MathWrite

Examples

April – May 2015

Purpose of this session

As a group we analyse and discuss a number of examples in mathematical writing. The emphasis will be on:

- Titles.
- Abstracts and keywords.
- Introductory section
  - Opening sentences.
  - Overview part.
- References.

Titles

What to look for:

- Mathematical expressions.
- Attractive?
- Does it say anything?
- Does the title communicate the author's intentions?
- How can it be improved?

Examples of titles

Solutions of the operator equation $A^*K + KA + KRK = -W$

Some estimates of solutions for the equations of motion of compressible viscous fluid in a 3D exterior domain

Numerical results for the CGBI method to viscous channel flow

Exact boundary controllability of thermo-elastic plates with variable coefficients

Abstracts and Keywords

Abstract. We consider a stochastic parabolic partial differential equation with unknown boundary conditions. We provide a state-space formulation of the problem using the semigroup approach. Introducing the Onsager-Machlup functional, we can formulate the maximum likelihood state estimation problem. The derived estimator has a recursive form.

Keywords: Parabolic; Recursive form.

A and K (2)

Abstract. The Dirac equation admits a natural time-invariant algebra **Q** of unbounded operators of the Hilbert space **H** $= L^2(\mathbb{R}^3, \mathbb{C}^4)$. Time-invariance means that ...

Keywords: Dirac equation; Hyperbolic system.

Abstract. this note concerns extending a nonlinear semigroup to a linear one after identifying points with corresponding Dirac measures. Generators of such linear extensions are characterized. A linear extension and hence the original semigroup is recovered from this generator by means of an exponential formula.

Keywords: Nonlinear semigroups; Linear extensions.

A and K (3)

Abstract: The *conjugate gradient boundary iteration* (CGBI) is a domain decomposition method for elliptic partial differential equations [1] [2] [3] [6] [7]. In the context of fractional step methods [10], a Navier-Stokes solver based on CGBI and a characteristic's method was constructed. In this article, numerical results for the incompressible flow past a backward facing step and past a cylinder (both modelled in 2d) are presented.

Introductory section

1. Introduction. Let X be a Banach space and $B(X)$ the space of all bounded linear operators on X. Let A be a (not necessarily densely defined) closed linear operator in X and $a \in L^1_{loc}(\mathbb{R}^+)$ be a nonnegative kernel $\not\equiv 0$ ...

0. Introduction. In [4] we introduced for the first time the so-called Wentzell boundary conditions for some classes of linear, or nonlinear, second order differential operator with domain in the space $C[0,1]$ of all real-valued continuous functions on $[0,1]$ ...

Here we examine a particular case where, in addition, the analyticity holds for ...

Introductory section continued

1. Introduction. A phenomenon observed in non-Newtonian fluids which is absent in Navier-Stokes fluids, is the presence of a normal stress component, additional to the pressure, determined by the velocity field. For second grade incompressible fluids which adhere to a boundary this component has been calculated explicitly by Berker [1], and the expression shows that for an incompressible Navier-Stokes flow, the component is zero.

A question to be asked then is: If the boundary is permeable *i.e.* allows motion of the fluid through it, is it possible for fluid to pass through the boundary as a result of fluid motion in the region bounded by the particular wall? . . .

Introductory section – more

. . . In Sec. 11 the jump conditions across a curve of discontinuity is derived based on a combination of conservation of mass and balance of linear momentum. In addition the jump in compression is related to the jump in velocity as is shown by considering the kinematic balance of volume (Sec. 3). Section 12 discusses the computation of jump curves and the consequential computation of discontinuous "solutions".

Section 13 is devoted to some very brief remarks on the problem of finding the long-term behaviour of the piston. For this purpose examples are used once again. From these it is evident that, with the piston being withdrawn, the limits set by the inequality constraint mentioned above, can be forced to the extreme.

References

There are iniquities in the example below. Can you spot some of them?

References

[1] H. Brézis and A. Pazy, Convergence and approximation of semigroups of nonlinear operators in Banach spaces, *J. Funct. Anal.*, **7**(1972), 63–74.

[2] P. R. Chernoff and J. E. Marsden, Properties of Infinite Dimensional Hamiltonian Systems, *Lecture Note in Math.*, **425**, Springer-Verlag, New York, 1974.

[3] A. J. Chorin, T. J. R. Hughes, M. F. McCracken and J. E. Marsden, Product formulas and numerical algorithms, *Comm. Pure Appl. Math.*, **31**(1978), 205–256.

[4] P. D. Lax and R. D. Richtmyer, Survey of the stability of linear finite difference equations, *Comm. Pure Appl. Math.*, **9**(1956), 267–293.

[5] J. E. Marseden, On product formulas for nonlinear semigroups, *J. Funct. Anal.*, **13**(1973), 51–72.

[6] I. Miyadera and Y. Kobayashi, Convergence and approximation of nonlinear semigroups, *Proceedings of Japan–France Seminar on Functional Analysis and Numerical Analysis*, Japan Society for the Promotion of Science, Tokyo, 277-295, 1978.

[7] I. Miyadera and S. Oharu, Approximation of semi-groups of nonlinear operators, *Tôhoku Math. J.*, **22**(1970), 24–47.

[8] R. D. Richtmyer and K. W. Morton, Difference Methods for Initial Value Problems, 2nd Edition, Wiley-Interscience, New York, 1967.

[9] Y. Yamada, On some quasilinear wave equations with dissipative terms, *Nagoya Math. J.*, **87**(1982), 17–39.

Style

Style is the way in which an author reveals acquired insights to others. An example:

Of the infinite dimensional Banach spaces, Hilbert space is the most closely related, especially in its elementary geometrical aspects, to the Euclidean or finite dimensional unitary spaces.

N. Dunford & J.T. Schwartz, *Linear Operators, Part 1: General Theory,* Second Printing, Interscience Publishers, Inc., New York, 1964.