MathCompetition UP

Department of Mathematics and Applied Mathematics

Departement Wiskunde en Toegepaste Wiskunde

GRADES 10 AND 11

27-31 JULY 2020

TIME: 2 HOURS

NO CALCULATORS.

GRADE 10 EN 11

27-31 JULIE 2020

TYD: 2 URE

GEEN SAKREKENAARS.

UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
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Denkleiers • Leading Minds • Dikgopolo tša Dihlalefi

Leading Minds

Question 1

How many integers (whole numbers) x satisfy $x^2 < 20$? (Some are negative.)

Vraag 1

Hoeveel heelgetalle x bevredig $x^2 < 20$? (Party is negatief.)

(**A**) 3 (**B**) 5 (**C**) 7 (**D**) 9 (**E**) 11

Question 2

$\dfrac{2^{20} \times 20^{40}}{200^{20}}$ equals

Vraag 2

$\dfrac{2^{20} \times 20^{40}}{200^{20}}$ is gelyk aan

(**A**) 2^{20} (**B**) 2^{40} (**C**) 2^{60} (**D**) 2^{80} (**E**) None of these/Geen van hierdie

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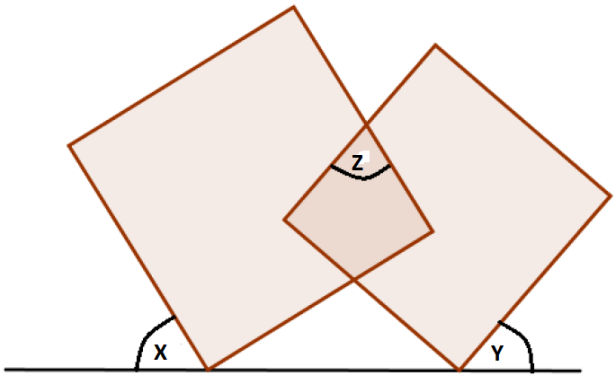
Question 3

The figure below contains two rectangles. If $X = 30°$ and $Y = 45°$, find Z in degrees.

Vraag 3

Die figuur hieronder bevat twee reghoeke. As $X = 30°$ en $Y = 45°$, bepaal Z in grade.



(**A**) 75° (**B**) 85° (**C**) 95° (**D**) 105° (**E**) 115°

Question 4

It is given that $a \oplus b = \frac{a \times b}{a - b}$. Find the value of $12 \oplus 3 - 2 \oplus 6$.

Vraag 4

Dit is gegee dat $a \oplus b = \frac{a \times b}{a - b}$. Bepaal die waarde van $12 \oplus 3 - 2 \oplus 6$.

(**A**) 1 (**B**) 7 (**C**) 14 (**D**) 24 (**E**) 27

Question 5

If $\dfrac{U + 2P}{3U + 4P} = 5$, find $\frac{U}{P}$ if $P \neq 0$ and $3U + 4P \neq 0$.

Vraag 5

As $\dfrac{U + 2P}{3U + 4P} = 5$, bepaal $\frac{U}{P}$ as $P \neq 0$ en $3U + 4P \neq 0$.

(**A**) $-\frac{9}{7}$ (**B**) $-\frac{7}{9}$ (**C**) $-\frac{7}{5}$ (**D**) $-\frac{5}{7}$ (**E**) None of these/Geen van hierdie

Question 6

Driving between two towns at $110\,km/h$ instead of $100\,km/h$ saves 9 minutes. What is the distance between the towns?

Vraag 6

As jy $110\,km/h$ ry in plaas van $100\,km/h$ spaar jy 9 minute. Wat is die afstand tussen die dorpe?

(**A**) $165\,km$ (**B**) $99\,km$ (**C**) $150\,km$ (**D**) $210\,km$ (**E**) $135\,km$

Question 7

Suppose for any real number x that $f(x-4) = 10x - 3$. What is $f(x+4)$?

Vraag 7

Vir enige reële getal x is $f(x-4) = 10x - 3$. Wat is $f(x+4)$?

(**A**) $10x + 34$ (**B**) $10x + 73$ (**C**) $10x + 77$ (**D**) $10x + 37$ (**E**) $10x + 43$

Question 8

Thabo looked at her watch and noticed that, at that moment, it was $9x$ minutes after 2 : 00 pm and $6x$ minutes before 3 : 00 pm for some value of x. What time was it at that moment?

Vraag 8

Thabo kyk na sy horlosie en let op dat dit $9x$ minute na 2 : 00 pm was en $6x$ minute voor 3 : 00 pm was vir 'n sekere x. Watter tyd van die dag was die oomblik?

(**A**) 2 : 32 (**B**) 2 : 30 (**C**) 2 : 35 (**D**) 2 : 40 (**E**) 2 : 36

Question 9

A square is divided into two equal rectangles. Each rectangle has a perimeter of 30 cm. What is the perimeter of the original square in cm?

Vraag 9

'n Vierkant word in twee ewe groot reghoeke verdeel. Elke reghoek het 'n omtrek van 30 cm. Wat is die omtrek van die oorspronklike vierkant in cm?

(**A**) 20 (**B**) 24 (**C**) 30 (**D**) 36 (**E**) 40

Question 10

Solve for x if $\sqrt{2x} + \sqrt{8x} = x$.

Vraag 10

Los op vir x as $\sqrt{2x} + \sqrt{8x} = x$.

(**A**) 16 (**B**) 10 (**C**) 18 (**D**) 8 (**E**) 12

Question 11

Triangle T has sides $3, 4$ and 5. Suppose triangle S is similar to triangle T. If one side of the triangle S is 60, what is the the smallest possible perimeter of triangle S?

Vraag 11

Driehoek T het sye $3, 4$ en 5. Veronderstel driehoek S is gelykvormig aan driehoek T. As een van die sye van driehoek S gelyk aan 60 is, wat is die kleinste moontlike omtrek van driehoek S?

(**A**) 120 (**B**) 144 (**C**) 240 (**D**) 180 (**E**) 240

Question 12

Phabi has a sequence where the nth term is of the form an^2+bn+c for some real numbers a, b, c. If the first three terms of the sequence is $401, 400, 407$, what is the fourth term in the sequence?

Vraag 12

Phabi het 'n reeks waar die $n$de term $an^2 + bn + c$ is vir sekere reële getalle a, b, c. Die eerste drie terms van die reeks is $401, 400, 407$. Wat is the vierde term in die reeks?

(**A**) 392 (**B**) 402 (**C**) 412 (**D**) 422 (**E**) 432

Question 13

In the addition calculation below each letter represents a different digit. What is C?

Vraag 13

In die optel berekening hieronder verteenwoordig elke letter 'n verskillende syfer. Wat is C?

$$\begin{array}{rl} & \text{AAB} \\ + & \text{ABB} \\ + & \text{AAC} \\ = & 2020 \end{array}$$

(**A**) 2 (**B**) 4 (**C**) 5 (**D**) 7 (**E**) None of these/Geen van hierdie

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Question 14

The graphs of straight lines $2y + x + 3 = 0$ and $3y + ax + 2 = 0$ are perpendicular. What is a?

Vraag 14

Die grafieke van reguit lyne $2y + x + 3 = 0$ en $3y + ax + 2 = 0$ is loodreg. Wat is a?

(**A**) -6 (**B**) 6 (**C**) $-2/3$ (**D**) $3/2$ (**E**) None of these/Geen van hierdie

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Question 15

In a bag there are n blue balls and n red balls. When three balls are drawn at random from the bag without replacement, the probability that all three balls are blue is $\frac{1}{9}$. What is n?

Vraag 15

In 'n sak is daar n blou balle en n rooi balle. Wanneer drie balle willekeurig gekies word sonder vervanging, is die waarskynlikheid dat al drie balle blou is $\frac{1}{9}$. Wat is n?

(**A**) 10 (**B**) 14 (**C**) 18 (**D**) 21 (**E**) 27

Question 16

The letters A, B, C, D and E are to be placed in the grid so that each of these letters appears exactly once in each row and exactly once in each column. Which letter will go in the square marked with $*$?

Vraag 16

Die letters A, B, C, D en E word in die vierkant gevul sodat elkeen van die letters presies eenkeer verskyn in elke ry verskyn en presies eenkeer in elke kolom verskyn. Watter letter sal in die vierkant gemerk met $*$ verskyn?

A				E
		C	A	
E		B	C	
	*			
B			D	

(**A**) A (**B**) B (**C**) C (**D**) D (**E**) E

Question 17

Thabo counted $2, 3, 4, 5, 6, 7, 8, 9, 20, 22, \ldots$ avoiding all numbers containing a 1. So the 9th number is 20. What is the 2020th number in his list?

Vraag 17

Thabo tell $2, 3, 4, 5, 6, 7, 8, 9, 20, 22, \ldots$ deur al die getalle te vermy wat 'n 1 bevat. So die 9de getal in die ry is 20. Wat is die 2020ste getal in die ry?

(**A**) 3792 (**B**) 3793 (**C**) 3794 (**D**) 3795 (**E**) 3796

Question 18

Given that a and b are integers, find $b-a$ such that $1+\sqrt{2}$ is a root of $x^4+ax+b=0$.

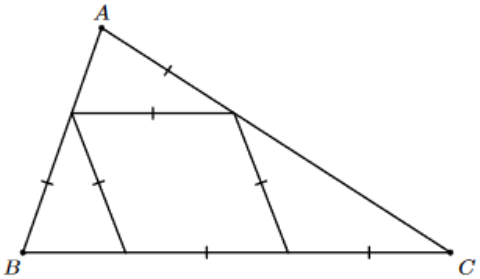
(**A**) 1 (**B**) 3 (**C**) 5 (**D**) 7 (**E**) 9

Vraag 18

Gestel dat a en b heelgetalle is. Bepaal $b-a$ as $1+\sqrt{2}$ 'n wortel van $x^4+ax+b=0$ is.

Question 19

Triangle ABC is subdivided into three isosceles triangles and a rhombus. What is the size of angle $B\hat{C}A$ in degrees?



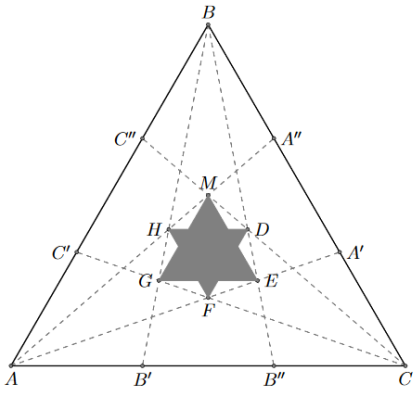
(**A**) 20° (**B**) 24° (**C**) 30° (**D**) 32° (**E**) 36°

Vraag 19

Driehoek ABC is verdeel in drie gelykbenige driehoeke en 'n ruit. Wat is die grootte van $B\hat{C}A$ in grade?

Question 20

Given an equilateral triangle $\triangle ABC$, consider the points A' and A'' that trisect line segment BC. Let B' and B'' trisect side AC and C' and C'' trisect AB. What percentage of the area of $\triangle ABC$ is the area of the shaded star?



(**A**) 6% (**B**) 7% (**C**) 8% (**D**) 9% (**E**) 10%

Vraag 20

Veronderstel $\triangle ABC$ is 'n gelyksydige driehoek waar punte A' en A'' die lynstuk BC in drie gelyke dele verdeel. Laat B' en B'' lynstuk AC in drie gelyke dele verdeel. Laat C' en C'' lynstuk AB in drie gelyke dele verdeel. Watter persentasie van die oppervlak van $\triangle ABC$ is die gekleurde area van die ster?