Mathematics Winter School worksheet

EUCLIDEAN GEOMETRY - MEMO

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Junior Tukkie

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QUESTION 1

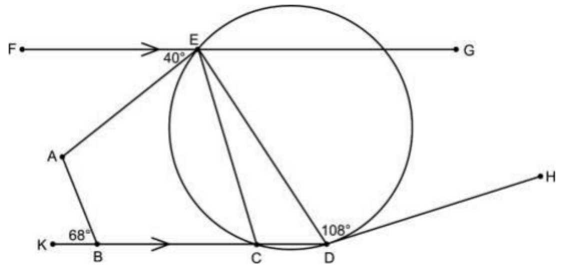
1.1	$\hat{P}_1+\hat{P}_2 = 90°$ ∠ in semi ⊙ $\hat{S} = 60°$ ∠s of Δ	✓ S✓R ✓S
1.1.2	$\hat{T}_4 = 85°$ vertically opposite ∠s $\hat{P}_1 = 35°$ ∠s of Δ $\hat{R}_3 = 35°$ ∠s in same segment	✓S ✓S ✓S✓R
1.1.3	$\hat{O}_1 = 70°$ ∠at centre … $\hat{N}_1 = 25°$ ∠s of Δ	✓S✓R ✓S✓R
1.1.4	$\hat{P}_2 = 55°$ $\angle s\ of\ \Delta$ $\hat{R}_4 = \hat{P}_2 = 55°$ tan-chord theorem	✓S ✓S✓R
1.2	$\hat{N}_1 \neq \hat{R}_3$ ∴ NT is not a tangent to that circle	✓S ($25° \neq 35°$) ✓Justification

QUESTION 2

2.1	$\hat{A}_1 = \hat{C}_2$ tan − chord $\hat{A}_1 = Z$ tan − chord $\hat{C}_2 = Z$ both $= \hat{A}_1$ $\therefore BC//RZ$ corr angles	✓S✓R ✓S/R ✓R
2.2	$\hat{Z} = \hat{P}$ ∠s in same segment $\hat{Z} = \hat{C}_2$ corr ∠s ∴ BC is a tangent converes tan − chord	✓S/R ✓S ✓R
2.3	$\hat{B}_1 = \hat{D}_2$ *ext* ∠ of a cyclic quad $\hat{R} = \hat{B}_1$ corr ∠s $BC//RZ$ $\hat{R} = \hat{D}_2$ $\hat{Z} = \hat{P}$ ∠s in same segment $\hat{A}_2 = \hat{C}_4$ 3rd ∠ $\Delta RZA///\Delta DPC$ AAA	✓S✓R ✓S/R ✓S/R ✓R
2.4	$\frac{ZA}{PC} = \frac{RA}{DC}$ *similar* Δ*s* $RA = \frac{PC \times ZA}{PC} \ldots (1)$ $\frac{ZA}{CA} = \frac{RA}{BA}$ $RA = \frac{BA \times ZA}{AC} \ldots (2)$ $\frac{\frac{PC \times ZA}{PC}}{} = \frac{BA \times ZA}{AC}$ $\frac{DC}{CP} \times \frac{AC}{AB} = 1$	✓S/R ✓$RA = \frac{PC \times ZA}{PC}$ ✓S/R ✓$RA = \frac{BA \times ZA}{AC}$ ✓simplification

QUESTION 3 3

		✓Construction

	 Construction $B\hat{A}E$=40°+68°=108° $alt\ \angle s,$ $E\hat{C}D$=108° $tan\text{–}chord$ $B\hat{A}E=E\hat{C}D$ $\therefore AECD\ is\ a\ cycli\ quad$ $converse{:}ext\ \angle{=}to\ interior\ opp\ \angle$	✓$B\hat{}AE$=40°+68°=108° ✓$alt\ \angle s,$ ✓$B\hat{}AE=E\hat{}CD$ ✓Reason

QUESTION 4

4.1.1	$B\widehat{A}E = 90°$ $\angle$ in semi circle	✓ S ✓R
4.1.2	$\widehat{E}_1 = 80°$ opp angles of cyclic quad	✓ S ✓R
4.1.3	$\widehat{D}_1 = 45°$ ext$\angle$ of Δ FED	✓ S ✓R
4.2	$\widehat{B}_1 = 35°$ Interior $\angle$ of Δ $\hat{F} = 35°$ given AB // CF alternate angle	✓ S ✓R ✓ S ✓R

QUESTION 5

5.1	$\widehat{R}_2 = \widehat{M}_2$ $alt\ \angle$s,, RE //TM $\widehat{R}_1 = \widehat{T}$ $corr\ \angle$s, RE //TM	✓ S ✓R ✓ S ✓R
5.2	$\frac{EM}{EG} = \frac{RT}{RG}$ line // one side of Δ $RT = RM$ $\angle$s opp = sides $\frac{EM}{EG} = \frac{RM}{RG}$	✓ S ✓R ✓ S ✓R
5.3	In ΔGYE: $\widehat{G} = \widehat{G}$ common angle $\widehat{E}_1 = \widehat{R}_1$ given $\widehat{Y}_1 = \hat{E}_1 + \widehat{E}_2$ 3rd angle $\Delta GYE///\Delta GER$ AAA	✓ S /R ✓ S /R ✓ S /R ✓R
5.4	$\frac{GY}{GE} = \frac{YE}{ER} = \frac{GE}{GR}$ equiangular Δs	✓R
5.5.1	$(RG)^2 = (GM)^2 - (RM)^2$ $pythag$ $= 100 - 36$ $RG = 8$	✓Sub into pythag ✓answer
5.5.2	$\frac{GE}{GM} = \frac{GR}{GT}$ line // one side of Δ	✓R

	$\frac{GE}{10} = \frac{8}{14}$ $GE = \frac{40}{7}$	✓GT=14 ✓substitution ✓answer

QUESTION 6

6.1	$\hat{B} = x - 25°$ tan − chord $\hat{A} = \hat{B} = x - 25°$ ∠s opp = sides $In\ \Delta ABD$: $2(x - 25°) + 3x = 180°$ int ∠s of a Δ $5x = 230°$ $x = 46°$	✓ S ✓R ✓ S /R ✓ S ✓R ✓answer
6.2	Join CD and Draw $C\hat{F}D$ $\hat{A}_1 = \hat{C}_1 + \hat{D}_1$ ext ∠ of a Δ $\hat{C}_1 = \hat{D}_1$ ∠s opp = sides $\therefore \hat{A}_1 = 2\hat{C}_1$ $\hat{C}_1 = \hat{F}_1$ tan chord $C\hat{B}D = 2\hat{F}$ ∠at centre ... $= 2\hat{C}$ $\hat{A}_1 = C\hat{B}D$	✓Construction ✓ S ✓R ✓$\hat{C}_1 = \hat{D}_1$ ✓$\hat{C}_1 = \hat{F}_1$

[11]

QUESTION 7

7.1	$\hat{O}_1 = \hat{F}_2 + \hat{R}_1$ ext ∠ of a Δ $\hat{O}_1 = 100°$	✓ S ✓R
7.2	$\hat{H}_1 = 50°$ ∠ at centre 2 × ...	✓ S ✓R
7.3	$\hat{T} = 130°$ opp ∠s of a cyclic quad	✓ S ✓R
7.4	$\hat{H}_2 + \hat{H}_1 = \hat{F}_2$ corr ∠s ; $AF//EH$ $\hat{H}_2 = 78° - 50°$ $\hat{H}_2 = 28°$	✓ S ✓R ✓ answer

QUESTION 8

8.1	In $\Delta\ PQR$ and ΔPSQ: $\hat{P} = \hat{P}$ common angle $\hat{Q} = 90°$ given $= \hat{S}_2$ $QS \perp PR$ $\Delta\ PQR///\Delta PSQ$ AAA	✓ S/R ✓ S/R ✓ S ✓R
8.2	$\frac{PQ}{PS} = \frac{PR}{PQ}$ $\Delta\ PQR///\Delta PSQ$ $PQ^2 = PS.PR$	✓ S/R
8.3	$\Delta\ PQR///\Delta QSR$ AAA	✓ S/R ✓ S

	$\frac{QR}{SR}=\frac{PR}{QR}$ $QR^2=SR.PR$	$QR^2=SR.PR$
8.4	$PQ^2+QR^2=PS.PR+SR.PR$ $=PR(PS+SR)$ $=PR.PR$ $=PR^2$	✓substitution ✓factorising ✓answer

QUESTION 9

9.1	Tan-chord	
9.2	In ΔABC and ΔADB: $\hat{A}_1=\hat{A}_1$ common angle $\hat{B}_1=\hat{D}_1$ proven in 9.1 $\hat{C}=\hat{D}$ 3rd angle $\Delta ABC///\Delta ADB$ AAA	✓ S/R ✓ S/R ✓ S/R ✓ R
9.3	$\hat{E}_2=\hat{F}_1$ $alt\ \angle s\ \ EA//GF$ $\hat{F}_1=\hat{D}_2$ ext ∠ of a cyclic quad $\hat{E}_2=\hat{D}_2$	✓ S ✓R ✓ S ✓R
9.4	In ΔAEC and ΔADE: $\hat{A}_2=\hat{A}_2$ common angle $\hat{E}_2=\hat{D}_2$ proven in 9.3 $\hat{C}=\hat{E}$ 3rd angle $\Delta AEC///\Delta ADE$ AAA $\frac{AE}{AD}=\frac{AC}{AE}$ $\Delta AEC///\Delta ADE$ $AE^2=AC.AD$	✓ S/R ✓ S/R ✓ S/R ✓ R ✓ S
9.5	$\frac{AB}{AD}=\frac{AC}{AB}$ $\Delta ABC///\Delta ADB$ $AB^2=AC.AD$ AE^2 from 9.4 $AB=AE$	✓ S ✓ S ✓ S