# 5 Maths in physics

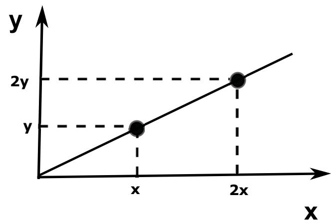
## 5.1 Graphs

In Physical Sciences we get to know and control our universe by identifying variables and examining the relationships between the variables.

Experiment → Data → Graph → Relationships → Equations

In an experiment one variable is changed (independent) and its effect on another variable is measured (dependent). **The results of an experiment can only be valid if all variables except the independent and dependent are kept constant.** The data is plotted and the shape of the graph is used to identify the relationships between the variables.

Example: A straight line through the origin shows that the variables are **directly proportional**. $y \propto x$
If x increases, then y increases **with the same factor**.

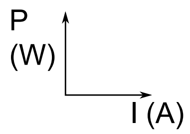



You can also work back from the equation to find the shape of the graph. The next two pages provide a summary of the shapes of graphs we often find in physics.

Example: Draw a graph of the power(**P**) dissipated by a resistor versus the current(**I**) through the resistor:

$\underbrace{\text{Power}}_{\text{y-axis}}$ versus $\underbrace{\text{current}}_{\text{x-axis}}$:

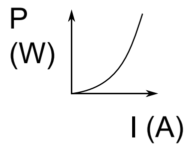
y-axis: dependent
x-axis: independent




Find the correct equation.
There are two equations that have both **P** and **I**: $\mathbf{P = IV}$ and $\mathbf{P = I^2R}$.

I need to use the equation in which all the other variables are constant.
The question referred to "a resistor" and therefore **R is constant** and therefore $\mathbf{P = I^2R}$ is the correct equation to use.

If **R** is constant in $\mathbf{P = I^2R}$, then $\mathbf{P \propto I^2}$
and the shape of the graph is a parabola.



## Graphs

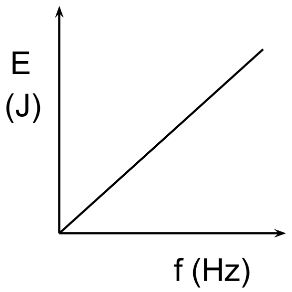
### Directly proportional
(Straight line through origin)

Example: The energy of the photon versus the frequency of the light:

$E = hf$

h is constant

$\therefore E \propto f$



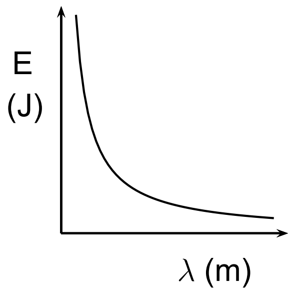

### Inversely proportional
(hyperbola)

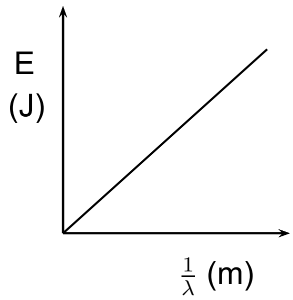
Example: The energy of a photon versus the wavelength of the light in a vacuum:

$E = \frac{hc}{\lambda}$

h and c are constant

$\therefore E \propto \frac{1}{\lambda}$



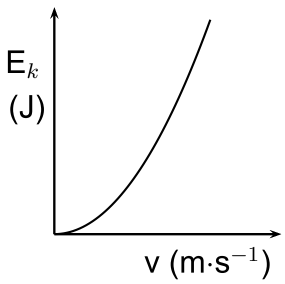

### Directly proportional to $x^2$
(Parabola)

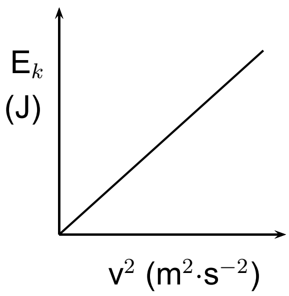
Example: The kinetic energy of an object versus the velocity at which it moves:

$E_k = \frac{1}{2} mv^2$

m is constant

$\therefore E_k \propto v^2$



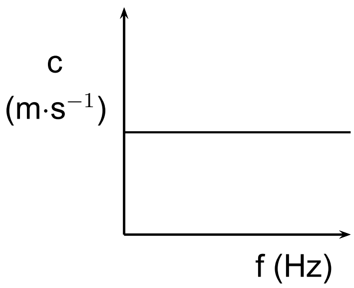




### Constant/Not dependent on
(Horisontal line)

Example: The speed of light in a vacuum versus the frequency of the light:

c is constant ($3 \times 10^8$ $m \cdot s^{-1}$)

$\therefore$ c is independent of f




If the constant is negative the graph turns upside down.

NEVER use the term *indirectly proportional*. The correct term is *inversely proportional*.

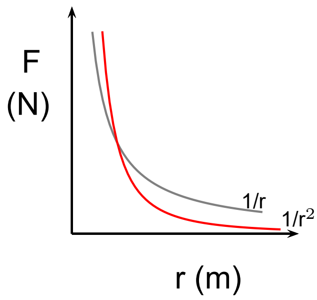
**Inversely proportional to $x^2$**

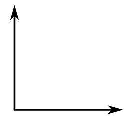
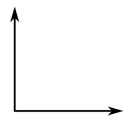
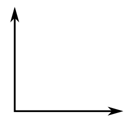
Example: The force of attraction between two objects versus the distance between their centres:

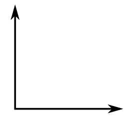
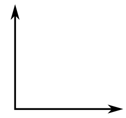
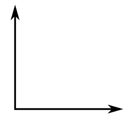
$$F = \frac{Gm_1m_2}{r^2}$$

G, $m_1$ and $m_2$ are constant

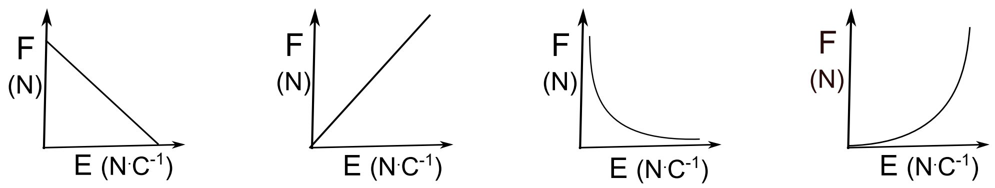
$\therefore F \propto \frac{1}{r^2}$




| | Draw the sketch graph of... (no values) | Equation Constant(s) Relationship | Graph |
|---|---|---|---|
| 1 | the acceleration of objects with different masses on a frictionless surface when the same horizontal force is exerted on them. (a versus m) | |  |
| 2 | the potential energy of objects versus their masses at a height of 2 m above the earth's surface. | |  |
| 3 | the mechanical energy of a ball falling versus the height of the ball. Ignore air resistance. | |  |

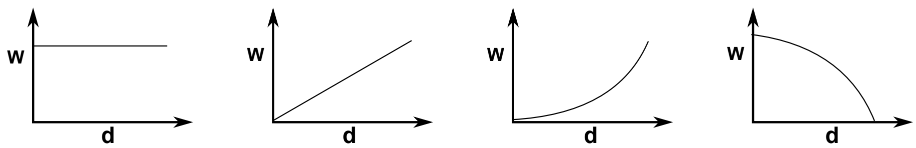
| | Draw the sketch graph of. . .<br>(no values) | Equation<br>Constant(s)<br>Relationship | Graph |
|---|---|---|---|
| 4 | the power dissipated by a man versus the time required for the man to do 200 J of work. | |  |
| 5 | the momentum versus the velocity of an object. | |  |
| 6 | the frequency observed by a stationary observer versus the frequency emitted by a sound source that is travelling at a constant velocity towards the observer. | |  |

Which ONE of the graphs below represents the correct relationship between force F on a charge and the electric f eld E? (Gr11 DOE 2016)




A crate is pulled to the right by a force that is exerted at a 30° angle to the surface. The crate experiences a kinetic frictional force.

The rope does work on the crate as it accelerates over the surface. Which one of the following graphs is the best representation of the work done by the applied force on the crate and the distance, d, that it moved?




A B C D

## 5.2 Equations: relationships between variables

Equations give the relationship between variables. It can be used to predict how a change in one or more variables will inf uence another variable. To identify the relationship between two variables in an equation **all the other variables in the equation must be constant**.

<table>
<tr><td colspan="2">Manipulation of equations</td></tr>
<tr><td colspan="2">A wave has wavelength $\lambda$, frequency f and speed v in a certain medium.<br>In another medium the same wave has a length $3\lambda$.<br>How does the speed of the same wave (frequency f) in the new medium<br>compare with the speed in the original medium?</td></tr>
<tr><td>Method 1<br>$$\begin{aligned} v_{old} &= f\lambda \\ v_{new} &= f(3\lambda) \\ &= 3\ (f\lambda) \\ &= 3\ v_{old} \end{aligned}$$</td><td>Method 2<br>$$\begin{aligned} v &= f\lambda \quad \text{and f is constant} \\ v &\propto \lambda \end{aligned}$$<br>$\lambda$ increases with factor 3<br>v increases with factor 3</td></tr>
<tr><td colspan="2">When a net force is applied to an object with mass m the object has an acceleration a.<br>What will the acceleration be when the same net force is applied to an object with mass 3m?</td></tr>
<tr><td>$$\begin{aligned} F_{net} &= m\,a \\ a_{old} &= \frac{F_{net}}{m} \\ a_{new} &= \frac{F_{net}}{3m} \\ &= \frac{1}{3}\left(\frac{F_{net}}{m}\right) \\ &= \frac{1}{3}\ a_{old} \end{aligned}$$</td><td>$$\begin{aligned} F_{net} &= m\,a \\ a &= \frac{F_{net}}{m} \\ a &\propto \frac{1}{m} \end{aligned}$$<br>m increases with factor 3<br>a decreases with factor 3<br>$$a_{new} = \frac{1}{3}\ a_{old}$$</td></tr>
</table>

QUESTION 4

4.1 Mars has a radius half of the earth's radius and a mass a tenth of the earth's mass. A hammer has a weight W on earth. What is the weight of the hammer on Mars?

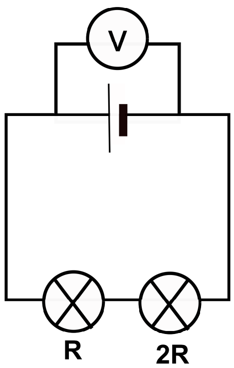
A. 1/40 W  B. 1/20 W  C. 4/10 W  D. 2/10 W

4.2 An object is dropped from rest and after falling a distance x, its momentum is p. Ignore the effects of air friction. The momentum of the object, after it has fallen a distance 2x, is . . .
(DOE Nov 2019 no.1.5)

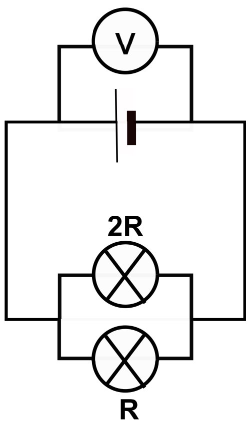
A p  B $\sqrt{2}$P  C p/2  D 2p

4.3 Two bulbs (resistance R and 2R) are connected. Which of the bulbs will glow brighter when . . .

a. they are connected in series?




b. they are connected in parallel?



## 5.3 Techniques for graph analysis

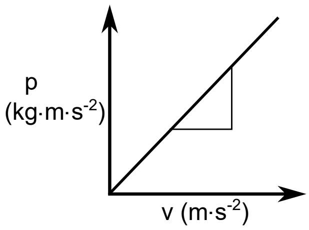
1 **Reading from graph**

If *one of the variables on the graph* is asked the answer can be read from the graph.

2 **Gradient of graph**

gradient = $\frac{\Delta\text{y-axis}}{\Delta\text{x-axis}}$

Example: Momentum versus velocity of an object:

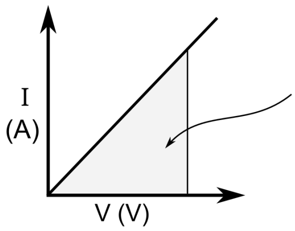



gradient = $\frac{\Delta p}{\Delta v} = m$

3 **Area under graph**

The variable represented by the area under the graph is determined by multiplying the variables of the axes.

Example: Current over a ohmic resistor versus the potential difference applied over it:




area = $I \times$ V = P

**Example: Analysis of movement graphs**

**Gradient of the graph**

gradient = $\frac{\Delta\text{y-axis}}{\Delta\text{x-axis}}$

Gradient of x:t-graph = $\frac{\Delta x}{\Delta t}$ = v
Gradient of v:t-graph = $\frac{\Delta v}{\Delta t}$ = a

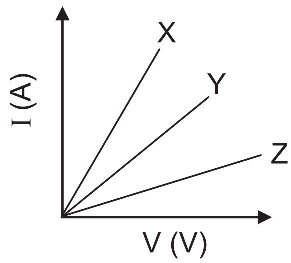
**Area under the graph**

Area: multiplying the variables of the axis.

Area under v:t-graph = $v.\Delta$t = $\Delta$x
Area under a:t-graph = $a.\Delta$t = $\Delta$v

1

1.8 Learners investigate the relationship between current (I) and potential difference (V) at a constant temperature for three different resistors, **X**, **Y** and **Z**. The resistances of **X**, **Y** and **Z** are $R_X$, $R_Y$ and $R_Z$ respectively. They obtain the graphs shown below. (Nov 2016)




Which ONE of the following conclusions regarding the resistances of the resistors is CORRECT?

A $R_z > R_Y > R_x$

B $R_X = R_Y = R_Z$

C $R_X > R_Y > R_Z$

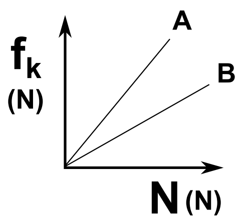
D $R_X > R_Y$ and $R_Y < R_Z$

2

Plastic containers with different masses are pulled over surface A.

The graph shows how the kinetic friction on the containers vary with the normal force acting on the containers.

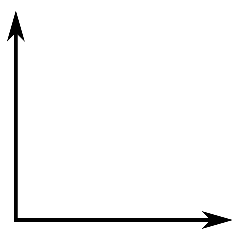
The experiment is repeated on another surface B. Is surface A smoother than surface B or not? Refer to the graph and explain in detail.




3

A constant current is sent through an ohmic resistor.

a. Draw a graph of the charge moving through the resistor versus the time that the current f ows through the resistor.



b. What does the gradient of the graph represent?

## 5.4 Straight line graphs with y-intercepts

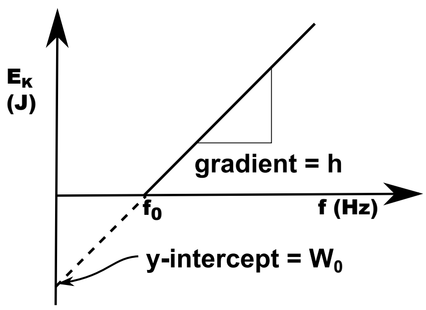
When an equation can be written in the form $\boxed{y = mx + c}$ and **c is not zero**, the graph is a straight line with m the gradient and **c the y-intercept**. In this case y is **not** directly proportional to x.

Example:
A graph of the $E_k$ of a photo-electron, released from a specif c metal, versus frequency of the light source:

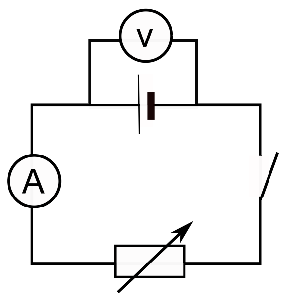
$$E = W_0 + E_K$$

$$hf = W_0 + E_k \quad \text{because} \quad E = hf$$

$$E_k = \underbrace{h}_{\text{gradient}} f - \underbrace{W_o}_{\text{y-intercept}}$$

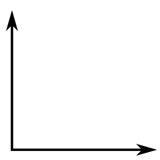



A circuit is set up to determine the emf and internal resistance of a cell. A rheostat is used to change the current.

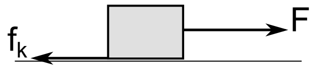



a. Draw a sketch graph to show how the potential difference over the cell (terminal potential difference) varies with current through the circuit.

b. Determine what is represented by the gradient and by the y-intercept of the graph.



## Newton's second law of motion: Graphs




An applied force is exerted on a box in a horizontal direction. The box experiences a kinetic frictional force. The magnitude of the applied force is changed and the acceleration is measured every time.

Draw a sketch graph of the acceleration versus the **net force** on the object.

i. What does the gradient of the line represent?

Draw a sketch graph of the **net force** on the object versus the acceleration.

i. What does the gradient of the line represent?

Draw a sketch graph of the **applied force** on the object versus the acceleration.

i. What does the y-intercept of the line represent?

ii. What does the gradient of the line represent?

Draw a sketch graph of the acceleration versus the **applied force** on the object.

i. What does the y-intercept of the line represent?

ii. What does the gradient of the line represent?