2 Memorandum: Work, energy and power

1.1

a.

$$\begin{aligned} W_g &= F_g \Delta x cos\Theta \\ &= (60)(9{,}8)(3)cos(90-40)^\circ \\ &= (60)(9{,}8)(3)cos(50))^\circ \\ &= 1133{,}88\ J \end{aligned}$$

b.

$$\begin{aligned} W_N &= N\Delta x cos90^\circ \\ &= 0\ J \end{aligned}$$

1.2

a.

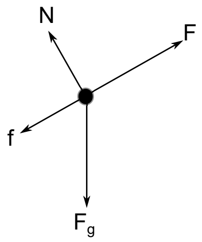
$$\begin{aligned} W_g &= F_g \Delta x cos\Theta \\ &= F_g \Delta x cos90^\circ \\ &= 0\ J \end{aligned}$$

b.

$$\begin{aligned} W_F &= F\Delta x cos90^\circ \\ &= 60(3{,}25)cos30^\circ \\ &= 168{,}87\ J \end{aligned}$$

1.3

Method 1:



$$\begin{aligned} W_N &= N\Delta x cos90^\circ \\ &= 0\ J \end{aligned}$$

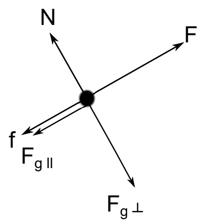
$$\begin{aligned} W_F &= F\Delta x cos\Theta \\ &= 8000(3)cos0^\circ \\ &= 24000\ J \end{aligned}$$

$$\begin{aligned} W_f &= f_k\Delta x cos\Theta \\ &= 20(3)cos180^\circ \\ &= -60\ J \end{aligned}$$

$$\begin{aligned} W_g &= F_g\Delta x cos\Theta \\ &= (1200)(9{,}8)(3)cos(90+30)^\circ \\ &= (1200)(9{,}8)(3)cos(120)^\circ \\ &= -17640\ J \end{aligned}$$

$$\begin{aligned} W_{net} &= W_F + W_f + W_g + W_N \\ &= 24000 - 60 - 17640 + 0 \\ &= 6300{,}00\ J \end{aligned}$$

Method 2:



$$\begin{aligned} F_{net} &= F + (-f) + (-F_{g||}) \\ &= 8000 - 20 - (1200 \times 98 \times sin30^\circ) \\ &= 2100{,}00\ N, up\ the\ slope \end{aligned}$$

$$\begin{aligned} W_{net} &= F_{net}\Delta x cos\Theta \\ &= (2100{,}00)(3)cos0^\circ \\ &= 6300{,}00\ J \end{aligned}$$

1.4

Option 1

$$\begin{aligned} W_{net} &= \Delta E_K \\ W_N + W_F + W_f + W_g &= \frac{1}{2}(m)(v_f^2 - v_i^2) \\ 0 + 50(2{,}60)cos30^\circ + 4{,}2(2{,}60)cos180^\circ + (5)(9{,}8)(2{,}60)cos90^\circ &= \frac{1}{2}(5)(v_f^2 - 0) \\ 0 + 112{,}58 - 10{,}92 + 0 &= \frac{1}{2}(5)(v_f^2 - 0) \\ 101{,}66 &= 2{,}5v_f^2 \\ v_f &= 6{,}38 \quad m\cdot s^{-1} \end{aligned}$$

1.4	**Option 2** $$\begin{aligned} W_{net} &= \Delta E_K \\ F_{net}\Delta x cos\theta &= \frac{1}{2}(m)(v_f^2 - v_i^2) \\ (F_x + (-f))(2{,}60)cos0^\circ &= \frac{1}{2}(5)(v_f^2 - 0) \\ ((50cos30^\circ) - 4{,}2)(2{,}60)cos0^\circ &= \frac{1}{2}(5)(v_f^2 - 0) \\ v_f &= 6{,}38 \quad m \cdot s^{-1} \end{aligned}$$
1.5	a. $$\begin{aligned} W_F &= F\Delta x cos\theta \\ &= (200)(4{,}8)cos0^\circ \\ &= 960{,}00\ J \end{aligned}$$
	b. The angle with the ground is 90 - 42 = 48° **Option 1** $$\begin{aligned} W_{net} &= \Delta E_K \\ W_N + W_F + W_f + W_g &= \frac{1}{2}(m)(v_f^2 - v_i^2) \\ 0 + 960{,}00 + 23(4{,}8)cos180^\circ + (720)(9{,}8)(4{,}8)cos(90-48)^\circ &= \frac{1}{2}(720)(v_f^2 - 0) \\ 0 + 960{,}00 + 23(4{,}8)cos180^\circ + (720)(9{,}8)(4{,}8)cos42^\circ &= \frac{1}{2}(720)(v_f^2 - 0) \\ 0 + 960{,}00 - 110{,}40 + 25169{,}42 &= 360v_f^2 \\ v_f &= 8{,}50 \quad m \cdot s^{-1} \end{aligned}$$
	Option 2 $$\begin{aligned} W_{net} &= \Delta E_K \\ F_{net}\Delta x cos\theta &= \frac{1}{2}(m)(v_f^2 - v_i^2) \\ (F + F_{g\|\|} + (-f))(4{,}8)cos0^\circ &= \frac{1}{2}(720)(v_f^2 - 0) \\ (200 - 720(9{,}8)sin48^\circ - 23)(4{,}8)cos0^\circ &= 360v_f^2 \\ v_f &= 8{,}50 \quad m \cdot s^{-1} \end{aligned}$$
2.1	a. **The principle of conservation of mechanical energy:** The total mechanical energy (sum of gravitational potential energy and kinetic energy) in an isolated system remains constant.
2.2	b. $$\begin{aligned} E_{mechi} &= E_{mechf} \\ mgh_i + \frac{1}{2}mv_i^2 &= mgh_f + \frac{1}{2}mv_f^2 \\ gh_i + \frac{1}{2}v_i^2 &= gh_f + \frac{1}{2}v_f^2 \\ 9{,}8(2) + 0 &= 9{,}8(0{,}5) + \frac{1}{2}v_f^2 \\ v_f &= 5{,}42\ m \cdot s^{-1} \end{aligned}$$

3.1	$$\begin{aligned} W_{nc} &= \Delta E_k + \Delta E_p \\ (W_N) + W_F + W_f &= \frac{1}{2}m(v_f^2 - v_i^2) + mg(h_f - h_i) \\ 0 + 0 + 20000(4)cos0^\circ + 1200(4)cos180^\circ &= \frac{1}{2}(2000)(3{,}1^2 - 3^2) + 2000(9{,}8)h \\ 80000 - 4800 &= 610 + 19600h \\ h &= 3{,}81\ m \end{aligned}$$
3.2	a. $$\begin{aligned} f_k &= \mu_k N \\ &= 0{,}18F_{g\perp} \\ &= 0{,}18(10 \times 9{,}8cos30^\circ) \\ &= 0{,}18(84{,}87) \\ &= 15{,}28\ N \end{aligned}$$ $$\begin{aligned} W_{net} &= \Delta E_k \\ (W_N) + W_F + W_g + W_f &= \frac{1}{2}m(v_f^2 - v_i^2) \\ 0 + F(3)cos0^\circ + (10 \times 9{,}8)(3)cos(90+30)^\circ + 15{,}28(3)cos180^\circ &= \frac{1}{2}(10)(3^2 - 2^2) \\ F &= 72{,}61\ N \end{aligned}$$
	b. $$\begin{aligned} \Delta t &= (\frac{v_i + v_f}{2})\Delta t \\ 3 &= (\frac{2+3}{2})\Delta t \\ \Delta t &= 1{,}20\ s \end{aligned}$$ $$\begin{aligned} P &= \frac{W}{\Delta t} \\ &= \frac{72{,}16(3)cos0^\circ}{1{,}20} \\ &= 181{,}53\ W \end{aligned}$$
4.1	$$\begin{aligned} W_{net} &= \Delta E_k \\ (W_N) + W_g + W_f &= \frac{1}{2}m(v_f^2 - v_i^2) \\ 0 + 0 + 10(2)cos180^\circ &= \frac{1}{2}(7{,}02)(0 - v_i^2) \\ v_i &= 2{,}39\ m \cdot s^{-1} \end{aligned}$$
4.2	**Principle of conservation of linear momentum**: The total linear momentum of a closed system remains constant (is conserved).
4.3	$$\begin{aligned} \Sigma p_i &= \Sigma p_f \\ p_{1i} + p_{2i} &= p_{1\&2} \\ mv_{1i} + mv_{2i} &= (m_1 + m_2)v_f \\ 0{,}02v_{1i} + 0 &= (7 + 0{,}02)(2{,}39) \\ v_{1i} &= 838{,}89\ m \cdot s^{-1} \end{aligned}$$

5.1	$$\begin{aligned} E_{mechi} &= E_{mechf} \\ mgh_i + \frac{1}{2}mv_i^2 &= mgh_f + \frac{1}{2}mv_f^2 \\ gh_i + \frac{1}{2}v_i^2 &= gh_f + \frac{1}{2}v_f^2 \\ (9{,}8)5 + 0 &= 0 + \frac{1}{2}v_f^2 \\ v_f &= 9{,}90\ m \cdot s^{-1} \end{aligned}$$
5.2	F_{net}=0 $\therefore$ a = 0 (Newton I) $\therefore$ v constant $\therefore$ E_k constant or F_{net}=0 $\therefore$ W_{net} = 0 (Newton I) $\therefore$ E_k constant
5.3	**A conservative force** is a force for which the work done in moving an object between two points is independent of the path taken. Examples are gravitational force, the elastic force in a spring and electrostatic forces (coulomb forces).
5.4	$$\begin{aligned} W_{nc} &= \Delta E_K + \Delta E_P \\ (W_N) + W_f &= \frac{1}{2}m(v_f^2 - v_i^2) + mg(h_f - h_i) \\ (0) + 18\Delta x cos180^\circ &= \frac{1}{2}(5)(0 - (9{,}90)^2) + 9{,}8(5)(3 - 0) \\ \Delta x &= 5{,}45\ m \\ sin\theta &= \frac{3}{5{,}45} \\ \theta &= 33{,}40^\circ \end{aligned}$$