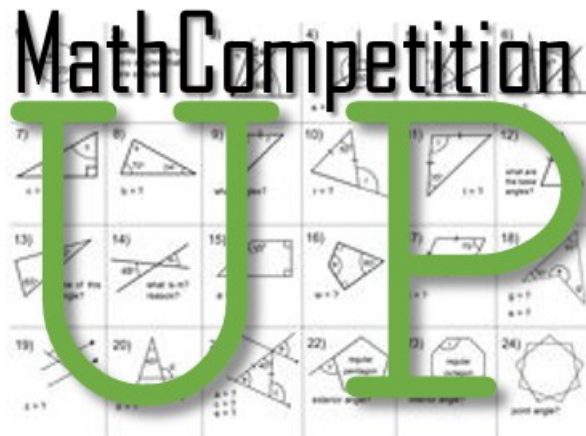


# Math Competition

# UJEP



Department of Mathematics and Applied  
Mathematics  
Departement Wiskunde en Toegepaste Wiskunde

**GRADES 10 AND 11**

**GRADE 10 EN 11**

31 July – 5 Aug 2017

31 July – 5 Aug 2017

TIME: 2 HOURS

TYD: 2 URE

©2012 OUTEURSREG VOORBEHOU, UNIVERSITEIT VAN PRETORIA  
©2012 COPYRIGHT RESERVED, UNIVERSITY OF PRETORIA



UNIVERSITEIT VAN PRETORIA  
UNIVERSITY OF PRETORIA  
YUNIBESITHI YA PRETORIA

Denkleiers • Leading Minds • Dikgopolo tša Dihlalefi

Leading Minds

## INSTRUCTIONS

- ◆ No calculators or other calculation aids are allowed.
- ◆ **Mark allocation**  
Every question counts 1 mark.  
Random guessing is not advisable, as the mark allocated to a question may be deducted for a wrong answer.
- ◆ Every question has five possible answers, (A) to (E).  
Only **ONE** answer is correct.  
Colour in the rectangle of the correct answer on the answer sheet.  
Do not colour outside the rectangle.  
Use a soft pencil.

### Example:

Suppose Question 21 reads:

The smallest integer larger than 1 is

(A) 0 (B) -1 (C) 1 (D) 2 (E) 3

The correct answer is 2, which is answer (D).

On the answer sheet you must colour in the rectangle (D) against Question 21.

Question 21 / Vraag 21  (A)  (B)  (C)  (D)  (E)

## INSTRUKSIES

- ◆ Geen sakrekenaars of ander rekenhulpmiddels word toegelaat nie.
- ◆ **Puntetoekenning**  
Elke vraag tel 1 punt.  
Raaiery word nie aanbeveel nie, aangesien die punt toegeken aan die vraag afgetrek mag word vir 'n verkeerde antwoord.
- ◆ Elke vraag het vyf moontlike antwoorde, (A) tot (E).  
Slegs **EEN** antwoord is korrek.  
Kleur die reghoek van die korrekte antwoord op die antwoordvel in.  
Moenie buite die reghoek inkleur nie.  
Gebruik 'n sagte potlood.

### Voorbeeld:

Gestel Vraag 21 is:

Die kleinste heelgetal groter as 1 is

(A) 0 (B) -1 (C) 1 (D) 2 (E) 3

Die korrekte antwoord is 2, en dit is antwoord (D).

Op die antwoordvel moet jy die reghoek (D) inkleur teenoor Vraag 21.

Question 21 / Vraag 21  (A)  (B)  (C)  (D)  (E)

---

**Question 1**

The average of 20, 17 and  $x$  is 21. What is  $x$ ?

- (A) 23      (B) 24      (C) 25      (D) 26      (E) 27
- 

**Vraag 1**

Die gemiddelde van 20, 17 en  $x$  is 21. Wat is  $x$ ?

- (A) 23      (B) 24      (C) 25      (D) 26      (E) 27
- 

**Question 2**

What is the degree of the polynomial  $((1 + 3x^2 - 6x)^3(5 + x^3 - x^4 + x^2)^2)^4$ ?

- (A) 36      (B) 48      (C) 56      (D) 96      (E) 216
- 

**Vraag 2**

Wat is die graad van die polinoom  $((1 + 3x^2 - 6x)^3(5 + x^3 - x^4 + x^2)^2)^4$ ?

- (A) 36      (B) 48      (C) 56      (D) 96      (E) 216
- 

**Question 3**

15% of a round cake is cut as shown in the figure. What is the size of the angle denoted by a question mark?



15% van 'n ronde koek is uitgesny soos getoon. Wat is die grootte van die hoek aangedui deur 'n vraagteken?

- (A) 30°      (B) 45°      (C) 48°      (D) 54°      (E) 60°
- 

**Question 4**

Which of the following inequalities is ALWAYS TRUE if  $U$  and  $P$  are real numbers where  $U < P$ ?

- (A)  $U^2 < P^2$   
(B)  $U + P < 2017$   
(C)  $U + 2017 < P$   
(D)  $U + 2017 > P$   
(E)  $U - 2017 < P$
- 

**Vraag 4**

Watter een van die volgende uitdrukkings is ALTYD WAAR as  $U$  en  $P$  reële getalle is met  $U < P$ ?

- (A)  $U^2 < P^2$   
(B)  $U + P < 2017$   
(C)  $U + 2017 < P$   
(D)  $U + 2017 > P$   
(E)  $U - 2017 < P$
-

---

**Question 5**

Odd numbers are arranged in the pattern below. Note that the number 39 is in the 1st column. In what column will the number 577 be?

1	3	5	7	9
19	17	15	13	11
21	23	25	27	29
39	37	35	33	31
41	43	45	47	49
59	57	55	53	51

**Vraag 5**

Onewe getalle word in die patroon hieronder gerangskik. Let op dat die getal 39 in die 1ste kolom is. In watter kolom is die getal 577?

- (A) 1            (B) 2            (C) 3            (D) 4            (E) 5
- 

**Question 6**

Thabo walks 5 metres North, then 5 metres East and then 7 metres North. How far is Thabo from his starting point?

**Vraag 6**

Thabo stap 5 meter Noord, dan 5 meter Oos en dan 7 meter Noord. Hoe ver is Thabo van sy beginpunt?

- (A) 11 m            (B) 12 m            (C) 13 m            (D) 14 m            (E) 17 m
- 

**Question 7**

If  $f(x) = x^2 + 3x + 7$ , what is  $\frac{f(x+2)-f(x)}{2}$ ?

**Vraag 7**

As  $f(x) = x^2 + 3x + 7$ , wat is  $\frac{f(x+2)-f(x)}{2}$ ?

- (A)  $2x + 2$             (B)  $2x + 3$             (C)  $2x + 4$             (D)  $2x + 5$             (E)  $2x + 6$
- 

**Question 8**

Sarah forms a triangle by drawing the three lines  $y = 8x + 8$ ,  $y = -2x + 8$  and  $y = 0$  in the Cartesian plane. What is the area of this triangle?

**Vraag 8**

Sarah vorm 'n driehoek deur die lyne  $y = 8x + 8$ ,  $y = -2x + 8$  en  $y = 0$  in die Cartesiese vlak te skets. Wat is die oppervlakte van die driehoek?

- (A) 24            (B) 20            (C) 18            (D) 16            (E) 12
-

---

**Question 9**

Suppose  $a$  is a real number such that  $3^a = 100$ . Which one of the following is true?

- (A)  $a < 3$       (B)  $3 < a < 4$       (C)  $4 < a < 5$       (D)  $5 < a < 33$       (E)  $33 < a$
- 

**Vraag 9**

Gestel  $a$  is a reële getal sodat  $3^a = 100$ . Watter een van die volgende is waar?

**Question 10**

Which one of the following equations has  $x = 2 - \sqrt{5}$  as a solution?

- (A)  $x^2 - x - 1 = 0$     (B)  $x^2 - 5x - 1 = 0$     (C)  $x^2 - 6x - 1 = 0$   
(D)  $x^2 - 3x - 1 = 0$     (E)  $x^2 - 4x - 1 = 0$
- 

**Vraag 10**

Watter een van die volgende vergelykings het  $x = 2 - \sqrt{5}$  as 'n oplossing?

**Question 11**

Suppose  $a, b$  and  $c$  are real numbers such that  $x^2 + 3x + 5 = a(x - 2)^2 + b(x - 2) + c$  for any real number  $x$ . Find  $3a + 2b + c$ .

- (A) 23      (B) 32      (C) 45      (D) 54      (E) 60
- 

**Vraag 11**

Veronderstel  $a, b$  en  $c$  is reële getalle sodat  $x^2 + 3x + 5 = a(x - 2)^2 + b(x - 2) + c$  vir enige reële getal  $x$ . Bereken  $3a + 2b + c$ .

**Question 12**

Make  $x$  the subject of the formula if

**Vraag 12**

Maak  $x$  die onderwerp van die formule as

$$\frac{x - y}{x + y - z} = y.$$

- (A)  $x = \frac{yz - y - y^2}{y - 1}$   
(B)  $x = \frac{y^2 + yz - y}{y - 1}$   
(C)  $x = \frac{y + yz - y^2}{y - 1}$   
(D)  $x = \frac{y^2 + y - yz}{y - 1}$   
(E)  $x = \frac{y^2 - y - yz}{y - 1}$
-

---

**Question 13**

What is an equivalent expression for  $\frac{x^2 - 3x - 18}{x^2 - 8x + 12} \cdot \frac{x^2 - 4x + 3}{x^2 + x - 2} \cdot \frac{x^2 - 4}{x^2 - 9}$ ?

- (A)  $\frac{x^2 + 6x + 5}{x^2 - 6x + 5}$     (B)  $\frac{x^2 + 9x + 20}{x^2 + 3x - 4}$     (C)  $\frac{x^2 + 3x - 4}{x^2 + 9x + 20}$   
(D)  $\frac{x^2 - 5x + 20}{x^2 + 6x + 9}$     (E) 1
- 

**Vraag 13**

Wat is 'n ekwivalente uitdrukking vir  $\frac{x^2 - 3x - 18}{x^2 - 8x + 12} \cdot \frac{x^2 - 4x + 3}{x^2 + x - 2} \cdot \frac{x^2 - 4}{x^2 - 9}$ ?

---

**Question 14**

Solve for  $x$  if  $\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} = 4$ .

- (A)  $\frac{5}{2}$     (B)  $\frac{9}{4}$     (C)  $\frac{13}{6}$     (D)  $\frac{17}{8}$     (E)  $\frac{21}{10}$
- 

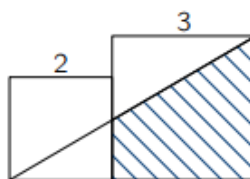
**Vraag 14**

Los op vir  $x$  as  $\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} = 4$ .

---

**Question 15**

Two squares of side length 2 cm and 3 cm are placed next to each other on a straight line as shown below. What is the area of the shaded trapezium?



- (A) 5 cm<sup>2</sup>    (B) 6,3 cm<sup>2</sup>    (C) 7 cm<sup>2</sup>    (D) 7,2 cm<sup>2</sup>    (E) 7,5 cm<sup>2</sup>
- 

**Vraag 15**

Twee vierkante van sy lengte 2 cm en 3 cm word langs mekaar gesit op 'n reguit lyn soos hieronder aangewys. Wat is die oppervlakte van die trapesium?

---

**Question 16**

A dark room contains 12 bottles of wine: 3 Merlot, 4 Cabernet and 5 Pinot Noir. If you choose 4 bottles at random from the room, what is the probability of getting at least one of each type?

- (A)  $\frac{5}{11}$     (B)  $\frac{6}{11}$     (C)  $\frac{5}{13}$     (D)  $\frac{6}{13}$     (E)  $\frac{7}{12}$
- 

**Vraag 16**

Daar is 12 wynbottels in 'n donker kamer: 3 Merlot, 4 Cabernet en 5 Pinot Noir. As jy 4 bottels willekeurig kies, wat is die waarskynlikheid dat jy ten minste een van elke tipe sal kry?

---

### Question 17

A positive integer is called **lucky** if you subtract two times one of its positive factors from the number and get 12. For example, 16 is **lucky** because  $16 - 2 \times 2 = 12$ . What is the sum of the digits of the biggest **lucky** number?

- (A) 7            (B) 8            (C) 9            (D) 10            (E) 11
- 

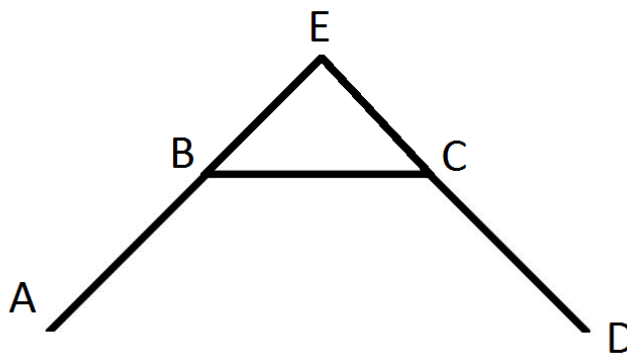
### Question 18

A triangle with sides 6, 9 and 12 has area  $A$  units. What is the area of a triangle with sides 8, 12 and 16 in terms of  $A$ ? (Hint: Think similar triangles.)

- (A)  $\frac{3}{2}A$             (B)  $\frac{4}{3}A$             (C)  $\frac{25}{16}A$             (D)  $\frac{16}{9}A$             (E)  $\frac{9}{4}A$
- 

### Question 19

$A, B, C$  and  $D$  are four consecutive vertices of a regular polygon. Suppose  $AB$  and  $DC$  are extended to meet at point  $E$ . If the size of  $\hat{BEC}$  is  $x$  degrees, how many vertices does the regular polygon have in terms of  $x$ ?



- (A)  $\frac{720}{180 - x}$             (B)  $\frac{720}{180 + x}$             (C)  $\frac{360}{90 + x}$             (D)  $\frac{360}{90 - x}$             (E)  $\frac{360}{x}$
- 

### Vraag 17

'n Positiewe heelgetal word **gelukkig** genoem, as jy twee keer 'n positiewe faktor van homself aftrek en 12 kry. Byvoorbeeld, 16 is **gelukkig** omdat  $16 - 2 \times 2 = 12$ . Wat is die som van die syfers van die grootste **gelukkige** getal?

- (A) 7            (B) 8            (C) 9            (D) 10            (E) 11
- 

### Vraag 18

'n Driehoek met sye 6, 9 en 12 het oppervlakte  $A$  eenhede. Wat is die oppervlakte van 'n driehoek met sye 8, 12 en 16 in terme van  $A$ ? (Wenk: Dink aan gelykvormige driehoeke.)

- (A)  $\frac{3}{2}A$             (B)  $\frac{4}{3}A$             (C)  $\frac{25}{16}A$             (D)  $\frac{16}{9}A$             (E)  $\frac{9}{4}A$
- 

### Vraag 19

$A, B, C$  en  $D$  is vier opeenvolgende hoekpunte van 'n reëlmatige veelhoek. Veronderstel  $AB$  en  $DC$  word verleng na punt  $E$ . As die grootte van  $\hat{BEC}$  gelyk is aan  $x$  grade, hoeveel hoekpunte het die veelhoek in terme van  $x$ ?

- (A)  $\frac{720}{180 - x}$             (B)  $\frac{720}{180 + x}$             (C)  $\frac{360}{90 + x}$             (D)  $\frac{360}{90 - x}$             (E)  $\frac{360}{x}$
-

---

**Question 20**

Twenty chairs are placed in a row. In how many ways can you paint exactly four of them blue and the others red, such that no two consecutive chairs are blue?

- (A) 1 820      (B) 2 380      (C) 3 060

**Vraag 20**

Twintig stoele word in 'n ry gepak. Op hoeveel maniere kan jy presies vier van hulle blou verf en die ander rooi, sodat geen twee opeenvolgende stoele blou is nie.

- (D) 3 876      (E) 4 845
-