gold is produced by the gold industry. Thus, gold faces a higher overall demand elasticity, despite having the same export elasticity as coal and other mining in its downward-sloping export demand curve. The increase in the foreign-currency export price of gold, driven by the appreciation of the local currency in the long run, leads to a decrease in demand for gold exports. Investment-related industries continue to show output deviations that are more negative than GDP as investment levels have not recovered sufficiently by the end of the simulation period. In general, output deviations for most industries are quite close to that of GDP in the long run. Gaps between individual industries and that of GDP strongly reflect changes in the long-run expenditure composition of GDP, taking into consideration the relevant demand and supply elasticities for different industries.

Figure 8 and 13 also show that illegal flows and employment levels are only slightly affected by the change in assumptions regarding the behaviour of low skilled legal workers. Illegal employment now falls by 32.1 per cent compared to 31.3 per cent before.

\textit{Policy Results – Additional Sensitivity Analysis}

ZAR-M features nested CES labour input demand equations. Analysis con-

\textsuperscript{15}Corresponding industry results for \textit{Scenario 1} are not shown in this paper. However, they can be explained by the same mechanisms described here for \textit{Scenario 2}. The poorer outcomes in terms of GDP and its macro components in \textit{Scenario 1} simply push down its industry level results relative to those in reported for \textit{Scenario 2}. The structure and pattern of the results remain virtually unchanged.
cerning my choice of substitution elasticities ($\sigma$) for labour demand by birthplace ($b$) and legal status ($s$) indicate both sets of results to be robust and not overly sensitive to realistic alternatives for these parameter values. Under Scenario 2, doubling $\sigma$ in the labour input demand equations only elevated the gain in overall legal employment from 2.0 to 2.3 per cent in 2020. Apart from a slight increase in the benefit to legal residents, no discernable change to the pattern and structure of results were evident.

Figure 14 illustrates this sensitivity analysis in terms of changes to employment and real wages for illegal migrants as a result of the policy shock. An increase in $\sigma$ flattens the demand curve (D$_1$ to D$_2$) for foreign-illegal workers, effectively allowing domestic-legal workers to become better substitutes. The greater reduction in illegal employment (L$_2$ to L$_3$) opens up even more vacancies for legal workers, whilst the reduced wage (RW$_2$ to RW$_3$) payable to illegal workers improves the welfare of local residents. It is also evident from Figure 14 that varying the key labour supply substitution elasticity ($\eta$), that is, the slope of the supply curve, would have very little impact on the simulation result given the nature of the policy shock described in this paper.

5 Concluding Remarks

This paper investigates the economic impacts of a policy-induced cut in illegal immigration to South Africa. The policy shock is designed to adversely affect the labour supply preferences of potential migrants for illegally moving to South Africa. The policy simulation is run under two different sets of assumptions. Scenario 1 incorporates the idea of equilibrium rates of unemployment. Scenario 2 abandons this optimising behaviour implied by equation (E3) for low skilled legal workers and allows increases in their demand to be met without much wage pressure.

Simulation results show that under Scenario 1, legal residents are worse off in terms of employment and welfare as a result of the cut in illegal immigration. The pattern of results closely mirror those produced in Dixon et al (2008) under similar labour market assumptions. Alternative assumptions introduced in Scenario 2 attempts to create a more realistic modelling environment for South Africa. Results under Scenario 2 show a much more positive impact on legal residents. Despite minor occupational-mix effects, overall employment for legal residents increase, with large gains for lower skilled workers. There are also short-term gains in private consumption, and a long-term increase in public consumption, both indicative of increases in overall welfare of legal residents.

The policy simulations conducted in this paper do not take any cost factors into consideration due to the large degree of uncertainty regarding the cost of implementation. For policies that focus solely on restricting inflows via supply side mechanisms, these implementation costs may be extensive. Combining such efforts in a cost-effective manner with policies that would reduce demand for illegal migrants may be more advantageous in terms of economic welfare to legal
residents. These alternative simulation scenarios will be investigated in future research.

Finally, I do not attempt to make any moral judgment in this paper on distressed workers who seek employment opportunities illegally. Workers who have legitimate cause for asylum are also not considered here. This paper merely aims to inform policy discussions on the economic consequences of reducing the inflow of illegal migrants to South Africa via supply side policies.

References


\[16\] Dixon et al (2011) suggests that implementing a demand-reducing tax instead of various supply-restricting actions would yield greater benefit to legal residents.


FIGURE 1  Labour Flows from Start-of-Year Categories to End-of-Year Activities

<table>
<thead>
<tr>
<th>Categories</th>
<th>Skilled local employment</th>
<th>Lower skilled local employment</th>
<th>Short-run local unemployment</th>
<th>Long-run local unemployment</th>
<th>Skilled migrant activity</th>
<th>Lower skilled migrant activity</th>
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<tr>
<td>Skilled local employment</td>
<td>1</td>
<td>zero</td>
<td>2</td>
<td>zero</td>
<td>5</td>
<td>zero</td>
</tr>
<tr>
<td>Lower skilled local</td>
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<td>1</td>
<td>2</td>
<td>zero</td>
<td>zero</td>
<td>5</td>
</tr>
<tr>
<td>unemployment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short-run local</td>
<td>1</td>
<td>1</td>
<td>zero</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>unemployment</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-run local</td>
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<td>1</td>
<td>zero</td>
<td>3</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>unemployment</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Skilled migrant</td>
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<td>zero</td>
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<td>market</td>
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</tbody>
</table>

FIGURE 2  Employment of Illegal Workers (Scenario 1)
FIGURE 12   Employment of Legal Workers (Scenario 2) (Percentage Deviation)

FIGURE 13   Employment and Real Wages (Scenario 2) (Percentage Deviation)
FIGURE 14  Sensitivity Analysis

Real Wage

RW

RW

RW

D

L

L

L

Labour

S

S

D

(high \sigma)

25

25

25
APPENDIX  Labour Market Equations in ZAR-M

Number of people in each category at the beginning of year $t$

\[
\text{CAT}_t (b,s,f) = \text{ACT}_{t-1} (b,s,f) \times 0.99 \quad \text{for all } b,s \text{ and } f \neq \text{new}
\]

\[
\text{CAT}_t (b,s, 'new') = \text{exogenous} \quad \text{for all } b,s
\]

Labour supply by people from categories to activities

\[
L_i (cat;act) = \text{CAT}_t (cat) \times \psi_i \left[ \text{PREF}_i (cat;act), \text{ATRW}_i (act) \right]
\]

\[
L_i (act) = \sum_{cat} L_i (cat;act) \quad \text{for all categories } cat \text{ & activities } act
\]

Labour demand and employment

\[
D_i (j) = Z_i (j) \times \psi_o \left[ BTRW_i (j), K_i (j), A_i (j) \right] \quad \text{for all local industries } j
\]

\[
BTRW_i (j) = \psi_w \left[ BTRW_i (b,s,o) \land b,s \text{ and local occupations } o \right] \quad \text{for all local industries } j
\]

\[
D_i (b,s,o,j) = D_i (j) \times \psi_o \left[ BTRW_i (bb,ss,o) \land bb,ss \text{ and local occupations } oo \right]
\]

\[
D_i (b,s,o) = \sum_j D_i (b,s,o,j) \quad \text{for all } b,s, \text{ local occupations } o
\]

\[
E_i (b,s,o) = D_i (b,s,o) \quad \text{for all } b,s, \text{ local occupations } o
\]

Relationship between after-tax and before-tax real wages

\[
\text{ATRW}_i (b,s,o) = BTRW_i (b,s,o) \times \left[ 1 - T_l (b,s) \right] \quad \text{for all } b,s, \text{ local occupations } o
\]

\[
\text{ATRW}_i (b,s,u) = BTRW_i^{\text{base}} (b,s) \times F_i (b,s) \quad \text{for all } b,s, \text{ unemployment } u
\]

Wage adjustment mechanism

\[
\frac{\text{ATRW}_i (b,s,o)}{\text{ATRW}_{i-1}^{\text{base}} (b,s,o)} - \frac{\text{ATRW}_{i-1} (b,s,o)}{\text{ATRW}_{i-1}^{\text{base}} (b,s,o)} = \beta(s,o) \left( \frac{D_i (b,s,o) - L_i (b,s,o)}{D_i^{\text{base}} (b,s,o) - L_i^{\text{base}} (b,s,o)} \right)
\]

\[
\text{for all } b,s, \text{ local occupations } o
\]

Vacancies and movement into employment activities

(area 1 in Figure 1)

\[
V_i (act) = E_i (act) - H_i (act;act) \quad \text{for all local employment activities } act
\]

\[
H_i (cat;act) = V_i (act) \times \frac{L_i (cat;act)}{\sum_{v=act} L_i (v;act)} \quad \text{for all categories } cat \neq \text{activities } act \&\text{ for all local employment activities } act
\]
\[ H_c \left( \text{cat}; \text{cat} \right) = \text{CAT}_c \left( \text{cat} \right) - \sum_{\text{act}=\text{cat}} H_c \left( \text{cat}; \text{act} \right) \quad \text{for all employment categories cat} \]

**APPENDIX**  
**Labour Market Equations in ZAR-M (continued)**

Movements into unemployment activities and migrant destinations  
(area 2 in Figure 1)

\[ H_c \left( \text{cat}; u \right) = \begin{cases} L \left( \text{cat}; u \right) + \left[ \mu \left( \text{cat} \right) \right] \text{CAT}_c \left( \text{cat} \right) & \text{for } S \text{ unemployment activity } u \\ 0 & \text{for } L \text{ unemployment activity } u \end{cases} \]

for all local employment categories cat

(area 3 in Figure 1)

\[ H_c \left( \text{cat}; u \right) = \begin{cases} 0 & \text{for } S \text{ unemployment activity } u \\ \text{CAT}_c \left( \text{cat} \right) - \sum_{\text{act}=\text{cat}} H_c \left( \text{cat}; \text{act} \right) & \text{for } L \text{ unemployment activity } u \end{cases} \]

for all local unemployment cat & all unemployment activities u

(area 4 in Figure 1)

\[ H_c \left( \text{cat}; u \right) = \begin{cases} \text{CAT}_c \left( \text{cat} \right) - \sum_{\text{act}=\text{cat}} H_c \left( \text{cat}; \text{act} \right) & \text{for cat dom,leg & } S \text{ unemp act } u \\ \text{CAT}_c \left( \text{cat} \right) - \sum_{\text{act}=\text{cat}} H_c \left( \text{cat}; \text{act} \right) & \text{for cat fgn,leg & aus migrant act } u \\ \text{CAT}_c \left( \text{cat} \right) - \sum_{\text{act}=\text{cat}} H_c \left( \text{cat}; \text{act} \right) & \text{for cat fgn,ill & zim migrant act } u \\ 0 & \text{otherwise} \end{cases} \]

for all new cat & all unemployment or migrant activities u

(area 5 in Figure 1)

\[ H_c \left( \text{cat}; u \right) = \begin{cases} L \left( \text{cat}; u \right) & \text{for all local cat} \\ \text{CAT}_c \left( \text{cat} \right) - \sum_{\text{act}=\text{cat}} H_c \left( \text{cat}; \text{act} \right) & \text{for all foreign cat} \end{cases} \]

for all non-new cat & migrant activities u

Completing the link from categories to activities

\[ \sum_{\text{cat}} H_c \left( \text{cat}; \text{act} \right) = E_c \left( \text{act} \right) \quad \text{for all local unemployment & migrant activities act} \]

**APPENDIX**  
**Notation in the Exposition of the Labour Market**

\[ \text{CAT}_t \left( b, s, f \right) \quad \text{Number of people at the start of year } t \text{ who are from birthplace } b \text{, have status } s \text{, and who performed labour force function } f \text{ in year } t-1 \]

\[ \text{ACT}_{-1} \left( b, s, f \right) \quad \text{Number of people in labour force activity } \left( b, s, f \right) \text{ during year } t-1 \]
\( L_t(cat;act) \) Labour supply of people in category \( cat \) to activity \( act \) during year \( t \) with both \( cat \) and \( act \) described by the dimension \((b,s,f)\)

\( \text{PREF}_t(cat;act) \) Variable reflecting the labour supply preferences of people in category \( cat \) for working in activity \( act \) during year \( t \)

\( D_t(b,s,o,j) \) Demand for labour inputs by industry \( j \) for employment activity \( o \) with \((b,s)\) characteristics

\( D_t(j) \) Total labour input to industry \( j \)

\( \psi \) Homothetic function

\( Z_t(j) \) Activity of industry \( j \)

\( K_t(j) \) Capital stock of industry \( j \)

\( A_t(j) \) Other variables that influence demand for labour in industry \( j \)

\( BTRW_t(j) \) Overall before-tax real wage to industries

\( BTRW_t(b,s,o) \) Before-tax real wage of workers in employment activity \( o \) with \((b,s)\) characteristics

\( ATRW_t(b,s,o) \) After-tax real wage of workers in employment activity \( o \) with \((b,s)\) characteristics

\( ATRW_t(b,s,u) \) After-tax real wage received by labour in unemployment activity \( u \) representing some form of social security payment

\( \beta(s,o) \) Positive parameter that controls the response of wage rates to gaps between labour demand and supply

\( TL_t(b,s) \) Labour tax rate applying to all \((b,s)\) workers in employment activity \( o \)

\( F_t(b,s) \) Fraction of \( BTRW_t^\text{ave}(b,s) \) received in unemployment activity \( u \)

\( E_t(act) \) Total employment in activity \( act \)

\( V_t(act) \) Vacancies in employment activity \( act \)

\( H_t(cat;act) \) Actual flow of people from start-of-year category \( cat \) to activity \( act \) during year \( t \)

\( \mu \) Fraction of people in employment category \( cat \) at the start of year \( t \) who become involuntarily unemployed during year \( t \)