ANALYTICAL AND NUMERICAL INVESTIGATIONS OF THE OPTIMAL CONTROL PROBLEM FOR SEMILINEAR SOBOLEV TYPE EQUATION

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Introduction

A lot of initial-boundary value problems for the equations and the systems of equations not resolved with respect to time derivative are considered in the framework of abstract Sobolev type equations

$$L\,\dot{x} = Mx + Bu, \quad \ker L \neq \{0\}, \tag{1}$$

$$L\,\dot{x} + Mx + \sum_{j=1}^{k} N_j(x) = u, \quad \ker L \neq \{0\}. \tag{2}$$

that make up the vast field of non-classical equations of mathematical physics. The Cauchy problem

$$x(0) - x_0 = 0 \tag{3}$$

for degenerate equations (1) or (2) is unsolvable for arbitrary initial values. We consider the Showalter – Sidorov problem

$$L(x(0) - x_0) = 0 \tag{4}$$

for degenerate equation (2), which is a natural generalization of the Cauchy problem. The optimal control problem

$$J(x,u) \to \inf,\, u \in \mathfrak{U}_{ad} \subset \mathfrak{U}, \tag{5}$$

for a Sobolev type equation was first considered by G.A. Sviridyuk and A.A. Efremov[1]. This research provided the basis for a branch of optimal control studies referring to linear and semi-linear Sobolev type equations.

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[1] Sviridyuk G.A., Efremov A.A. Optimal Control of Sobolev Type Linear Equations with Relativity p-Sectorial Operators. *Differential Equations*, 1995, vol. 31, no. 11, pp. 1882–1890.

Structure

- Phase space morphology
- The Showalter – Sidorov problem for the abstract Sobolev type semi-linear equation
- Optimal control problem for the abstract Sobolev type semi-linear equation
- Decomposition method
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Thematic justification

Systematic study of initial-boundary value problems for the equations not resolved with respect to time derivative began in the '40s of the last century with the works of S.L. Sobolev. Currently, they constitute an independent part of theory of nonclassical equations of mathematical physics.

S.L. Sobolev

A.I. Kozhanov, G.V. Demidenko, A.G. Sveshnikov, M.O. Korpusov, and others

N.A. Sidorov, B.V. Loginov, A.V. Sinitsyn, M.V. Falaleev, and others
I.V. Mel'nikova, A.I. Filinkov, M.A. Al'shanskiy, and others

Yu.E. Boyarintsev, V.F. Chistyakov, A.A. Shcheglova, A.V. Keller, and others

R. Showalter, A. Favini, A. Yagi, and others

G.A. Sviridyuk, T.G. Sukacheva, A.V. Keller, V.E. Fedorov, A.A. Zamyshlyaeva, S.A. Zagrebina, and others

Phase space morphology

Let

(i) $\mathcal{H} = (\mathcal{H}; \langle \cdot, \cdot \rangle)$ be a real separable Hilbert space;

(ii) $(\mathfrak{H}, \mathfrak{H}^*)$ and $(\mathfrak{B}_j, \mathfrak{B}_j^*), j = \overline{1, k}, k \in \mathbb{N}$ be dual pairs of reflexive Banach spaces;

(iii) $L \in \mathcal{L}(\mathfrak{H}; \mathfrak{H}^*)$ be a linear continuous selfadjoint positive semidefinite Fredholm operator with orthonormal set of characteristic vectors $\{\varphi_k\}$ making a basis in a space $\mathfrak{H}$;

(iv) $M \in \mathcal{L}(\mathfrak{H}; \mathfrak{H}^*)$ be a linear continuous symmetric 2-coercive operator;

(v) $N_j \in C^r(\mathfrak{B}_j; \mathfrak{B}_j^*), r \geq 1, j = \overline{1, k},$ be s-monotone (i.e. $\left\langle N_y' x, x \right\rangle > 0,\ \forall x, y \in \mathfrak{B}_j \backslash \{0\}$) and p_j-coercive operators (i.e. $\langle N(x), x \rangle \geq C_N \|x\|^p$ and $\|N(x)\|_* \leq C^N \|x\|^{p-1}$ for some constants $C_N,\ C^N \in \mathbb{R}_+$ and $p \in [2, +\infty)$ and for any $x \in \mathfrak{B}$, where $\|\cdot\|$, $\|\cdot\|_*$ are the norms in the spaces $\mathfrak{B}$ and $\mathfrak{B}^*$ respectively) with a symmetric Frechet derivative.

Phase space morphology

Let the embeddings

$$\mathfrak{H} \hookrightarrow \mathfrak{B}_k \hookrightarrow ... \hookrightarrow \mathfrak{B}_1 \hookrightarrow \mathcal{H} \hookrightarrow \mathfrak{B}_1^* \hookrightarrow ... \hookrightarrow \mathfrak{B}_k^* \hookrightarrow \mathfrak{H}^* \tag{6}$$

be dense and continuous. Let the following condition be fulfilled:

$$(\mathbb{I} - Q)u \text{ does not depend on } t \in (0, T), \tag{7}$$

here, the operator Q is a projector applied along $\operatorname{coker} L$ into $\operatorname{im} L$. Consider a set

$$\mathfrak{M} = \begin{cases} \{x \in \mathfrak{H} : (\mathbb{I} - Q)Mx + (\mathbb{I} - Q) \sum\limits_{j=1}^{k} N_j(x) = (\mathbb{I} - Q)u\}, \text{ if } \ker L \neq \{0\}; \\ \mathfrak{H}, \quad \text{if} \quad \ker L = \{0\}. \end{cases}$$

Theorem 1.

If the condition (7) is fulfilled, then the set $\mathfrak{M}$ is a Banach C^r-manifold diffeomorphically projecting along $\ker L$ into $\operatorname{coim} L$ everywhere except perhaps for zero point.

Showalter – Sidorov problem

Set $\{\varphi_k\}$ of eigenvectors of operator L is total in $\mathfrak{H}$, therefore, we seek Galerkin approximation to the solution of problem (2), (4) in the form

$$x^m(t) = \sum_{k=1}^{m} a_k(t)\varphi_k, \quad m > \dim\ker L.$$

Here the coefficients $a_k = a_k(t)$, $k = 1, ..., m$, are defined by the next problem

$$\langle L\dot{x}^m, \varphi_k\rangle + \left\langle Mx^m + \sum_{j=0}^{k} N(x^m), \varphi_k \right\rangle = \langle u, \varphi_k\rangle, \tag{8}$$

$$\langle L(x^m(0) - x_0), \varphi_k\rangle = 0,\ k = 1, ..., m. \tag{9}$$

Let $T_m \in \mathbb{R}_+$, $T_m = T_m(x_0)$, $\mathfrak{H}^m = \operatorname{span}\{\varphi_1,\ \varphi_2, ...,\ \varphi_m\}$.

Lemma 1.

For any $x_0 \in \mathfrak{H}$ there exists a unique local solution $x^m \in C^r(0, T_m; \mathfrak{H}^m)$ of problem (8), (9).

Showalter – Sidorov problem

Define P as a projector along $\ker L$ into $\mathrm{coim}\, L$. Let $x^1 = Px \in \mathrm{coim}\, L$. Make a space

$$\mathfrak{X} = \{x|\ x \in L_\infty(0,T;\mathrm{coim}\ L) \cap L_{p_k}(0,T;\mathfrak{B}_k),\quad \dot{x}^1 \in L_2(0,T;\mathrm{coim}\, L)\}.$$

$$L(x(0)-x_0)=0,\ L\,\dot{x} + Mx + \sum_{j=1}^{k} N_j(x) = u. \qquad (10)$$

Definition 1.

Vector function $x \in \mathfrak{X}$ is called a weak solution of equation (2) if it satisfies the following equality:

$$\int_0^T \varphi(t)\left[\left\langle L\frac{d}{dt}x,\psi\right\rangle + \left\langle Mx + \sum_{j=1}^{k} N_j,\psi\right\rangle\right]dt = \int_0^T \varphi(t)\,\langle u,\psi\rangle\,dt\quad \forall\psi\in\mathfrak{H},\ \forall\varphi\in L_2(0,T).$$

The following theorem establishes the convergence of the approximate solutions to the precise solution.

Theorem 2.

For any $x_0 \in \mathfrak{H},\ T \in \mathbb{R}_+,\ u \in L_2(0,T;\mathfrak{H}^*)$ there exists a unique weak solution $x \in \mathfrak{X}$ of problem (10).

Optimal control problem

Construct a space $\mathfrak{U} = L_2(0,T;\mathfrak{H}^*)$ and define there $\mathfrak{U}$ nonempty closed convex set $\mathfrak{U}_{ad}$. Consider an optimal control problem

$$L\,\dot{x} + Mx + \sum_{j=1}^{k} N_j(x) = u,\ \ L(x(0)-x_0)=0,\ \ J(x,u)\to \inf,\ \ u\in\mathfrak{U}_{ad}, \qquad (11)$$

where a penalty functional is defined as follows

$$J(x,u) = \beta\int_0^T \|x(t)-z_d(t)\|^{p_k}_{\mathfrak{B}_k}\,dt + (1-\beta)\int_0^T \|u(t)\|^2_{\mathfrak{H}^*}\,dt,\ \ \beta\in(0,1),$$

where z_d is the target state of the system.

Definition 2.

A pair $(\tilde{x},\tilde{u})\in\mathfrak{X}\times\mathfrak{U}_{ad}$ is called *a solution of optimal control problem* (11), if

$$J(\tilde{x},\tilde{u}) = \inf_{(x,u)} J(x,u),$$

where all pairs $(x,u)\in\mathfrak{X}\times\mathfrak{U}_{ad}$ satisfy problem (10); and a vector function $\tilde{u}$ is called *an optimal control* in problem (11).

Theorem 3.

For any $x_0\in\mathfrak{H}$, $T\in\mathbb{R}_+$ there exists a solution of problem (11).

Decomposition method

The optimal control problem (11):

$$L(x(0)-x_0)=0,\ L\ \dot{x}\ +Mx+\sum_{j=1}^{k}N_j(x)=u,\ J(x,u)\to\inf,\ u\in\mathfrak{U}_{ad}$$

is equivalent to a problem

$$\begin{gathered}L\ \dot{x}\ +Mx+\sum_{j=1}^{k}N_j(v)=u,\quad x(u,v)=v,\\ L(x(0)-x_0)=0,\quad u\in\mathfrak{U}_{ad},\quad v\in L_{p_k}(0,T;\mathfrak{B}_k),\end{gathered} \tag{12}$$

$$\begin{gathered}J_\theta(x,u,v)=\theta\cdot\beta\int_0^T\|x(t)-z_d(t)\|_{\mathfrak{B}_k}^{p_k}\,dt+\\ +(1-\theta)\cdot\beta\int_0^T\|v(t)-z_d(t)\|_{\mathfrak{B}_k}^{p_k}\,dt+(1-\beta)\int_0^T\|u(t)\|_{\mathfrak{H}^*}^{2}\,dt\to\inf.\end{gathered} \tag{13}$$

Theorem 4.

For any $x_0\in\mathfrak{H}$, $T\in\mathbb{R}_+$ there exists a solution of problem (12), (13).

Penalization method

Consider an optimal control problem

$$L\,\dot{x} + Mx + \sum_{j=1}^{k} N_j(v) = u, \quad L(x(0) - x_0) = 0,$$
$$u \in \mathfrak{U}_{ad}, \quad v \in L_{p_k}(0,T;\mathfrak{B}_k), \tag{14}$$

$$J_\theta^\varepsilon(x,u,v) \to \inf,$$

where a penalty functional is set as follows

$$J_\theta^\varepsilon(x,u,v) = \theta \cdot \beta \int_0^T \|x(t) - z_d(t)\|_{\mathfrak{B}_k}^{p_k}\, dt + (1-\theta)\cdot\beta \int_0^T \|v(t) - z_d(t)\|_{\mathfrak{B}_k}^2\, dt +$$
$$+(1-\beta)\int_0^T \|u(t)\|_{\mathfrak{H}^*}^2\, dt + r_\varepsilon \int_0^T \|x(t,v,u) - v(t)\|_{\mathcal{H}}^2\, dt, \ \theta \in (0,1),$$

where $x(t,v,u)$ is a solution of problem (14), and the penalty parameter $r_\varepsilon \to +\infty$ at $\varepsilon \to 0+$.

Theorem 5.

For any $x_0 \in \mathfrak{H}$, $T \in \mathbb{R}_+$, $\varepsilon > 0$ there exists a solution $(x_\varepsilon, v_\varepsilon, u_\varepsilon)$ of problem (14).

Penalization method

$$L(x(0) - x_0) = 0,\ L\,\dot{x} + Mx + \sum_{j=1}^{k} N_j(x) = u,\ J(x,u) \to \inf,\ u \in \mathfrak{U}_{ad}. \tag{11}$$

$$\begin{gathered} L\,\dot{x} + Mx + \sum_{j=1}^{k} N_j(v) = u, \quad L(x(0) - x_0) = 0, \\ u \in \mathfrak{U}_{ad}, \quad v \in L_{p_k}(0,T;\mathfrak{B}_k), \\ J_\theta^\varepsilon(x,u,v) \to \inf. \end{gathered} \tag{14}$$

Theorem 6.

For any $x_0 \in \mathfrak{H}$, $T \in \mathbb{R}_+$ and $\varepsilon \to 0+$ at there exists a sequence $\{v_\varepsilon, u_\varepsilon\}$ satisfying

$$v_\varepsilon \to \tilde{v}, \quad u_\varepsilon \to \tilde{u},$$

where a pair $(\tilde{v},\tilde{u}) = (\tilde{x},\tilde{u})$ is a solution of problem (11).

Oskolkov mathematical model of nonlinear filtration

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with the boundary $\partial\Omega$ of class C^∞.

In cylinder $\Omega \times \mathbb{R}_+$ consider the Oskolkov equation for nonlinear filtration

$$(\lambda - \Delta)x_t - \alpha\Delta x + |x|^{p-2}x = u, \ p \geq 2, \ \alpha \in \mathbb{R}_+, \lambda \in \mathbb{R} \tag{15}$$

with Dirichlet condition

$$x(s,t) = 0, \ (s,t) \in \partial\Omega \times \mathbb{R}_+ \tag{16}$$

and Showalter – Sidorov condition

$$(\lambda - \Delta)(x(s,0) - x_0(s)) = 0, \ s \in \Omega. \tag{17}$$

Unknown function $x(s,t)$ corresponds to the pressure of filtrating liquid; parameters α, λ characterize viscous and elastic characteristics of the liquid respectively; term $u(s,t)$ corresponds to the external influence. The Oskolkov model of nonlinear filtration (15), (16) describes the process of filtration of viscoelastic incompressible fluid[2]. The control of external influence (for example, sources and sinks) in this model is aimed at achieving the desired pressure of the liquid in layer with lowest cost.

[2] A.P. Oskolkov. Initial-boundary value problems for equations of motion of nonlinear viscoelastic fluids / A.P. Oskolkov // Notes from Academic seminars of Leningrad Department of Steklov Institute of Mathematics. – 1985. – V. 147. – P. 110–119.

Showalter – Sidorov problem for the Oskolkov mathematical model of nonlinear filtration

Let $\mathfrak{H} = \overset{\circ}{W}{}_2^1(\Omega)$, $\mathfrak{B} = L_p(\Omega)$, $\mathcal{H} = L_2(\Omega)$. Denote as $\{\varphi_i\}$ a sequence of eigenfunctions of the Dirichlet homogeneous problem for Laplace operator $(-\Delta)$ in the domain Ω, and $\{\lambda_i\}$ is a corresponding sequence of eigenvalues, numerated nondecreasingly with regard to multiplicity. Construct Galerkin approximations of the solution

$$x_m(s,t) = \sum_{i=1}^{m} a_i(t)\varphi_i(s),\ m > \dim\ker L, \tag{18}$$

where coefficients $a_i = a_i(t),\ i = 1, ..., m$, are defined by the system of equations

$$\int_\Omega \left(\lambda x_{mt}\varphi_i(s) + \nabla x_{mt}\cdot\nabla\varphi_i(s) + \alpha\nabla x_m\cdot\nabla\varphi_i(s) + |x_m|^{p-2}x_m\varphi_i(s)\right)ds = \int_\Omega u\varphi_i(s)ds, \tag{19}$$

and the Showalter – Sidorov condition

$$\int_\Omega \left[\lambda\left(x_m(s,0) - x_0(s)\right)\varphi_i(s) + \nabla\left(x_m(s,0) - x_0(s)\right)\cdot\nabla\varphi_i(s)\right]ds = 0. \tag{20}$$

Optimal control problem for the Oskolkov mathematical model of nonlinear filtration

$$
\begin{gathered}
x(s,t)=0,\ (s,t)\in\partial\Omega\times\mathbb{R}_+,\ (\lambda-\Delta)(x(s,0)-x_0(s))=0,\\
(\lambda-\Delta)x_t-\alpha\Delta x+|x|^{p-2}x=u,\ p\geq 2,\ \alpha\in\mathbb{R}_+,\ \lambda\in\mathbb{R}.
\end{gathered}
\tag{21}
$$

Theorem 7.

If $\lambda\geq-\lambda_1,\ \alpha\in\mathbb{R}_+$ and $n>2$, $2\leq p\leq 2+\frac{4}{n-2}$ or $n=2$, $p\in(1,+\infty)$, then for any $x_0\in\mathfrak{H},\ T\in\mathbb{R}_+,\ u\in L_2(0,T;\mathfrak{H}^*)$ there exists a unique weak solution $x\in\mathfrak{X}$ of problem (21).

$$
J(x,u)=\frac{1}{2}\int_0^T\|x-z_d\|^2_{\overset{\circ}{W}{}_2^1(\Omega)}dt+\frac{N}{2}\int_0^T\|u\|^2_{W_2^{-1}(\Omega)}dt.
$$

Choose $\mathfrak{U}_{ad}\subset L_2(0,T;W_2^{-1}(\Omega))$ is nonempty closed convex set.

Theorem 8.

If $\lambda\geq-\lambda_1,\ \alpha\in\mathbb{R}_+$ and $n>2$, $2\leq p\leq 2+\frac{4}{n-2}$ or $n=2$, $p\in(1,+\infty)$, then for any $x_0\in\mathfrak{H},\ T\in\mathbb{R}_+$ there exists an optimal control of solutions for problem (5), (21).

Necessary condition for the existence of optimal control problem

Theorem 9.

Let $\lambda \geq -\lambda_1,\ \alpha \in \mathbb{R}_+$ and $n > 2$, $2 \leq p \leq 2 + \frac{4}{n-2}$ or $n = 2$, $p \in (1, +\infty)$, if u is an optimal control for problem (5), then there exists a vector function

$$y \in L_\infty(0, T; \operatorname{coim} L) \cap L_2(0, T; \mathfrak{H})$$

satisfying the following

$$(\lambda - \Delta)x_t - \alpha\Delta x + |x|^{p-2}x = u,$$

$$(-\lambda + \Delta)y_t - \alpha\Delta y + (p-1)|x|^{p-2}y = (-\Delta)(x(u) - z_d), (s,t) \in Q_T,$$

$$x(s,t) = y(s,t) = 0,\ (s,t) \in \partial\Omega \times (0,T),$$

$$(\lambda - \Delta)(x(s,0) - x_0(s)) = 0,\ (-\lambda + \Delta)y(s,T) = 0,\ s \in \Omega,$$

$$\int\limits_{Q_T} (y + N(-\Delta)^{-1}(u))(v - u)dsdt \geq 0 \ \ \forall v \in \mathfrak{U}_{ad}.$$

Numerical method algorithm

Despite the high coverage research of optimal control problems for distributed systems, questions of optimal control of solutions for degenerate nonlinear systems remain insufficiently studied. The most difficult of these is the construction of efficient numerical methods for solving optimal control problems. Here is an algorithm of the numerical method. Consider the problem

$$L\,\dot{x} + Mx + \sum_{j=1}^{k} N_j(x) = u, \;\; L(x(0) - x_0) = 0 \tag{22}$$

and the equivalent problem

$$L\,\dot{x} + Mx + \sum_{j=1}^{k} N_j(v) = u, \;\; v = x, \;\; L(x(0) - x_0) = 0. \tag{23}$$

An approximate solution of problem (23) will be sought in the form

$$\begin{gathered} \tilde{x}(s,t) = x^m(s,t) = \sum_{i=1}^{m} a_i(t)\varphi_i(s), \;\; \tilde{v}(s,t) = v^m(s,t) = \sum_{i=1}^{m} v_i(t)\varphi_i(s), \\ \tilde{u}(s,t) = \sum_{i=1}^{m} < u(s,t), \varphi_i(s) > \varphi_i(s) = \sum_{i=1}^{m} u_i(t)\varphi_i(s). \end{gathered} \tag{24}$$

Substitute Galerkin sums (24) in the first equation (23). Then the scalar multiply the resulting equation on eigenfunctions $\varphi_i(s), i = 1, ..., m,$ and obtain the system of algebraic-differential equations

$$\langle Lx_t^m, \varphi_i \rangle + \langle Mx^m, \varphi_i \rangle + \left\langle \sum_{j=1}^{k} N_j(v^m), \varphi_i \right\rangle = \langle u^m, \varphi_i \rangle \,, \;\; i = 1, ..., m. \tag{25}$$

Numerical method algorithm

Depending on the parameter equation of the system will be differential or algebraic. We are going to find variables $v_i(t),\ i = 1, ..., m$, $u_i(t),\ i = r, ..., m$ in the form

$$v_i(t, N) = \sum_{n=1}^{N} b_n \sin(\frac{\pi n t}{l}),\ u_i(t, N) = \sum_{n=1}^{N} c_n \sin(\frac{\pi n t}{l}) \tag{26}$$

or

$$v_i(t, N) = \sum_{n=0}^{N} b_n t^n,\ u_i(t, N) = \sum_{n=0}^{N} c_n t^n, \tag{27}$$

choosing the coefficients b_n and c_n in order that the functions $v_i(t, N)$, $u_i(t, N)$ afford a minimum to the functional

$$\begin{gathered} J_\theta^\varepsilon(\tilde{x}, \tilde{v}, \tilde{u}) = \theta \cdot \beta \int_0^T \|\tilde{x}(t) - z_d(t)\|_{\mathfrak{B}_k}^{p_k} dt + (1 - \theta) \cdot \beta \int_0^T \|\tilde{v}(t) - z_d(t)\|_{\mathfrak{B}_k}^{p_k} dt + \\ +(1 - \beta) \int_0^T \|\tilde{u}(t)\|_{\mathfrak{H}^*}^2 dt + r_\varepsilon \int_0^T \|\tilde{x}(t, \tilde{v}, \tilde{u}) - \tilde{v}(t)\|_{\mathcal{H}}^2 dt. \end{gathered} \tag{28}$$

By this means, the problem is reduced to the extremum seeking for a function with $2N\ (2(N+1))$ variables.

Computational experiments

Example. It is required to find an approximate solution of the optimal control problem:

$$(\lambda-\Delta)x_t(s_1,s_2,t)-\alpha\Delta x(s_1,s_2,t)+|x(s_1,s_2,t)|^{p-2}x(s_1,s_2,t)=u(s_1,s_2,t),$$
$$(\lambda-\Delta)(x(s_1,s_2,0)-x_0(s_1,s_2))=0,\ (s_1,s_2)\in\Omega,$$
$$x(s_1,s_2,t)=0,\ (s_1,s_2,t)\in\partial\Omega\times(0,T)$$

for $n=2$, $p=4$, $\lambda=-2$, $\alpha=1$, $T=1$, $\theta=\frac{1}{2}$, $\beta=\frac{99}{100}$, $\varepsilon=\frac{1}{10}$, $m=2$, $N=3$, with initial function

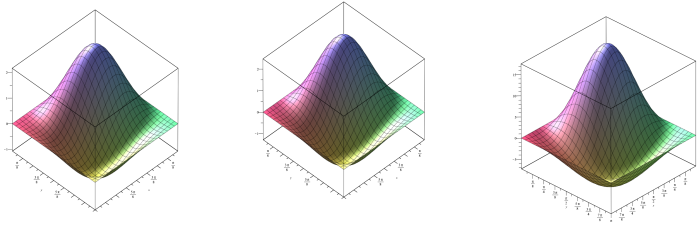
$$x_0=\frac{2}{\pi}\left((\sin s_1\sin s_2+2\sin(2s_1)\sin(s_2)+\sin(2s_1)\sin(2s_2)+2\sin(s_1)\sin(2s_2)\right)$$

and desired system state

$$z_d(s_1,s_2,t)=\frac{2}{\pi}\left(\sin s_1\sin s_2+(t+2)\sin(s_1)\sin(2s_2)+\sin(2s_1)\sin(2s_2)+\right.$$
$$\left.+(t^2+2)\sin(2s_1)\sin(s_2)\right).$$

Using software we obtained the control coefficients and the functional value $J_\varepsilon=7.259320$.

Computational experiments



а) б) в)

Fig. 1. The diagram of the computational problem solution at the time $t = 1$ for: a) function $x(s_1, s_2, 1)$; b) function $v(s_1, s_2, 1)$; c) function $u(s_1, s_2, 1)$

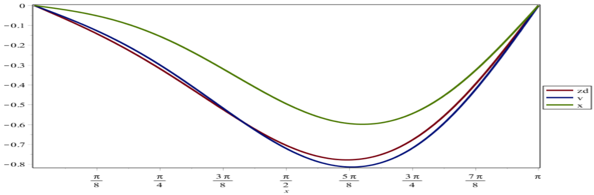


Fig. 2. The diagram of the computational problem solution at the time $t = 1$ and at $s_2 = \frac{7\pi}{8}$

Thank you for attention!