

# Math Competition

# UJEP

Department of Mathematics and Applied  
Mathematics  
Departement Wiskunde en Toegepaste Wiskunde

**GRADES 10 AND 11**

**GRADE 10 EN 11**

29 July – 4 Aug 2019

29 Julie – 4 Aug 2019

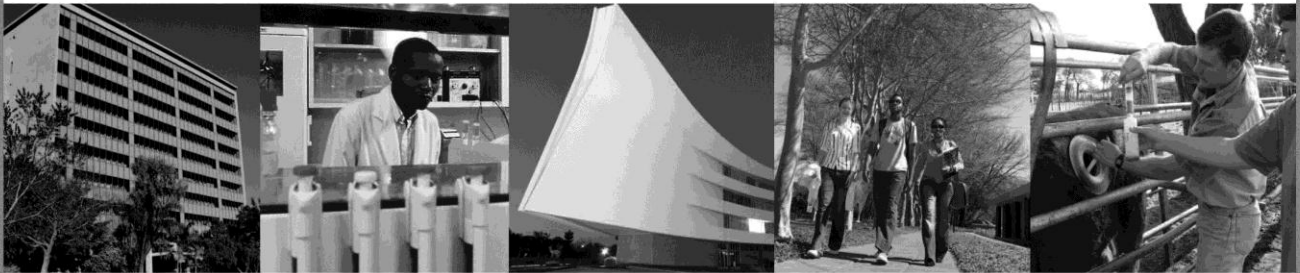
TIME: 2 HOURS

TYD: 2 URE

NO CALCULATORS.

GEEN SAKREKENAARS.

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Leading Minds

## INSTRUCTIONS

- ◆ No calculators or other calculation aids are allowed.
- ◆ **Mark allocation**  
Every question counts 1 mark.  
Random guessing is not advisable, as the mark allocated to a question may be deducted for a wrong answer.
- ◆ Every question has five possible answers, (A) to (E).  
Only **ONE** answer is correct.  
Colour in the rectangle of the correct answer on the answer sheet.  
Do not colour outside the rectangle.  
Use a soft pencil.

### Example:

Suppose Question 21 reads:

The smallest integer larger than 1 is

(A) 0    (B) -1    (C) 1    (D) 2    (E) 3

The correct answer is 2, which is answer (D).

On the answer sheet you must colour in the rectangle (D) against Question 21.

Question 21 / Vraag 21    (A)    (B)    (C)    (D)    (E)

## INSTRUKSIES

- ◆ Geen sakrekenaars of ander rekenhulpmiddels word toegelaat nie.
- ◆ **Puntetoekenning**  
Elke vraag tel 1 punt.  
Raaiery word nie aanbeveel nie, aangesien die punt toegeken aan die vraag afgetrek mag word vir 'n verkeerde antwoord.
- ◆ Elke vraag het vyf moontlike antwoorde, (A) tot (E).  
Slegs **EEN** antwoord is korrek.  
Kleur die reghoek van die korrekte antwoord op die antwoordvel in.  
Moenie buite die reghoek inkleur nie.  
Gebruik 'n sagte potlood.

### Voorbeeld:

Gestel Vraag 21 is:

Die kleinste heelgetal groter as 1 is

(A) 0    (B) -1    (C) 1    (D) 2    (E) 3

Die korrekte antwoord is 2, en dit is antwoord (D).

Op die antwoordvel moet jy die reghoek (D) inkleur teenoor Vraag 21.

Question 21 / Vraag 21    (A)    (B)    (C)    (D)    (E)

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**Question 1**

When a positive integer  $n$  is multiplied by 7, the product is less than 2019. What is the largest possible value of  $n$ ?

- (A) 285      (B) 286      (C) 287      (D) 288      (E) 289
- 

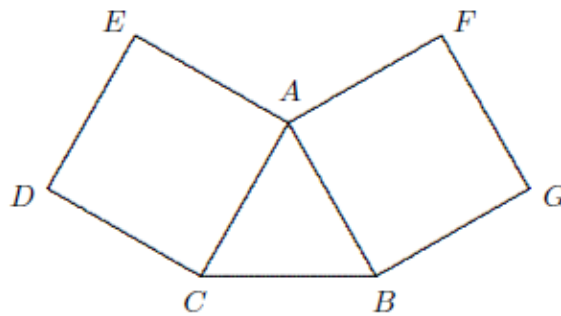
**Question 2**

If  $a = 2b$  and  $c = 3a$ , then  $a + b + c$  equals

- (A)  $5b$       (B)  $6b$       (C)  $7b$       (D)  $8b$       (E)  $9b$
- 

**Question 3**

In the figure below,  $ABC$  is an equilateral triangle and  $ACDE$  and  $AFGB$  are squares. What is the measure of  $\angle EAF$ ?



- (A)  $90^\circ$       (B)  $120^\circ$       (C)  $105^\circ$       (D)  $150^\circ$       (E)  $135^\circ$
- 

**Question 4**

Solve for  $x$  if  $\sqrt{1 + \sqrt{2 + x}} = 3$ .

- (A) 34      (B) 47      (C) 62      (D) 79      (E) 98
- 

**Question 5**

A cube has volume  $V$  and surface area  $S$ . Which equation is true?

- (A)  $\frac{V^2}{S^3} = 36$       (B)  $\frac{S^3}{V^2} = 36$       (C)  $\frac{V^2}{S^3} = 1$       (D)  $\frac{V^2}{S^3} = 216$       (E)  $\frac{S^3}{V^2} = 216$

**Vraag 1**

Wanneer 'n positive heelgetal  $n$ , met 7 vermenigvuldig word, is die produk kleiner as 2019. Wat is die grootste moontlike waarde vir  $n$ ?

- (A) 285      (B) 286      (C) 287      (D) 288      (E) 289
- 

**Vraag 2**

As  $a = 2b$  en  $c = 3a$ , dan is  $a + b + c$  gelyk aan

- (A)  $5b$       (B)  $6b$       (C)  $7b$       (D)  $8b$       (E)  $9b$
- 

**Vraag 3**

In die figuur hieronder is  $ABC$  'n gelyksydige driehoek.  $ACDE$  en  $AFGB$  is vierkante. Wat is die grootte van  $\angle EAF$ ?

**Vraag 4**

Los op vir  $x$  as  $\sqrt{1 + \sqrt{2 + x}} = 3$ .

- (A) 34      (B) 47      (C) 62      (D) 79      (E) 98
- 

**Vraag 5**

'n Kubus het volume  $V$  en buite-oppervlak  $S$ . Watter vergelyking is waar?

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**Question 6**

Thabo runs one kilometer at a speed of 6 km/h and then the next kilometer at 8 km/h. What is his average speed for the 2 kilometer jog?

- (A) 7 km/h      (B)  $\sqrt{48}$  km/h      (C) 7.2 km/h      (D)  $\frac{48}{7}$  km/h      (E) 7.5 km/h
- 

**Question 7**

Solve for  $x$  if  $\frac{1 + \frac{2}{x}}{3 + \frac{4}{x}} = 5$ .

- (A)  $-\frac{9}{7}$       (B)  $-\frac{5}{3}$       (C)  $-\frac{11}{9}$       (D)  $-\frac{13}{11}$       (E)  $-\frac{7}{5}$
- 

**Question 8**

Suppose  $a + b = U$  and  $ab = P$ , what is  $a^2 + b^2$  in terms of  $U$  and  $P$ ?

- (A)  $U^2$       (B)  $U^2 - P$       (C)  $U^2 + P$       (D)  $U^2 + 2P$       (E)  $U^2 - 2P$
- 

**Question 9**

The angles of a triangle are in the ratio 2 : 3 : 4. What is the size of the biggest angle?

- (A)  $50^\circ$       (B)  $60^\circ$       (C)  $70^\circ$       (D)  $80^\circ$       (E)  $90^\circ$
- 

**Question 10**

Let  $f(x) = x^2 + 2x + 3$ , then  $\frac{f(x+2h) - f(x-h)}{h}$  equals

- (A)  $6x + 5h + 6$   
(B)  $6x + 3h + 6$   
(C)  $5x + 3h + 6$   
(D)  $3x + 5h + 6$   
(E)  $5x + 5h + 6$

**Vraag 6**

Thabo hardloop een kilometer teen 'n spoed van 6 km/h en dan die volgende kilometer teen 8 km/h. Wat is sy gemiddelde spoed vir die 2 kilometer draf?

- (A) 7 km/h      (B)  $\sqrt{48}$  km/h      (C) 7.2 km/h      (D)  $\frac{48}{7}$  km/h      (E) 7.5 km/h
- 

**Vraag 7**

Los op vir  $x$  as  $\frac{1 + \frac{2}{x}}{3 + \frac{4}{x}} = 5$ .

- (A)  $-\frac{9}{7}$       (B)  $-\frac{5}{3}$       (C)  $-\frac{11}{9}$       (D)  $-\frac{13}{11}$       (E)  $-\frac{7}{5}$
- 

**Vraag 8**

Gestel  $a + b = U$  en  $ab = P$ , wat is  $a^2 + b^2$  in terme van  $U$  en  $P$ ?

- (A)  $U^2$       (B)  $U^2 - P$       (C)  $U^2 + P$       (D)  $U^2 + 2P$       (E)  $U^2 - 2P$
- 

**Vraag 9**

Die hoeke van 'n driehoek is in die verhouding 2 : 3 : 4. Wat is die grootte van die grootste hoek?

- (A)  $50^\circ$       (B)  $60^\circ$       (C)  $70^\circ$       (D)  $80^\circ$       (E)  $90^\circ$
- 

**Vraag 10**

Laat  $f(x) = x^2 + 2x + 3$ , dan is  $\frac{f(x+2h) - f(x-h)}{h}$  gelyk aan

- (A)  $6x + 5h + 6$   
(B)  $6x + 3h + 6$   
(C)  $5x + 3h + 6$   
(D)  $3x + 5h + 6$   
(E)  $5x + 5h + 6$

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**Question 11**

Mrs. Venter's class is voting for Jack or Jill to be captain. After counting 60% of the votes, if Jack gets the remainder of the votes, he will win by 29 votes. However if Jill gets the remainder of the votes, she will win by 23 votes. How big is Mrs. Venter's class?

- (A) 50      (B) 52      (C) 55      (D) 60      (E) 65
- 

**Question 12**

A triangle has vertices  $A(0; 0)$ ,  $B(20; 19)$  and  $C(10; 29)$ . What is the area of the triangle?

- (A) 175      (B) 185      (C) 195      (D) 205      (E) *None of these/Geen van hierdie*
- 

**Question 13**

Nxolo wrote down a sequence. If she adds up the first  $n$  numbers of the sequence, she gets  $2n^2 + 3n + 4$ . What is the tenth number in her sequence?

- (A) 45      (B) 49      (C) 33      (D) 41      (E) 37
- 

**Question 14**

A regular hexagon has perimeter 12 cm, what is the area of the hexagon in  $\text{cm}^2$ ?

- (A)  $3\sqrt{3}$       (B)  $4\sqrt{3}$       (C)  $6\sqrt{3}$       (D)  $8\sqrt{3}$       (E)  $9\sqrt{3}$
- 

**Question 15**

In a jug is 3 blue marbles and 4 red marbles. If you draw two marbles, without looking, what is the chances they are the same colour?

- (A)  $\frac{3}{7}$       (B)  $\frac{4}{7}$       (C)  $\frac{5}{7}$       (D)  $\frac{6}{7}$       (E) *None of these/Geen van hierdie*
- 

**Vraag 11**

Mev. Venter se klas stem vir Jack of Jill om klaskaptein te wees. Nadat 60% van die stemme getel is, gaan Jack wen met 29 stemme as hy die res van die stemme kry. Maar as Jill die res van die stemme kry, gaan sy met 23 stemme wen. Hoe groot is Mev. Venter se klas?

**Vraag 12**

'n Driehoek het hoekpunte  $A(0; 0)$ ,  $B(20; 19)$  en  $C(10; 29)$ . Wat is die oppervlakte van die driehoek?

**Vraag 13**

Nxolo het 'n getalry neergeskryf. As sy die eerste  $n$  getalle van die reeks bymekaar tel, kry sy  $2n^2 + 3n + 4$ . Wat is die tiende getal in haar ry?

**Vraag 14**

'n Reëlmatige seshoek het 'n omtrek van 12 cm. Wat is die oppervlakte van die seshoek in  $\text{cm}^2$ ?

**Vraag 15**

In 'n beker is 3 blou albasters en 4 rooi albasters. As jy twee albasters kies, sonder om te kyk, wat is die kans dat hulle dieselfde kleur sal wees?

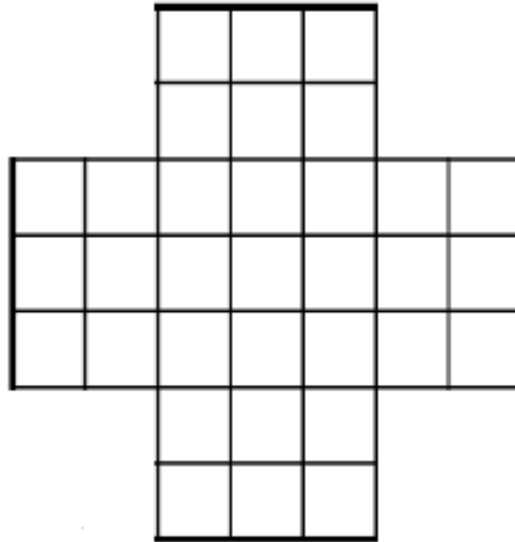
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**Question 16**

How many different rectangles (of all sizes) are there in this figure? Some do overlap.

**Vraag 16**

Hoeveel verskillende reghoeke (van alle groottes) is daar in hierdie figuur? Party oorvleuel wel.



- (A) 298      (B) 300      (C) 302      (D) 304      (E) 306
- 

**Question 17**

Ann, Ben and Con are three people. One of them is a liar who always tells lies, another is a saint who always tells the truth, and the third is a switcher who sometimes tells the truth and sometimes lies. They make the following statements:

Ann: I am the liar.  
Ben: Ann is the liar.  
Con: I am not the liar.

Which statement is true?

**Vraag 17**

Ann, Ben en Con is drie persone. Een van hulle is 'n leuenaar en vertel altyd leuens. Een is 'n engel en praat altyd die waarheid, terwyl die derde deurmekaar is en partykeer leuens vertel en partykeer die waarheid praat. Hulle maak die volgende bewerings:

Ann: Ek is 'n leuenaar.  
Ben: Ann is 'n leuenaar.  
Con: Ek is nie 'n leuenaar nie.

Watter bewering is waar?

- (A) Ann is a liar. / Ann is 'n leuenaar.  
(B) Ben is a switcher. / Ben is deurmekaar.  
(C) Con is a switcher. / Con is deurmekaar.  
(D) Ann is a saint. / Ann is 'n engel.  
(E) Con is a saint. / Con is 'n engel.

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**Question 18**

Suppose  $a, b, c$  are real numbers. If  $2^a = 3^b = 6^c$ , then

- (A)  $a + b + c = abc$
- (B)  $a^3 + b^3 + c^3 = abc$
- (C)  $ac + ab = bc$
- (D)  $ab + bc = ac$
- (E)  $bc + ac = ab$

**Vraag 18**

Veronderstel  $a, b, c$  is reële getalle. As  $2^a = 3^b = 6^c$ , dan is

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**Question 19**

What is the biggest real value for  $x$  such that  $(1 + x^2)^{3/2} + (1 - x^2)^{3/2} = 3x^2$ ?

- (A)  $\frac{\sqrt{3}}{\sqrt{2}}$       (B)  $\frac{\sqrt{2}}{\sqrt{3}}$       (C) 1      (D)  $\frac{\sqrt[4]{3}}{\sqrt{2}}$       (E)  $\frac{\sqrt[4]{2}}{\sqrt{3}}$

**Vraag 19**

Wat is grootste reële waarde vir  $x$  sodat  $(1 + x^2)^{3/2} + (1 - x^2)^{3/2} = 3x^2$ ?

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**Question 20**

Some persons in a party shake hands with each other. The following information is known:

- Each person shakes hands with exactly 20 persons.
- For each pair of persons who shake hands with each other, there is exactly 1 other person who shake hands with both of them.
- For each pair of persons who do not shake hands with each other, there are exactly 6 other persons who shake hands with both of them.

Determine the number of persons at the party.

- (A) 51      (B) 61      (C) 71      (D) 81      (E) 91

**Vraag 20**

Party persone by 'n partytjie skud hande met mekaar. Die volgende inligting is bekend:

- Elke persoon skud hande met presies 20 persone.
- Vir elke paar persone wat hande met mekaar skud, is daar presies 1 ander persoon wat met albei van hulle hande skud.
- Vir elke paar persone wat nie mekaar hande skud nie, is daar presies 6 ander persone wat met beide van hulle hande skud.

Bepaal die aantal persone by die partytjie?