

Mathematics Winter School worksheet

# EUCLIDEAN GEOMETRY



UNIVERSITEIT VAN PRETORIA  
UNIVERSITY OF PRETORIA  
YUNIBESITHI YA PRETORIA



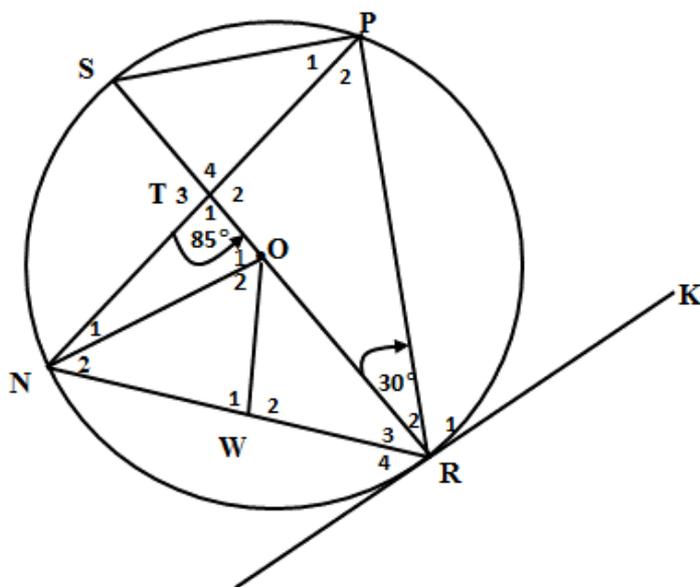
*Supported by:*

*Out of the Ordinary®*



**QUESTION 1 LIMPOPO 2016 PRELIM**

The vertices of  $\triangle PNR$  lie on the circumference of the circle O. Diameter SR and chord NP intersect at T.  $\hat{R}_2 = 30^\circ$  and  $\hat{T}_1 = 85^\circ$ . RK is a tangent to the circle at R.

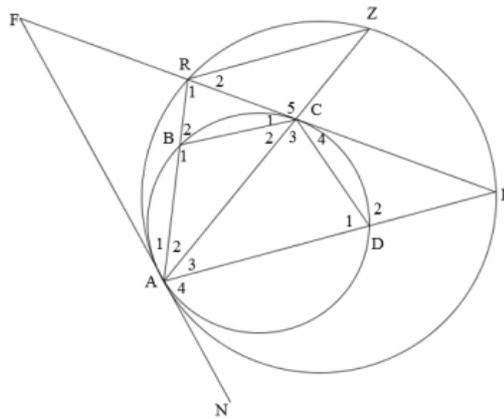


- 1.1 Determine, stating reasons, the size of:
- 1.1.1  $\hat{S}$  (3)
  - 1.1.2  $\hat{R}_3$  (4)
  - 1.1.3  $\hat{N}_1$  (4)
  - 1.1.4  $\hat{R}_4$  (3)
- 1.2 Determine whether NT is a tangent to the circle through N, O and R. Justify your answer. (2)

**[16]**

**QUESTION 2 EC 2016 PRELIM**

FAN is a common tangent to the smaller circle ABCD and the larger circle ARZP. FP is a tangent to the smaller circle at C. The straight line ABR meets the larger circle at R.



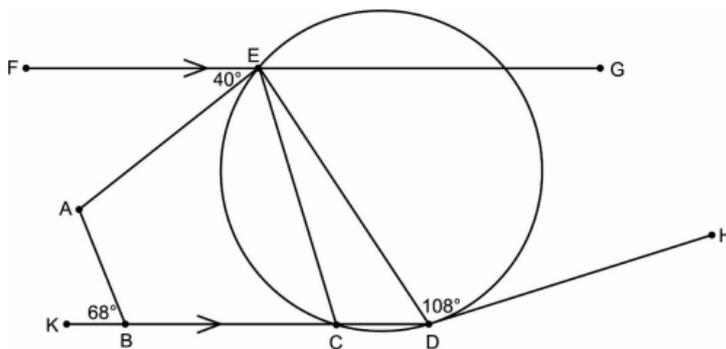
- 2.1 Prove that  $BC \parallel RZ$ . (4)
- 2.2 Hence, prove that BC is a tangent to circle ACP. (3)
- 2.3 Prove that  $\triangle RZA \parallel \triangle DPC$ . (5)
- 2.4 Hence, show that  $\frac{DC}{CP} \times \frac{AC}{AB} = 1$ . (5)

[17]

**QUESTION 3 IEB 2019 Supplementary**

In the diagram below, E, C and D are points on the circle.

- DH is a tangent to the circle at the point D with  $\widehat{HDE} = 108^\circ$
- DC is produced and  $DCBK \parallel GEF$
- A is a point between the parallel lines with  $\widehat{FEA} = 40^\circ$  and  $\widehat{ABK} = 68^\circ$

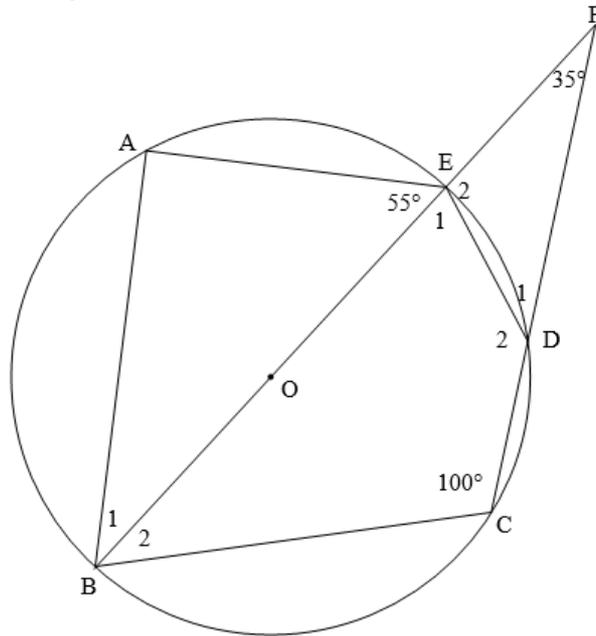


Prove that AECD is a cyclic quadrilateral.

(6)

**QUESTION 4 SEP 2016 WC**

In the diagram, O is the centre of the circle. A, B, C, D and E are points on the circumference of the circle. Chords BE and CD produced meet at F.  $\hat{C} = 100^\circ$ ,  $\hat{F} = 35^\circ$  and  $\hat{AEB} = 55^\circ$ .



4.1 Calculate, giving reasons, the size of each of the following angles:

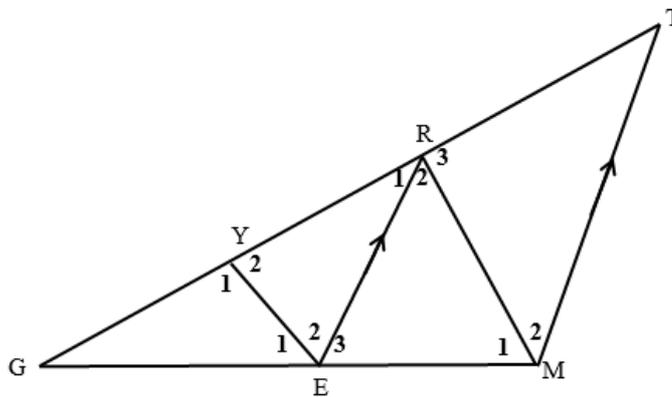
- 4.1.1  $\hat{A}$  (2)
- 4.1.2  $\hat{E}_1$  (3)
- 4.1.3  $\hat{D}_1$  (2)

4.2 Prove, giving reasons,  $AB \parallel CF$

(3)  
[10]

**QUESTION 5 GAU 2015 PRELIM**

In  $\triangle RGM$ ,  $\hat{R}_1 = \hat{R}_2 = \hat{E}_1$ . T lies on GR produced  $TM \parallel RE$ .



5.1 Give with reasons, two other angles which are equal to  $\hat{R}_1$ . (4)

5.2 Prove that  $\frac{EM}{EG} = \frac{RM}{RG}$ . (4)

5.3 Prove that  $\triangle GYE \parallel \triangle GER$  (4)

5.4 Hence, prove that  $\frac{EG}{EY} = \frac{RG}{RE}$  (1)

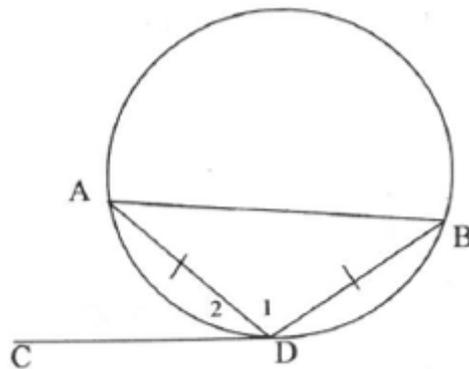
5.5 It is further given that  $\widehat{GRM} = 90^\circ$ ,  $RM = 6$  and  $GM = 10$ .  
Calculate the length of:

5.5.1  $RG$  (2)

5.5.2  $GE$  (4)

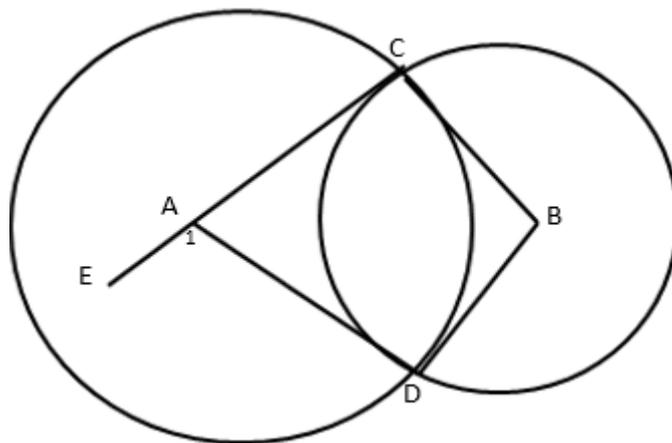
**QUESTION 6 ASHTON INTERNATIONAL 2017**

In the diagram below,  $\widehat{ADB} = 3x$ ,  $\widehat{ADC} = x - 25^\circ$  and  $AD = BD$ .



6.1 Determine, with reasons, the numerical value(s) of  $x$  for which  $CD$  is a tangent to the circle. (5)

6.2 Two circles, centres  $A$  and  $B$  intersect at  $C$  and  $D$ .  $CA$  is produced to point  $E$ .  $AC$  and  $AD$  are tangents to the smaller circle at  $C$  and  $D$  respectively.



Prove that  $\widehat{A_1} = \widehat{CBD}$  (5)

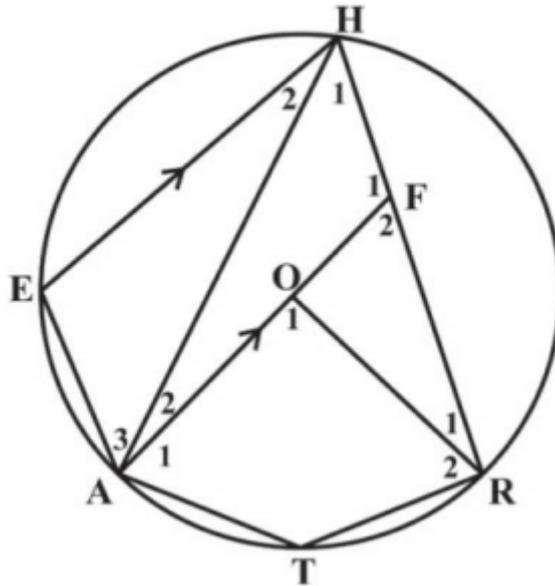
(Hint: Construct lines to complete the diagram. With the addition of these lines, you will be able to make use of the facts that:

- $EC$  and  $AD$  are tangents.
- $B$  is the centre of the smaller circle.

**QUESTION 7 St DAVIDS INANDA 2017**

In the diagram O is the centre of the circle HEATER. AOF is parallel to EH.

$\hat{F}_2 = 78^\circ$  and  $\hat{R}_1 = 22^\circ$ .

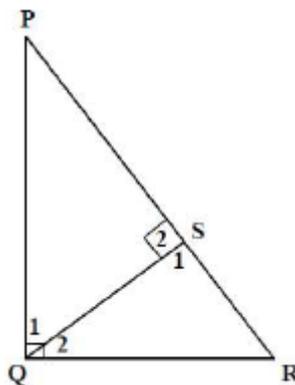


Calculate, with reasons, the sizes of:

- 7.1  $\hat{O}_1$  (2)
- 7.2  $\hat{H}_1$  (2)
- 7.3  $\hat{T}$  (2)
- 7.4  $\hat{H}_2$  (3)

**QUESTION 8**

In the diagram below,  $\Delta PQR$  is a right angled and  $QS \perp PR$ . Prove geometrical reasons for all statements when answering the following.



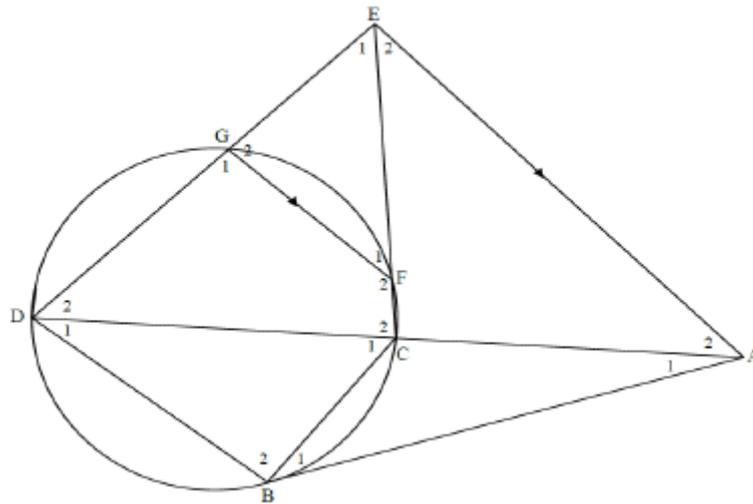
- 8.1 Prove that  $\Delta PQR \sim \Delta PSQ$  (4)
- 8.2 Hence, deduce that  $PQ^2 = PS \cdot PR$ . (1)

8.3 Hence write down a similar product for  $QR^2$ . (3)

8.4 Hence: Prove that  $PR^2 = PQ^2 + QR^2$ . (3)

QUESTION 9

In the diagram, DGFC is a cyclic quadrilateral and AB is a tangent to the circle at B. Chords DB and BC are drawn. DG and CF produced to meet at E and DC is produced to A.  $EA \parallel GF$ .



9.1 Give a reason why  $\hat{B}_1 = \hat{D}_2$  (1)

9.2 Prove that  $\triangle ABC \parallel \triangle ADB$ . (4)

9.3 Prove that  $\hat{E}_2 = \hat{D}_2$ . (4)

9.4 Prove that  $AE^2 = AD \times AC$ . (5)

9.5 Hence, deduce that  $AE = AB$ . (3)