Riesz spaces and equilibrium theory

Supervisors: Dr Mokhwetha Mabula and Miss Magadi Mabe

An ordered normed space $(X, \|.\|, \leq)$ is called a normed Riesz space if X is a Riesz space (that is, for $x, y \in X$, $\sup\{x, \}$ and $\inf\{x, y\}$ exist in X) and the norm $\|.\|$ is such that $|x| \leq |y|$ implies that $\|x\| \leq \|y\|$ for all $x, y \in X$. Let X and Y be two ordered vector spaces. An operator $T : X \to Y$ is called order preserving if $x \leq_X y$ implies that $Tx \leq_y Ty$ for all $x, y \in X$. The order $\leq_X$ on X is called representable if there exists a order preserving functional on X, and such functional are called utility functions in economics.

Definition 0.1. *Let $(X, \|.\|)$ be an ordered normed space with order induced by a cone $C = X^+$. The space $(X, \|.\|, C)$ is commodity space of a pure exchange economy.*

Definition 0.2. *Let X and Y be two vector spaces. The dual pair $\langle X, Y\rangle$ is a pair of vector spaces together with bilinear map $\langle ., . \rangle : X \times Y \to \mathbb{R}$ such that $\langle x, y\rangle = 0$ for all $x \in X \Rightarrow y = 0$ and $\langle x, y\rangle = 0$ for all $y \in Y \Rightarrow x = 0$.*

A normed space X with its normd dual X^* gives example of such pair.

Definition 0.3. *Let X be a normed space and X^* norm dual of X. The dual pair $\langle X, X^*\rangle$ is called commodity-price duality, where X is commodity space and X^* is price space.*

The following will be discussed;

(i) Topology induced by positive cone X^+ and existence of a quasi-equilibrium without assumption that utility space is closed.

(ii) Continuity of supporting prices with respect to the norm of commodity space.

(iii) The proof of second welfare theorem.

(iv) The existence of equilibrium.

Items (iii) and (iv) depend on item (ii).