Hons essay projects in the programme of Mathematics

Zeilberger's Algorithm. Supervisor: Dr AS Jooste

A sequence of real polynomials $\{P_n\}_{n=0}^{N}$, $N \in \mathbb{N} \cup \{\infty\}$, where P_n is of exact degree n, is orthogonal on the interval (a, b), with respect to the weight function $w(x) > 0$, if, for $m, n = 0, 1, \ldots N$,

$$\int_a^b P_n(x)P_m(x)\ w(x)dx = \begin{cases} 0 & \text{if } m \neq n, \\ d_n^2 & \text{if } m = n, \end{cases}$$

where $d_n^2 = \int_a^b P_n^2(x)\ w(x)d(x) \neq 0$.

Zeilberger's algorithm [Koepf] generates a recurrence equation for a set of orthogonal polynomials, with respect to some discrete variable for a sum under consideration. In this project the student will

- give a short background on orthogonal polynomials and their properties, focussing on the three-term recurrence relation in general;
- study the principles used in Zeilberger's algorithm;
- apply the algorithm in order to find some recurrence relations (not necessarily three-term recurrence relations) in order to show how Zeilbergers's algorithm can be applied.

In doing this project, you will gain some basic knowledge in the fields of orthogonal polynomials and Computer-Algebra and the study will contribute to a strong foundation in mathematics. You will get to know the different programs (written in, e.g., Maple or Mathematica) that can be used to obtain results, as well as the extent to which Computer-Algebra can be used to get results in this field.

References (and further reading):

G.E. Andrews, R. Askey and R. Roy. Special Functions. Encyclopedia of Mathematics and its Applications. Cambridge University Press, Cambridge, 1999.

T.S. Chihara. An introduction to Orthogonal Polynomials. Gordon and Breach, New York, 1998.

[Koepf] W. Koepf, Hypergeometric Summation, Al algorithmic approach to summation and special function identities (second edition). Springer-Verlag, London, 2014.