Hons essay projects in the programme of Applied Mathematics

Stochastic processes and their connection with orthogonal polynomials.(Supervisor: Dr AS Jooste)

A sequence of real polynomials $\{P_n\}_{n=0}^{N}$, $N \in \mathbb{N} \cup \{\infty\}$, where P_n is of exact degree n, is orthogonal on the interval (a, b), with respect to the weight function $w(x) > 0$, if, for $m, n = 0, 1, \ldots N$,

$$\int_a^b P_n(x)P_m(x)\ w(x)dx = 0 \ \text{ if } \ m \neq n.$$

This project will commence with a literature study in order to identify the classes of orthogonal polynomials that can be associated with Stochastics. It will be expected from the student to study the background of the identified systems of polynomials, as well as to discuss their different properties (e.g., three-term recurrence relation, the difference equation they satisfy, etc.) and the behaviour of their zeros. The main purpose of the study will be to illustrate how these systems of polynomials are applied to Stochastics.

In this study the student will gain some basic knowledge in the field of orthogonal polynomials; the computer program *Mathematica* can be a useful aid. It will therefore not only contribute to a strong foundation in mathematics, but he will also gain some numerical expertise.

References (and further reading):

T.S. Chihara. An introduction to Orthogonal Polynomials. Gordon and Breach, New York, 1998.
W. Schoutens. Stochastic processes and orthogonal polynomials. Lecture Notes in Statistics, 146. New York, Springer (2000).