

Mathematics Winter School worksheet

EUCLIDEAN GEOMETRY



UNIVERSITEIT VAN PRETORIA
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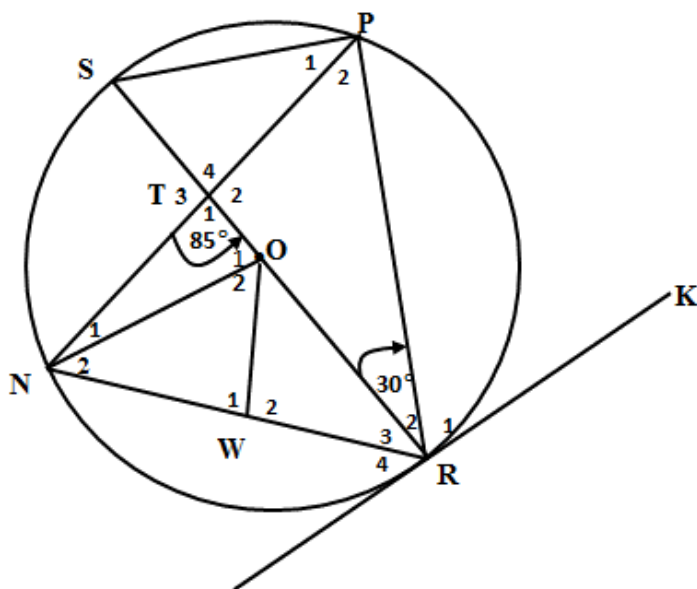
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QUESTION 1 LIMPOPO 2016 PRELIM

The vertices of $\triangle PNR$ lie on the circumference of the circle O. Diameter SR and chord NP intersect at T. $\hat{R}_2 = 30^\circ$ and $\hat{T}_1 = 85^\circ$. RK is a tangent to the circle at R.

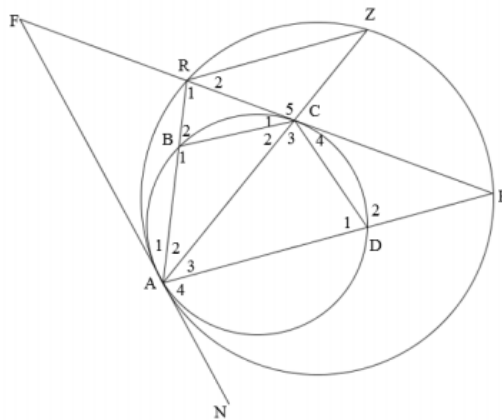


- 1.1 Determine, stating reasons, the size of:
 - 1.1.1 \hat{S} (3)
 - 1.1.2 \hat{R}_3 (4)
 - 1.1.3 \hat{N}_1 (4)
 - 1.1.4 \hat{R}_4 (3)
- 1.2 Determine whether NT is a tangent to the circle through N, O and R. Justify your answer. (2)

[16]

QUESTION 2 EC 2016 PRELIM

FAN is a common tangent to the smaller circle ABCD and the larger circle ARZP. FP is a tangent to the smaller circle at C. The straight line ABR meets the larger circle at R.



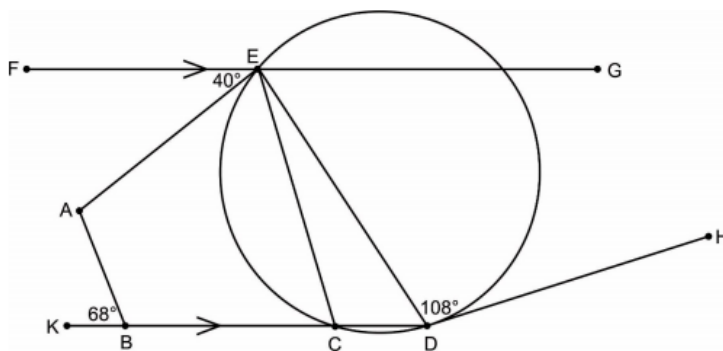
- 2.1 Prove that $BC \parallel RZ$. (4)
- 2.2 Hence, prove that BC is a tangent to circle ACP. (3)
- 2.3 Prove that $\triangle RZA \parallel \triangle DPC$. (5)
- 2.4 Hence, show that $\frac{DC}{CP} \times \frac{AC}{AB} = 1$. (5)

[17]

QUESTION 3 IEB 2019 Supplementary

In the diagram below, E, C and D are points on the circle.

- DH is a tangent to the circle at the point D with $\widehat{HDE} = 108^\circ$
- DC is produced and $DCBK \parallel GEF$
- A is a point between the parallel lines with $\widehat{FEA} = 40^\circ$ and $\widehat{ABK} = 68^\circ$

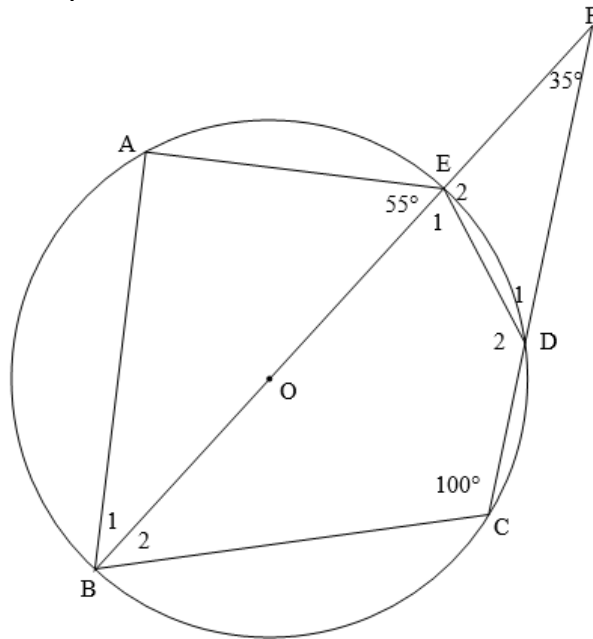


Prove that AECB is a cyclic quadrilateral.

(6)

QUESTION 4 SEP 2016 WC

In the diagram, O is the centre of the circle. A, B, C, D and E are points on the circumference of the circle. Chords BE and CD produced meet at F. $\hat{C} = 100^\circ$, $\hat{F} = 35^\circ$ and $\hat{AEB} = 55^\circ$.



4.1 Calculate, giving reasons, the size of each of the following angles:

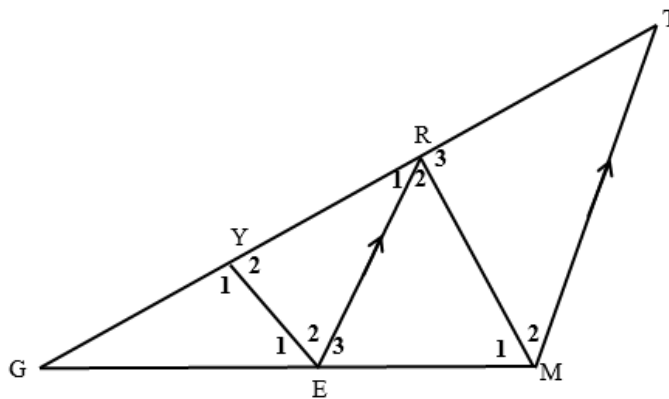
- 4.1.1 \hat{A} (2)
- 4.1.2 \hat{E}_1 (3)
- 4.1.3 \hat{D}_1 (2)

4.2 Prove, giving reasons, $AB \parallel CF$

(3)
[10]

QUESTION 5 GAU 2015 PRELIM

In ΔRGM , $\hat{R}_1 = \hat{R}_2 = \hat{E}_1$. T lies on GR produced $TM \parallel RE$.



5.1 Give with reasons, two other angles which are equal to \hat{R}_1 . (4)

5.2 Prove that $\frac{EM}{EG} = \frac{RM}{RG}$. (4)

5.3 Prove that $\Delta GYE \parallel \Delta GER$ (4)

5.4 Hence, prove that $\frac{EG}{EY} = \frac{RG}{RE}$ (1)

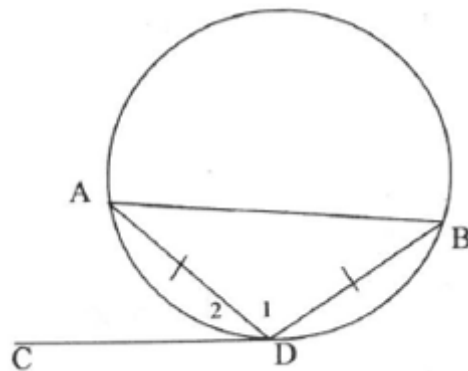
5.5 It is further given that $\widehat{GRM} = 90^\circ$, $RM = 6$ and $GM = 10$.
Calculate the length of:

5.5.1 RG (2)

5.5.2 GE (4)

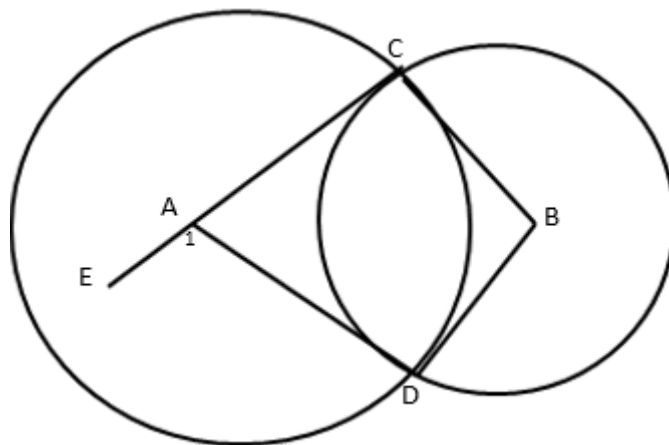
QUESTION 6 ASHTON INTERNATIONAL 2017

In the diagram below, $\widehat{ADB} = 3x$, $\widehat{ADC} = x - 25^\circ$ and $AD = BD$.



6.1 Determine, with reasons, the numerical value(s) of x for which CD is a tangent to the circle. (5)

6.2 Two circles, centres A and B intersect at C and D. CA is produced to point E. AC and AD are tangents to the smaller circle at C and D respectively.



Prove that $\widehat{A_1} = \widehat{CBD}$ (5)

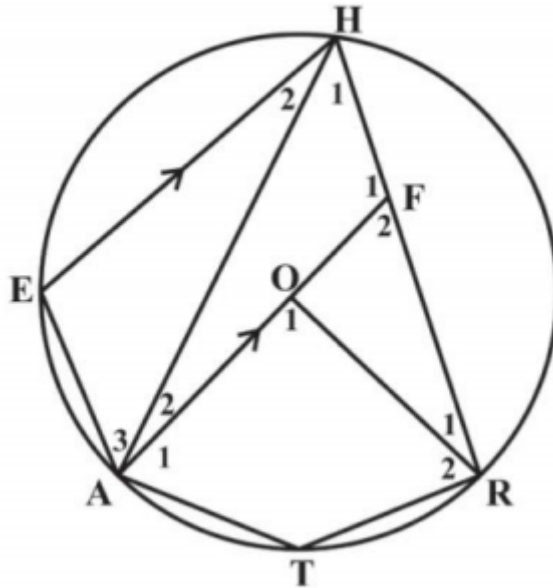
(Hint: Construct lines to complete the diagram. With the addition of these lines, you will be able to make use of the facts that:

- EC and AD are tangents.
- B is the centre of the smaller circle.

QUESTION 7 St DAVIDS INANDA 2017

In the diagram O is the centre of the circle HEATER. AOF is parallel to EH.

$\hat{F}_2 = 78^\circ$ and $\hat{R}_1 = 22^\circ$.

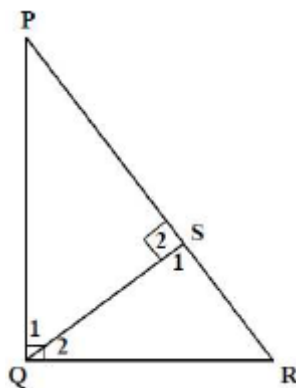


Calculate, with reasons, the sizes of:

- 7.1 \hat{O}_1 (2)
- 7.2 \hat{H}_1 (2)
- 7.3 \hat{T} (2)
- 7.4 \hat{H}_2 (3)

QUESTION 8

In the diagram below, ΔPQR is a right angled and $QS \perp PR$. Prove geometrical reasons for all statements when answering the following.



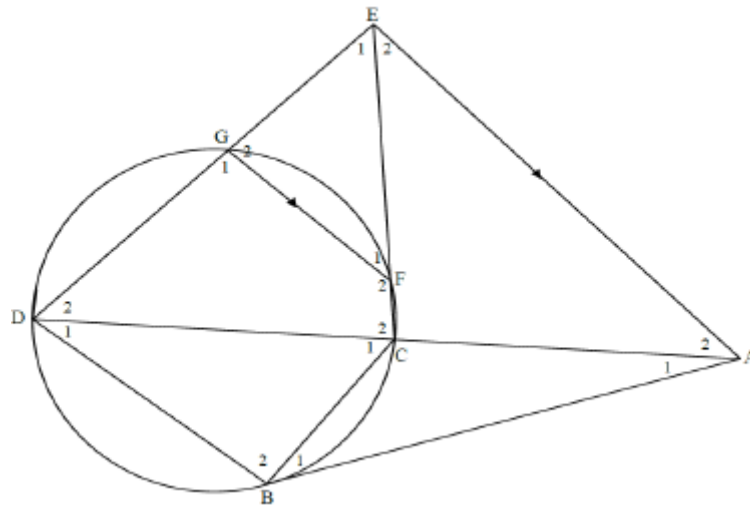
- 8.1 Prove that $\Delta PQR \sim \Delta PSQ$ (4)
- 8.2 Hence, deduce that $PQ^2 = PS \cdot PR$. (1)

8.3 Hence write down a similar product for QR^2 . (3)

8.4 Hence: Prove that $PR^2 = PQ^2 + QR^2$. (3)

QUESTION 9

In the diagram, DGFC is a cyclic quadrilateral and AB is a tangent to the circle at B. Chords DB and BC are drawn. DG and CF produced to meet at E and DC is produced to A. $EA \parallel GF$.



9.1 Give a reason why $\hat{B}_1 = \hat{D}_2$ (1)

9.2 Prove that $\triangle ABC \parallel \triangle ADB$. (4)

9.3 Prove that $\hat{E}_2 = \hat{D}_2$. (4)

9.4 Prove that $AE^2 = AD \times AC$. (5)

9.5 Hence, deduce that $AE = AB$. (3)