

Mathematics Winter School worksheet

# FUNCTIONS, GRAPHS AND INVERSES



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### QUESTION 1

Consider the function  $f(x) = 4^{-x} - 2$ .

- 1.1 Determine the coordinates of the intercepts of  $f$  with the axes. (3)
- 1.2 Sketch the graph of  $f$  showing the intercepts with the axes and the horizontal asymptote. (3)
- 1.3 Write down the equation of  $g$  if  $g$  is the graph of  $f$  shifted 3 units right and 2 units up. (2)
- [8]**

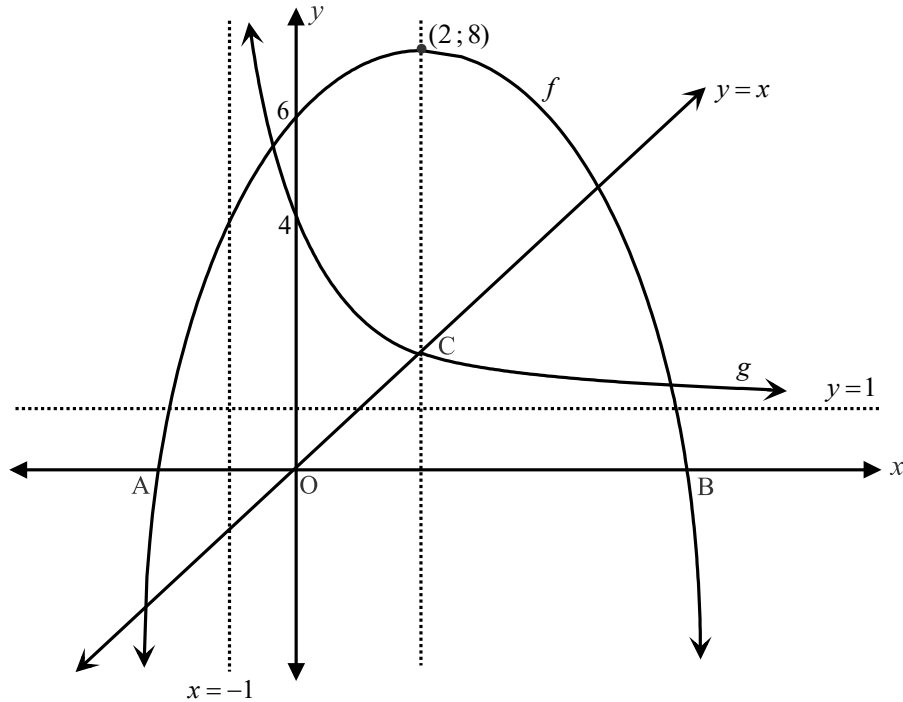
### QUESTION 2

Given:  $f(x) = \log_p x$  and  $g(x) = -3x + 6$

- 2.1 Show that  $p = 3$  if the graph of  $f$  passes through the point  $(9; 2)$ . (2)
- 2.2 Sketch the graphs of  $y = f^{-1}(x)$  and  $y = g(x)$  on the same set of axes. Indicate the intercepts with the axes and the point of intersection of both graphs. (4)
- 2.3 Determine graphically the solution of the equation:  $\frac{1}{3}f^{-1}(x) = 2 - x$  (2)
- 2.4 Write down the domain of  $f$ . (1)
- [9]**

**QUESTION 3**

Given:  $f(x) = ax^2 + bx + c$  and  $g(x) = \frac{3}{x+p} + q$  for all  $x > -1$ . The graph of  $f$  cuts the  $x$ -axis at A and B and the  $y$ -axis at 6. It has its turning point at (2; 8). The graph of  $g$  cuts the  $y$ -axis at 4 and intersects the line  $y = x$  at C. The line  $y = 1$  is an asymptote of  $g$ .



- 3.1 Show that  $a = -\frac{1}{2}$ ,  $b = 2$  and  $c = 6$  by determining the equation of  $f$ . (3)
- 3.2 Determine the length of AB. (4)
- 3.3 Write down the equation of  $g$ . (1)
- 3.4 Determine the coordinates of C. (5)
- 3.5 Determine graphically the values of  $x$  for which
- 3.5.1  $g(x) - x < 0$  (2)
- 3.5.2  $f'(x) \cdot g(x) < 0$  (1)
- 3.6 For which values of  $k$  will the equation  $-\frac{1}{2}x^2 + 2x = k - 6$  have equal roots. (2)

**[18]**

**QUESTION 4**

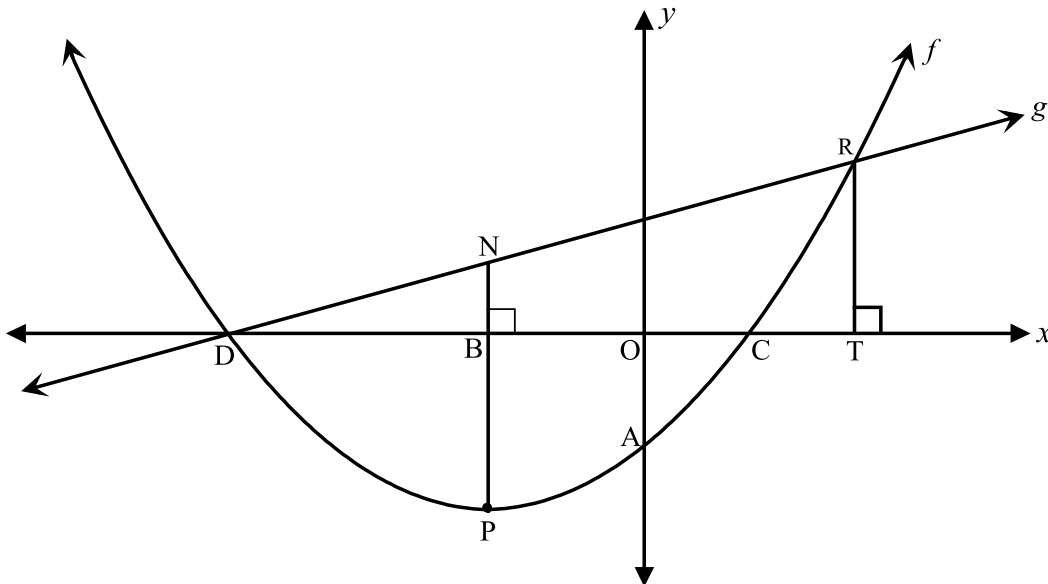
Given:  $f(x) = \frac{2}{x-1} + 2$  and  $g(x) = -2^{x+1} + 4$ .

- 4.1 Write down the asymptotes of the graph of  $f$ . (2)
- 4.2 Determine the intercepts with the axes for the graph of  $f$ . (1)
- 4.3 Determine the intercepts with the axes for the graph of  $f$ . (4)
- 4.4 Sketch the graphs of  $f$  and  $g$  on the same set of axes. (6)

[13]

### QUESTION 5

In the diagram below are the graphs of  $f(x) = ax^2 + bx + c$  and  $g(x) = x + 4$ .  
 P is the turning point of  $f$  and R is the point of intersection of  $f$  and  $g$ .

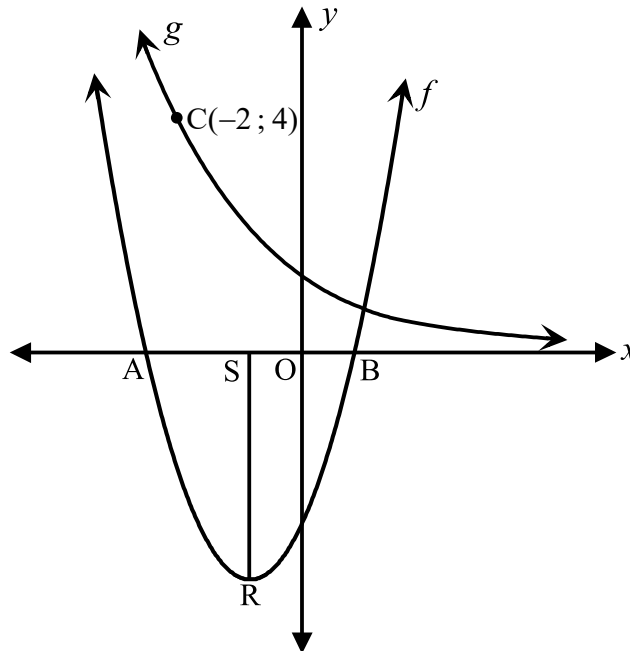


- 5.1 If  $OA = 4$  units,  $OB = 1\frac{1}{2}$  units and  $OC = 1$  unit, determine:
- 5.1.1 the coordinates of D. (1)
- 5.1.2 the value of  $a$ ,  $b$  and  $c$ . (4)
- 5.1.3 the coordinates of R. (5)
- 5.1.4 the length of NP. (4)
- 5.2 Determine graphically the values of  $x$  for which  $f(x) \cdot g(x) > 0$ . (1)
- 5.3 Determine the values of  $k$  for which  $f(x) + k = 0$  has two negative, unequal roots. (2)

[17]

### QUESTION 6

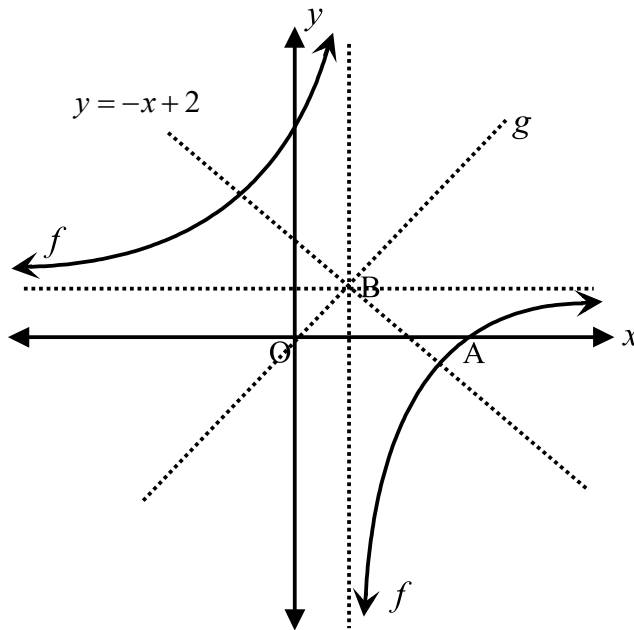
The graphs of  $f(x) = 2x^2 + 4x - 6$  and  $g(x) = a^x$  are represented in the sketch below. A and B are the  $x$ -intercepts of  $f$  and R is the turning point of  $f$ . The point  $C(-2; 4)$  is a point on the graph of  $g$ .



- 6.1 Show that  $a = \frac{1}{2}$ . (2)
- 6.2 Determine the length of AB. (3)
- 6.3 Determine the length of SR. (5)
- 6.4 Write down the equation of  $h$ , if  $h$  is the reflection of  $f$  in the  $y$ -axis. Express your answer in the form  $h(x) = a(x + p)^2 + q$ . (2)
- 6.5 Write down the equation of  $g^{-1}$  in the form  $y = \dots$  (2)
- 6.6 Sketch the graph of  $y = g(x)^{-1}$  on a set of axes. (2)
- 6.7 Determine the values of  $x$  for which:
- 6.7.1  $g(x)^{-1} \geq -2$  (3)
- 6.7.2  $x \cdot f'(x) < 0$  (2)
- [21]**

**QUESTION 7**

The diagram below shows the hyperbola defined by  $f(x) = \frac{-4}{x+p} + 1$ . The lines  $g$  and  $y = -x + 2$  are axes of symmetry of  $f$  and intersect at  $B$ , the point of intersection of the asymptotes.



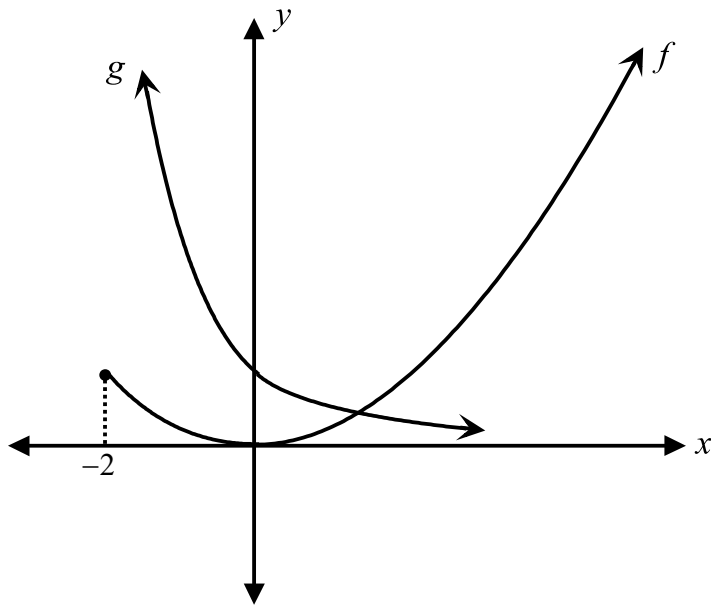
- 7.1 Determine the equation of  $f$  and hence the value of  $p$ . (4)
- 7.2 Determine the equation of  $g$ , the other axis of symmetry of  $f$ . (2)
- 7.3 Write down the domain of  $f$ . (2)
- 7.4 Suppose that the graph of  $f$  is shifted left so that  $A$  coincides with the origin. Determine the equation of the vertical asymptote of the newly formed graph. (3)
- [11]**

**QUESTION 8**

- 8.1 The graph of a parabola  $f$  has  $x$ -intercepts at  $x = 1$  and  $x = 5$ .  
The line  $g(x) = 4$  is a tangent to  $f$  at  $P$ , the turning point of  $f$ .  
Sketch the graph of  $f$ , clearly showing the intercepts with the axes and the coordinates of the turning point. (5)

8.2 The diagram below shows the graph of the following functions:

$$f(x) = \frac{1}{4}x^2 \text{ where } x \geq -2 \text{ and } g(x) = \left(\frac{1}{3}\right)^x.$$



8.2.1 Determine the inverse of  $f$  in the form  $y = \dots$  (2)

8.2.2 Sketch the graph of  $f^{-1}$ . (2)

8.2.3 Write down the range of  $f^{-1}$ . (2)

8.2.4 Determine the inverse of  $g$  in the form  $y = \dots$  (2)

8.2.5 For which values of  $x$  will  $g^{-1} \geq -1$ ? (3)

8.3 Given:  $f(x) = \frac{2}{x+1} + 1$

8.3.1 Find the equation of the horizontal asymptote of the graph of  $g$  if  $g(x) = f(x) - 2$  (2)

8.3.2 Find the equation of the vertical asymptote of the graph of  $h$  if  $h(x) = f(x - 2)$  (2)

**[20]**

### QUESTION 9

Given:  $f(x) = \log_a x$  passes through the point (9;2).

- 9.1 Show that  $a = 3$ . (2)
- 9.2 Determine the equation of  $f^{-1}(x)$  in the form  $y = \dots$  (2)
- 9.3 Show that the point (3;1) lies both on  $f(x)$  and  $g(x) = \frac{3}{x}$  (2)
- 9.4 Sketch  $f(x)$ ,  $g(x)$  and  $f^{-1}(x)$  on the same set of axes. (4)
- [10]**

### QUESTION 10

Given that  $f(x) = a^x$  which passes through the point  $\left(2; \frac{1}{4}\right)$

- 10.1 Prove that  $a = \frac{1}{2}$ . (2)
- 10.2 Determine the equation of  $y = f^{-1}(x)$  in the form  $y = \dots$  (2)
- 10.3 Determine the equation of  $y = h(x)$  where  $h(x)$  is a reflection of  $f(x)$  in  $x$  -axis. (2)
- 10.4 Sketch the graphs of  $f$ ,  $f^{-1}$  and  $h$  on the same set of axes, showing all intercepts with the axes. (4)
- [10]**