Hons essay projects in the programme of Mathematics

The use of orthogonal polynomials in Approximation theory: Dr AS Jooste

A sequence of real polynomials $\{P_n\}_{n=0}^{N}$, where N can be at infinity and each polynomial P_n is of exact degree n, is orthogonal on the interval (a, b), with respect to the weight function $w(x) > 0$, if, for $m, n = 0, 1, \dots N$,

$$\int_a^b P_n(x)P_m(x)\ w(x)dx = \begin{cases} 0 & \text{if } m \neq n, \\ d_n^2 & \text{if } m = n, \end{cases}$$

and $d_n^2 = \int_a^b P_n^2(x)\ w(x)d(x) \neq 0$.

Orthogonal polynomials are widely used in Approximation theory. We will start this project doing a literature study, summarizing the uses of the orthogonal polynomials systems belonging to the Askey scheme, in Approximation theory.

An essay on this topic will typically entail the following:

(1) A short introduction on orthogonal polynomials in general, focussing on their properties;

(2) A summary of the above-mentioned literature study;

(3) A section where we will choose one specific sequence of orthogonal polynomials and do a more in depth study of its applications in (especially) approximation theory.

In doing this project, you will gain some basic knowledge in the field of Orthogonal Polynomials and Approximation Theory and the study will contribute to a strong foundation in mathematics. We will also use the programs Maple or Mathematica in order to experience how Computer-Algebra can be used to get results in this field. This study will form the foundation for further studies, since there are numerous open problems in the particular field.

Further reading:

G.E. Andrews, R. Askey and R. Roy. Special Functions. Encyclopedia of Mathematics and its Applications. Cambridge University Press, Cambridge, 1999.

T.S. Chihara. An introduction to Orthogonal Polynomials. Gordon and Breach, New York, 1998.

W. Schoutens. Stochastic processes and orthogonal polynomials. Lecture Notes in Statistics, 146. New York, Springer (2000).