New best proximity point and pair results, measure of noncompactness and their application to the system of equations

P.R. Patle[*], M. Gabeleh[†]M.E. Samei,[‡]S.P. Moshokoa[§]

Abstract

In this paper the primary motive is to establish new best proximity point (pair) theorems with the utilization of techniques such as measure of noncompactness and several auxiliary functions. The obtained results are then applied to demonstrate existence of optimum solutions of a system of fractional order q-differential equations.

Key Words and Phrases: Best proximity point (pair), measure of noncompactness, q-fractional differential equation, q-Caputo derivative
Mathematics Subject Classification (2010): Primary 47H10; Secondry 34A08, 47H08, 47H09.

References

[1] A.A. Akhmerov, M.I. Kamenskii, A.S. Potapov, A.E. Rodkina, B.N. Sadovski, Measure of noncompactness and condensing operators, Springer Basel AG, (1992).

[2] A. Ambrosetti, Un teorema di esistenza per Ie equazioni differenziali negli spazi di Banach, Rend. Sem. Mat. Padova, 39 (1967), 349-361.

[3] J. Banas, K. Goebel, Measure of Noncompactness in Banach Spaces// Lecture notes in Pure and Appl Math, Vol 60 New York: Dekker, 1980.

[4] J. Banas, M. Jleli, M. Mursaleen, B. Samet and C. Vetro (Editors), Advanaces in nonlinear analysis via the concept of measure of Noncompactness, Springer, Singapore 2017.

[5] G. Darbo, Punti uniti in transformazioni a codominio non compatto (Italian), Rend. Sem. Math. Univ. Padova 24 (1955), 84–92.

---

[*]Department of Mathematics, School of Applied Sciences, VIT-AP University Vijaywada India; email:pradip.patle12@gmail.com,

[†]Department of Mathematics, Ayatollah Boroujerdi University, Boroujerd, Iran; email: gabeleh@abru.ac.ir, gab.moo@gmail.com,

[‡]Department of Mathematics, Faculty of Basic Science, Bu-Ali Sina University, Hamedan, Iran; email: mesamei@basu.ac.ir,

[§]Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria-0008, South Africa; email: moshokoasp@tut.ac.za

[6] A. Das, B.C. Deuri, Solution of Hammerstein type integral equation with two variables via a new fixed point theorem, The Journal of Analysis, (2022) https://doi.org/10.1007/s41478-022-00537-4

[7] R. Diaz, E. Pariguan, On hypergeometric functions and Pochhammer k-symbol, Divulgaciones Matemáticas, 15(2) (2007) 179–192.

[8] A. A. Eldred, W. A. Kirk, P. Veeramani, Proximal normal strucuture and relatively nonexpansive mappings, Studia. Math., 171 (2005), 283-293.

[9] M. Gabeleh, A characterization of proximal normal structures via proximal diametral sequences, J. Fixed Point Theory Appl. 19 (2017), 2909–2925.

[10] M. Gabeleh, M. Asadi, E. Karapinar, Best proximity results on condensing operators via measure of noncompactness with application to integral equations, Thai Journal of Mathematics, 18(3), (2020), 1519—1535.

[11] M. Gabeleh and J. Markin, Optimum solutions for a system of differential equations via measure of noncompactness, Indagationes Mathematicae, (2018), https://doi.org/10.1016/j.indag.2018.01.008.

[12] M. Gabeleh, D. K. Patel and P.R. Patle, Darbo type best proximity point (pair) results using measure of noncompactness with application, Fixed Point Theory, 23 (1) (2022), 247–266, DOI: 10.24193/fpt-ro.2022.1.16

[13] M. Gabeleh, C. Vetro, A new extension of Darbo's fixed point theorem using relatively Meir-Keeler condensing operators, Bull. Aust. Math. Soc., (2018), https://doi.org/10.1017/S000497271800045X.

[14] P.R. Patle, M. Gabeleh and V. Rakočević, On new classes of cyclic (noncyclic) condensing operators with applications, Journal of Nonlinear and Convex Analysis, 23 (7), (2022), 1335-1351

[15] A.A. Kilbas, H.M. Srivastava, J. Trujillo, Theory and Applications of Fractional Differential Equations, North-Holland Mathematics Studies, Elsevier, Amsterdam (2006)

[16] H. K. Nashine, R. Arab , P. R. Patle, D.K. Patel, Best proximity point results via measure of noncompactness and application, Numerical Functional Analysis and Optimization, (2021) DOI: 10.1080/01630563.2021.1884570

[17] P.R. Patle, M. Gabeleh and V. Rakočević, Sadovskii Type Best Proximity Point (Pair) Theorems with an Application to Fractional Differential Equations, Mediterr. J. Math. 19, 141 (2022). https://doi.org/10.1007/s00009-022-02058-7

[18] P.R. Patle, D.K. Patel, R. Arab, Darbo type best proximity point results via simulation function with application, Afrika Matematika, 31 (2020), 833-–845.

[19] P. M. Rajkovic, S. D. Marinkovic and M. S. Stankovic, Mean-value theorems in q-calculus, Matematicki vesnik, 54, 2002, 171–178.

[20] B.N. Sadovskii, Limit-compact and condensing operators (Russian), Uspehi Mat. Nauk, 27 (1972), 81–146.

[21] C.C. Tisdell, On the application of sequential and fixed point methods to fractional differential equations of arbitrary order, J. Integral Equations Appl., **24** (2012), 283-319.

[22] Y. Tang, T. Zhang, A remark on the q-fractional order differential equation, Applied Mathematics and Computation 350 (2019) 198–208.