Quaternion linear algebra

Supervisor: Marten Wortel

The quaternions form a four-dimensional algebra over the reals with basis $\{1, i, j, k\}$ subject to the rules $i^2 = j^2 = k^2 = -1$ and

$$ij = k,\ ji = -k,\ jk = i,\ kj = -i,\ ki = j,\ ik = -j.$$

They were invented in 1843 by Hamilton and applied to mechanics, and nowadays they have many applications, for example in the classification theory of Jordan algebras.

The quaternions form a division ring, i.e., a ring that satisfies all the axioms of a field except commutativity, so in this project we will investigate to what extend we can do linear algebra over the quaternions (instead of over a field). We will finish by proving the spectral theorem for normal matrices of quaternions.

A student attempting this project should be interested in and have a good grasp of abstract algebra.