

Investigating the influence of the number of Sylow subgroups

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Let G be a finite group. For a prime divisor p of $|G|$, we write $n_p(G)$ for the number of Sylow p -subgroups of G .

Theorem. [1] Let G be a finite group. If $n_3(G) \leq 7$ and $n_5(G) \leq 1455$, then G is solvable.

The goal of this project is to answer the questions listed below.

1. Is the converse of the above theorem necessarily true?
2. Can the bounds in the above theorem be improved for finite nilpotent groups?
3. What can be said about a similar kind of result for finite simple groups?

A solid background in abstract algebra is needed; in particular, the candidate would have mastered WTW 381. The candidate will also be expected to learn how to use the computer algebra package GAP[2] whilst working on this project.

References

- [1] C. S. Anabanti, A. Moretó, M. Zarrin, “*Influence of the number of Sylow subgroups on solvability of finite groups*”, *C. R. Math. Acad. Sci. Paris* **358** (2020), 1227–1230.
- [2] The GAP Group, *GAP – Groups, Algorithms, and Programming, Version 4.12.2*; 2022, <https://www.gap-system.org>.