Projective Geometry and Relevance Logic

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Models of Projective Geometry one can define a natural ternary 'colinearity' relation $a*b*c$ which means that points a, b, c lie on the same line. The relational models of the 'Routley-Meyer' semantics for Relevance logic also feature a more opaque ternary relation meant to capture $\rightarrow$. Alasdair Urquhart showed in [1] that there is a simple way to construct from a model of Projective Geometry a model of the Relevance logic **KR**, such that the two ternary relations are intimately related. He used this construction to prove that **KR** is undecidable. These ideas lead Urquhart to prove in [2] that the principal Relevance logic **R** is also undecidable, and to prove that **R** does not have the interpolation property [3].

This project will expand the details left to the reader in the paper [1] (a significant task). This includes:

- showing that **KR** is complete with respect to a certain class of relational structures,

- elaborating the connection between projective spaces and modular geometric lattices, and

- how undecidability is translated between these classes.

References

[1] Urquhart, A. 1983. 'Relevant Implication and Projective Geometry'. *Logique et Analyse* 26: 345–57.

[2] Urquhart, A. 1984. 'The Undecidability of Entailment and Relevant Implication'. *The Journal of Symbolic Logic* 49: 1059–73.

[3] Urquhart, Alasdair. 1993. 'Failure of Interpolation in Relevant Logics'. *Journal of Philosophical Logic* 22: 449–79.